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457.646 Topics in Structural Reliability

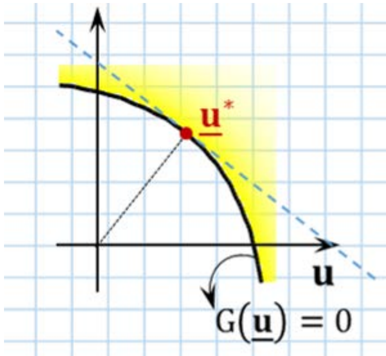
In-Class Material: Class 11

◎ Hasofer-Lind Reliability Index, β_{HL} (contd.)

② Nonlinear Limit-State Function

Transform $g(\quad)$ to $G(\quad)$ by

$$\begin{cases} \mathbf{X} = \\ \mathbf{u} = \end{cases}$$



- suppose one can find $\mathbf{u}^*$
- Linearize $G(\mathbf{u})$ at $\mathbf{u} =$

$\rightarrow$ $G(u) \overset{FO}{\simeq} G(\quad) + \qquad (\qquad)$

$= \qquad\qquad = 0$

Reliability index

Try $\dfrac{\mu_G}{\sigma_G} \cong \dfrac{\mu_G^{FO}}{\sigma_G^{FO}}$?

$\mu_G^{FO} =$

$\sigma_G^{2FO} = \qquad\qquad = \| \qquad \|^2$

$\therefore \dfrac{\mu_G^{FO}}{\sigma_G^{FO}} = \qquad = \qquad =$

In summary, the "distance" between the origin and the design point $\mathbf{u}^*$ in $\mathbf{u}$-space gives reliability index based on first-order approximation

★ **Note!**
$$\begin{cases} MVFOSM & \dfrac{\mu^{FO}}{\sigma^{FO}} \text{ at } \mathbf{x} = \\ HL & \dfrac{\mu^{FO}}{\sigma^{FO}} \text{ at } \mathbf{u} = \end{cases}$$

※ Procedure : i) Transform $g(\mathbf{x})$ to $G(\mathbf{u})$ using $\mathbf{x} =$

ii) Find

iii) Find at

iv) $\beta_{HL} =$

※ Description of β_{HL} in $\mathbf{x}$ space?

$$\boxed{\nabla G(\mathbf{u}^*)(\mathbf{u}-\mathbf{u}^*) = 0} \quad \underset{\mathbf{x}=}{\overset{\mathbf{u}=}{\rightleftarrows}} \quad \boxed{\nabla g(\mathbf{x}^*)(\mathbf{x}-\mathbf{x}^*) = 0}$$

Approx. Limit state space in $\mathbf{u}$

Proof :

$\nabla_{\mathbf{x}} g(\mathbf{x}^*) = \nabla_{\mathbf{u}} G(\mathbf{u}^*) \times$

$=$

$\mathbf{x}^* =$

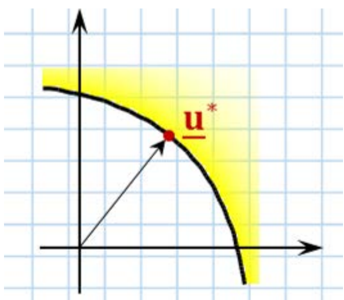
$\mathbf{x} =$

$\therefore g^{FO} = \nabla g(\mathbf{x}^*)(\mathbf{x}-\mathbf{x}^*)$

$$\boxed{\beta_{HL} = \frac{\mu_g^{FO}}{\sigma_g^{FO}} = \frac{\qquad}{\sqrt{\qquad}}}$$ FO at $\underline{\mathbf{x}} =$

Cf. $$\beta_{MVFOSM} = \frac{\mu_g^{FO}}{\sigma_g^{FO}} = \frac{g(\mathbf{M}_{\mathbf{x}})}{\sqrt{\nabla g(\mathbf{M}_{\mathbf{x}}) \sum_{\mathbf{xx}} \nabla g(\mathbf{M}_{\mathbf{x}})^{\mathrm{T}}}}$$ FO at $\underline{\mathbf{x}} =$

③ Finding the design point $\mathbf{u}^*$



$\mathbf{u}^* = \operatorname{argmin}\{ \quad | \quad \}$

Then evaluate $\hat{\boldsymbol{\alpha}} =$ at

And compute $\beta_{HL} = \hat{\boldsymbol{\alpha}}\mathbf{u}^*$

$\Rightarrow$ constrained nonlinear optimization problem

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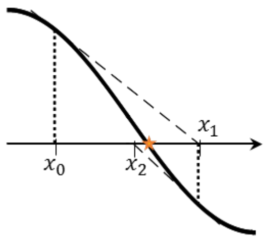
Reviews on optimization algorithm of finding $\mathbf{u}^*$

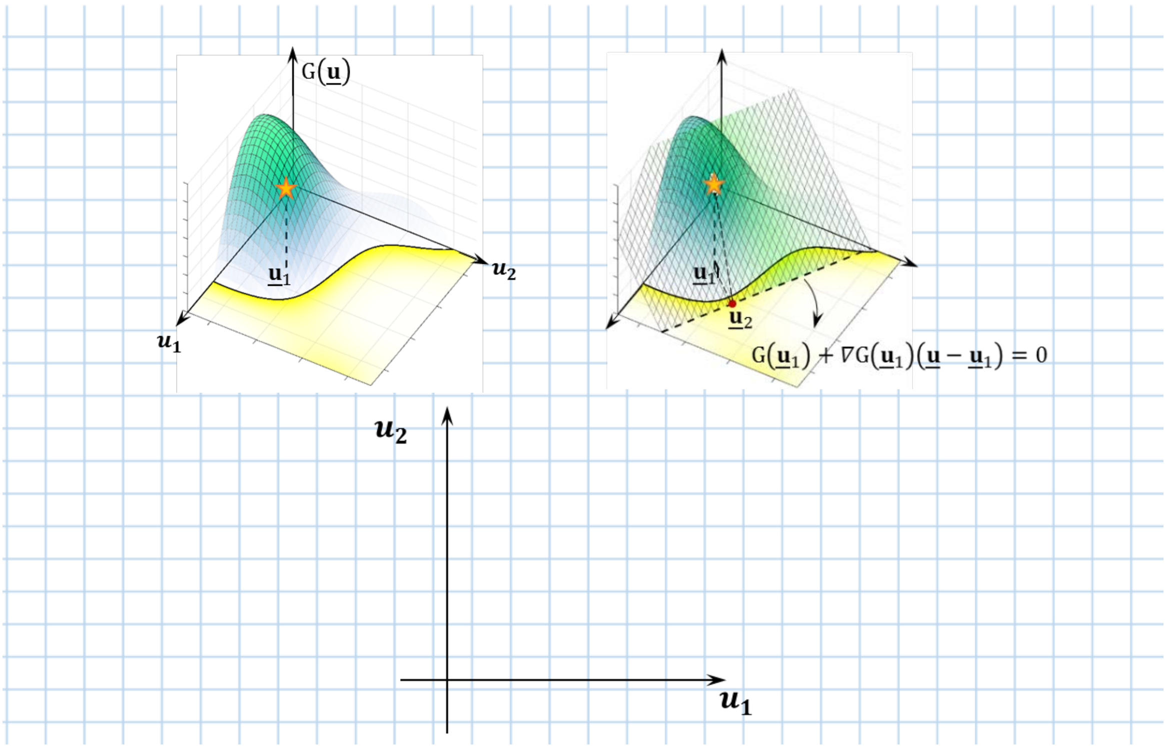
- Liu & ADK (1990)
- Papaioannou et al. (2010)

HL-RF, SQP, GP, DFO



a) HL-RF algorithm (Rackwitz & Fissler 1978)

"Newton-Raphson-like algorithm" solve $f(x)=0$ for $x=x^*$?



$\mathbf{u}_1$: initial point (e.g $\mathbf{u}_1$ = $\mathbf{M}_\mathbf{u} = \mathbf{0}$)

$\mathbf{u}_2 = (\qquad\qquad) \times (\qquad\qquad)$

$=$

$=$

$\mathbf{u}_{i+1} =$

To update $\mathbf{u}_i$ to , $\mathbf{u}_{i+1}$, one needs

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$$G(\mathbf{u}_i) =$$

$$\nabla_{\mathbf{u}} G(\mathbf{u}_i) =$$

Iterate until 1)

2)