Seoul National University
Dept. of Civil and Environmental Engineering

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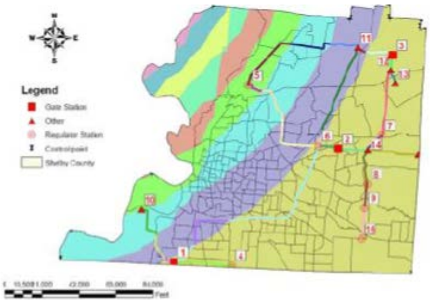
457.646 Topics in Structural Reliability

In-Class Material: Class 25

VIII-3. Random fields

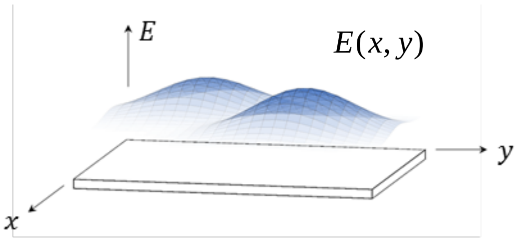
~ Random quantity distributed over _______________ field (space or time)

Ex1) Spatial Distribution of Random Ground Motion Intensity)



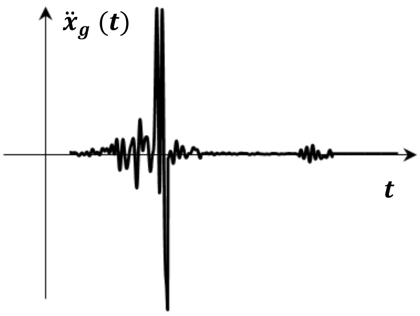
(Song & Ok, 2010)

Ex2) Spatial distribution of material property (Young's Modulus)



Ex3) Ground acceleration time history $\ddot{x}_g(t)$

e.g.



random process, stochastic process

⇒ (   ) # of random variables

⇒ (   ) representation is required

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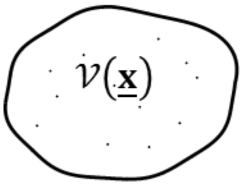
◎ Discretization of Random field → Random vector



→ $\mathbf{v} = \{v_1, \cdots, v_n\}^T$

Random n-vector

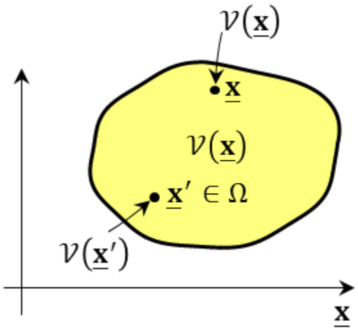
$$
? \begin{cases}
\mathbf{M}_{\mathbf{v}} = E[\mathbf{v}] = \{\mu_{v_i}\} \\
\mathbf{\Sigma}_{\mathbf{vv}} = E[(\mathbf{v}-\mathbf{M}_v)(\mathbf{v}-\mathbf{M}_v)^T] \\
\quad = \mathbf{D}_{\mathbf{v}}\mathbf{R}_{\mathbf{vv}}\mathbf{D}_{\mathbf{v}} \qquad \text{covariance matrix} \\
\text{where} \quad D_v = \text{diag}[\sigma_{v_i}] \\
\qquad R_{vv} = [\rho_{v_i v_j}] \\
f_{\mathbf{v}}(\mathbf{v}) \rightarrow \text{joint PDF of } \mathbf{v}
\end{cases}
$$

◎ Theoretical Representation of R.F

$v(\mathbf{x}),\ \mathbf{x} \in \Omega$ random field in domain Ω

Partial descriptors:

$$
\begin{cases}
\mu(\mathbf{x}) : \text{mean function} \quad E[v(\mathbf{x})] \\
\sigma^2(\mathbf{x}) : \text{variance function} \quad E[v^2(\mathbf{x})] - \mu^2(x) \\
\rho(\mathbf{x}, \mathbf{x}') : \text{correlation coefficient function} \quad \rho_{v(\mathbf{x})v(\mathbf{x}')}
\end{cases}
$$



For Gaussian R.F. the above gives a complete specification

For Nataf R.F., also specify $F_v(v;\mathbf{x})$

For general RF's, specify joint PDF of ( ) and ( )

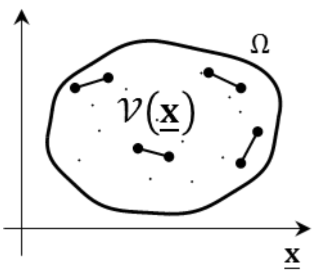
for, $x, x' \in \Omega$, $f_{vv}(v(x), v(x'))$

e.g. ______________ Random field

~ ______________ does not change over the domain Ω

$v(\mathbf{x})$, $x \in \Omega$

$$
\begin{cases}
\mu(\mathbf{x}) = \\
\sigma^2(\mathbf{x}) = \\
\rho(\mathbf{x}, \mathbf{x}') = \\
F(v;\mathbf{x}) =
\end{cases}
$$



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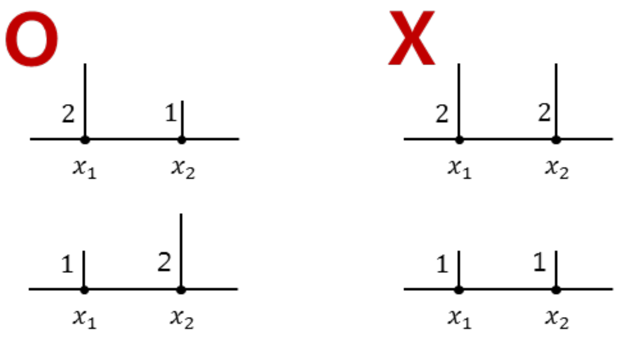
Note; This doesn't mean $v(\mathbf{x}) = v$ (not constant over the domain)

Scenario 1

Scenario 2

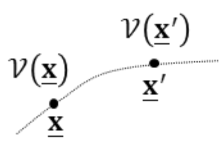


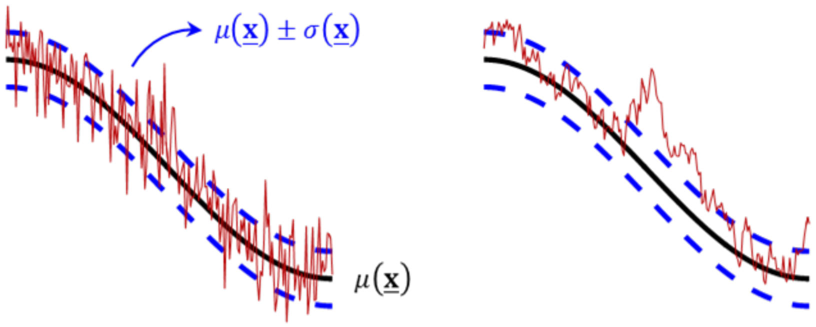
$\mu_{x_1} = 1.5 \; \mu_{x_2} = 1.5$

$\mu(\mathbf{x}) = \mu = 1.5$

$v(\mathbf{x}) = v$

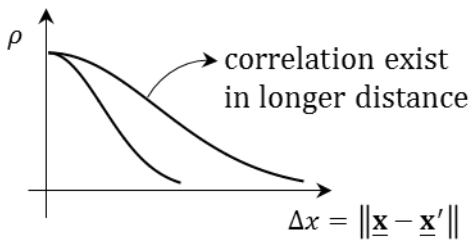
Q: Correlation Function $\rho(\mathbf{x}, \mathbf{x}')$ ←meaning?



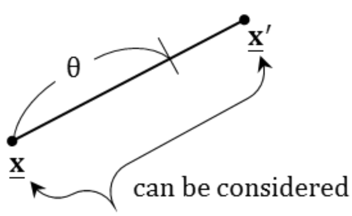


How to capture this from $\rho(\mathbf{x}, \mathbf{x}')$?

◎ Correlation length



$$\theta = \int_0^\infty \rho(\Delta x) dx$$



~ measure of the distance over which significant loss of correlation occurs

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Examples

- $\rho(\Delta x) = \exp\left(-\dfrac{\Delta x}{a}\right)$

$$\theta = \int_0^\infty \exp\left(-\frac{\Delta x}{a}\right) d\Delta x$$
$$= -a \exp\left(-\frac{\Delta x}{a}\right)\Big|_0^\infty = a$$

- $\rho(\Delta x) = \exp\left(-\dfrac{\Delta x^2}{a^2}\right)$

$$\theta = \int_0^\infty \exp\left(-\frac{\Delta x^2}{a^2}\right) d\Delta x$$
$$= \frac{1}{2}\int_{-\infty}^\infty \exp\left(-\frac{\Delta x^2}{a^2}\right) d\Delta x$$
$$= \frac{1}{2}\sqrt{\pi} a \qquad \theta \propto a$$