Cryogenic Engineering

Chapter 3.

Gas Liquefaction System

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3.1 System performance parameters

- **Gas – liquefaction systems**

  The systems that can produce low temperatures required for liquefaction.

- **System performance parameters**

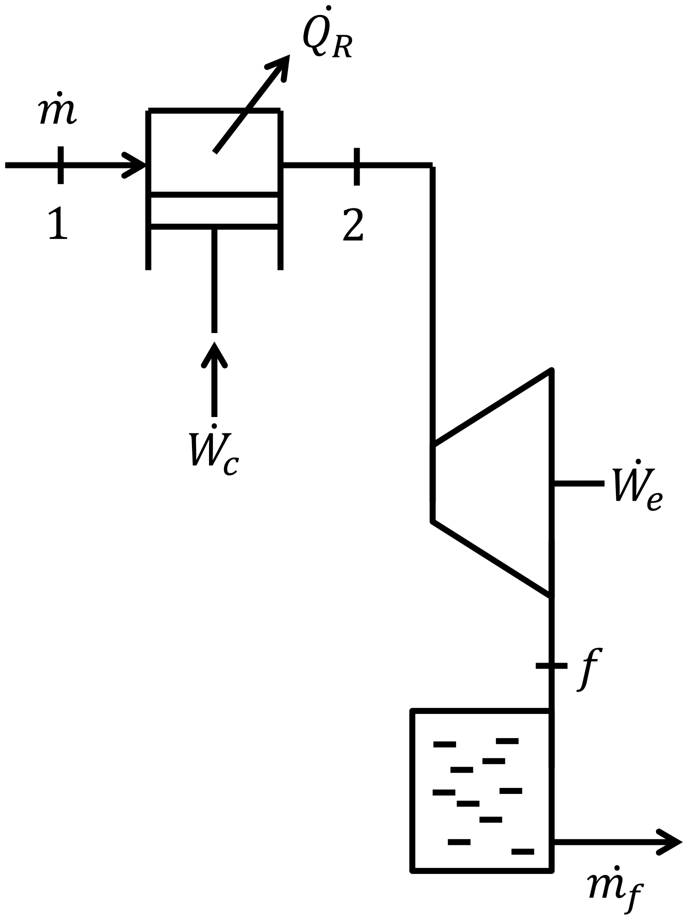
  - $\dfrac{Work\ required}{Unit\ mass\ of\ gas\ compressed} = -\dfrac{\dot{W}}{\dot{m}}$
  - $\dfrac{Work\ required}{Unit\ mass\ of\ gas\ liquefied} = -\dfrac{\dot{W}}{\dot{m}_f}$
  - $Liquefiecd\ fraction\ of\ the\ total\ flow\ of\ gas = y = -\dfrac{\dot{m}_f}{\dot{m}}$
  - $Figure\ of\ Merit(FoM) = \dfrac{Theoretical\ work}{Actual\ work} = -\dfrac{\dot{W}_i}{\dot{W}}$ (0~1)

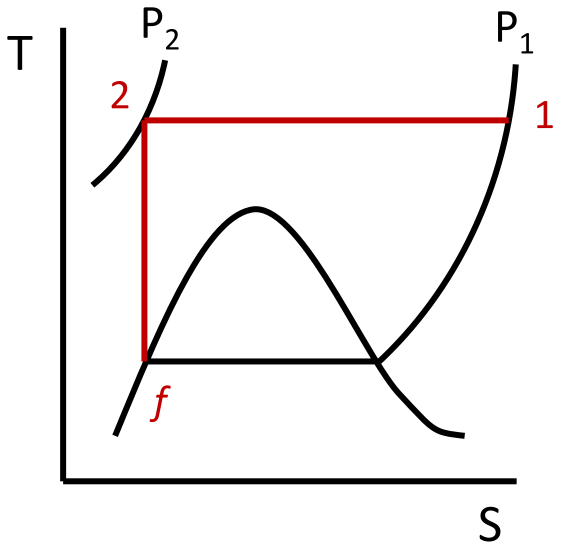
3.1 System performance parameters

+ Additional performance parameters

- Compressor adiabatic efficiency
- Expander adiabatic efficiency
- Compressor mechanical efficiency
- Expander mechanical efficiency
- HX effectiveness
- Pressure drop
- Heat transfer to the system

3.2 Thermodynamically ideal cycle (Liquefaction system - open cycle)





1-2: Isothermal compression
2-f: Isentropic expansion(expander)

In this case, P_2 = 70 ~ 80 GPa for N_2

(too high!)

3.2 Thermodynamically ideal cycle (Liquefaction system - open cycle)

- **Apply 1st and 2nd laws of thermodynamics to the system**

1st law of thermodynamics:

$$\dot{Q}_{net} - \dot{W}_{net} = \sum_{outlets} \dot{m}\left(h + \frac{v^2}{2g_c} + \frac{gz}{g_c}\right) - \sum_{inlets} \dot{m}\left(h + \frac{v^2}{2g_c} + \frac{gz}{g_c}\right)$$

$$\rightarrow \dot{Q}_{net} - \dot{W}_{net} = \sum_{outlets} \dot{m}h - \sum_{inlets} \dot{m}h$$

$$\rightarrow \dot{Q}_R - \dot{W}_i = \dot{m}(h_f - h_1) = -\dot{m}(h_1 - h_f)$$

$$(\dot{W}_i = W_e - W_c)$$

3.2 Thermodynamically ideal cycle (Liquefaction system - open cycle)

- **Apply 1st and 2nd laws of thermodynamics to the system**

2nd law of thermodynamics:

$$ds = \left.\frac{\delta q}{T}\right)_{rev,ideal}$$

$$\rightarrow \quad \delta q = T \cdot ds$$

$$\rightarrow \quad \dot{Q}_R = \dot{m}T_1(S_2 - S_1) = -\dot{m}T_1(S_1 - S_f)$$

3.2 Thermodynamically ideal cycle (Liquefaction system - open cycle)

- Apply 1st and 2nd laws of thermodynamics to the system

From 1st and 2nd laws of thermodynamics,

$$\dot{Q}_R - \dot{W}_i = \dot{m}(h_f - h_1) = -\dot{m}(h_1 - h_f) \quad \cdots \quad 1st\ law$$

$$\dot{Q}_R = \dot{m}T_1(S_2 - S_1) = -\dot{m}T_1(S_1 - S_f) \quad \cdots \quad 2nd\ law$$

$$\rightarrow \quad -\frac{\dot{\boldsymbol{W}}_{\boldsymbol{i}}}{\dot{\boldsymbol{m}}} = \boldsymbol{T}_1(\boldsymbol{S}_1 - \boldsymbol{S}_f) - (\boldsymbol{h}_1 - \boldsymbol{h}_f) = -\frac{\dot{\boldsymbol{W}}_{\boldsymbol{i}}}{\dot{\boldsymbol{m}}_{\boldsymbol{f}}}$$

$$*\ \dot{m} = \dot{m}_f \quad \rightarrow \quad liquid\ yield\ \ \boldsymbol{y} = \frac{\dot{m}_f}{\dot{m}} = \mathbf{1}$$

3.2 Thermodynamically ideal cycle (Liquefaction system - open cycle)

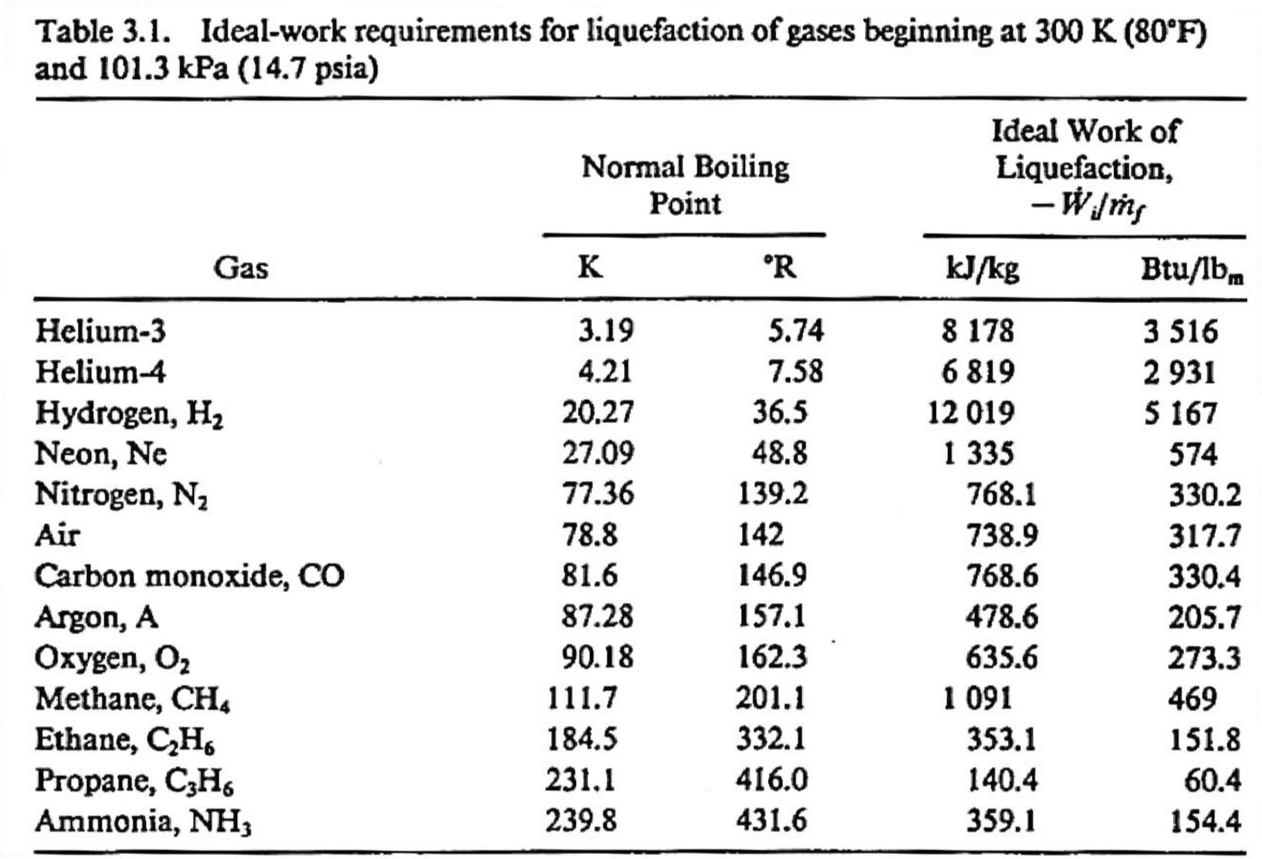
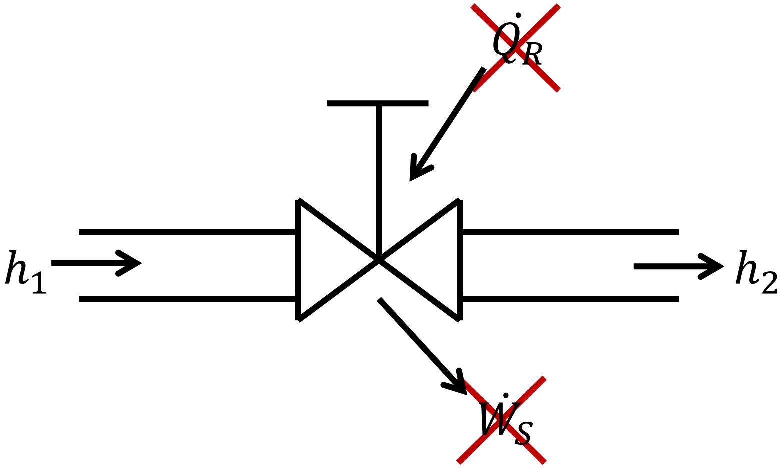


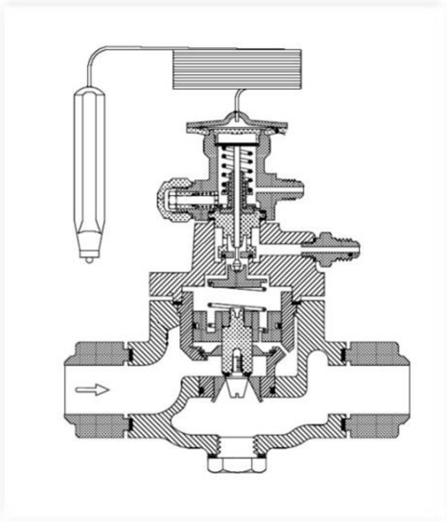
Table 3.1. Ideal-work requirements for liquefaction of gases beginning at 300 K (80°F) and 101.3 kPa (14.7 psia)

Gas	Normal Boiling Point		Ideal Work of Liquefaction, $-\dot{W}_i/\dot{m}_f$	
	K	°R	kJ/kg	Btu/lb$_m$
Helium-3	3.19	5.74	8 178	3 516
Helium-4	4.21	7.58	6 819	2 931
Hydrogen, H_2	20.27	36.5	12 019	5 167
Neon, Ne	27.09	48.8	1 335	574
Nitrogen, N_2	77.36	139.2	768.1	330.2
Air	78.8	142	738.9	317.7
Carbon monoxide, CO	81.6	146.9	768.6	330.4
Argon, A	87.28	157.1	478.6	205.7
Oxygen, O_2	90.18	162.3	635.6	273.3
Methane, CH_4	111.7	201.1	1 091	469
Ethane, C_2H_6	184.5	332.1	353.1	151.8
Propane, C_3H_6	231.1	416.0	140.4	60.4
Ammonia, NH_3	239.8	431.6	359.1	154.4

3.3 Joule – Thomson effect (Expansion device – valve)

- **Expansion valve**





1st law of thermodynamics:

$$\dot{Q}_R - \dot{W}_S = \sum_{outlets} \dot{m}\left(h + \frac{v^2}{2g_c} + \frac{gz}{g_c}\right) - \sum_{inlets} \dot{m}\left(h + \frac{v^2}{2g_c} + \frac{gz}{g_c}\right)$$

$$\rightarrow \quad 0 = \dot{m}(h_2 - h_1)$$

$$\rightarrow \quad \boldsymbol{h_2 = h_1}$$

3.3 Joule – Thomson effect (Expansion device – valve)

- **Joule – Thomson coefficient**

$$\mu_{JT} = \left.\frac{\partial T}{\partial P}\right)_h$$

→ Change in temperature due to a change in pressure at constant enthalpy

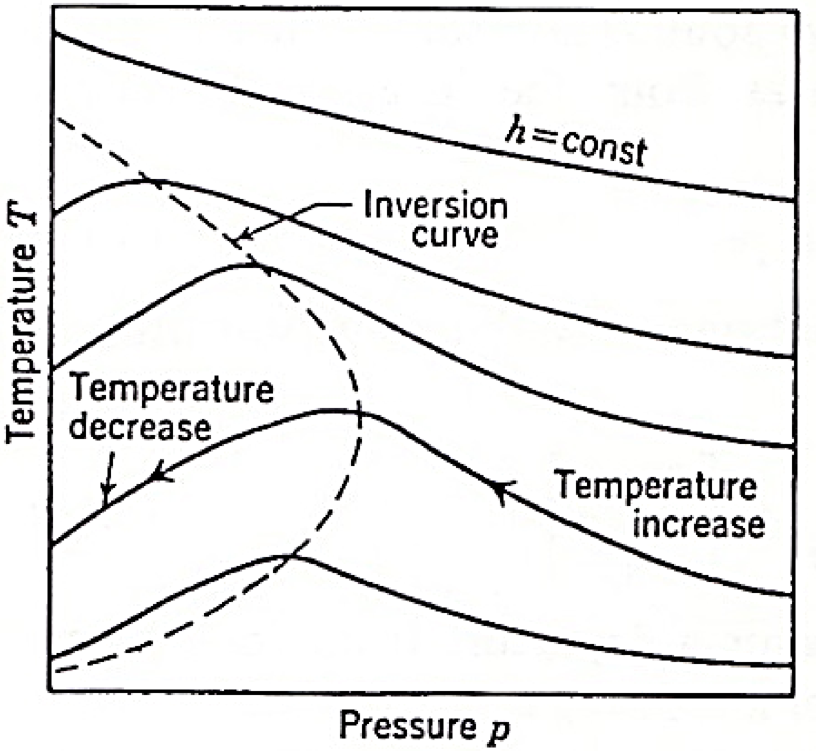
(Slope of isenthalpic line)

3.3 Joule – Thomson effect (Expansion device – valve)

- **Joule – Thomson coefficient**

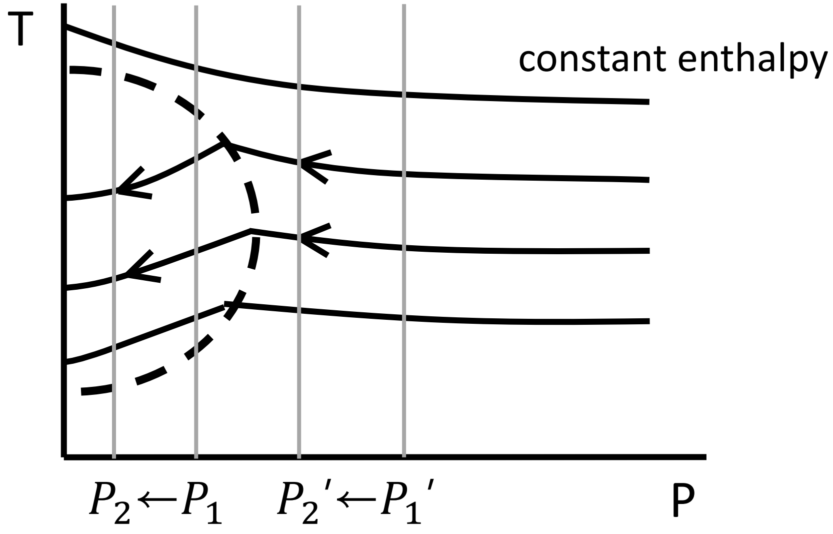
$$\mu_{JT} = \left.\frac{\partial T}{\partial P}\right)_h$$

→ Slope of isenthalpic line



Isenthalpic expansion of a real gas.

3.3 Joule – Thomson effect (Expansion device – valve)



$P_2 \leftarrow P_1$: Temperature ↓ if Pressure ↓ → $\mu_{JT} > 0$

$P_2' \leftarrow P_1'$: Temperature ↑ if Pressure ↓ → $\mu_{JT} < 0$

3.3 Joule – Thomson effect (Expansion device – valve)

- **Joule – Thomson coefficient**

$$\mu_{JT} = \left.\frac{\partial T}{\partial P}\right)_h = -\left.\frac{\partial T}{\partial h}\right)_P \left.\frac{\partial h}{\partial P}\right)_T$$

from basic thermodynamics (Van Wylen and Sonntag, 1976),

$$dh = \left.\frac{\partial h}{\partial T}\right)_P dT + \left.\frac{\partial h}{\partial P}\right)_T dP = C_P\, dT + \left[v - T\left.\frac{\partial v}{\partial T}\right)_P\right] dP$$

$$\rightarrow \quad \left.\frac{\partial h}{\partial T}\right)_P = C_P, \qquad \left.\frac{\partial h}{\partial P}\right)_T = v - T\left.\frac{\partial v}{\partial T}\right)_P$$

3.3 Joule – Thomson effect (Expansion device – valve)

- **Joule – Thomson coefficient**

$$\mu_{JT} = \left.\frac{\partial T}{\partial P}\right)_h = -\left.\frac{\partial T}{\partial h}\right)_P \left.\frac{\partial h}{\partial P}\right)_T$$

$$= \frac{1}{C_P}\left[T\left.\frac{\partial v}{\partial T}\right)_P - v\right]$$

For an ideal gas,

$$\left.\frac{\partial v}{\partial T}\right)_P = \frac{R}{P} = \frac{v}{T}$$

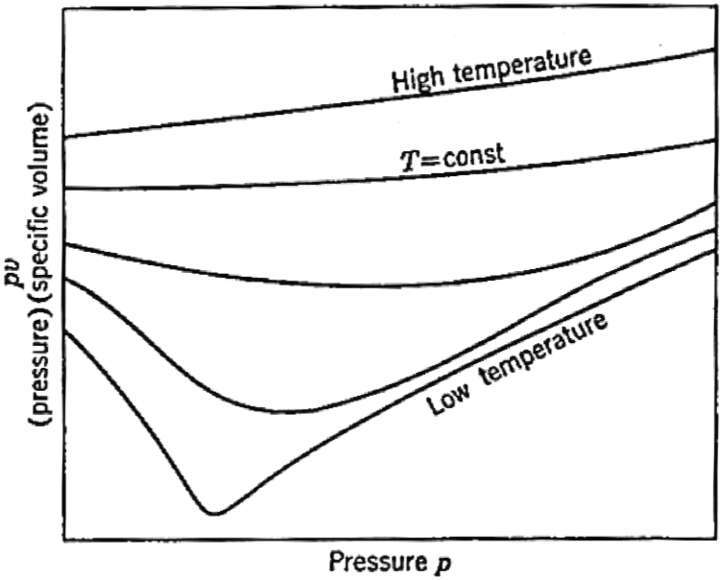
$$\mu_{JT} = \frac{1}{C_P}\left[T\left.\frac{\partial v}{\partial T}\right)_P - v\right] = 0$$

3.3 Joule – Thomson effect (Expansion device – valve)

- **Joule – Thomson coefficient**

from $h = u + Pv$,

$$\mu_{JT} = \frac{1}{C_P}\left[T\left.\frac{\partial v}{\partial T}\right)_P - v\right] = -\frac{1}{C_P}\left\{\left.\frac{\partial u}{\partial P}\right)_T + \left.\frac{\partial (Pv)}{\partial P}\right]_T\right\}$$



Variation of the product Pv

with pressure and temperature for a real gas.

3.3 Joule – Thomson effect (Expansion device – valve)

- **Van der Waals gas**

Equation of state (EOS)

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT$$

- **Joule – Thomson coefficient for van der Waals gas**

$$\mu_{JT} = \frac{(2a/RT)(1 - b/v)^2 - b}{C_P\left[1 - \left(\frac{2a}{vRT}\right)\left(1 - \frac{b}{v}\right)^2\right]}$$

for large value of the specific volume,

$$\mu_{JT} = \frac{1}{C_P}\left(\frac{2a}{RT} - b\right)$$

3.3 Joule – Thomson effect (Expansion device – valve)

- **Inversion curve for van der Waals gas**

The inversion curve is represented by all points at which the Joule-Thomson coefficient is zero.

$$\mu_{JT} = \frac{(2a/RT)(1-b/v)^2 - b}{C_P\left[1-\left(\frac{2a}{vRT}\right)\left(1-\frac{b}{v}\right)^2\right]} = 0 \quad \rightarrow (2a/RT)(1-b/v)^2 - b = 0$$

- **Inversion temperature for van der Waals gas**

$$T_i = \frac{2a}{bR}\left(1-\frac{b}{v}\right)^2$$

3.3 Joule – Thomson effect (Expansion device – valve)

$$T_i = \frac{2a}{bR}\left(1 - \frac{b}{v}\right)^2$$

$$T_{i,max} = \frac{2a}{bR} < T_{room} : He, H_2, Ne$$

→ Cannot produce low T with expansion valve alone.
(Require expander, turbine)

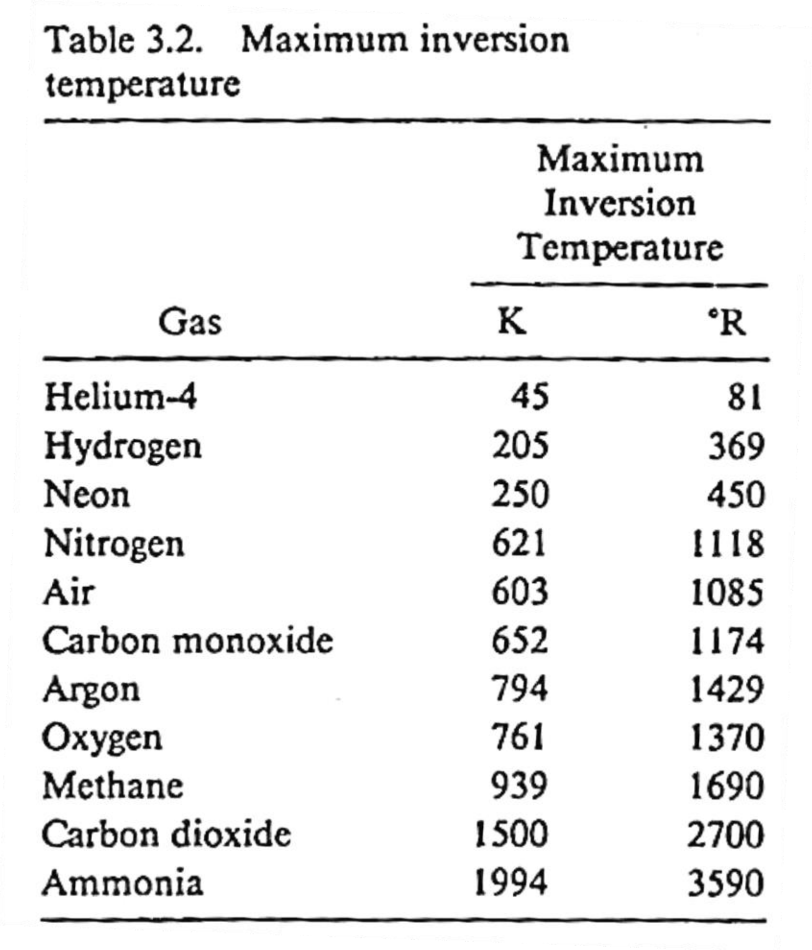


Table 3.2. Maximum inversion temperature

	Maximum Inversion Temperature	
Gas	K	°R
Helium-4	45	81
Hydrogen	205	369
Neon	250	450
Nitrogen	621	1118
Air	603	1085
Carbon monoxide	652	1174
Argon	794	1429
Oxygen	761	1370
Methane	939	1690
Carbon dioxide	1500	2700
Ammonia	1994	3590

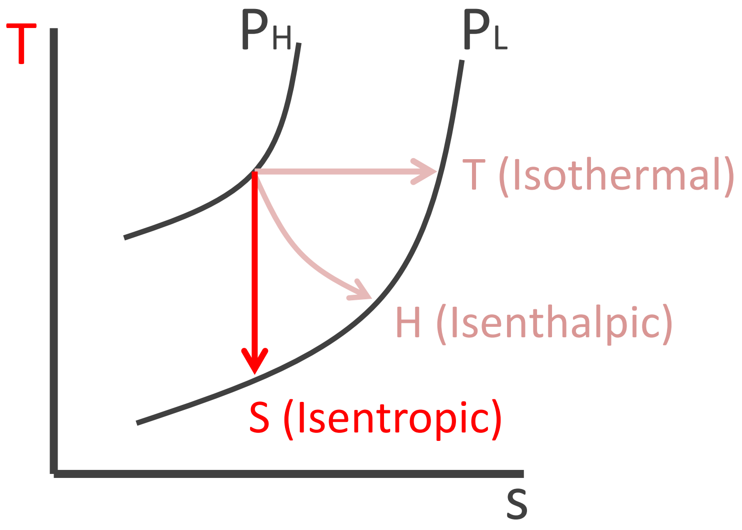
3.4 Adiabatic expansion

- **Work producing device : expansion engine (turbine)**

Adiabatic Expansion : Most effective means of lowering T of the gas

(※ Adiabatic + Reversible = Isentropic!)

Problem : 2 Phase Mixture in an expander!



3.4 Adiabatic expansion

- **Isentropic expansion coefficient, μs**

$$\mu_s = \left(\frac{\partial T}{\partial P}\right)_s = -\left(\frac{\partial T}{\partial s}\right)_p \left(\frac{\partial s}{\partial P}\right)_T = \frac{T}{C_p}\left(\frac{\partial v}{\partial T}\right)_P$$

$$C_P = \left(\frac{\partial h}{\partial T}\right)_P = \left(\frac{\partial Q}{\partial T}\right)_p \; , Q = TdS$$

Maxwell's Relation

$$\left\{\begin{aligned} -\left(\frac{\partial S}{\partial P}\right)_T &= \left(\frac{\partial V}{\partial T}\right)_P \\ \left(\frac{\partial T}{\partial V}\right)_S &= -\left(\frac{\partial p}{\partial S}\right)_V \\ \left(\frac{\partial T}{\partial p}\right)_S &= +\left(\frac{\partial V}{\partial S}\right)_p \\ \left(\frac{\partial S}{\partial V}\right)_T &= +\left(\frac{\partial p}{\partial T}\right)_V \end{aligned}\right.$$

3.4 Adiabatic expansion

- **Isentropic expansion coefficient, μ_s**

$$\mu_s = \left(\frac{\partial T}{\partial P}\right)_s = -\left(\frac{\partial T}{\partial s}\right)_p \left(\frac{\partial s}{\partial P}\right)_T = \frac{T}{C_p}\left(\frac{\partial v}{\partial T}\right)_P$$

Volume expansion coefficient $\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P$ → $= \frac{T}{C_p}\beta v$

For Ideal gas :

$Pv = RT$

→ $\left(\frac{\partial v}{\partial T}\right)_P = \frac{R}{P} = \frac{v}{T}$ → $= \frac{v}{C_p}$

For Van der Waals gas :

$(P + \frac{a}{v^2})(v - b) = RT$

→ $$= \frac{v(1 - \frac{b}{v})}{C_p[1 - \left(\frac{2a}{vRT}\right)\left(1 - \frac{b}{v}\right)^2]}$$

3.4 Adiabatic expansion

- **Methods of cooling**

- External work method :
  energy is removed as external work

- Internal work method (Expansion Valve) :
  do not remove energy from gas
  but moves molecules farther apart

Liquefaction systems for gases

= A big purpose for learning cryogenic systems!

(Korea's energy supply is heavily dependent on import by ships!)

3.5 **Linde-Hampson**

3.9 **Claude**
3.10 Kapitza
3.11 Heylandt

3.6 Precooled Linde
3.8 **Cascade**
3.13 LNG (Cascade)

3.7 Linde **dual-P**
3.12 Claude dual-P

3.14 Comparison of Liquefaction Systems

(For Ne, H_2, He)
3.15 Linde
3.16 Claude
3.17 He system
3.19 Collins He system

3.18 Ortho-para H_2 Conversion

3.4 Adiabatic expansion

Why are H_2, He and Ne special?

Inversion Temperature
= boundary of semicircle of figure 3.2

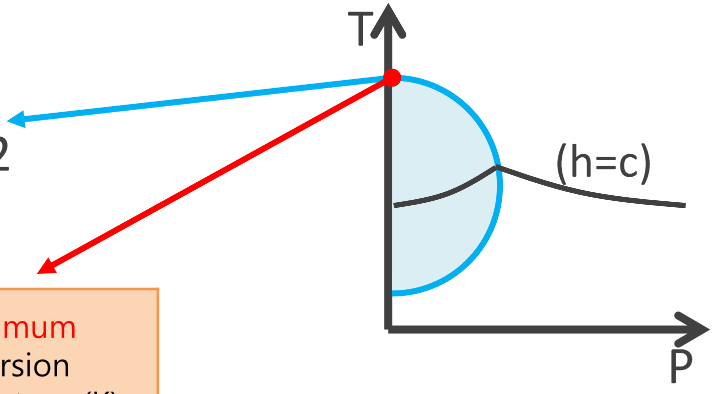


Figure 3.2
(Chapter 3.3)

Very Low!

Gas	Maximum Inversion Temperature (K)
[He] Helium-4	45
[H_2] Hydrogen	205
[Ne] Neon	250
[N_2] Nitrogen	621
[N_2+O_2] Air	603
[CO] Carbon monoxide	652
[Ar] Argon	794
[O_2] Oxygen	761
[CH_4] Methane	939
[CO2] Carbon Dioxide	1500
[NH_3] Ammonia	1994

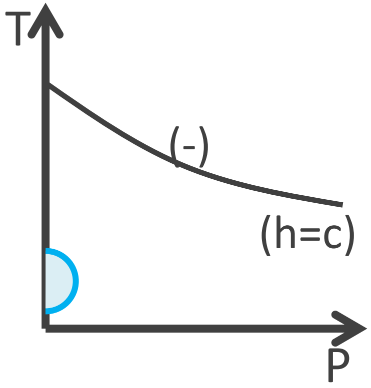
3.4 Adiabatic expansion

Why are H_2, He and Ne special?

In semicircle, $\mu JT > 0$
= Isenthalpic expansion (J-T Valve) is cooling!

Out of semicircle, $\mu JT < 0$
= Isenthalpic expansion (J-T valve) is heating!

= We cannot make liquid by J-T valve!
= To make liquid, we must applicate expander or dual-pressure or pre-cooling.



Case of H2, He, Ne
(Circle is very small)

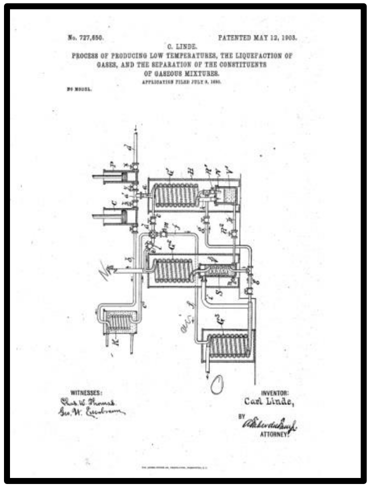
3.5 Simple Linde-Hampson System

- **History**

The most simple gas liquefaction/separation system.

This system is based on the Joule-Thomson effect.
Chapter 3.3. When a (non-ideal) gas expands, it will cold down (below inversion temperature). This is why our whistle is colder than our body temperature!

William Hampson and Carl von Linde
independently filed for patent
of the cycle in 1895.



No. 727,650. PATENTED MAY 12, 1903.

C. LINDE.

PROCESS OF PRODUCING LOW TEMPERATURES, THE LIQUEFACTION OF
GASES, AND THE SEPARATION OF THE CONSTITUENTS
OF GASEOUS MIXTURES.

APPLICATION FILED JULY 8, 1895.

NO MODEL.

WITNESSES:
Chas. W. Thomas
Geo. W.

INVENTOR:
Carl Linde,
BY
ATTORNEY.

3.5 Simple Linde-Hampson System

- **History**

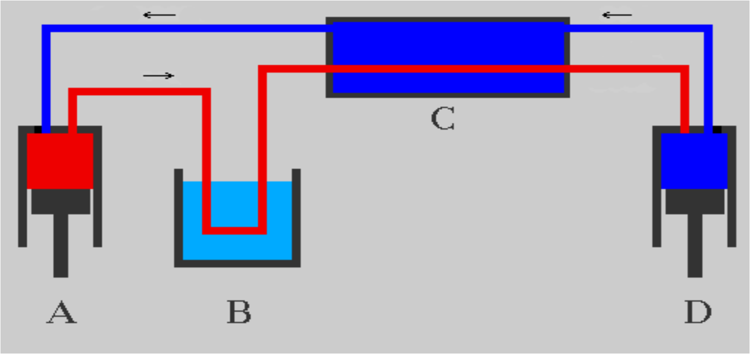
Brother founds

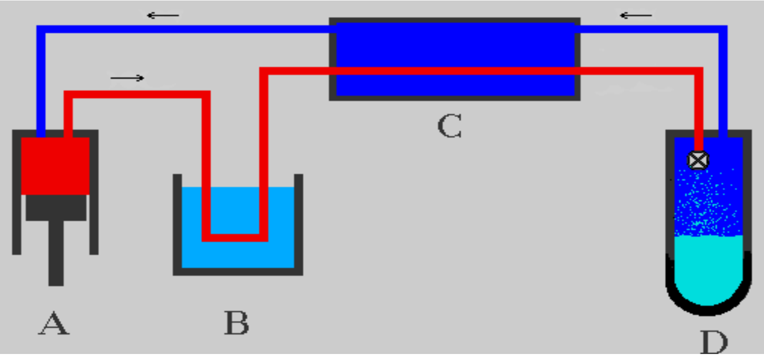
SIEMENS

He founds himself

Linde

Enhance from Siemens cycle(1857) to Linde-Hampson cycle(1895)





A : Compression
B : Cooling
C : Cooling (HX)
D : Expansion
C : Heating (HX)

A : Compression
B : Cooling
C : Cooling (HX)
D : Joule-Thompson Orifice
(Reservoir + Expansion Valve)
C : Heating (HX)

3.5 Simple Linde-Hampson System

- **History**

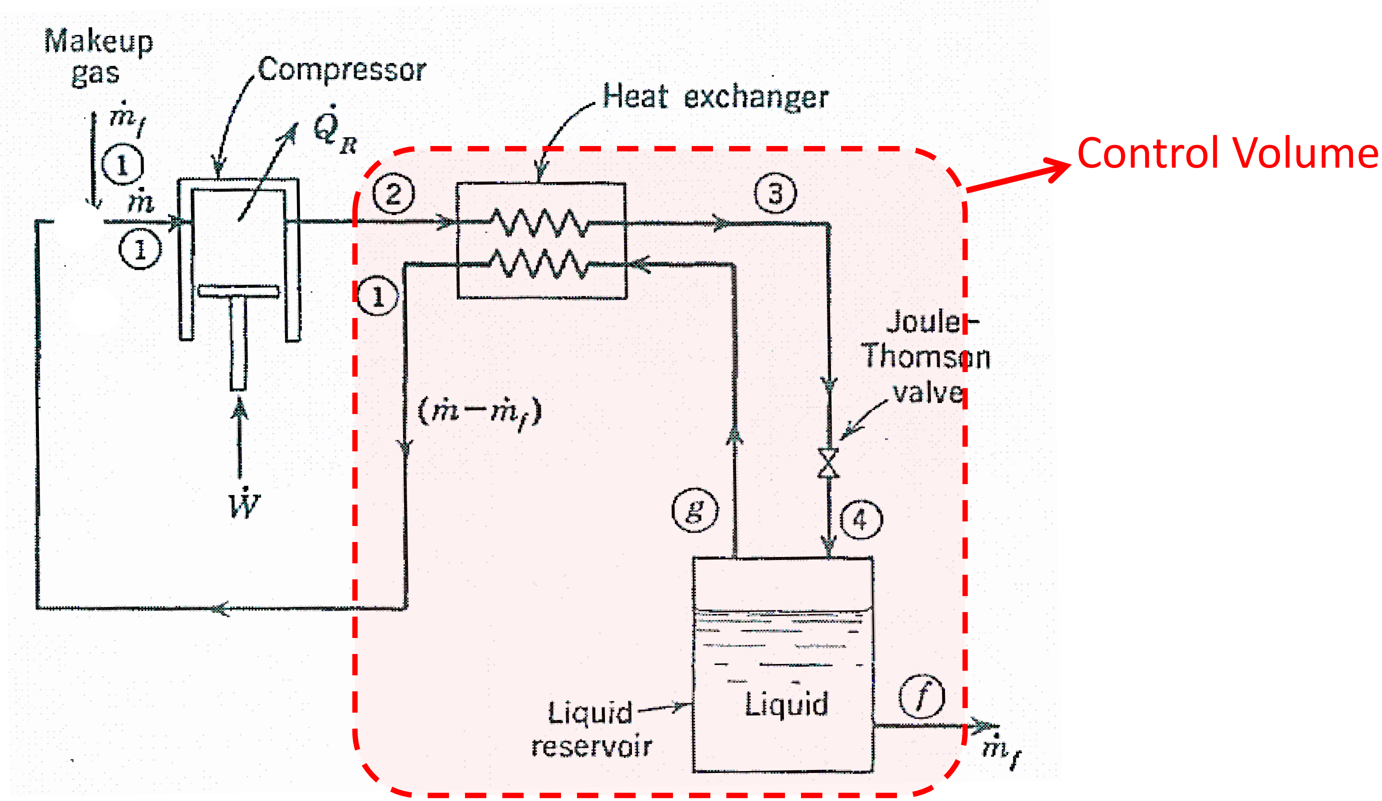
Siemens cycle has poor efficiency and is only efficient for high temperature gases. But Linde-Hampson cycle can easily collect liquid and cool dramatically.

Heike Kamerlingh Onnes made 'liquid helium' (1908)
by this Linde-Hampson system and found 'super-conductivity'.

And He got Nobel prize in Physics (1913)!

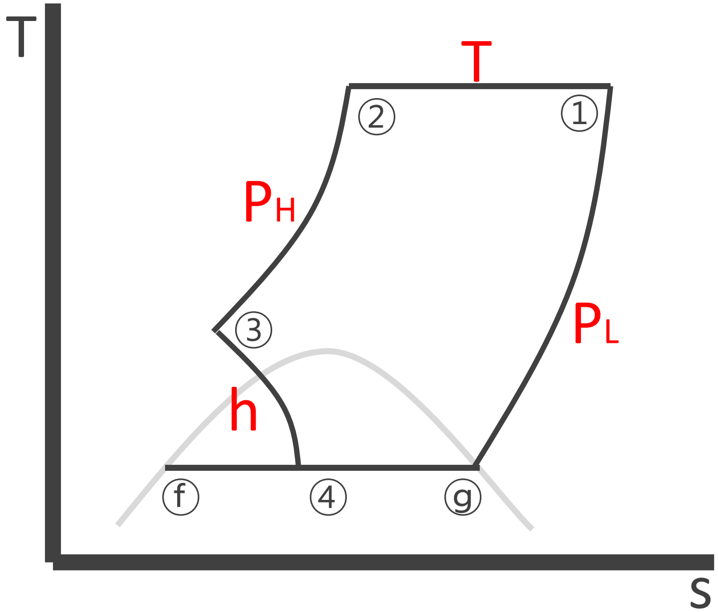
3.5 Simple Linde-Hampson System

- **Diagram**



3.5 Simple Linde-Hampson System

- **Diagram**



Normal room : T_1=300K, P_1=1bar

Liquid N_2 : T_4=77K

Constant T : Isothermal

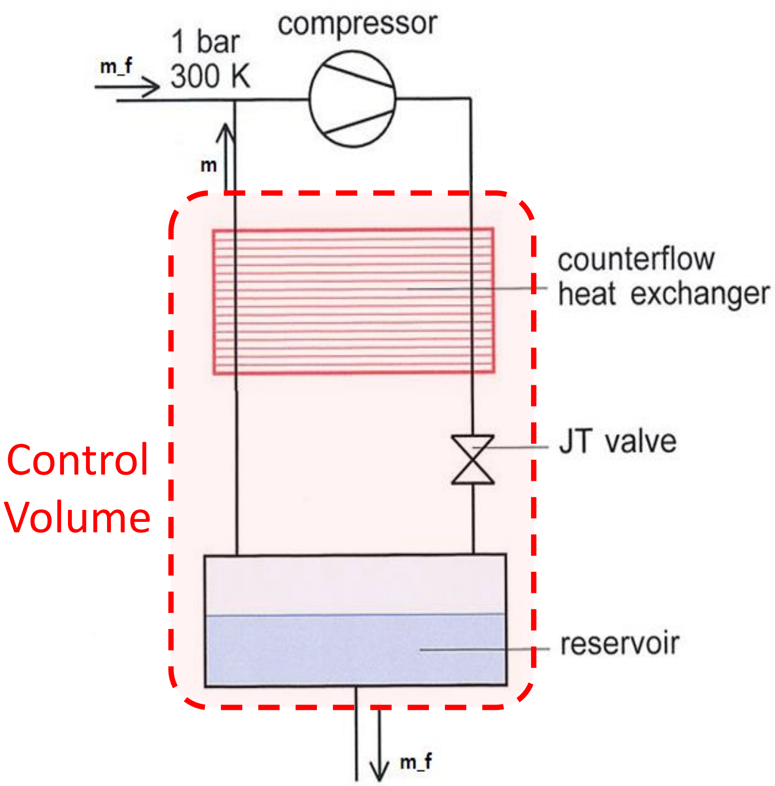
Constant P : Isobaric

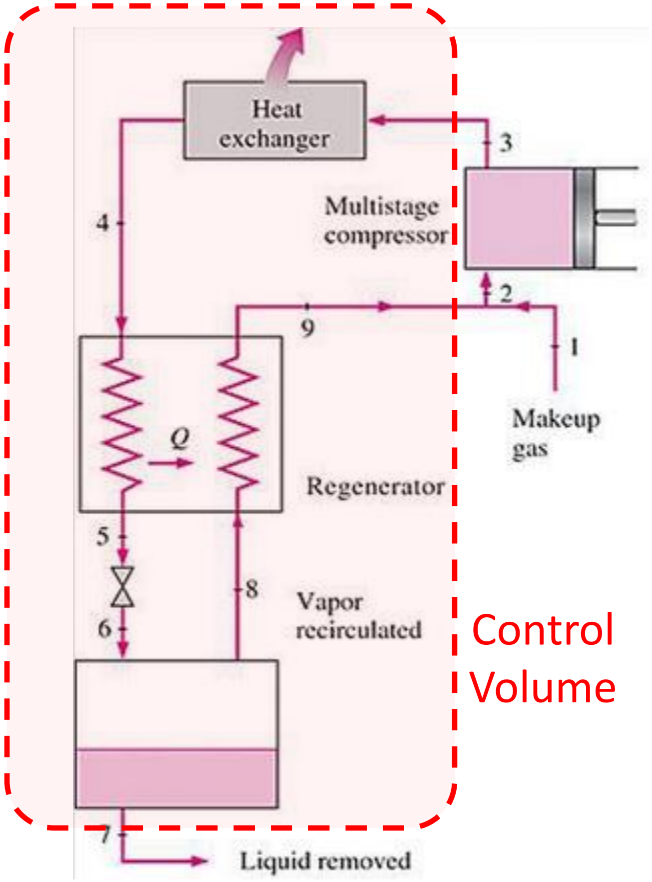
Constant h : Isenthalpic

3.5 Simple Linde-Hampson System

- **Diagram**

Another form of diagrams





3.5 Simple Linde-Hampson System

- **Diagram**

Assumption

- Reversible pressure drop
- No heat in-leak
  (Reversible isothermal process in compressor)
- 100% effective heat exchanger

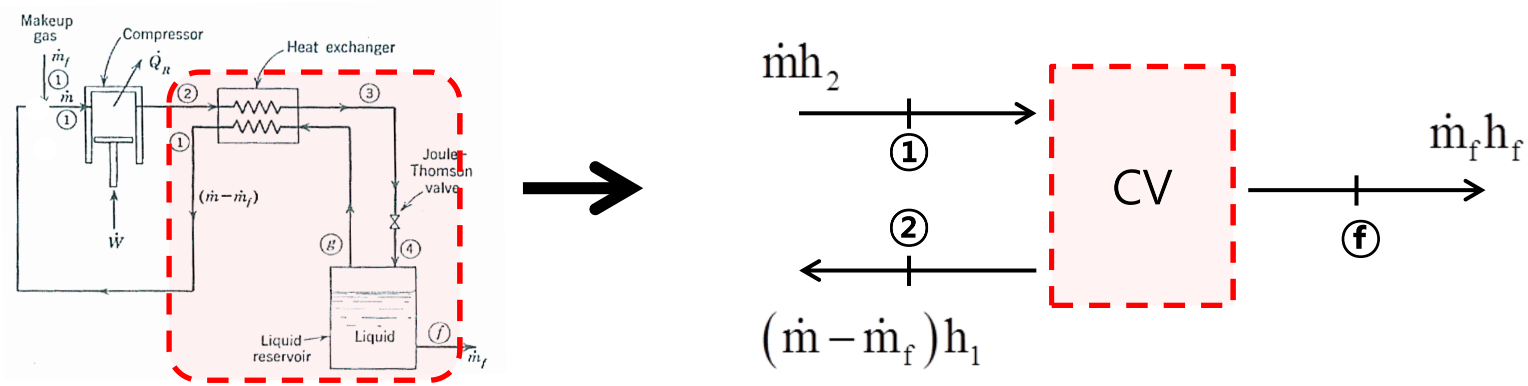
Quality $x = \frac{(mass\ of\ sat.gas)}{(total\ mass)}$

(※ Volume of gas is very larger than liquid)

3.5 Simple Linde-Hampson System

- **Questions**

Q1. We can choose P2 in the system. Which is best P2?



At control volume, $\dot{m}h_2 = (\dot{m} - \dot{m}_f)h_1 + \dot{m}h_f$

Liquid yield $$y = \frac{m_f}{m} = \frac{h_1 - h_2}{h_1 - h_f}$$

: Fraction of gas flow that is liquefied

3.5 Simple Linde-Hampson System

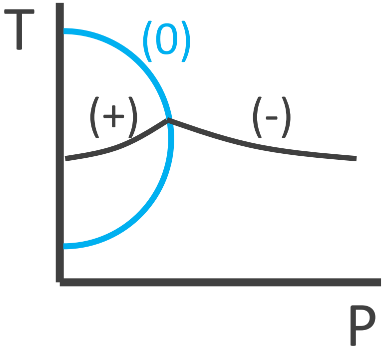
- **Questions**

Best P2 is $\frac{\partial y}{\partial P_2} = 0$

→ (h_1, h_f is fixed value) $\frac{\partial h_2}{\partial P_2} = 0$

→ $\frac{\partial h}{\partial P} = -\mu_{JT} C_P$

→ (C_P is positive number) $\mu_{JT} = 0$

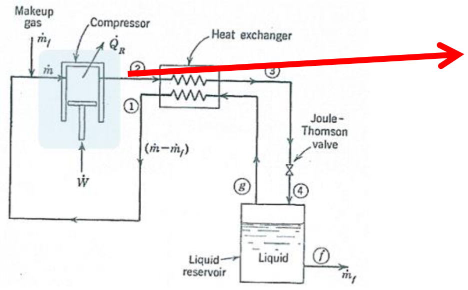


→ μ_{JT} is slope at T-P diagram.
So, semicircle in T-P graph means best P_2!
(Inversion Curve) (Chapter 3.3)

3.5 Simple Linde-Hampson System

- **Questions**

Q2. How much works do we need for running this system?



At compressor,

$$\dot{Q}_R - \dot{W} = \dot{m}(h_2 - h_1)$$

$$※\ \dot{Q}_R = \dot{m}T_1(s_1 - s_2)$$

$$-\frac{\dot{W}}{\dot{m}} = T_1(s_1 - s_2) - (h_1 - h_2)$$

$$-\frac{\dot{W}}{\dot{m}_f} = -\frac{\dot{W}}{\dot{m}y} = (\frac{h_1 - h_f}{h_1 - h_2})[T_1\,(s_1 - s_2) - (h_1 - h_2)]$$

3.5 Simple Linde-Hampson System

- **Questions**

Q3. What will happen in real system without assumption?

Reversible pressure drop
→ P_2 will be lower.

No heat in-leak
Reversible isothermal process in compressor
→ Q_{elec} will be added. And y will be lower.

$$\dot{m}h_2 + Q_{elec} = (\dot{m} - \dot{m}_f)h_1 + \dot{m}h_f$$

3.5 Simple Linde-Hampson System

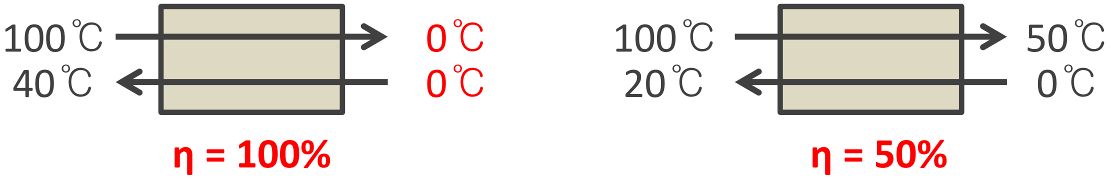
- **Questions**

100% effective heat exchanger

→ Temperature difference at both side will be lower.

To keep temperature difference, we need input more pump work.

(To enlarge mass flow rate)



3.5 Simple Linde-Hampson System

- **It's not for Ne/H2/He !** → **We will learn later... (Chapter 3.15~3.19)**

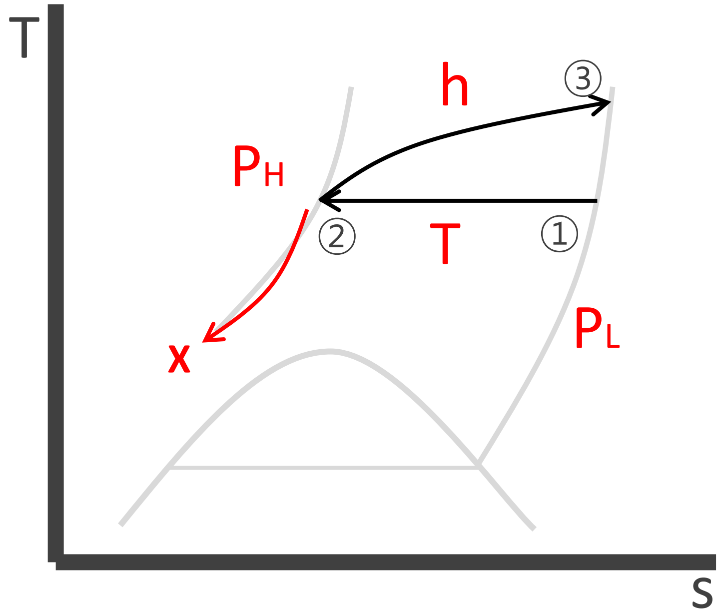


Figure 3.6

Reason 1.

Gas	M.I.T. (K)
[He] Helium-4	45
[H2] Hydrogen	205
[Ne] Neon	250

Maximum inversion temp. << room temp.

→ Their expansion = heating!

→ Gas in HX warmed rather than cooled!

3.5 Simple Linde-Hampson System

- **It's not for Ne/H2/He !** → **We will learn later… (Chapter 3.15~3.19)**

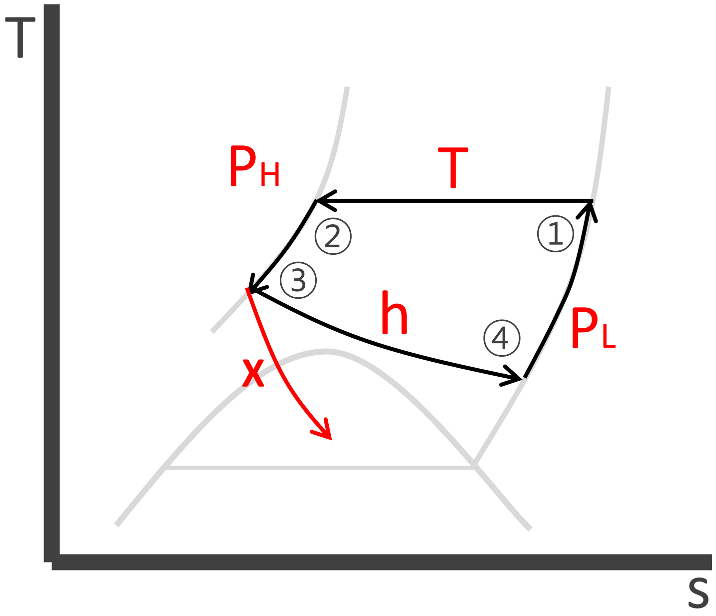


Figure 3.7

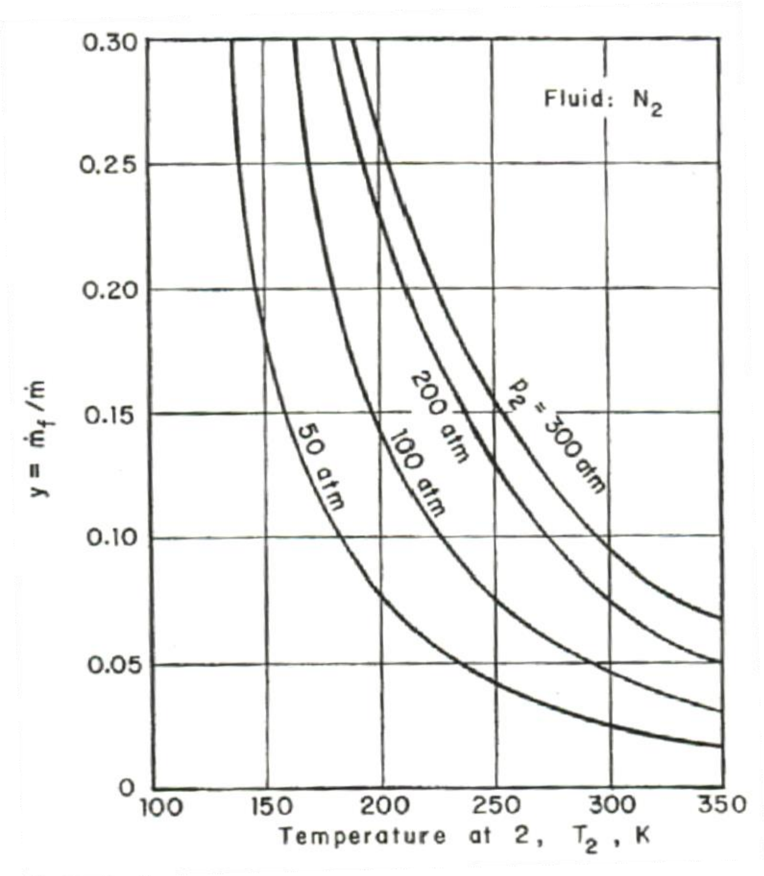
$$y = \frac{m_f}{m} = \frac{h_1 - h_2}{h_1 - h_f}$$

Reason 2.

Liquid yield(y) is negative. (h1 < h2)

→ Even if we could attain low temperature, no gas would be liquefied.

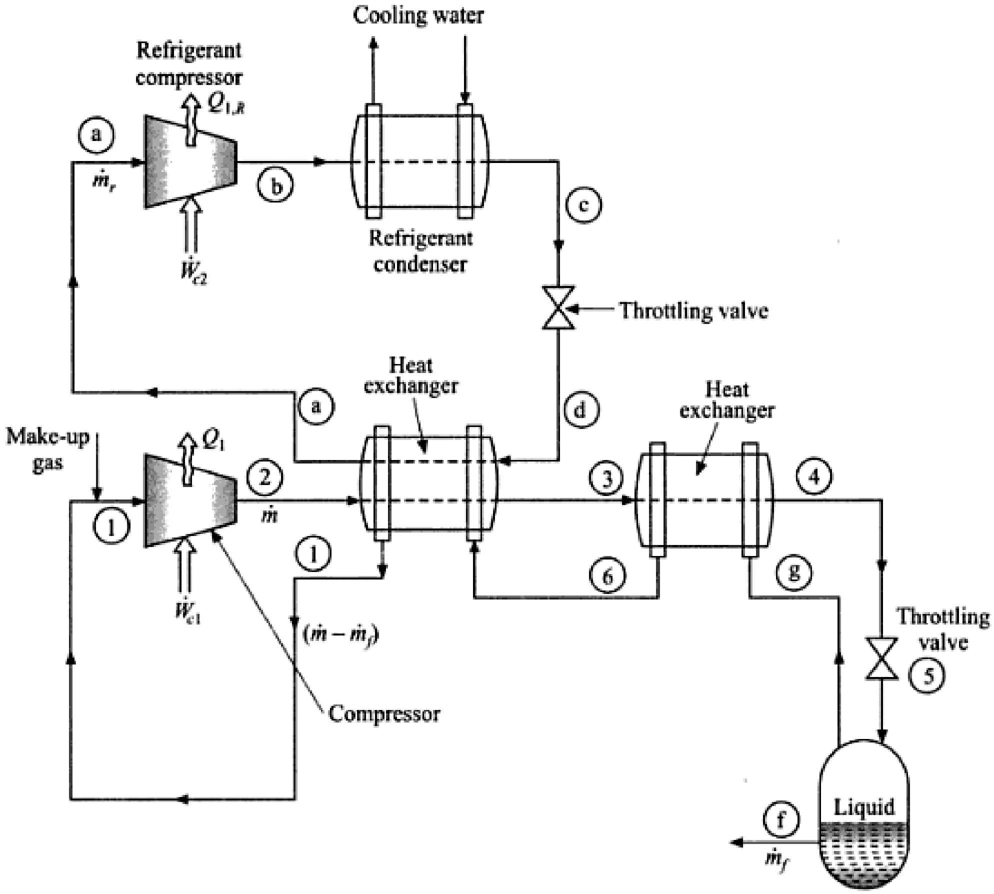
3.6 Precooled Linde-Hampson system



Liquid yield versus compressor temperature
for a Linde-Hampson system
using nitrogen as the working fluid

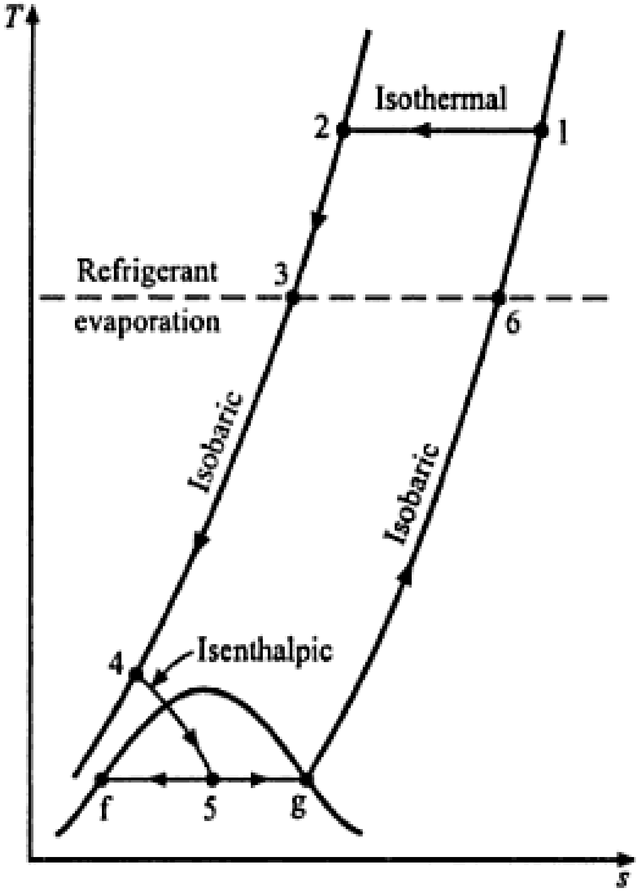
- It is apparent that the performance of a Linde-Hampson system could be improved if the gas entered the heat exchanger at a temperature lower than ambient temperature

3.6 Precooled Linde-Hampson system



Precooled Linde-Hampson system

3.6 Precooled Linde-Hampson system



Precooled Linde-Hampson system T-S diagram

3.6 Precooled Linde-Hampson system

- **Liquid yield**

Applying the First Law for steady flow to the heat exchanger, the two liquid receivers, and the two expansion valves.

$$\dot{m}h_2 + \dot{m}_r h_d = \left(\dot{m} - \dot{m}_f\right)h_1 + \dot{m}_r h_a + \dot{m}_f h_f$$

$$y = \frac{\dot{m}_f}{\dot{m}} = \frac{h_1 - h_2}{h_1 - h_f} + r\frac{h_a - h_d}{h_1 - h_f}$$

$$r = \dot{m}_r / \dot{m}$$

r is the refrigerant mass flow rate ratio
$\dot{m}_r$ is the mass flow rate of the auxiliary refrigerant
$\dot{m}$ is the total mass flow rate through the high pressure compressor

3.6 Precooled Linde-Hampson system

- **Liquid yield**

The second term of liquid yield represents the improvement in liquid yield that is obtained through the use of precooling.

$$y = \frac{\dot{m}_f}{\dot{m}} = \frac{h_1 - h_2}{h_1 - h_f} + r\frac{h_a - h_d}{h_1 - h_f}$$

3.6 Precooled Linde-Hampson system

- **Limit of the liquid yield 1**

From the Second Law of Thermodynamics, T_3 and T_6 cannot be lower than the boiling point of the auxiliary refrigerant at point d

3.6 Precooled Linde-Hampson system

- **Limit of the liquid yield 1**

  : The maximum liquid yield

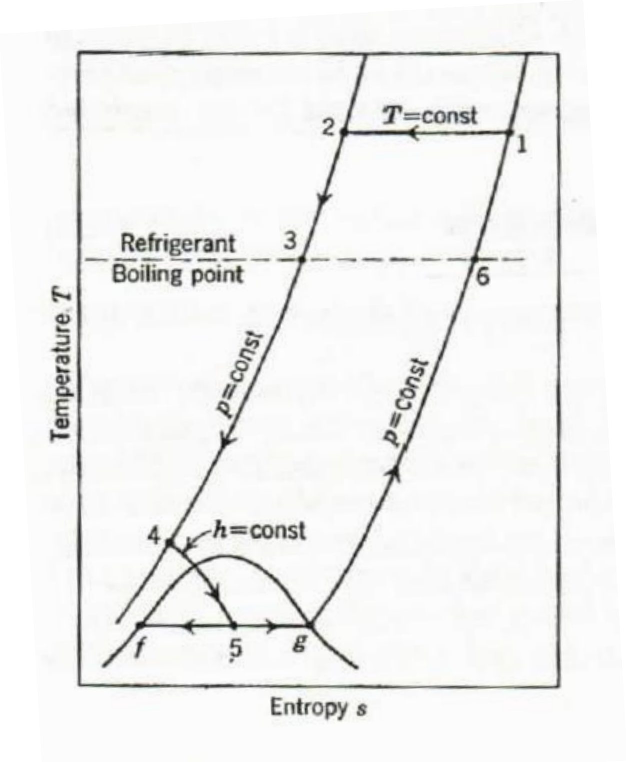
With a suitable value of the refrigerant flow-rate ratio r, liquid yield could have a value of 1, which means 100 percent for the liquid yield.

$$y_{max} = \frac{h_6 - h_3}{h_6 - h_f}$$

3.6 Precooled Linde-Hampson system

- **Limit of the liquid yield 1**

  : The maximum liquid yield



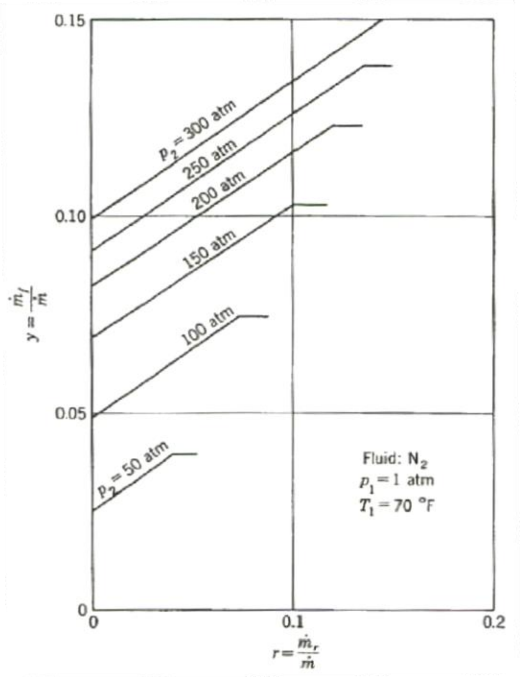
Precooled Linde-Hampson cycle

h_3 and h_6 are taken at the temperature of the boiling refrigerant at point d)

$$y_{max} = \frac{h_6 - h_3}{h_6 - h_f}$$

3.6 Precooled Linde-Hampson system

- **Limit of the liquid yield 2**



Liquid yield versus refrigerant flow rate ratio for the precooled Linde-Hampson system using nitrogen as the working fluid

If the refrigerant flow rate ratio were too large, the liquid at point d would not be completely vaporized, and liquid would enter the refrigerant compressor.

3.6 Precooled Linde-Hampson system

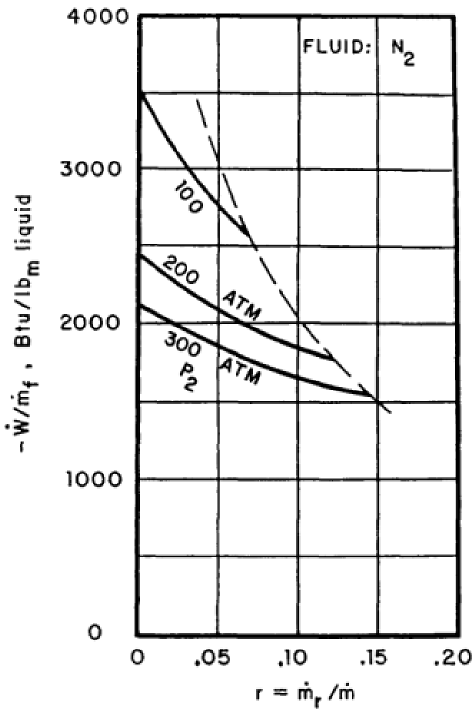
- **The work requirement**

  - If the main compressor is reversible and isothermal and the auxiliary compressor is reversible and adiabatic.
  - The last term represents the additional work requirement for the auxiliary compressor. (usually on the order of 10 percent of the total work)

$$-\frac{\dot{W}}{\dot{m}} = T_1(s_1 - s_2) - (h_1 - h_2) + r(h_b - h_a)$$

3.6 Precooled Linde-Hampson system

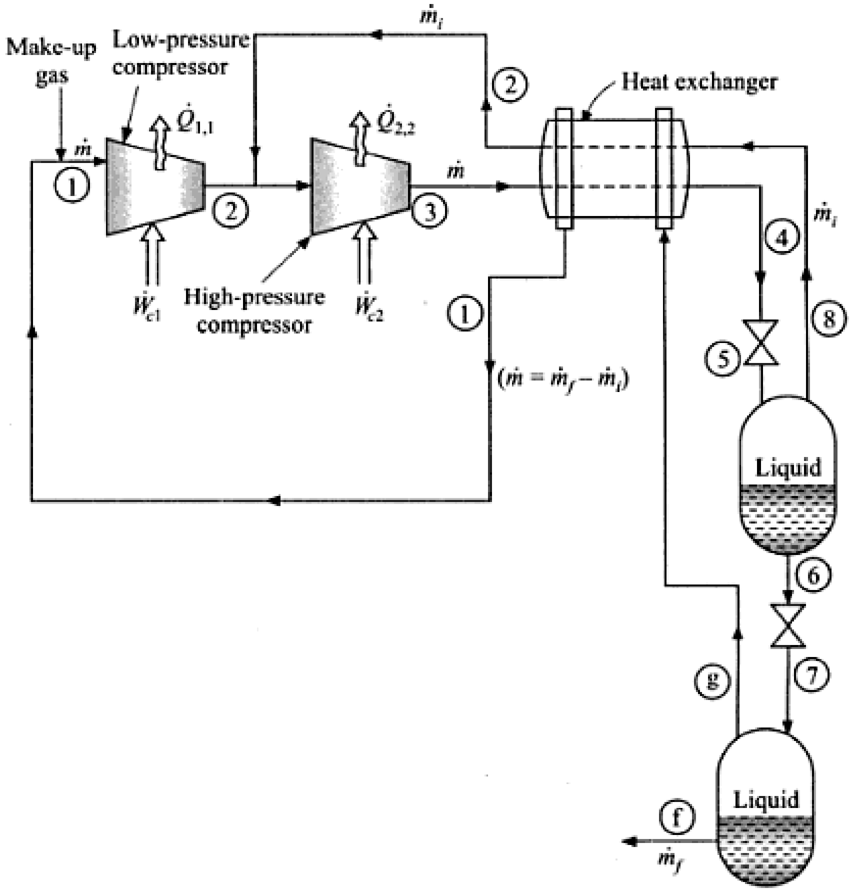
- **The work requirement**



Work required to liquefy a unit mass of nitrogen in a precooled Linde-Hampson system

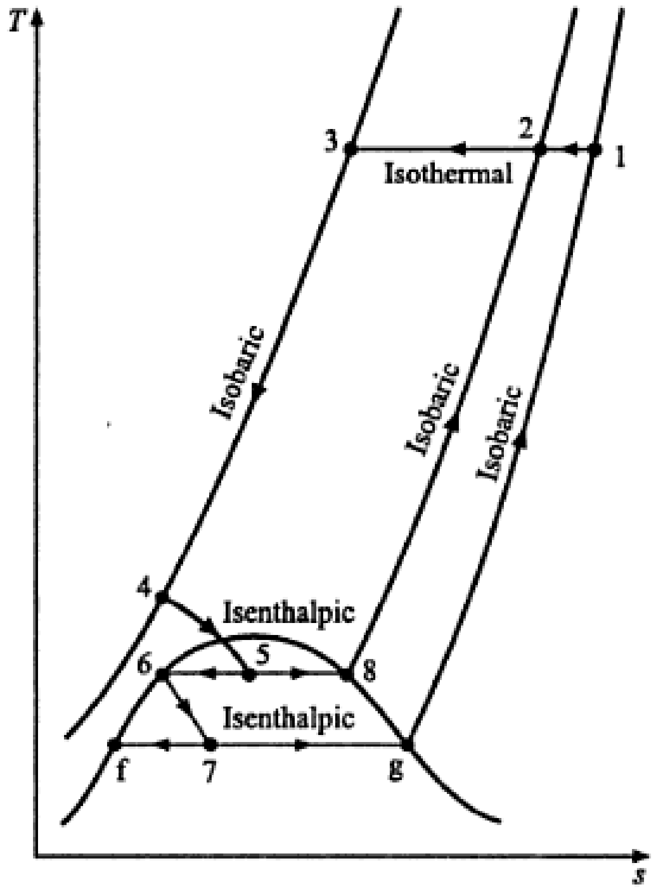
The increase in liquid yield more than offsets the additional work requirement, however, so that the work requirement per unit mass of gas liquefied is actually less for the precooled system than for the simple system.

3.7 Linde dual-pressure cycle



Linde dual-pressure system

3.7 Collins helium-liquefaction system



Linde dual-pressure system T-S diagram

3.7 Linde dual-pressure cycle

- **Liquid yield**

Applying the First Law for steady flow to the heat exchanger, the two liquid receivers, and the two expansion valves.

$$\dot{m}h_3 = \dot{m}_f h_f + \dot{m}_i h_2 + \left(\dot{m} - \dot{m}_i - \dot{m}_f\right)h_1$$

$$y = \frac{\dot{m}_f}{\dot{m}} = \frac{h_1 - h_3}{h_1 - h_f} - i\frac{h_1 - h_2}{h_1 - h_f}$$

$$i = \dot{m}_i / \dot{m}$$

i is the intermediate pressure stream flow rate ratio
$\dot{m}_i$ is the mass flow rate of the intermediate pressure stream at point 8
$\dot{m}$ is the total mass flow rate through the high pressure compressor

3.7 Linde dual-pressure cycle

- **Liquid yield**

- This modification reduces the liquid yield somewhat.
- The second term of liquid yield represents the reduction in the liquid yield below that of the simple system because of splitting the flow at the intermediate pressure liquid receiver.

$$y = \frac{\dot{m}_f}{\dot{m}} = \frac{h_1 - h_3}{h_1 - h_f} - i\,\frac{h_1 - h_2}{h_1 - h_f}$$

3.7 Linde dual-pressure cycle

- **The work requirement**

Applying the First Law for steady flow to the two compressors.

$$\left(\dot{Q}_{R1}-\dot{W}_{C1}\right)+\left(\dot{Q}_{R2}-\dot{W}_{C2}\right)=\dot{m}h_3-\left(\dot{m}-\dot{m}_i\right)h_1-\dot{m}_i h_2$$

$$\dot{Q}_{R1}=-\left(\dot{m}-\dot{m}_i\right)T_1\left(s_1-s_2\right)$$
$$\dot{Q}_{R2}=-\dot{m}T_1\left(s_2-s_3\right)$$

$$-\frac{\dot{W}}{\dot{m}}=\left[T_1\left(s_1-s_3\right)-\left(h_1-h_3\right)\right]-i\left[T_1\left(s_1-s_2\right)-\left(h_1-h_2\right)\right]$$

3.7 Linde dual-pressure cycle

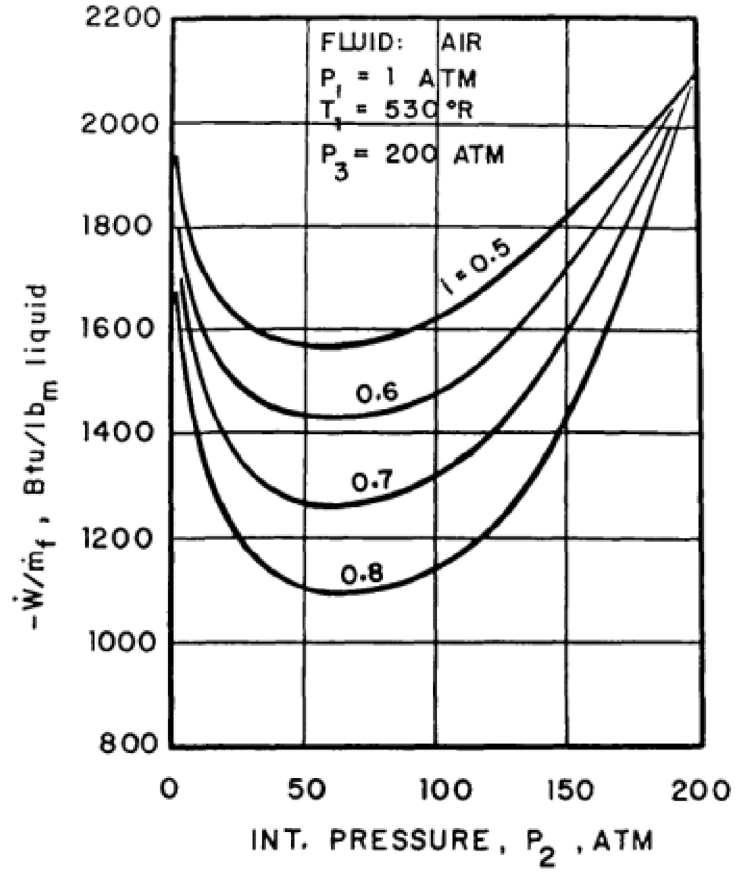
- **The work requirement**

This modification reduces the total work required.

The work requirement is reduced below that of the simple system by the amount given by the second bracketed term.

$$-\frac{\dot{W}}{\dot{m}}=\left[T_1\left(s_1-s_3\right)-\left(h_1-h_3\right)\right]-i\left[T_1\left(s_1-s_2\right)-\left(h_1-h_2\right)\right]$$

3.7 Linde dual-pressure cycle



Work required to liquefy a unit mass of air
in the Linde dual-pressure system

3.7 Linde dual-pressure cycle

- **Optimal intermediate pressure**

  - As ‘Work required to liquefy a unit mass of air in the Linde dual-pressure system’ shows, there is an optimum intermediate pressure p_2 for a given intermediate stream mass flow rate ratio, which makes the work requirements a minimum.

  - Typical air liquefaction plants operate with $i = 0.8$, $p_3 = 200 atm$, p_2 $between$ 40 and 50 atm.

3.8 Cascade system

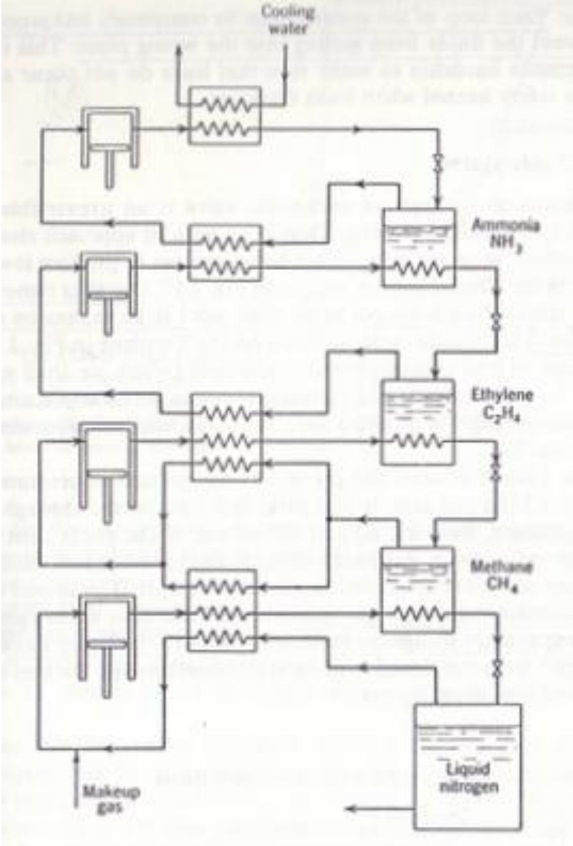


Fig. 3.16. The cascade system.

- The cascade system is an extension of the precooled system
- There are refrigeration system chain of ammonia – ethylene – methane -nitrogen
- From a thermodynamic point of view, the cascade system in very desirable for liquefaction because it approaches the ideal reversible system more closely than any other discussed thus far0

3.8 Cascade system

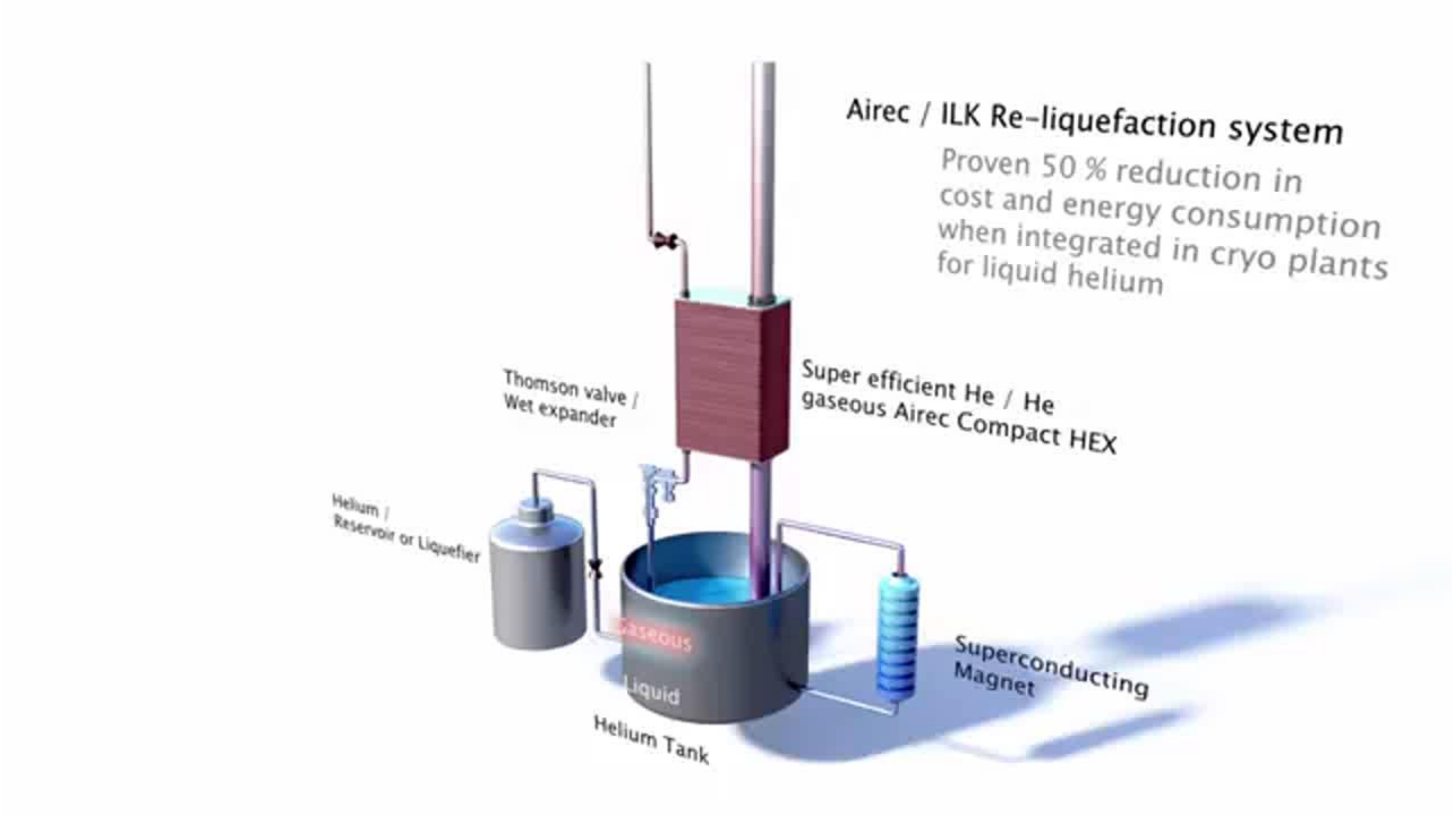
$$\dot{m}_f h_f + (\dot{m} - \dot{m}_f)h_1 - \dot{m}h_2 + \sum_{i=1}^{n_{comp}} \dot{m}_{x,i}\left(h_{a,i} - h_{b,i}\right) + \dot{m}_{x,n}\left(h_{b,n} - h_{c,n}\right) = 0$$

Liquid yield : $y = \dfrac{h_1 - h_2}{h_1 - h_f} + \displaystyle\sum_{i=1}^{n\_comp} x_i \dfrac{h_{i,i} - h_{e,i}}{h_1 - h_f} + \dot{m}_{x,n}\left(h_{b,n} - h_{c,n}\right)$

$$-\dot{W} = \dot{m}[(h_2 - h_1) - T_1(h_2 - h_1)]h_f + \sum_{i=1}^{n\_comp} \dot{m}_{x,i}\left(h_{e,i} - h_{i,i}\right)$$

Power per mass flow : $-\dfrac{\dot{W}}{\dot{m}} = [(h_2 - h_1) - T_1(h_2 - h_1)]h_f + \displaystyle\sum_{i=1}^{n\_comp} x_i\left(h_{e,i} - h_{i,i}\right)$

Cryogenic system video

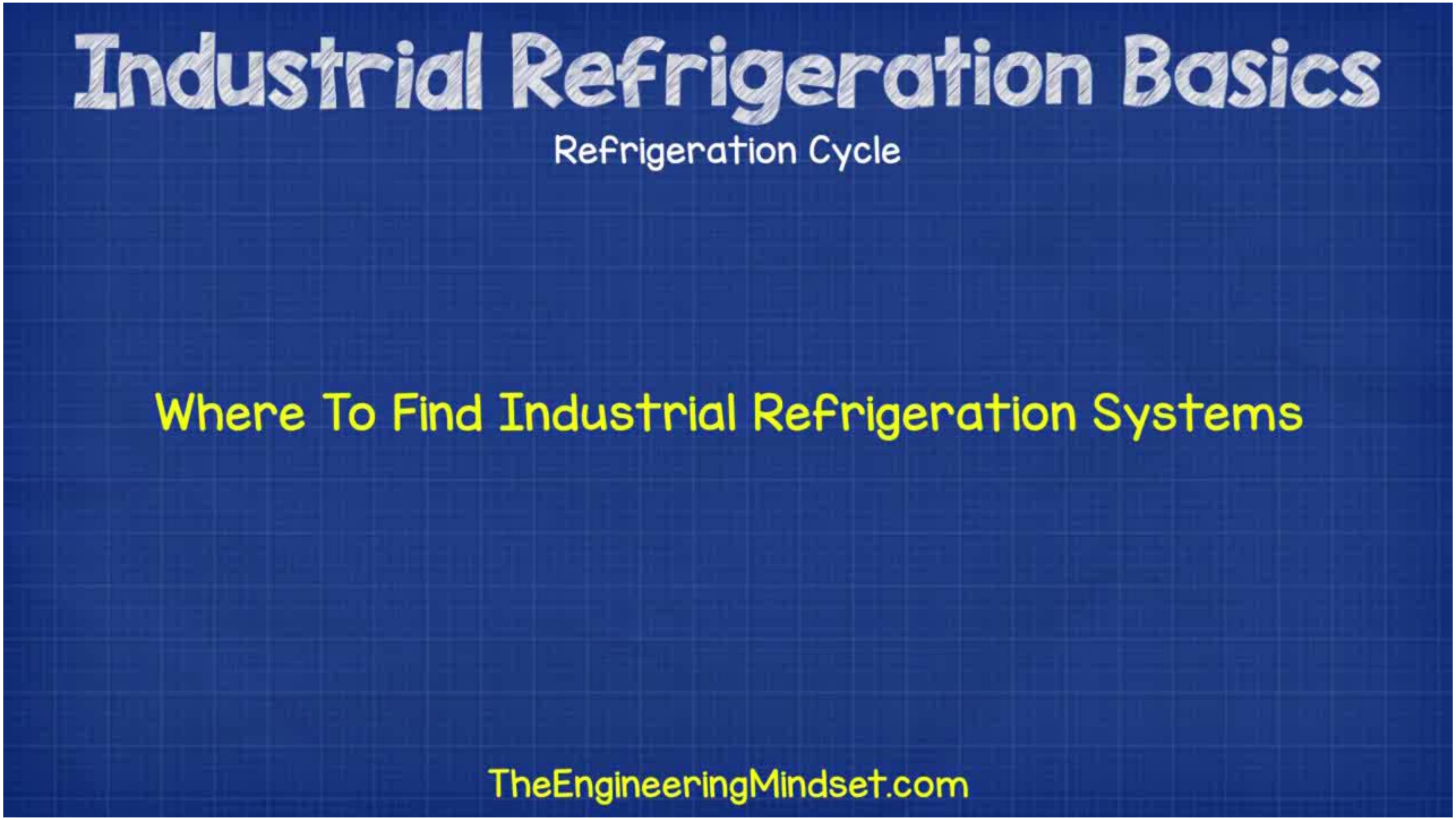


Airec / ILK Re-liquefaction system

Proven 50 % reduction in cost and energy consumption when integrated in cryo plants for liquid helium

Cryogenic system video

Cryogenic system video



Industrial Refrigeration Basics

Refrigeration Cycle

Where To Find Industrial Refrigeration Systems

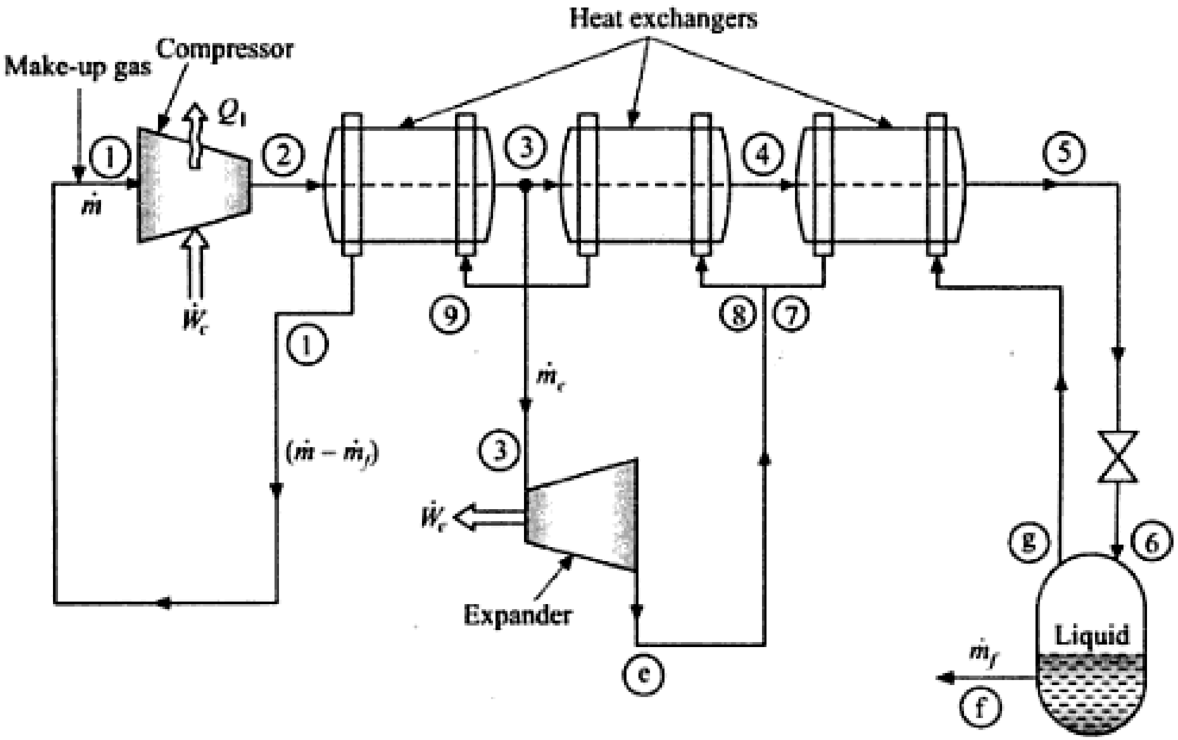
TheEngineeringMindset.com

3.9 Claude system

Georges Claude(1870-1960)

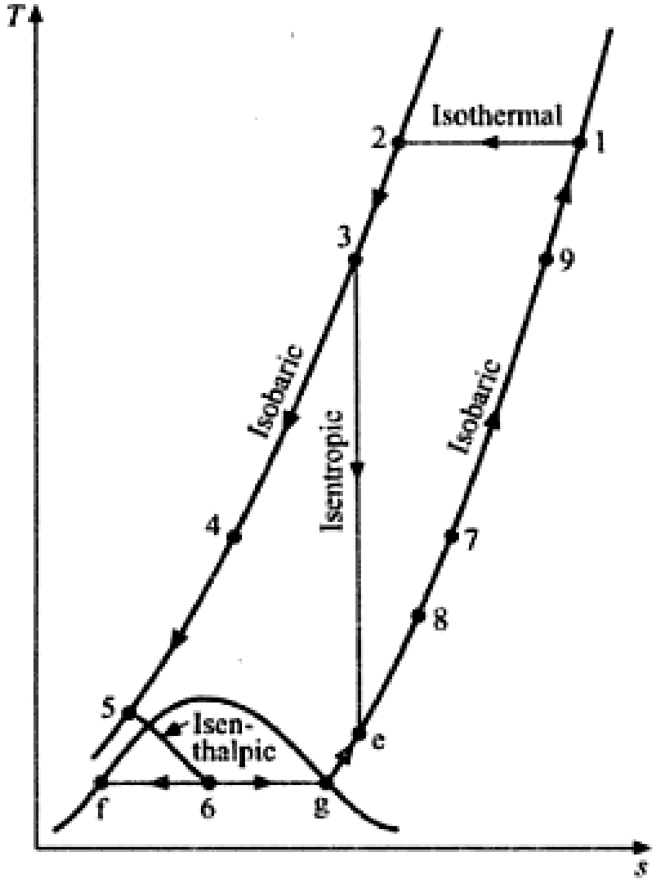
In 1902 Claude devised what is now known as the Claude system for liquefying air. The system enabled the production of industrial quantities of liquid nitrogen, oxygen, and argon

3.9 Claude system



Claude system

3.9 Claude system



Claude system T-S diagram

3.9 Claude system

- **Valve VS Expansion engine or expander**

  - The expansion through an expansion valve is an irreversible process. Thus if we wish to approach closer to the ideal performance, we must seek a better process.

  - If the expansion engine is reversible and adiabatic, the expansion process in isentropic, and a much lower temperature is attained than an isenthalpic expansion.

3.9 Claude system

- **Why we can not eliminate the expansion valve?**

The expansion valve could not be eliminated because of the problem of two-phase flow within the engine cylinder or turbine blade flow passages.

3.9 Claude system

- **Liquid yield**

Applying the First Law for steady flow to the heat exchangers, the expansion valve, and the liquid receiver as a unit, for no external heat transfer.

$$\dot{m}h_2 + \dot{m}_e h_e = \left(\dot{m} - \dot{m}_f\right)h_1 + \dot{m}_e h_3 + \dot{m}_f h_f$$

$$y = \frac{\dot{m}_f}{\dot{m}} = \frac{h_1 - h_2}{h_1 - h_f} + x\frac{h_3 - h_e}{h_1 - h_f}$$

$$x = \dot{m}_e / \dot{m}$$

x is the fraction of the total flow that passes through the expander

$\dot{m}_e$ is the mass flow rate of fluid through expander

$\dot{m}$ is the total mass flow rate through the high pressure compressor

3.9 Claude system

- **Liquid yield**

The second term of liquid yield represents the improvement in performance over the simple Linde Hampson system.

$$y = \frac{\dot{m}_f}{\dot{m}} = \frac{h_1 - h_2}{h_1 - h_f} + x\frac{h_3 - h_e}{h_1 - h_f}$$

3.9 Claude system

- **The work requirement**

The work requirement per unit mass compressed is exactly the same as that for the Linde Hampson system if the expander work is not utilized to help in the compression.

$$-\frac{\dot{W}}{\dot{m}} = T_1(s_1 - s_2) - (h_1 - h_2)$$

The work requirement per unit mass compressed for the Linde Hampson system

3.9 Claude system

- **The work requirement**

If the expander work is used to aid in compression, then the net work requirement is given by

$$-\frac{\dot{W}}{\dot{m}} = -\frac{\dot{W}_c}{\dot{m}} - \frac{\dot{W}_e}{\dot{m}}$$

$$\frac{\dot{W}_e}{\dot{m}} = x\left(h_3 - h_e\right)$$

$$\frac{\dot{W}_c}{\dot{m}} = \left(h_2 - h_1\right) + T_1\left(s_1 - s_2\right)$$

The net work is given by

$$-\frac{\dot{W}}{\dot{m}} = \left[T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)\right] - x\left(h_3 - h_e\right)$$

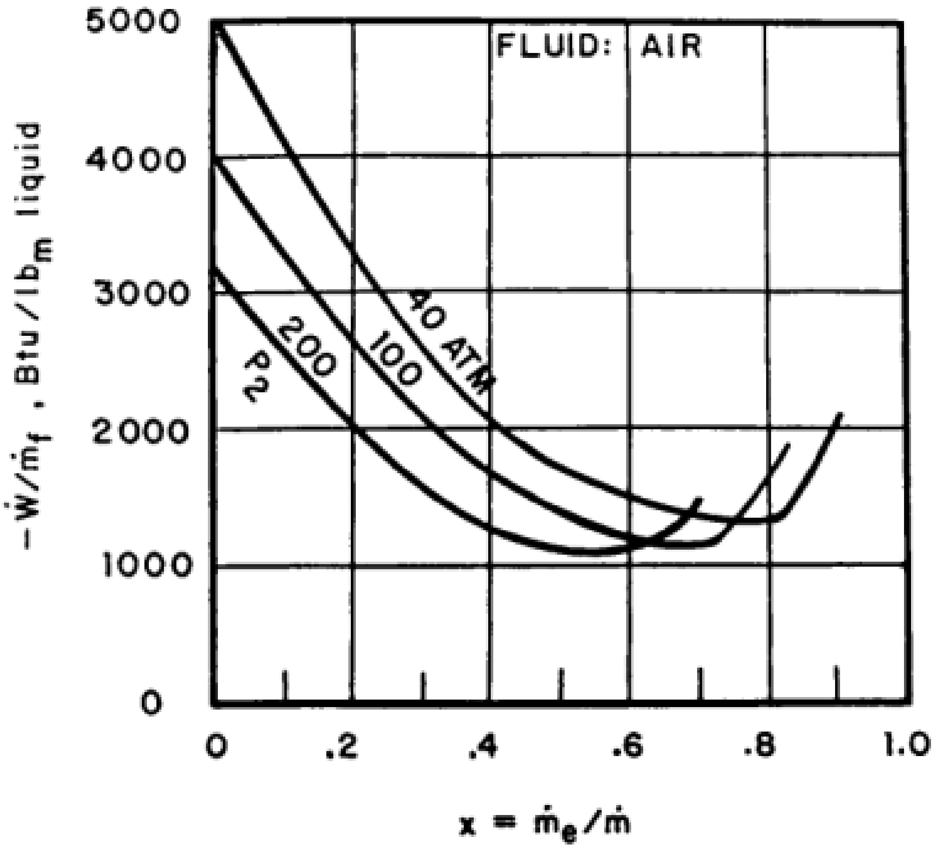
3.9 Claude system

- **The work requirement**

The last term is the reduction in energy requirements due to the utilization of the expander work output.

$$-\frac{\dot{W}}{\dot{m}}=\left[T_1\left(s_1-s_2\right)-\left(h_1-h_2\right)\right]-x\left(h_3-h_e\right)$$

3.9 Claude system



Work required to liquefy a unit mass of air in the Claude system

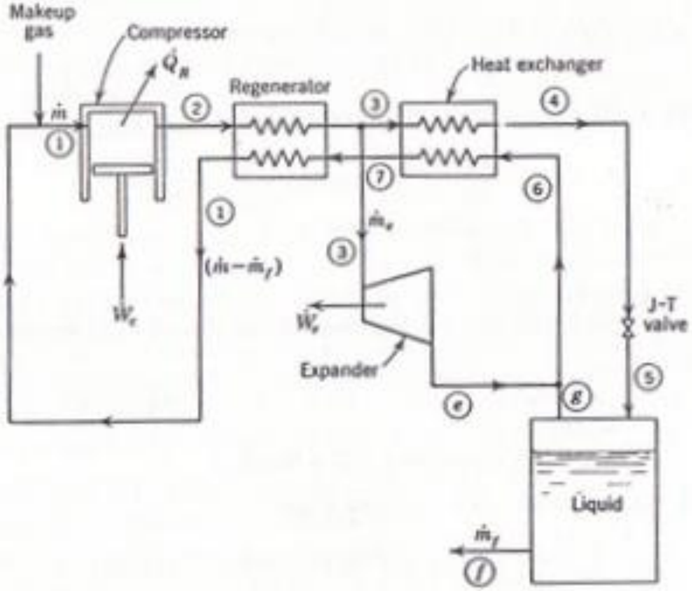
3.9 Claude system

- **Smallest work requirement per unit mass liquefied**

  - There is a finite temperature at point 3 that will yield the smallest work requirement per unit mass liquefied.

  - As the high pressure is increased, the minimum work requirement per unit mass liquefied decreases.

3.10 Kapitza system



- Modified Claude system which eliminate the third low temperature heat exchanger
- A rotary expansion engine was used instead of a reciprocating expander
- Kapitza system usually operated at relatively low pressures-on the order of 700kPa

3.10 Kapitza system

$$\dot{m}_f h_f + (\dot{m} - \dot{m}_f)h_1 - \dot{m}h_2 + \dot{m}_x(h_3 - h_e) = 0$$

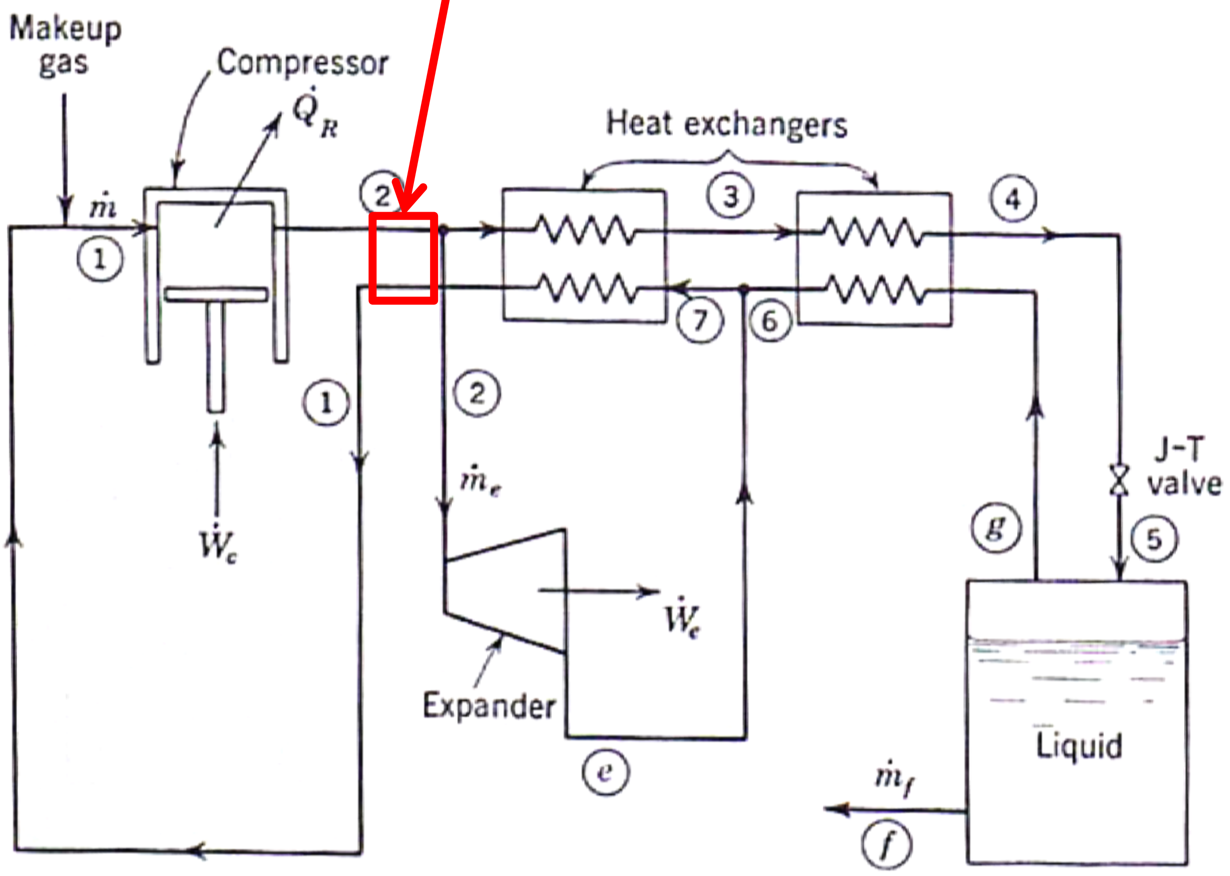
Liquid yield : $y = \dfrac{h_1 - h_2}{h_1 - h_f} + x\dfrac{h_3 - h_e}{h_1 - h_f}$

$$-\dot{W} = \dot{m}[(h_2 - h_1) - T_1(h_2 - h_1)]h_f + \dot{m}_x(h_e - h_3)$$

Power per mass flow : $-\dfrac{\dot{W}}{\dot{m}} = [(h_2 - h_1) - T_1(h_2 - h_1)]h_f + x(h_e - h_3)$

3.11 Heylandt System

(Claude system has one more HX at here)



3.11 Heylandt System

For a high pressure (Approximately 20 MPa ≒ 200 atm)
For an expansion-engine (flow-rate ratio of approximately 0.6)

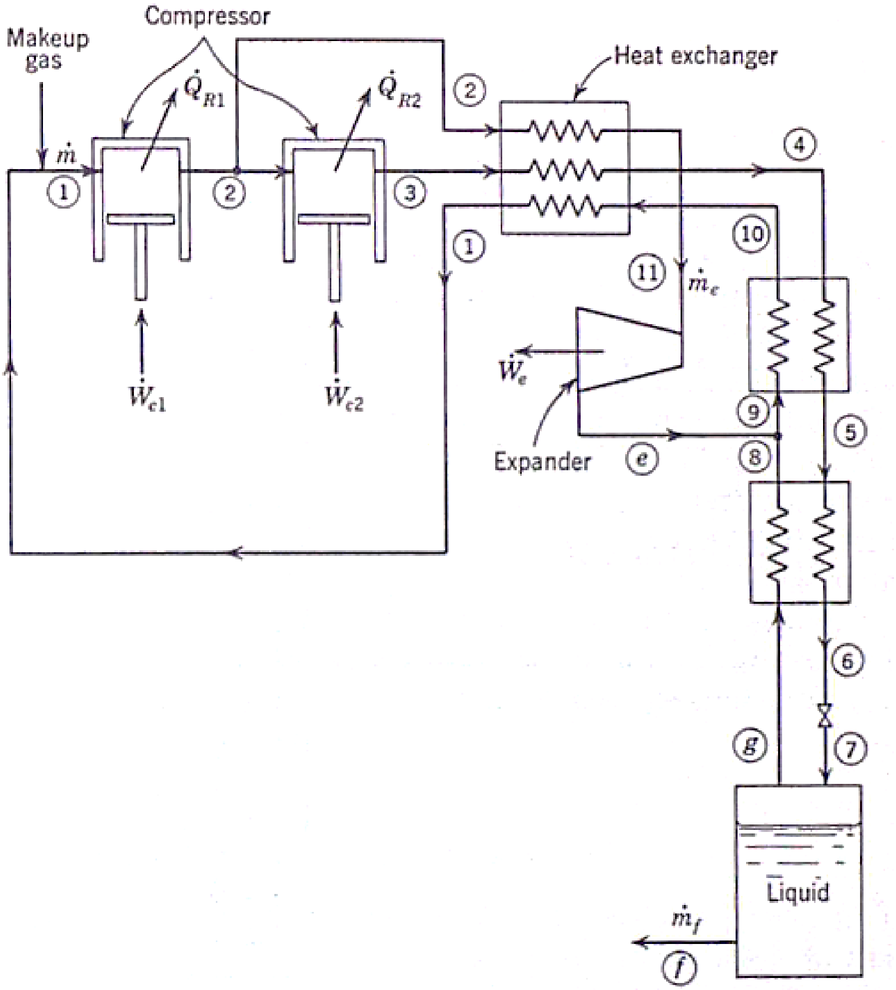
The optimum value of T before expansion = near ambient T
→ So, it can eliminate first HX in the Claude system by compressing!
(∴ Modified Claude system)

Advantage
→ The lubrication problems in the expander are easy to solve!
(Because T of expander is very low)

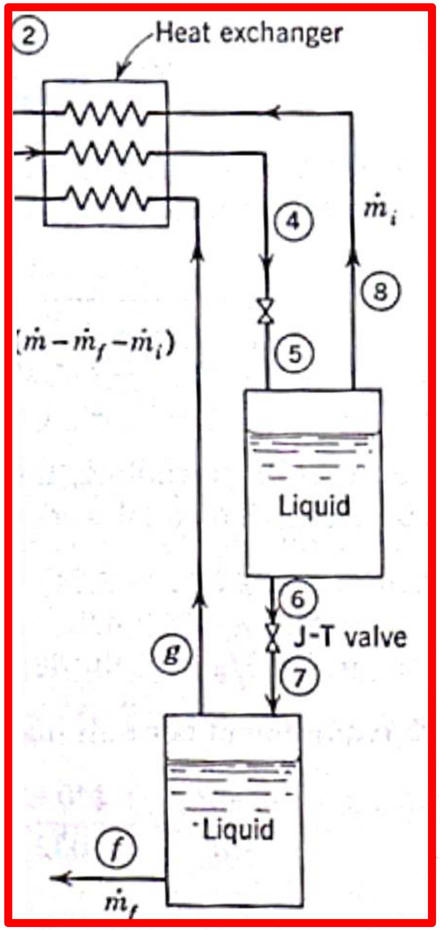
※ Contribution of expander and expansion valve is nearly equal.
(At original Claude system, expander makes more contribution)

3.12 Other liquefaction systems using expanders

Dual-pressure Claude System



Linde Dual-pressure
(Chapter 3.7)



3.12 Other liquefaction systems using expanders

… It is similar to the Linde dual-pressure system. **(Chapter 3.7)**
(A reservoir is replaced by expander and two HX)

Advantage
Gas through expander is compressed to some intermediate P.
→ Work requirement per unit mass of gas liqefied is reduced.

※ If nitrogen compressed from 1 atm to 35 atm, optimum performance is attained when 75 percent of flow diverted through the expander.

3.13 Liquefaction systems for LNG

3.13 Liquefaction systems for LNG

- LNG liquefaction

Natural gas

- Made up of several components; methane, ethane, propane, and nitrogen
 (Condense at different temperature levels.)
- Require mixed refrigerant liquefaction system.

3.13 Liquefaction systems for LNG

- Mixed refrigerant cascade (MRC) system

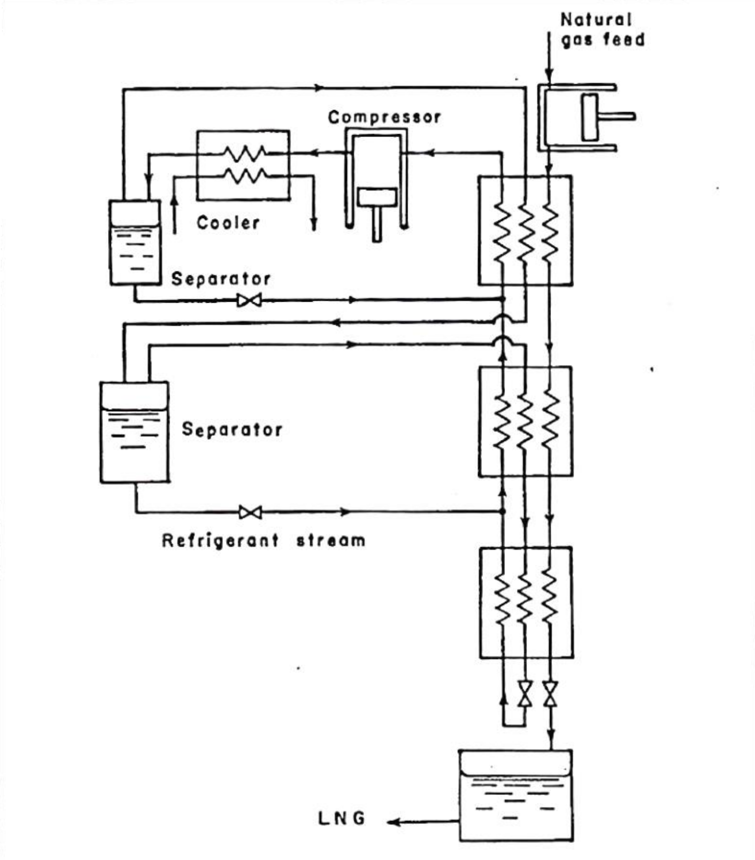


Fig. 3.23. Mixed refrigerant cascade system used for liquefaction of natural gas.

- Single compressor is used for circulating the mixed refrigerant gas
- Natural gas feed to the MRC system at 3.9~5.3 MPa
- Not applicable for liquefaction of pure gases or mixtures such as air (only two components)

3.14 Comparison of liquefaction systems

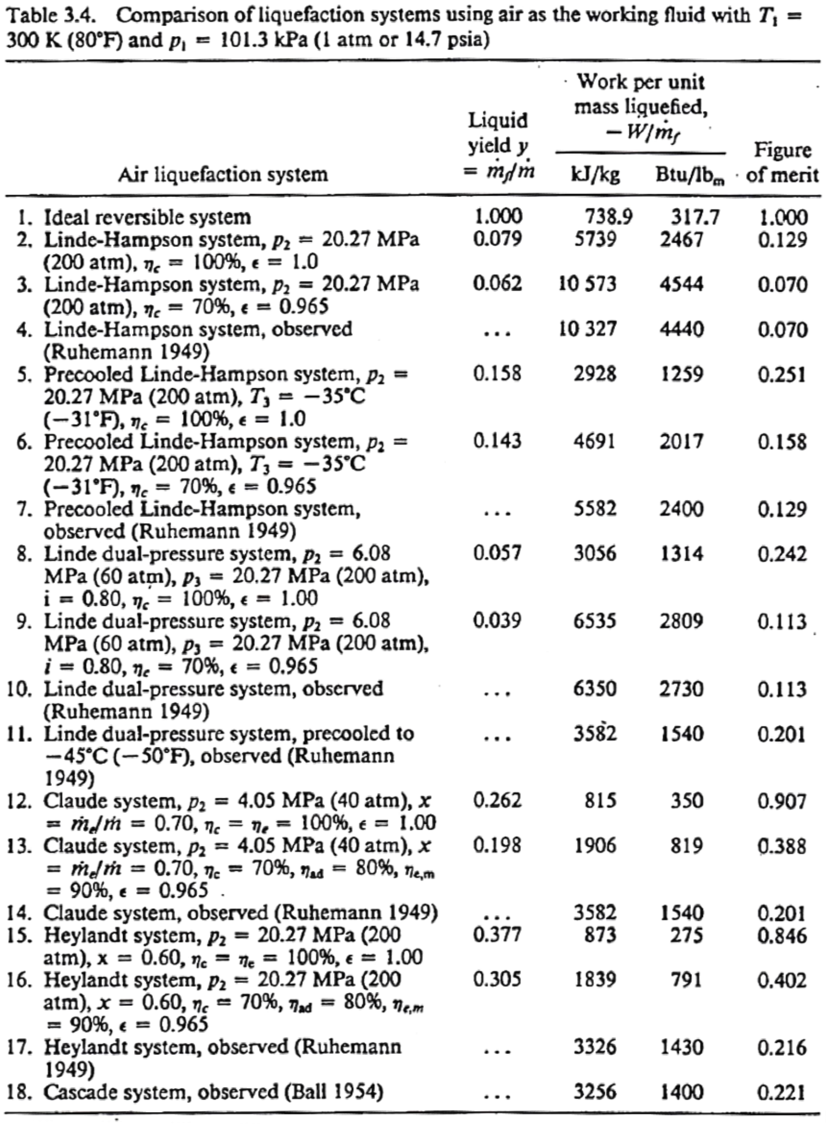
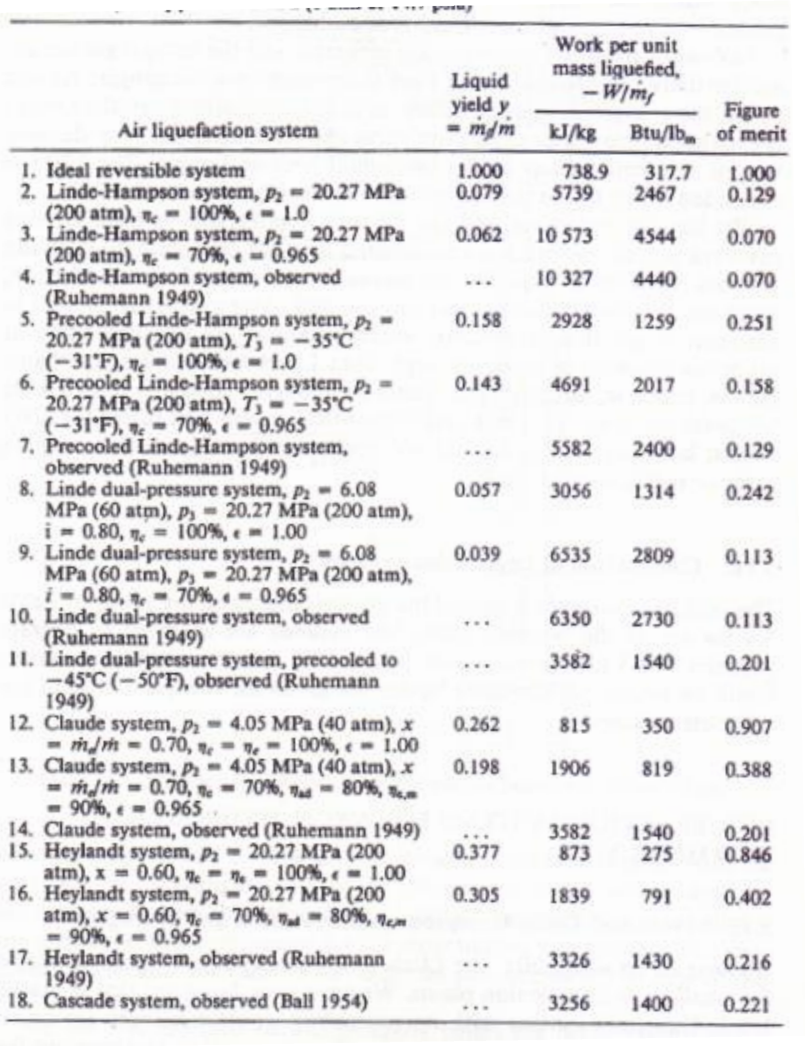


Table 3.4. Comparison of liquefaction systems using air as the working fluid with T_1 = 300 K (80°F) and p_1 = 101.3 kPa (1 atm or 14.7 psia)

Air liquefaction system	Liquid yield $y = \dot{m}_f/\dot{m}$	Work per unit mass liquefied, $-W/\dot{m}_f$		Figure of merit
		kJ/kg	Btu/lb_m	
1. Ideal reversible system	1.000	738.9	317.7	1.000
2. Linde-Hampson system, p_2 = 20.27 MPa (200 atm), η_c = 100%, ϵ = 1.0	0.079	5739	2467	0.129
3. Linde-Hampson system, p_2 = 20.27 MPa (200 atm), η_c = 70%, ϵ = 0.965	0.062	10 573	4544	0.070
4. Linde-Hampson system, observed (Ruhemann 1949)	. . .	10 327	4440	0.070
5. Precooled Linde-Hampson system, p_2 = 20.27 MPa (200 atm), T_3 = −35°C (−31°F), η_c = 100%, ϵ = 1.0	0.158	2928	1259	0.251
6. Precooled Linde-Hampson system, p_2 = 20.27 MPa (200 atm), T_3 = −35°C (−31°F), η_c = 70%, ϵ = 0.965	0.143	4691	2017	0.158
7. Precooled Linde-Hampson system, observed (Ruhemann 1949)	. . .	5582	2400	0.129
8. Linde dual-pressure system, p_2 = 6.08 MPa (60 atm), p_3 = 20.27 MPa (200 atm), i = 0.80, η_c = 100%, ϵ = 1.00	0.057	3056	1314	0.242
9. Linde dual-pressure system, p_2 = 6.08 MPa (60 atm), p_3 = 20.27 MPa (200 atm), i = 0.80, η_c = 70%, ϵ = 0.965	0.039	6535	2809	0.113
10. Linde dual-pressure system, observed (Ruhemann 1949)	. . .	6350	2730	0.113
11. Linde dual-pressure system, precooled to −45°C (−50°F), observed (Ruhemann 1949)	. . .	3582	1540	0.201
12. Claude system, p_2 = 4.05 MPa (40 atm), $x = \dot{m}_e/\dot{m}$ = 0.70, $\eta_c = \eta_e$ = 100%, ϵ = 1.00	0.262	815	350	0.907
13. Claude system, p_2 = 4.05 MPa (40 atm), $x = \dot{m}_e/\dot{m}$ = 0.70, η_c = 70%, η_{ad} = 80%, $\eta_{e,m}$ = 90%, ϵ = 0.965	0.198	1906	819	0.388
14. Claude system, observed (Ruhemann 1949)	. . .	3582	1540	0.201
15. Heylandt system, p_2 = 20.27 MPa (200 atm), x = 0.60, $\eta_c = \eta_e$ = 100%, ϵ = 1.00	0.377	873	275	0.846
16. Heylandt system, p_2 = 20.27 MPa (200 atm), x = 0.60, η_c = 70%, η_{ad} = 80%, $\eta_{e,m}$ = 90%, ϵ = 0.965	0.305	1839	791	0.402
17. Heylandt system, observed (Ruhemann 1949)	. . .	3326	1430	0.216
18. Cascade system, observed (Ball 1954)	. . .	3256	1400	0.221

3.15 Precooled Linde-Hampson system for neon and hydrogen



Air liquefaction system	Liquid yield $y = \dot{m}_f/\dot{m}$	Work per unit mass liquefied, $-W/\dot{m}_f$ kJ/kg	Work per unit mass liquefied, $-W/\dot{m}_f$ Btu/lb$_m$	Figure of merit
1. Ideal reversible system	1.000	738.9	317.7	1.000
2. Linde-Hampson system, $p_2 = 20.27$ MPa (200 atm), $\eta_c = 100\%$, $\epsilon = 1.0$	0.079	5739	2467	0.129
3. Linde-Hampson system, $p_2 = 20.27$ MPa (200 atm), $\eta_c = 70\%$, $\epsilon = 0.965$	0.062	10 573	4544	0.070
4. Linde-Hampson system, observed (Ruhemann 1949)	...	10 327	4440	0.070
5. Precooled Linde-Hampson system, $p_2 =$ 20.27 MPa (200 atm), $T_3 = -35$°C (−31°F), $\eta_c = 100\%$, $\epsilon = 1.0$	0.158	2928	1259	0.251
6. Precooled Linde-Hampson system, $p_2 =$ 20.27 MPa (200 atm), $T_3 = -35$°C (−31°F), $\eta_c = 70\%$, $\epsilon = 0.965$	0.143	4691	2017	0.158
7. Precooled Linde-Hampson system, observed (Ruhemann 1949)	...	5582	2400	0.129
8. Linde dual-pressure system, $p_2 = 6.08$ MPa (60 atm), $p_3 = 20.27$ MPa (200 atm), $i = 0.80$, $\eta_c = 100\%$, $\epsilon = 1.00$	0.057	3056	1314	0.242
9. Linde dual-pressure system, $p_2 = 6.08$ MPa (60 atm), $p_3 = 20.27$ MPa (200 atm), $i = 0.80$, $\eta_c = 70\%$, $\epsilon = 0.965$	0.039	6535	2809	0.113
10. Linde dual-pressure system, observed (Ruhemann 1949)	...	6350	2730	0.113
11. Linde dual-pressure system, precooled to −45°C (−50°F), observed (Ruhemann 1949)	...	3582	1540	0.201
12. Claude system, $p_2 = 4.05$ MPa (40 atm), $x = \dot{m}_e/\dot{m} = 0.70$, $\eta_c = \eta_e = 100\%$, $\epsilon = 1.00$	0.262	815	350	0.907
13. Claude system, $p_2 = 4.05$ MPa (40 atm), $x = \dot{m}_e/\dot{m} = 0.70$, $\eta_c = 70\%$, $\eta_{ad} = 80\%$, $\eta_{e,m} = 90\%$, $\epsilon = 0.965$	0.198	1906	819	0.388
14. Claude system, observed (Ruhemann 1949)	...	3582	1540	0.201
15. Heylandt system, $p_2 = 20.27$ MPa (200 atm), $x = 0.60$, $\eta_c = \eta_e = 100\%$, $\epsilon = 1.00$	0.377	873	275	0.846
16. Heylandt system, $p_2 = 20.27$ MPa (200 atm), $x = 0.60$, $\eta_c = 70\%$, $\eta_{ad} = 80\%$, $\eta_{e,m} = 90\%$, $\epsilon = 0.965$	0.305	1839	791	0.402
17. Heylandt system, observed (Ruhemann 1949)	...	3326	1430	0.216
18. Cascade system, observed (Ball 1954)	...	3256	1400	0.221

Table 3.4. Comparison of liquefaction systems using air as the working fluid with $T_1 = 300$ K (80^{o}F) and $p_1 = 101.3$ kPa (1 atm or 14.7 psia)

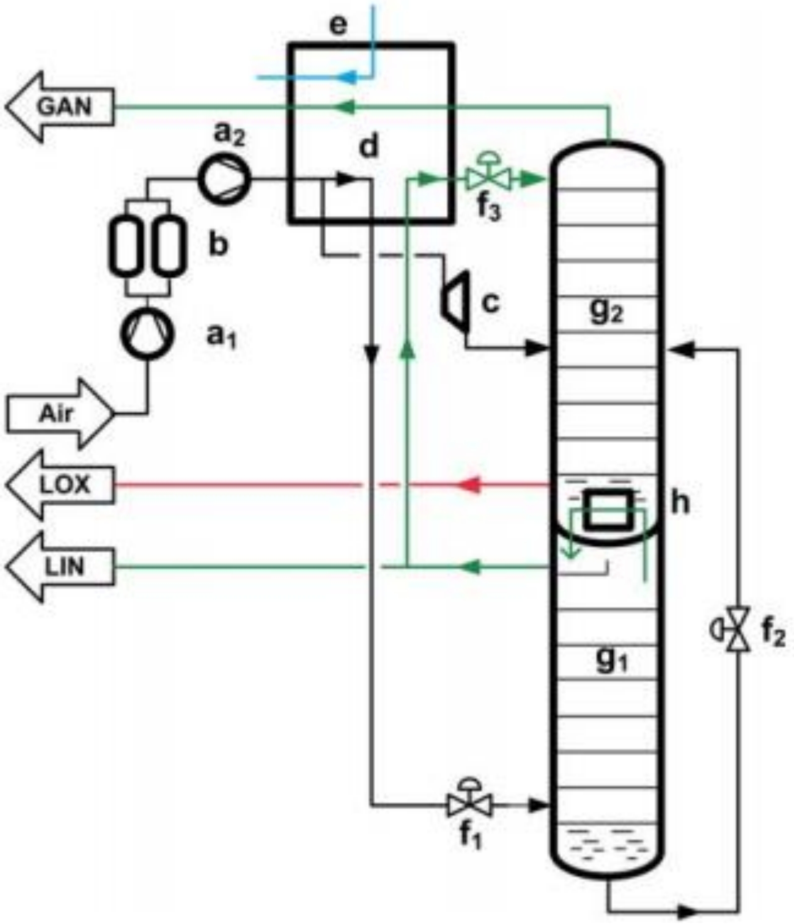
3.15 Precooled Linde-Hampson system for neon and hydrogen

The Linde-Hampson system is quite desirable for small-scale liquefaction plants.

However, the basic Linde-Hampson system with no precooling would not work for neon, hydrogen, or helium

Because the maximum inversion temperature for these gases is below ambient temperature.

3.15 Precooled Linde-Hampson system for neon and hydrogen



Heylandt air-separation system

a) Compressor stages,

b) Carbon dioxide and water removal,

c) Expander,

d) Main heat exchanger,

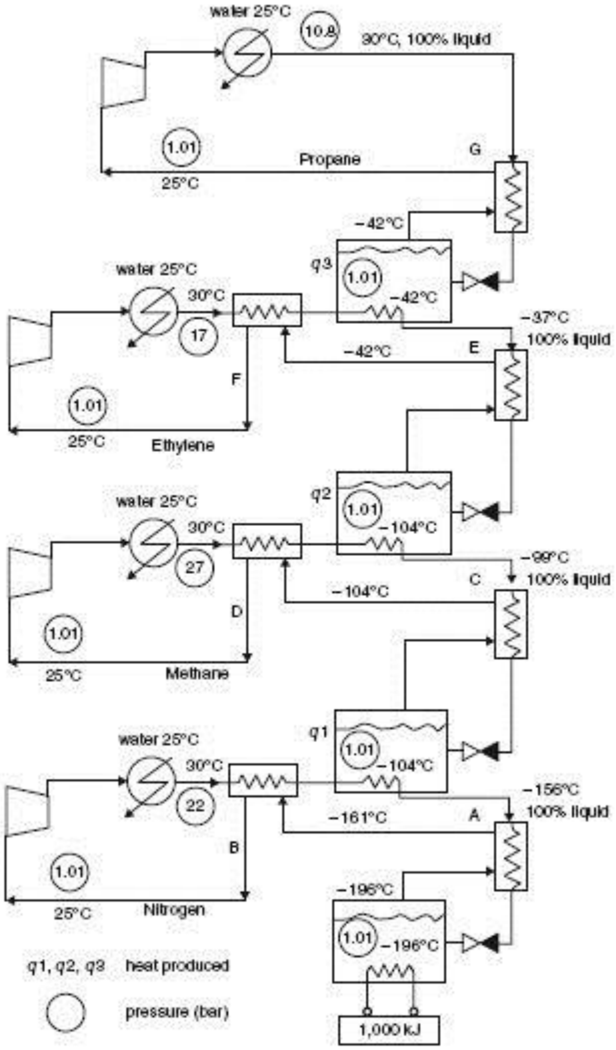
e) Optional external refrigerant stream,

f) J-T valves,

g) Separation columns,

h) Condenser-reboiler

3.15 Precooled Linde-Hampson system for neon and hydrogen



Cascade system

3.15 Precooled Linde-Hampson system for neon and hydrogen

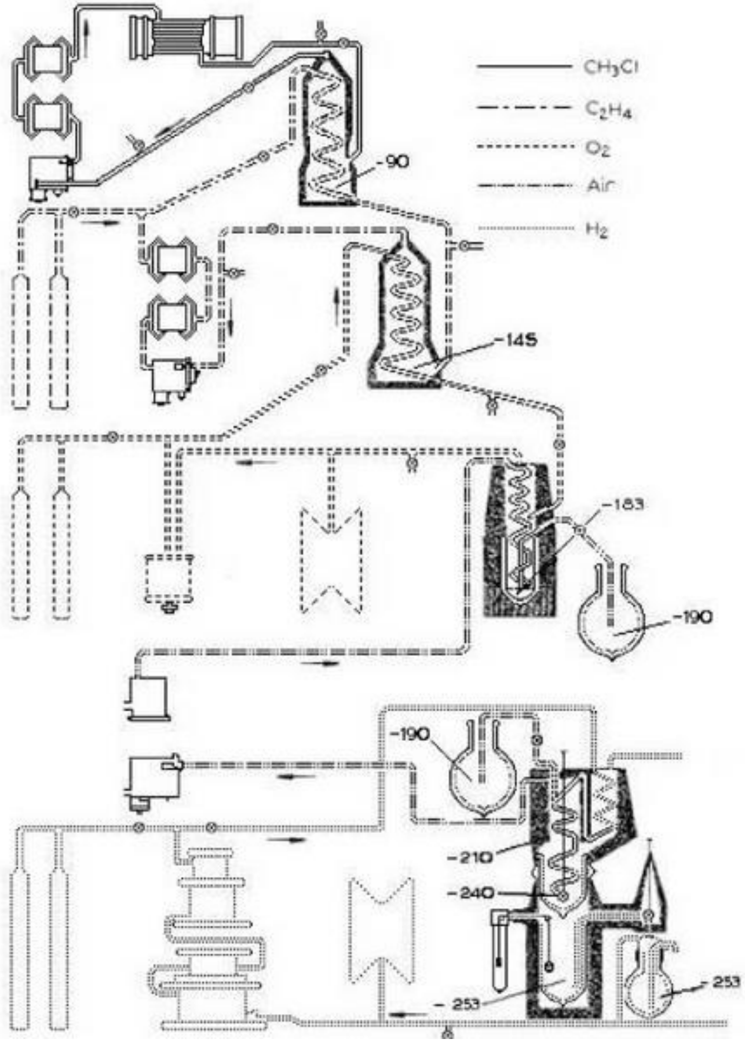
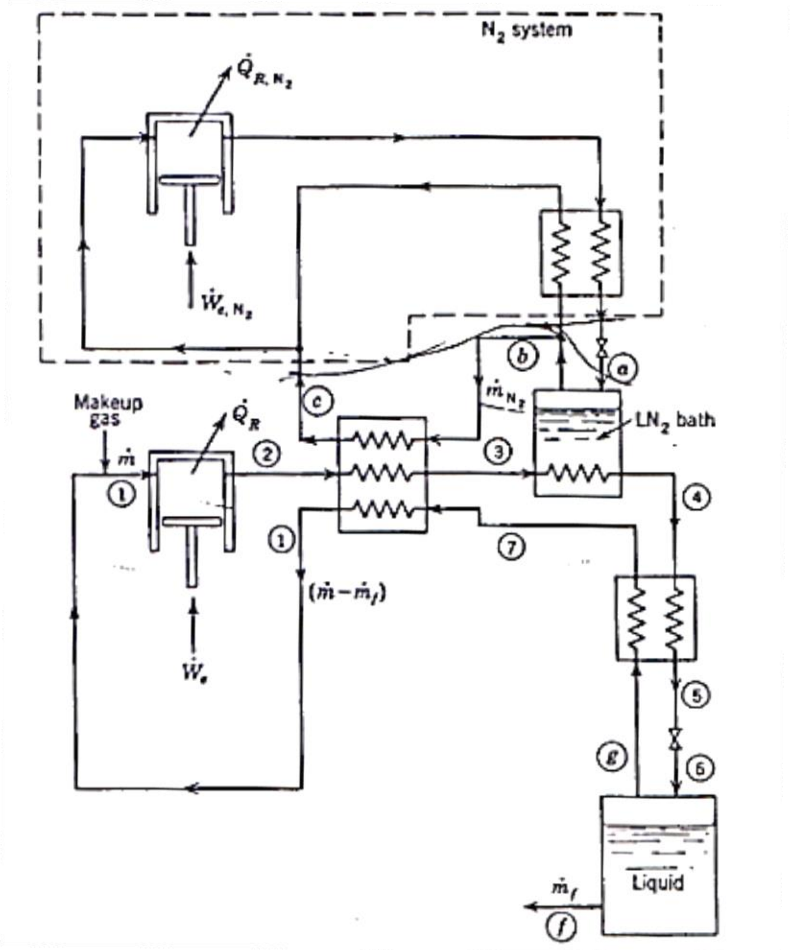


Diagram for the cascade for liquid gases from 1906

3.15 Precooled Linde-Hampson system for neon and hydrogen



Liquid-nitrogen-precooled Linde-Hampson system for neon or Hydrogen

3.15 Precooled Linde-Hampson system for neon and hydrogen

$$y = \frac{h_7 - h_4}{h_7 - h_f}$$

$$0 = \dot{m}_{N_2} h_c + \left(\dot{m} - \dot{m}_f\right) h_1 + \dot{m}_f h_f - \dot{m}_{N_2} h_a - \dot{m} h_2$$

$\dot{m}_{N_2}$: the mass flow rate of liquid nitrogen boiled away to precool the incoming hydrogen or neon

$\dot{m}$: the mass flow rate of hydrogen or neon through the compressor

$\dot{m}_f$: the mass flow rate of hydrogen or neon which is liquefied

3.15 Precooled Linde-Hampson system for neon and hydrogen

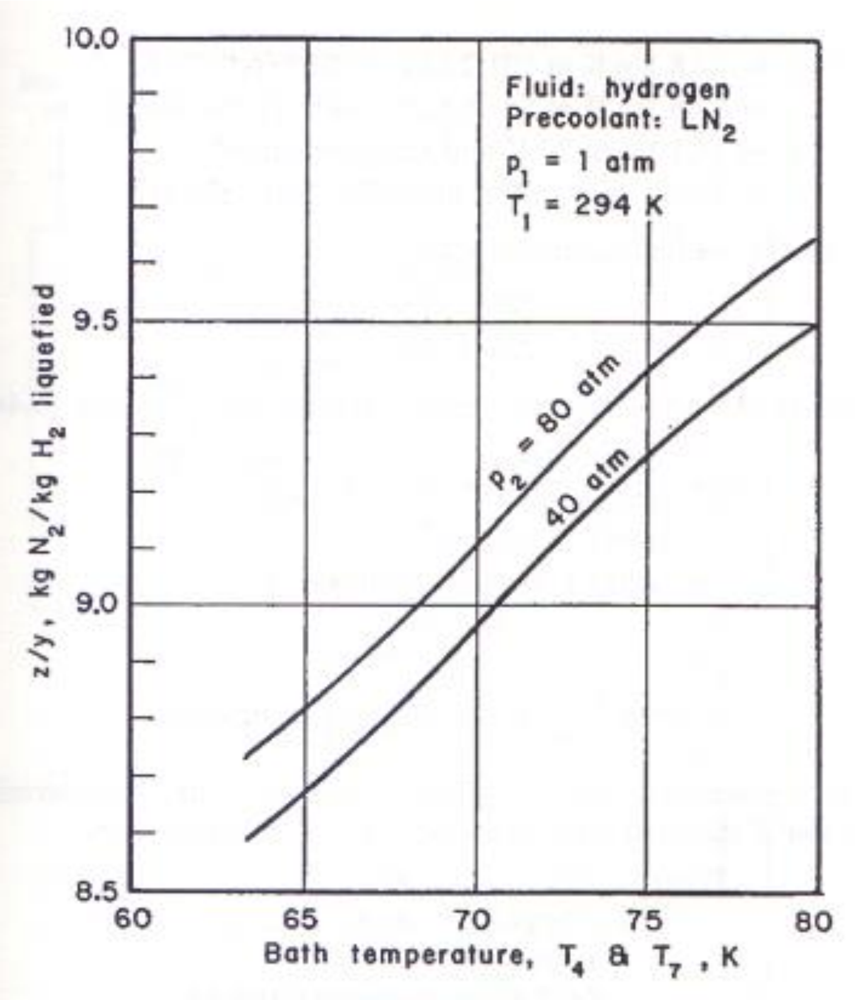
The nitrogen boil-off rate per unit mass of hydrogen or neon compressed

$$z = \frac{\dot{m}_{N_2}}{\dot{m}}$$

$$z = \frac{h_2 - h_1}{h_c - h_a} + y\frac{h_1 - h_f}{h_c - h_a}$$

$$\frac{\dot{m}_{N_2}}{\dot{m}_f} = \frac{\dot{m}_{N_2}/\dot{m}}{\dot{m}_f/\dot{m}} = \frac{z}{y}$$

3.15 Precooled Linde-Hampson system for neon and hydrogen



Nitrogen boil-off per unit mass of hydrogen produced for the liquid-nitrogen-precooled Linde-Hampson system as a function of the liquid-nitrogen bath temperature.

3.15 Precooled Linde-Hampson system for neon and hydrogen

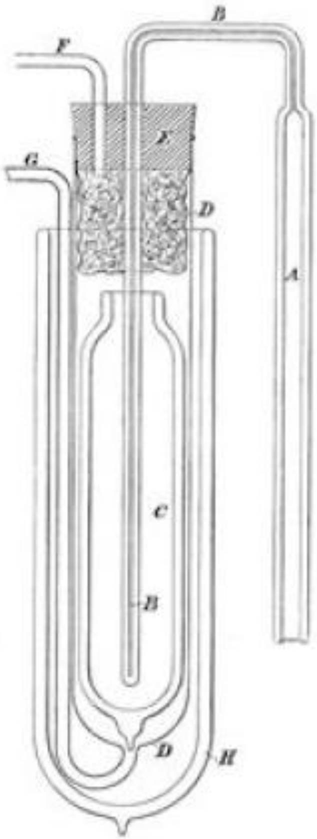


Figura 4 - Hydrogen liquefier used by Travers. Credit: Ref. [27].

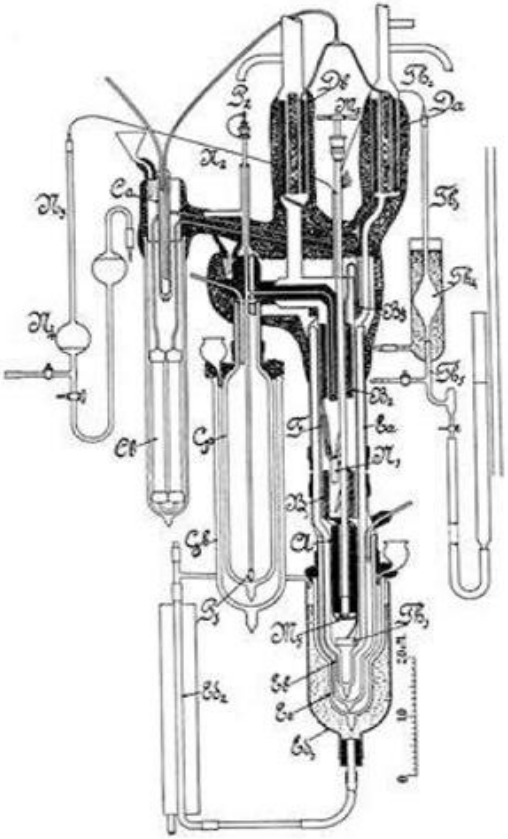
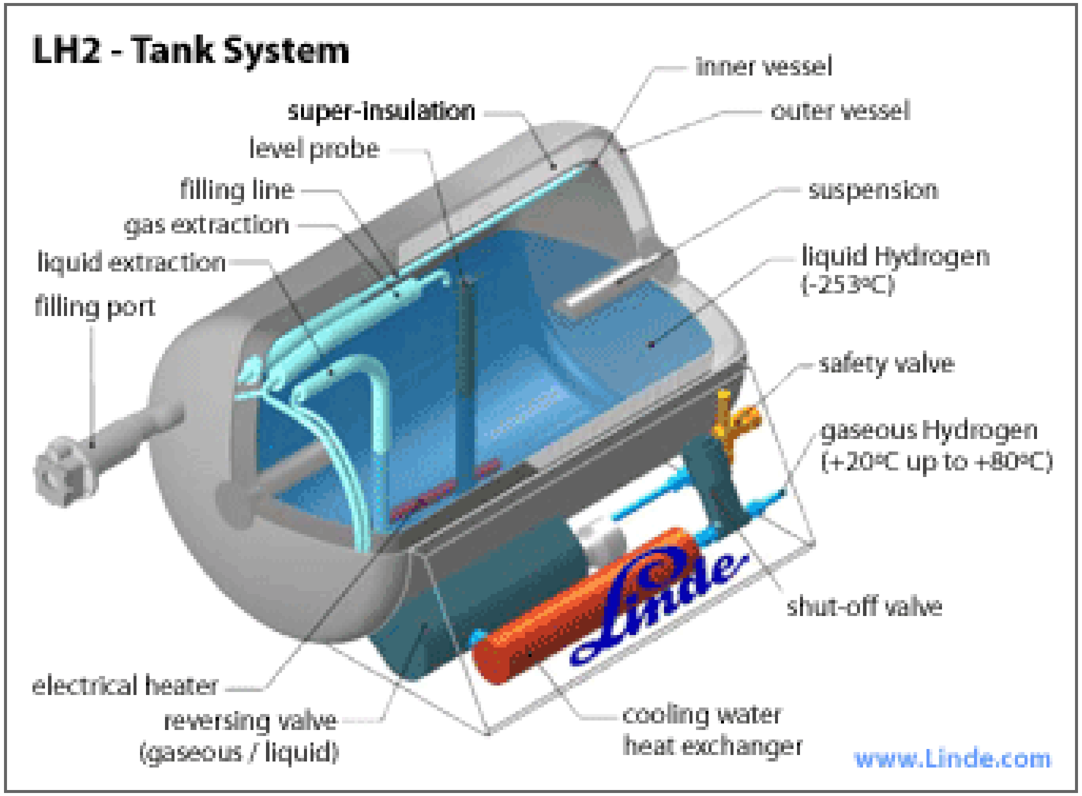


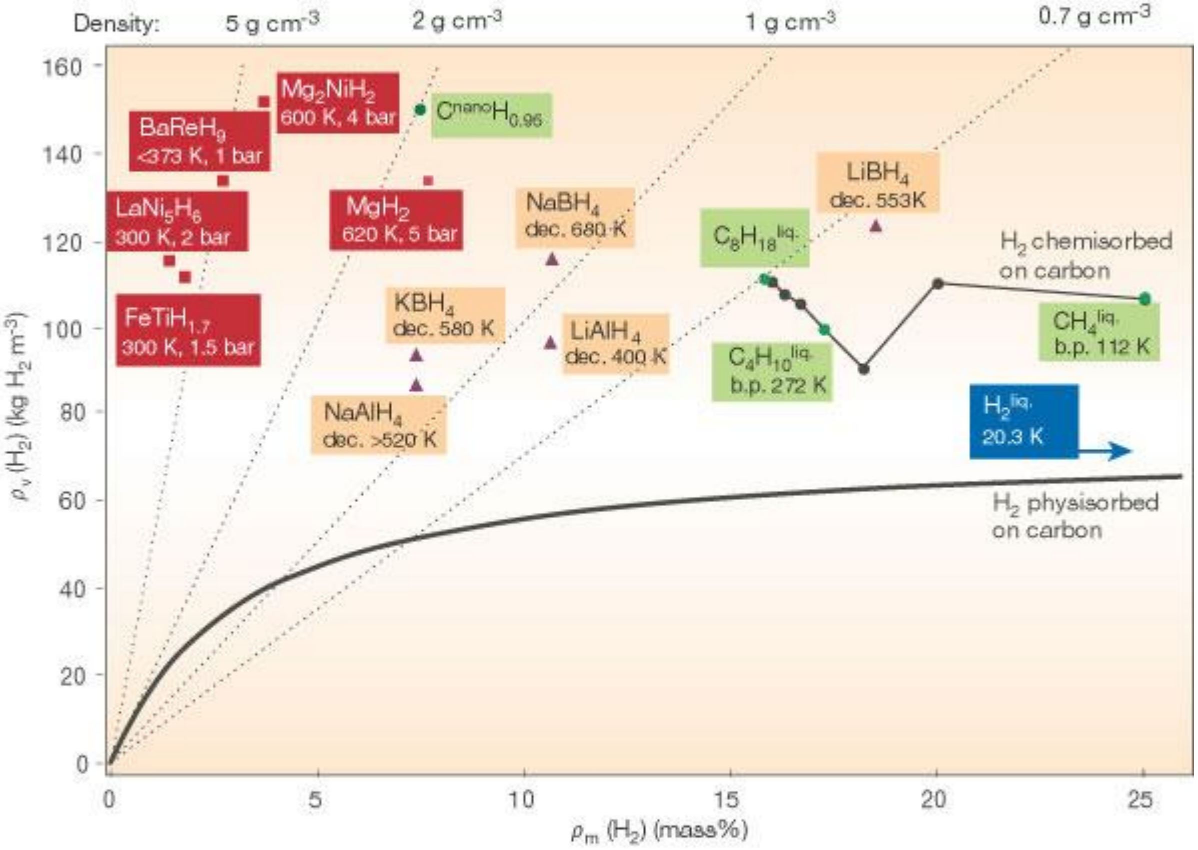
Figura 6 - Helium liquefier. Credit: Ref. [35].

3.15 Precooled Linde-Hampson system for neon and hydrogen



Liquid hydrogen storage tank system, horizontal mounted with double gasket and dual seal

3.15 Precooled Linde-Hampson system for neon and hydrogen



A complete survey plot of hydrogen storage in metal hydrides and carbon-based materials

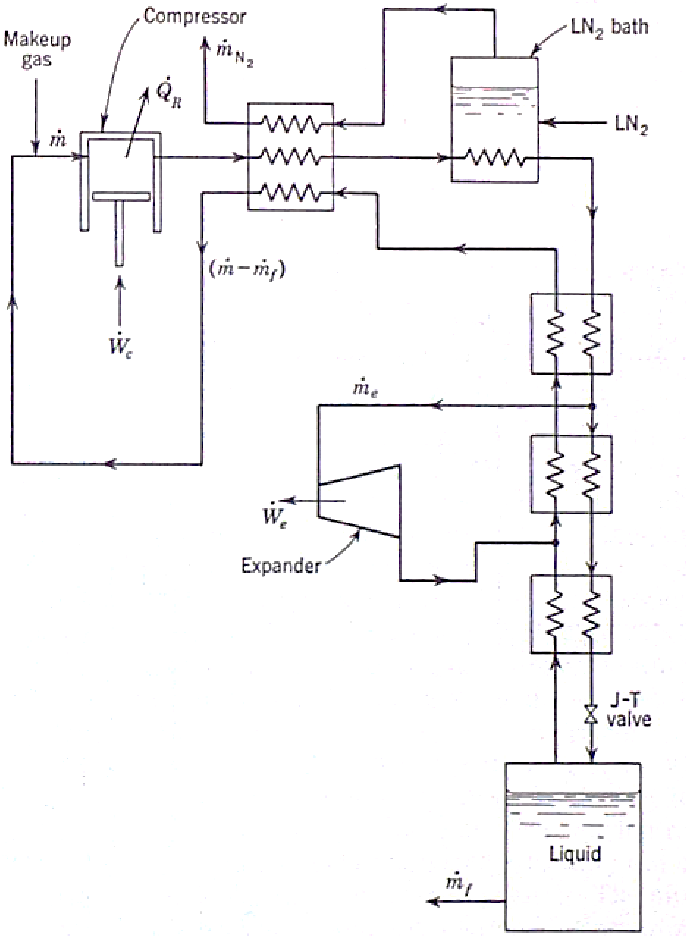
3.15 Precooled Linde-Hampson system for neon and hydrogen

Created by NPO “GELIYMASH”, liquefiers of helium with liquid helium capacity of 700 l/h were delivered to the biggest European helium center - Orenburg Gas Processing Plant.

3.15 Precooled Linde-Hampson system for neon and hydrogen

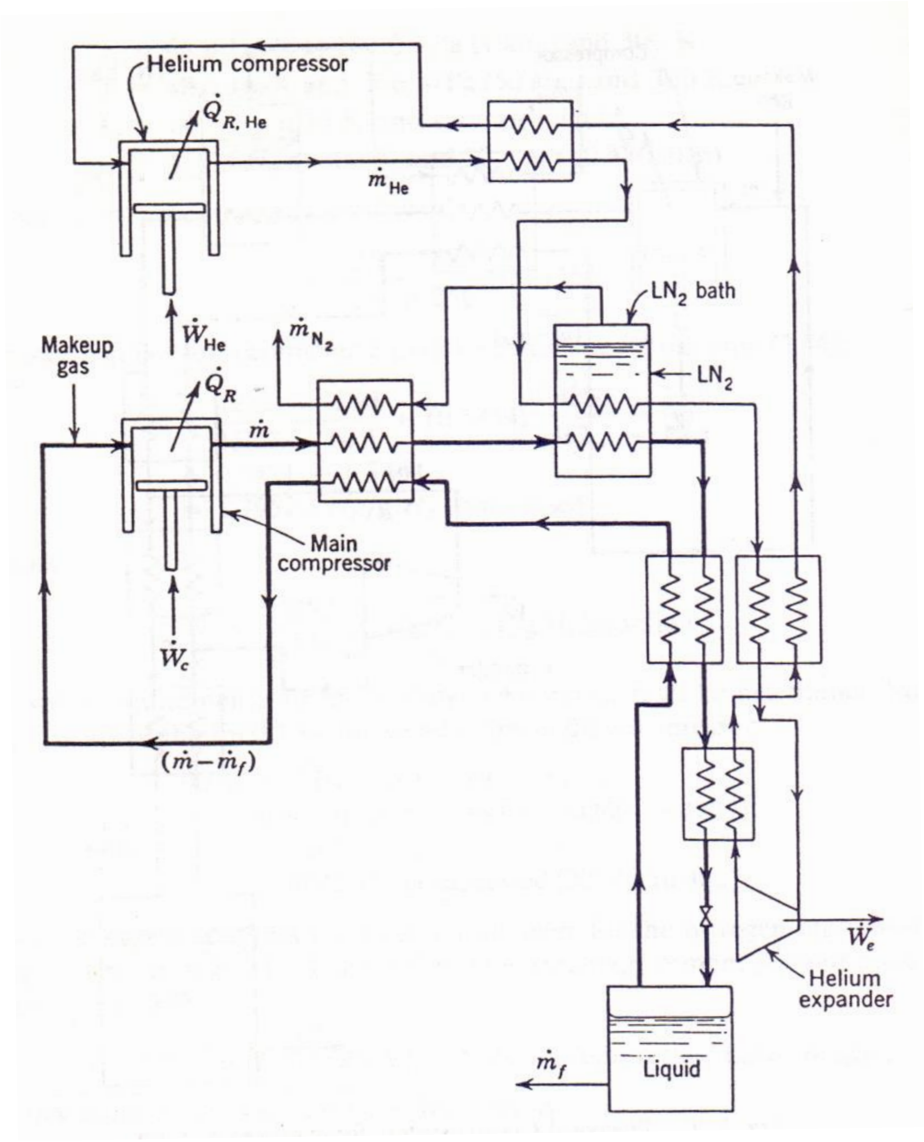
One of the latest cryogenic helium plants created by NPO "GELIYMASH" was delivered to the Corporation "TVEL" in 2007 for testing superconducting elements that were designed for the European international project of a thermonuclear reactor.

3.16 Claude system for hydrogen or neon



Precooled Claude system for hydrogen or neon.

3.17 Helium-Refrigerated hydrogen-liquefaction system



3.17 Helium-Refrigerated hydrogen-liquefaction system

An auxiliary helium-gas refrigerator can be used to condense hydrogen or neon

Liquid neon

3.17 Helium-Refrigerated hydrogen-liquefaction system

- Advantage
  - Relatively low pressures can be used
  - The compressor size can be reduced (although two compressors are required)
  - The pipe thickness can be reduced
  - The hydrogen or neon need be compressed only to a pressure high enough to overcome the irreversible pressure drops through the heat exchangers and piping in an actual system

3.17 Helium-Refrigerated hydrogen-liquefaction system

Usually adequate for the hydrogen loop
Pressure : $300\ kPa\ to\ 800\ kPa$

The system is relatively insensitive to the pressure level of the helium refrigerator

Helium-gas pressure : $1\ MPa$
Work requirements to liquefied: $11000\ kJ/kg$ (approximately)
Figure of Merit : 0.11

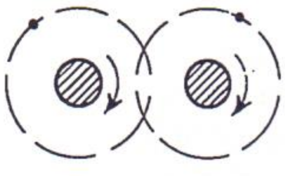
3.18 Ortho-para-hydrogen conversion in the liquefier

N.B.P. = $20.3K$

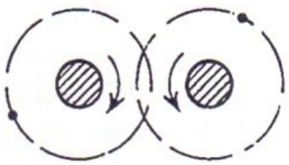
Types of hydrogen molecules

$ortho - H_2$
(Spins aligned, high energy)

$para - H_2$
(Spins aligned, high energy)



Ortho-hydrogen



Para-hydrogen

3.18 Ortho-para-hydrogen conversion in the liquefier

$$ortho - H_2 \rightarrow para - H_2 + \ \Delta\alpha \ (heat\ of\ conversion)$$

$$70.3kJ/kg$$

Latent heat $44.3kJ/kg$

At high temperature is a mixture of $75\%\ ortho - H_2$ and $25\%\ para - H_2$

As temperature is cooled to the normal boiling point of hydrogen, the $ortho - H_2$ concentration decreases from $75\ to\ 0.2\%$

3.18 Ortho-para-hydrogen conversion in the liquefier

The changeover from $ortho -$ to $para - hydrogen$ involves a heat of conversion that is greater than the heat of vaporization of $para - hydrogen$

Therefore serious boil-off losses will result unless measures are taken to prevent it.

3.18 Ortho-para-hydrogen conversion in the liquefier

A catalyst may be used to speed up the conversion process, while the heat of conversion is absorbed in the liquefaction system before the liquid is stored in the liquid receiver.

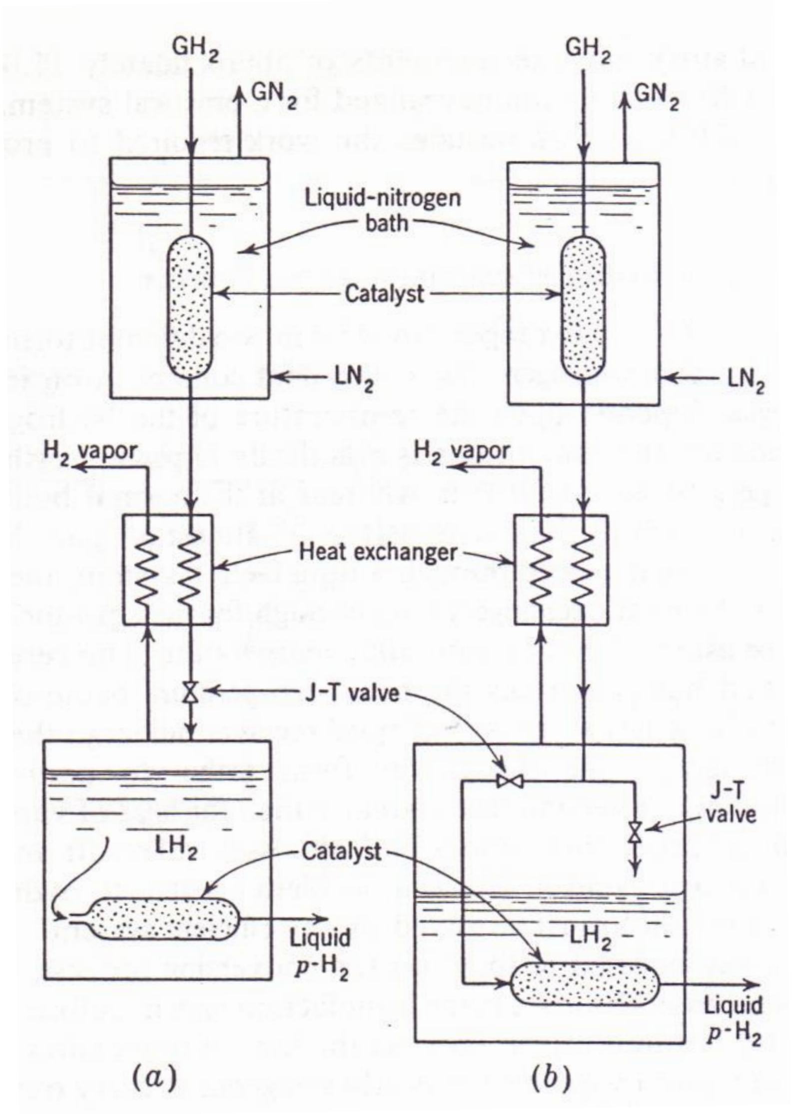
At $70\ K$: 55 to $60\%\ para - hydrogen$

3.18 Ortho-para-hydrogen conversion in the liquefier

The changeover from $ortho-$ to $para-hydrogen$ involves a heat of conversion that is greater than the heat of vaporization of $para-hydrogen$

Therefore serious boil-off losses will result unless measures are taken to prevent it.

3.18 Ortho-para-hydrogen conversion in the liquefier



3.18 Ortho-para-hydrogen conversion in the liquefier

- Two possible arrangements for ortho-para conversion

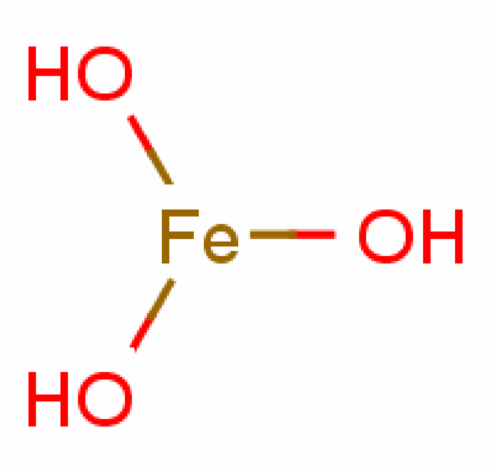
In the first arrangement, the hydrogen is passed through the catalyst in the liquid-nitrogen bath, expanded through the expansion valve into the liquid receiver, and drawn through a catalyst bed before passing into a storage vessel. The hydrogen that is evaporated due to the heat of conversion flows back through the heat exchanger and furnishes additional refrigeration to the incoming stream

3.18 Ortho-para-hydrogen conversion in the liquefier

The second arrangement is similar to the first one, except that the high-pressure stream is divided into two parts before the expansion valve. One part is expanded through an expansion valve and flows though a catalyst bed immersed in a liquid-hydrogen bath. The other part of the high-pressure stream is expanded through another expansion valve into the liquid receiver to furnish refrigeration for the catalyst bed.

3.18 Ortho-para-hydrogen conversion in the liquefier

- Catalysts

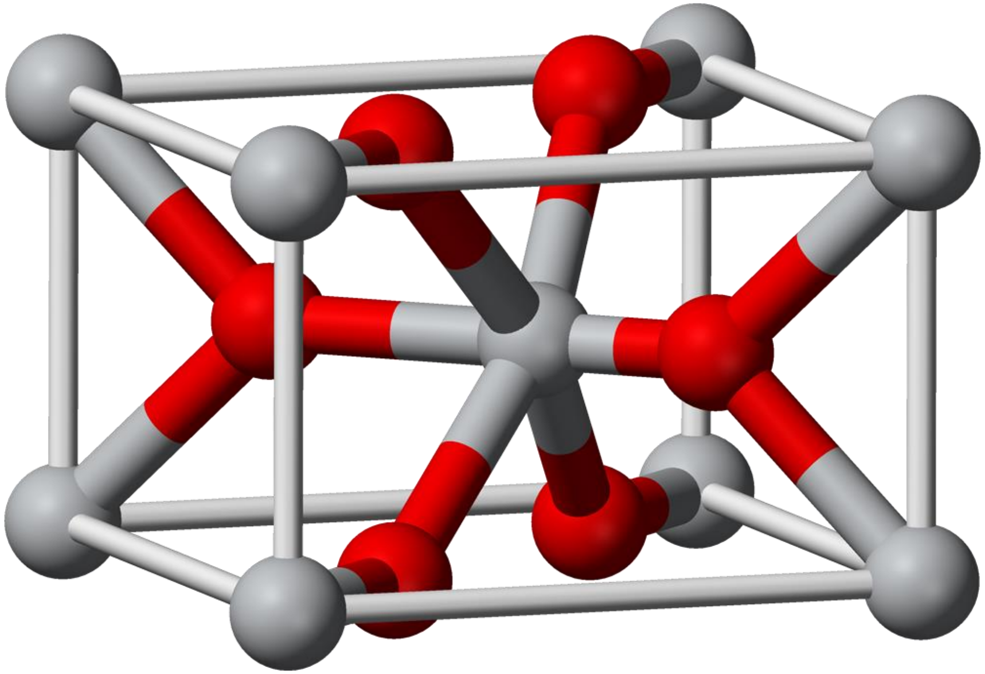


hydrous ferric oxide

Nickel-based catalyst

3.18 Ortho-para-hydrogen conversion in the liquefier

- Catalysts



Chromic oxide

3.19 Collins helium-liquefaction system

Helium ? It is one of the most difficult gases to liquefy!

- Different type He liquefaction system

Collins type He liquefier (Japan)

Nippon Sanso LHE60 (Japan, 1971)

3.19 Collins helium-liquefaction system

Helium ? It is one of the most difficult gases to liquefy!

- Different type He liquefaction system

PSI He liquefier(University of Florida)

Linde L70 He liquefier (Portugal)

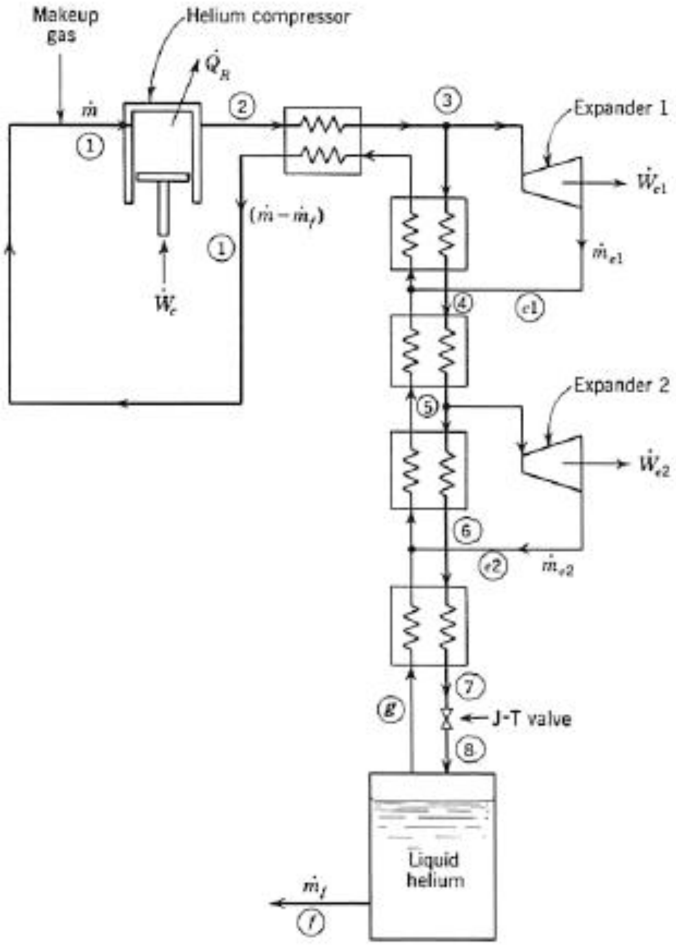
3.19 Collins helium-liquefaction system

Helium refrigerator-liquefier at MIT

Samuel C. Collins (MIT) designed Helium liquefier (1952)

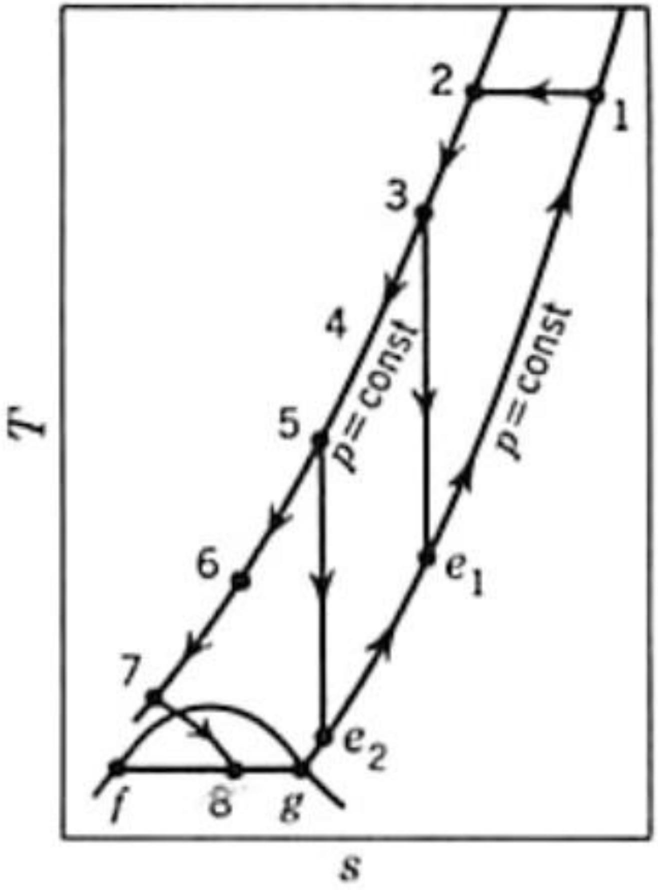
It is an extension of the Claude system

3.19 Collins helium-liquefaction system



Collins helium-liquefaction system

3.19 Collins helium-liquefaction system



Collins helium-liquefaction system T-S diagram

3.19 Collins helium-liquefaction system

- **Liquid yield**

Consisting of all components except the helium compressor and the expansion engines

$$y = \frac{h_1 - h_2}{h_1 - h_f} + x_1 \frac{\Delta h_{e_i}}{h_1 - h_f} + x_2 \frac{\Delta h_{e_2}}{h_1 - h_f}$$

$$x_1 = \dot{m}_{e_1}/\dot{m}$$
$$x_2 = \dot{m}_{e_2}/\dot{m}$$
$$\Delta h_{e_i} = enthalpy\ change\ of\ fluid\ passing\ through\ expander\ 1$$
$$\Delta h_{e_2} = enthalpy\ change\ of\ fluid\ passing\ through\ expander\ 2$$
$$\dot{m}_{e_1}, \dot{m}_{e_2} = mass\ flow\ rates\ of\ fluid\ through\ expander\ 1\ and\ 2, respectively$$

3.19 Collins helium-liquefaction system

- **Liquid yield**

For more than two engines?

→ Additional term similar to the second term for each expander would be added!

3.19 Collins helium-liquefaction system

- **Increase the Collins liquefier performance?**

- By using liquid-nitrogen precooling bath

  → Cool-down time would be reduced from 4h to 2h

  → Liquid yield can be tripled

But,
Pre coolant bath is not required because it doesn't dependent solely on the Joule-Thomson effect!

3.20 Simon helium-liquefaction system

- **System Characteristics**
  - Liquefy small quantities of helium
  - Not operate as a steady-flow system
  - It produces helium in a batch process

3.20 Simon helium-liquefaction system

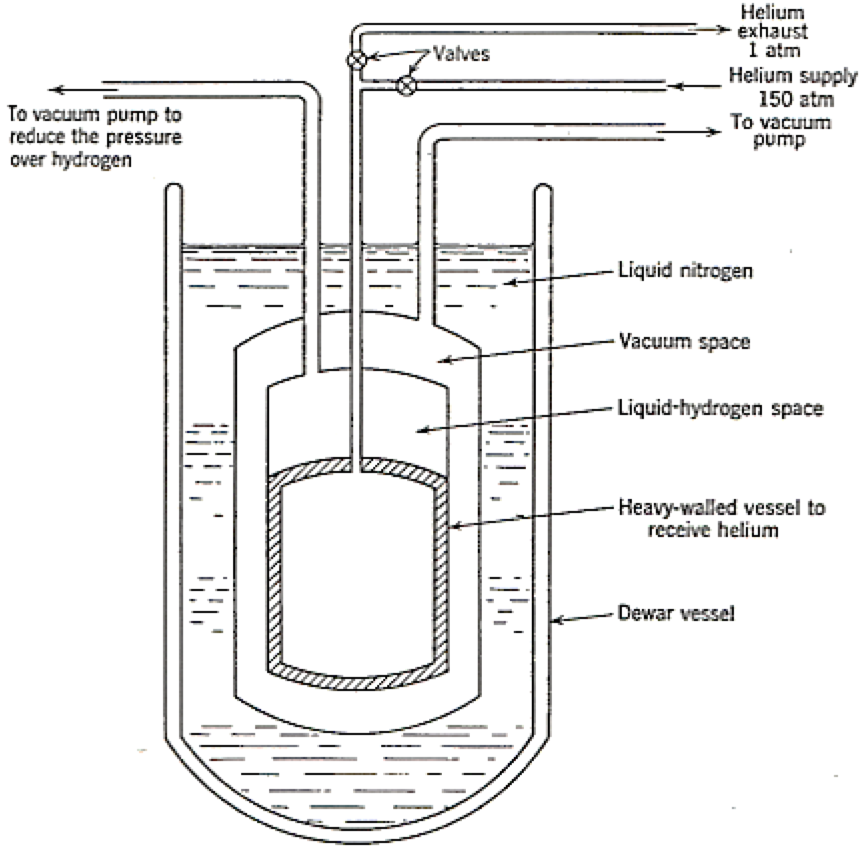
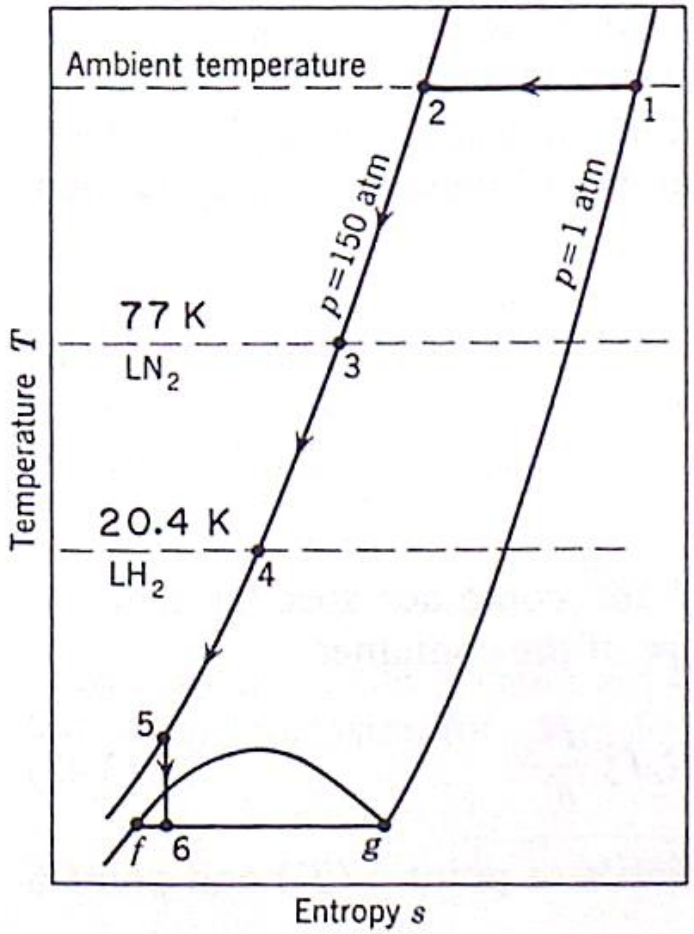


Fig. 3.30. Simon helium-liquefaction system.

3.20 Simon helium-liquefaction system



Simon helium-liquefaction system T-S diagram

3.20 Simon helium-liquefaction system

- **Simon helium-liquefaction system process**

Process 1-2 : Helium gas is introduced into the heavy-wall container.
(Pressure : 10~15 MPa, Ambient temperature)

Process 2-3 :
- Liquid Nitrogen is introduced into the enclosing bath.
- Cools the entire container and contents (77K)
- Vacuum space is filled with helium gas.
  (act as heat transfer medium)
- At the completion of the process, inner vessel is thermally isolated

3.20 Simon helium-liquefaction system

- **Simon helium-liquefaction system process**

Process 3-4 : Liquid hydrogen is introduced into the upper part of the inner container. Inner container and contents are further cooled to liquid-hydrogen temperature. (20.4K)

Process 4-5 :
- The pressure above the liquid hydrogen is reduced to approximately 0.23kPa.
- Liquid hydrogen boils as the pressure is lowered until the hydrogen solidifies.
- At the end of the process, the solid hydrogen is at approximately 10K.

3.20 Simon helium-liquefaction system

- **Simon helium-liquefaction system process**

Process 5-6 :
- The pressure of the gaseous helium is reduced to atmosphere pressure by allowing the helium to be released to a gas holder external to the system.
- The helium that finally remains in the inner container does work against the helium gas that is discharged.
- The discharged gas removes energy from the system, thereby lowering the temperature of the remaining helium.
- At the end of this process, 75~100% is liquid helium.

3.20 Simon helium-liquefaction system

If heat is transferred reversibly,

$$dQ = m_c T_c ds_c = m_c c_c dT_c$$

$m_c = mass\ of\ container$
$T_c = container\ temperature$
$c_c = container\ specific\ heat$

3.20 Simon helium-liquefaction system

Substituting the Debye expression for the container specific heat,

$$ds_c = 233.78 R_c {T_c}^2 \frac{dT_c}{{\theta_D}^3}$$

Integration,

$$\Delta s_c = -\frac{77.93 R_c}{{\theta_D}^3}\left({T_5}^3 - {T_f}^3\right)$$

3.20 Simon helium-liquefaction system

There is no external heat transfer from the surroundings to the inner vessel,

$$(m_6 - m_5)s_5 - 0 = (m_6 s_6 - m_5 s_5) + m_c \Delta s_c$$

$m_6 = total\ mass\ of\ helium\ in\ the\ vessel\ after\ expansion$
$m_5 = total\ mass\ of\ helium\ in\ the\ vessel\ before\ the\ expansion$
$s_5, s_6 = specific\ entropy\ values\ at\ points\ 5\ and\ 6, respectively$

3.20 Simon helium-liquefaction system

There is no external heat transfer from the surroundings to the inner vessel,

$$(m_6 - m_5)s_5 - 0 = (m_6 s_6 - m_5 s_5) + m_c \Delta s_c$$

$m_6 = total\ mass\ of\ helium\ in\ the\ vessel\ after\ expansion$
$m_5 = total\ mass\ of\ helium\ in\ the\ vessel\ before\ the\ expansion$
$s_5, s_6 = specific\ entropy\ values\ at\ points\ 5\ and\ 6, respectively$

Solving for s_6,

$$s_6 = s_5 - \left(\frac{m_c}{m_6}\right)\Delta s_c$$

3.20 Simon helium-liquefaction system

Write the entropy s_6 in terms of the liquid yield

$\mathrm{y} = m_f/m_6,$

$$s_6 = ys_f + (1-y)s_g$$

$s_f = entropy\ of\ the\ saturated\ liquid\ at\ the\ final\ condition$
$s_g = entropy\ of\ the\ saturated\ vapor\ at\ the\ final\ condition$

3.20 Simon helium-liquefaction system

We can solve for the liquid yield,

$$y = \frac{\left(s_g - s_5\right) + (\frac{m_c}{m_6})\Delta s_c}{s_g - s_f}$$

3.20 Simon helium-liquefaction system

The helium specific volume,

$$v_6 = yv_f + (1-y)v_g = v_g - y\left(v_g - v_f\right) = V/m_6$$

$$V = container\ internal\ volume$$

3.20 Simon helium-liquefaction system

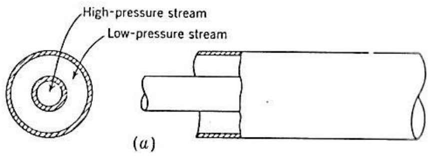
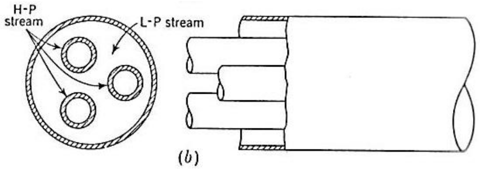
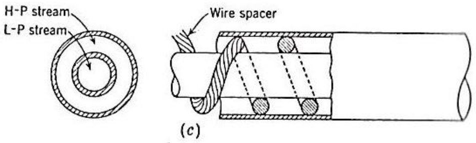
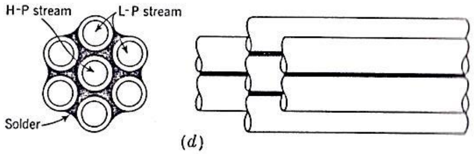
Fraction volume that is filled with liquid helium,

$$\frac{V_f}{V} = \frac{m_f v_f}{m_6 v_6} = \frac{y v_f}{v_6}$$

Substituting for the specific volume v_6 we can obtain,

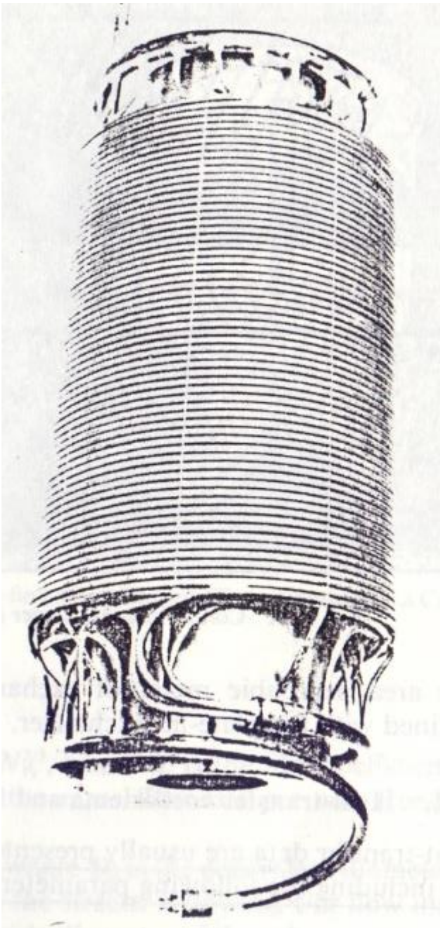
$$\frac{V_f}{V} = \frac{y}{\left(\frac{v_g}{v_f}\right) - y[\left(\frac{v_g}{v_f}\right) - 1}$$

3.21 Type of heat exchagners

Single tube Linde exchanger	High pressure stream for inner circle because of its high density	
Multiple tube Linde exchanger	For multi heat exchanging ex) Linde dual pressure system	
Single tube Linde exchanger with a wire spacer	wire spacer causes turbulent flow which increases heat exchange rate while also increases pressure drop	
Bundle type heat exchanger	bundle of circular tube with an ensure of good thermal contact	

3.21 Type of heat exchangers

- **Giauque Hampson exchanger**



The heat exchanger which consists of helixes of small diameter tubes through which the high pressure stream flows.

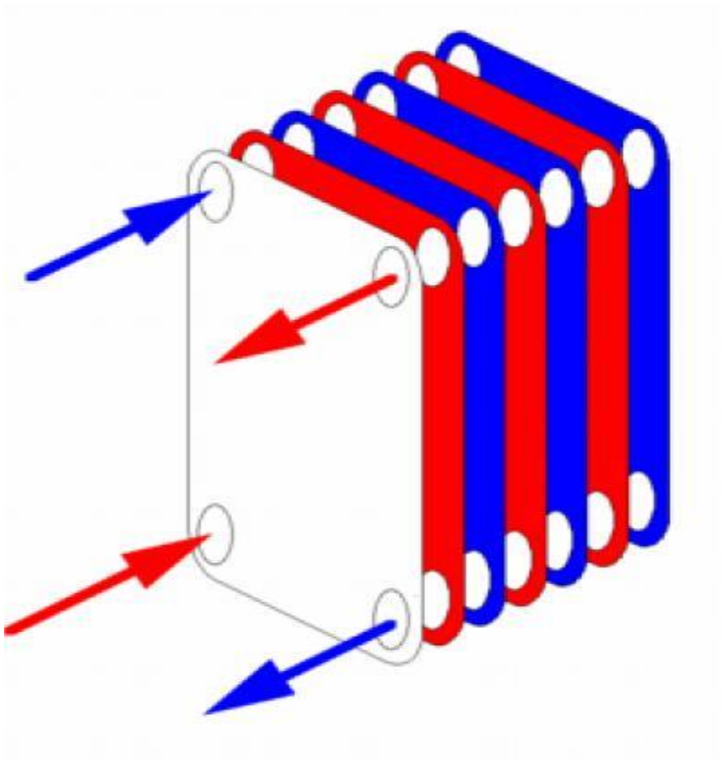
3.21 Type of heat exchangers

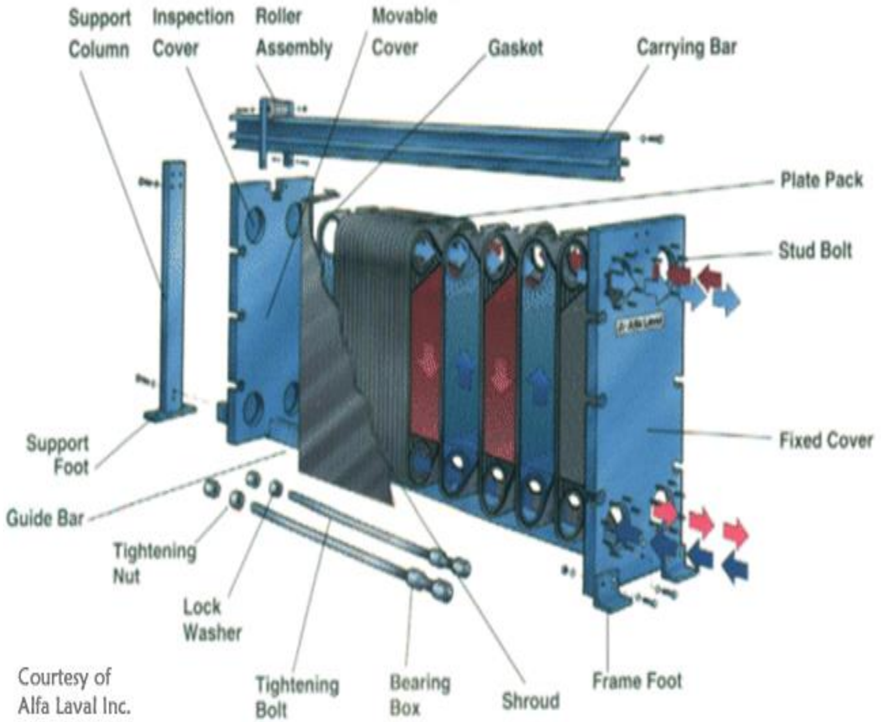
- **Types of Heat Exchangers**

1. Plate and Frame Exchangers
2. Spiral Heat Exchangers
3. Plate-Fin Heat Exchangers
4. Air coolers
5. Double Pipe Exchangers
6. Shell and Tube Exchangers

3.21 Type of heat exchangers

1.Plate and Frame Exchangers

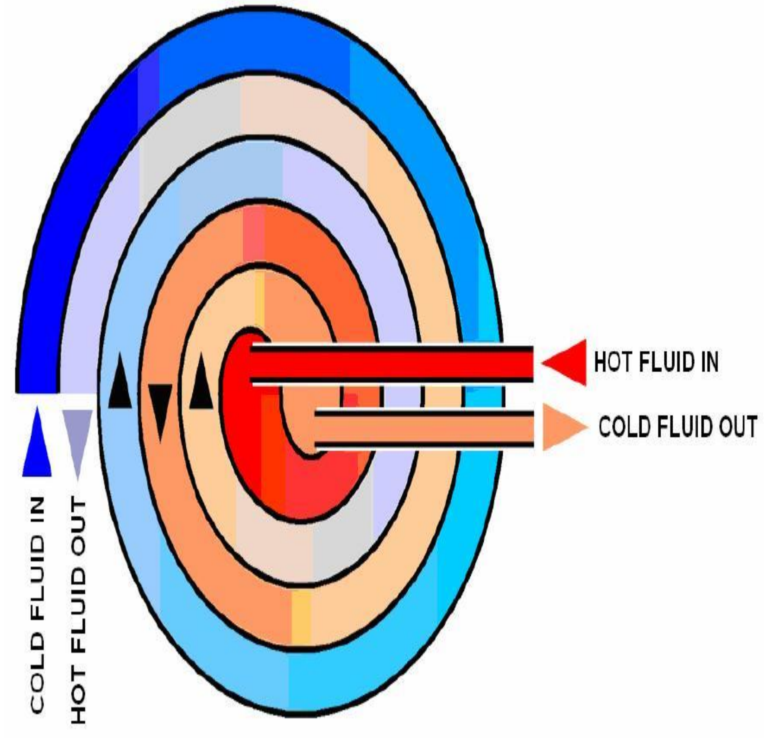




Courtesy of
Alfa Laval Inc.

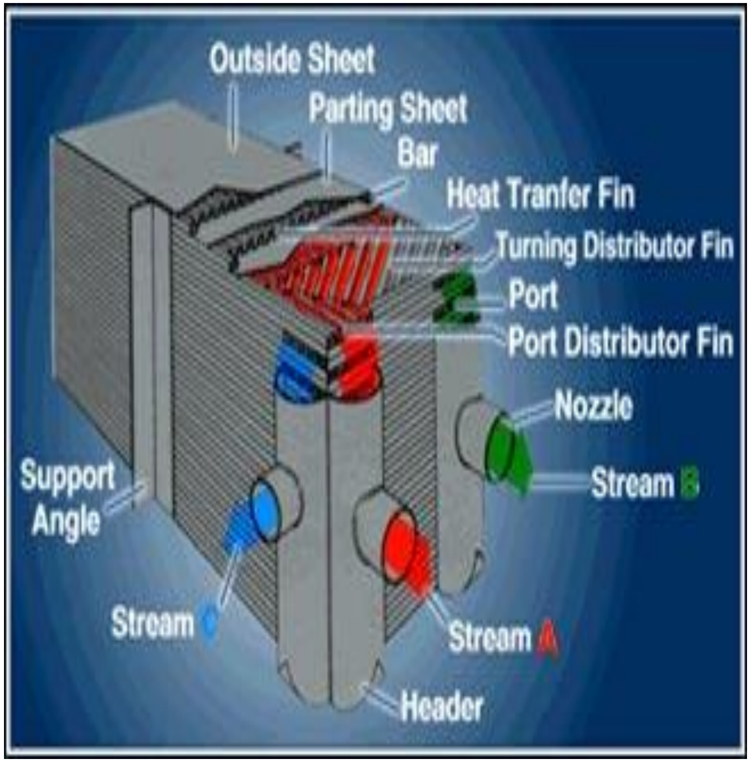
3.21 Type of heat exchangers

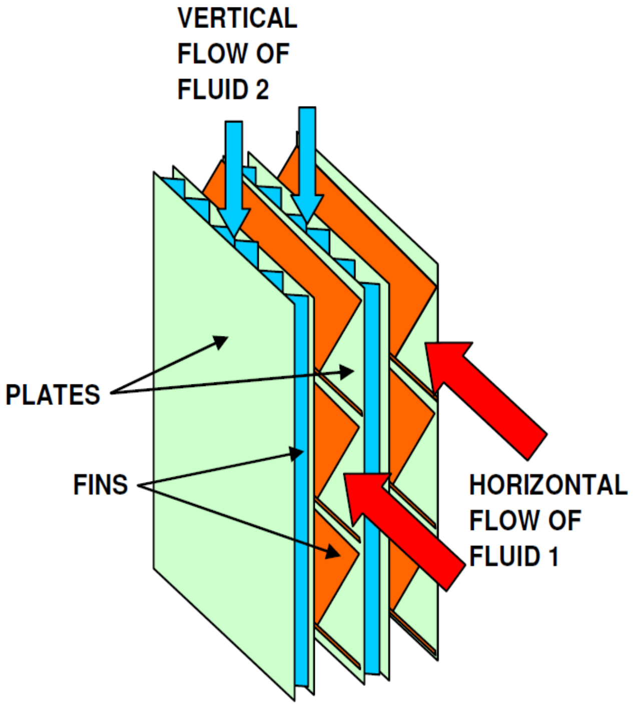
2. Spiral Heat Exchangers



3.21 Type of heat exchangers

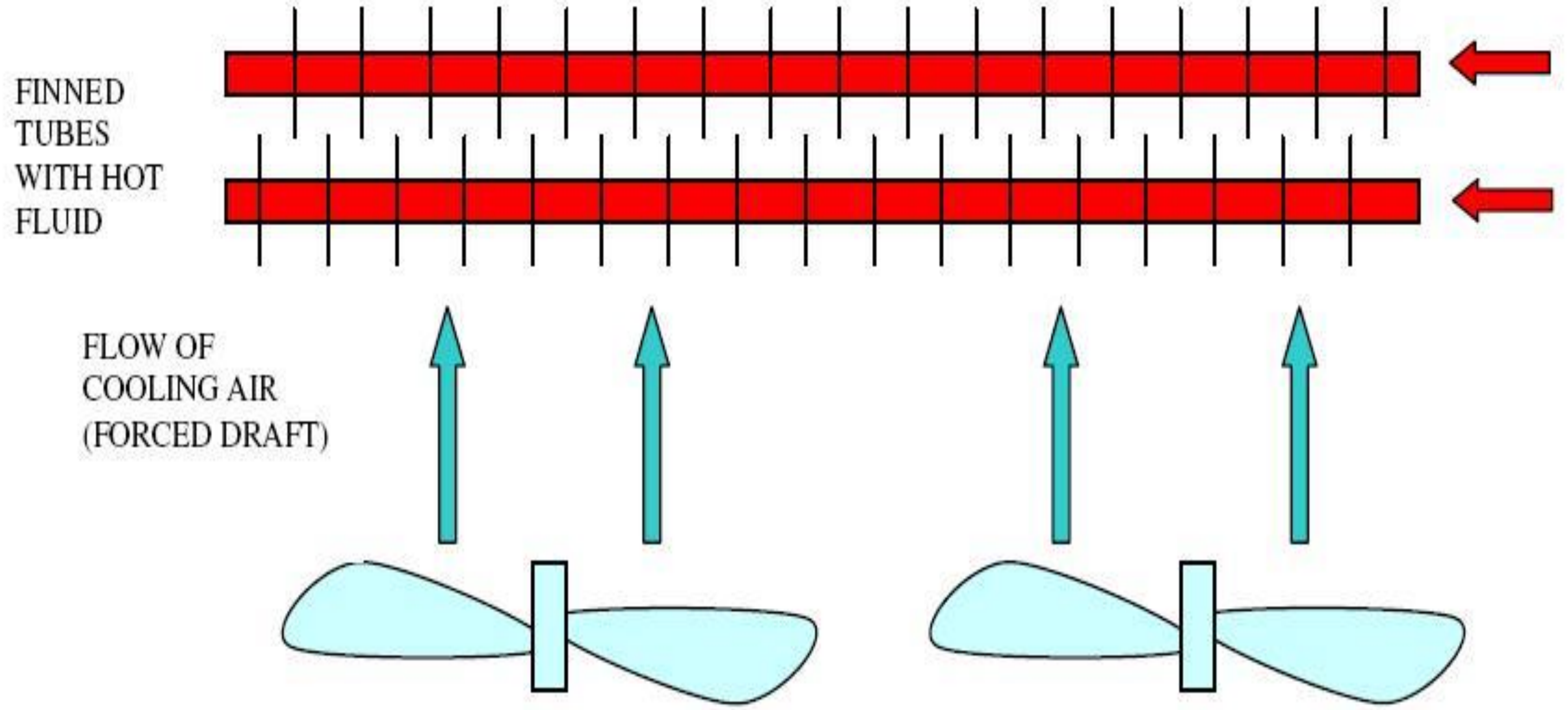
3. Plate-Fin Heat Exchangers





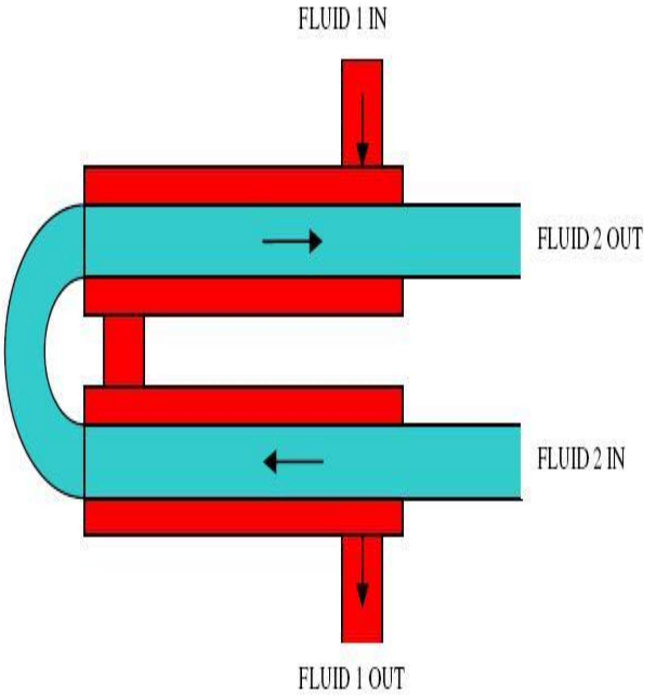
3.21 Type of heat exchangers

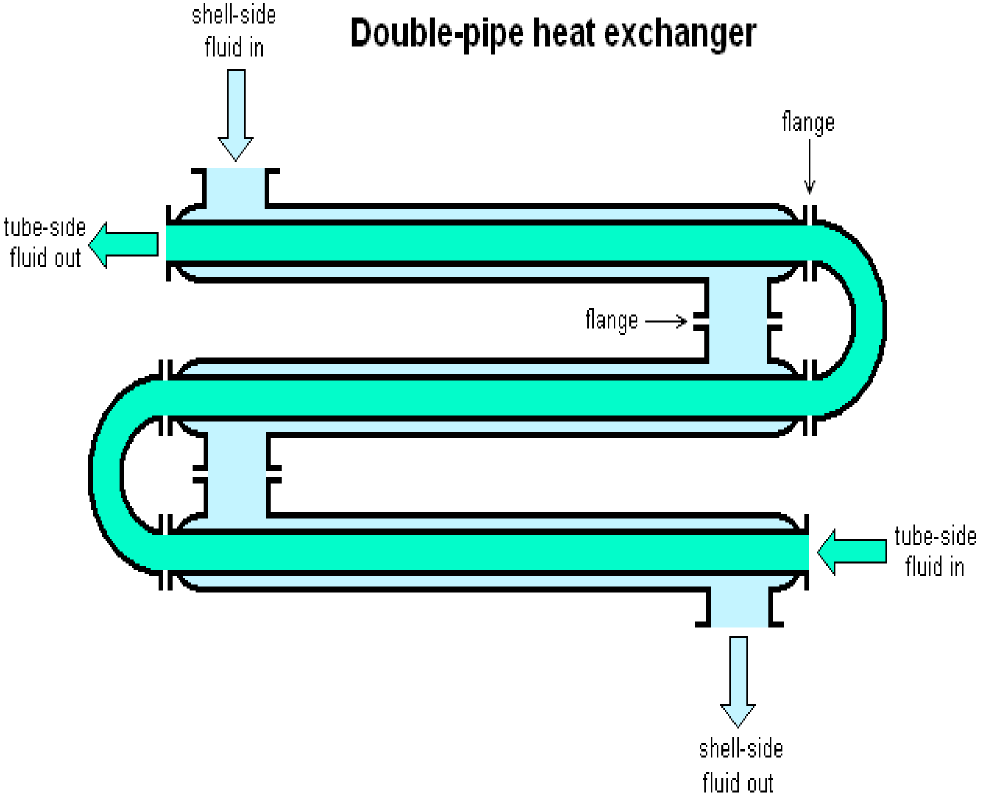
4. Air coolers



3.21 Type of heat exchangers

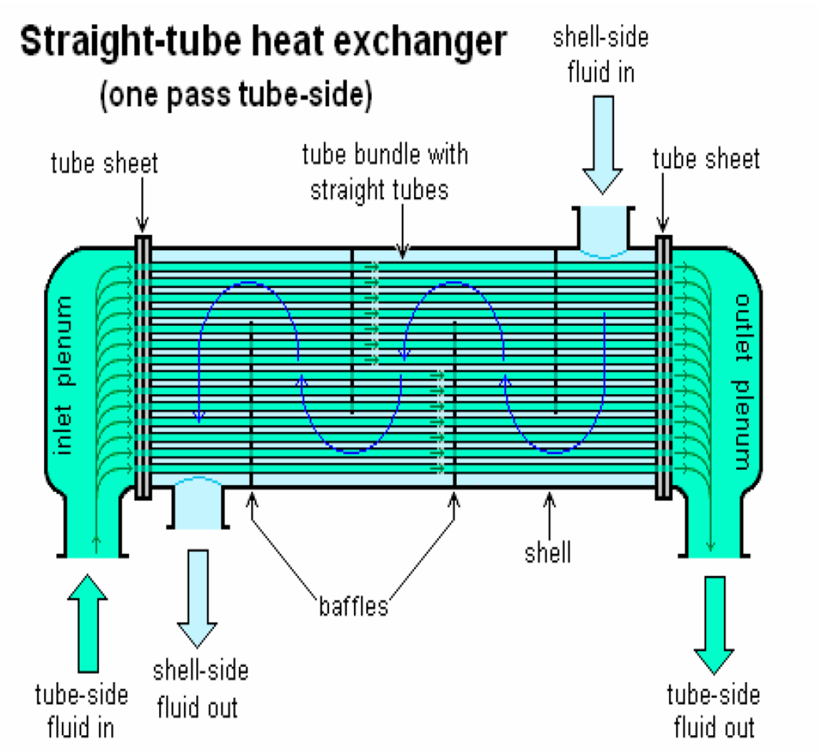
5. Double Pipe Exchangers

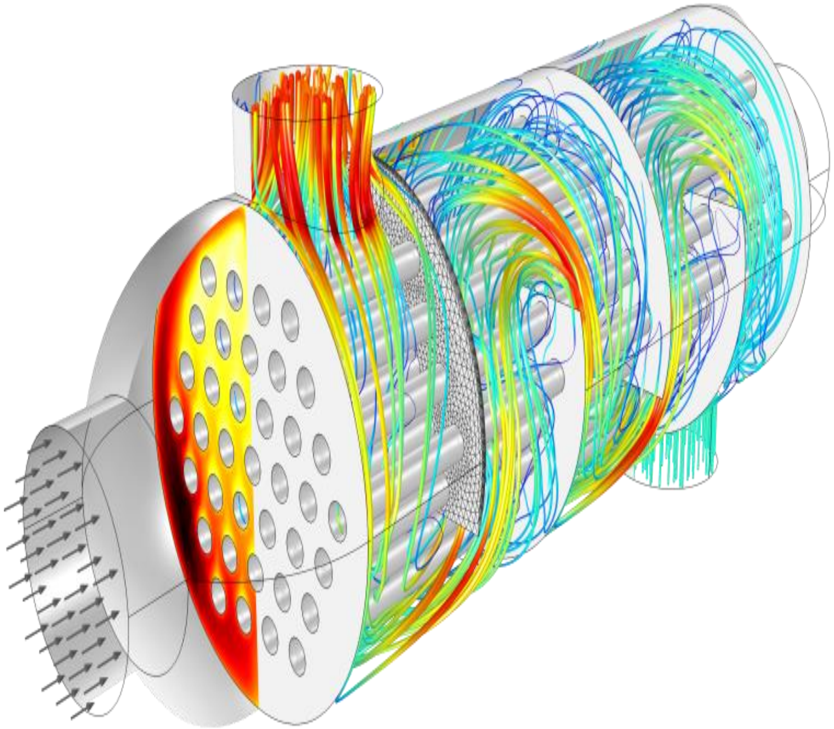




3.21 Type of heat exchangers

6. Shell and Tube Exchangers





3.22 Heat-transfer coefficients and friction factors

- **Heat-transfer data are usually presented in terms of dimensionless moduli, including the following parameters.**

1. Prandtl number : $N_{\mathrm{Pr}} = {}^{\mu C_p}/_{k_t}$

2. Nusselt number : $N_{\mathrm{Nu}} = {}^{h_c D_e}/_{k_t}$

3. Colburn J-factor : $j_H = (\frac{h_c}{G C_p}) N_{\mathrm{Pr}}^{2/3}$

4. Reynolds number : $N_{\mathrm{Re}} = \frac{D_e G}{\mu}$

5. Friction factor : $f = (\frac{\Delta p}{L})(\frac{G^2}{2 g \rho D_e})$

3.23 Fin effectiveness

- **Fin effectiveness**

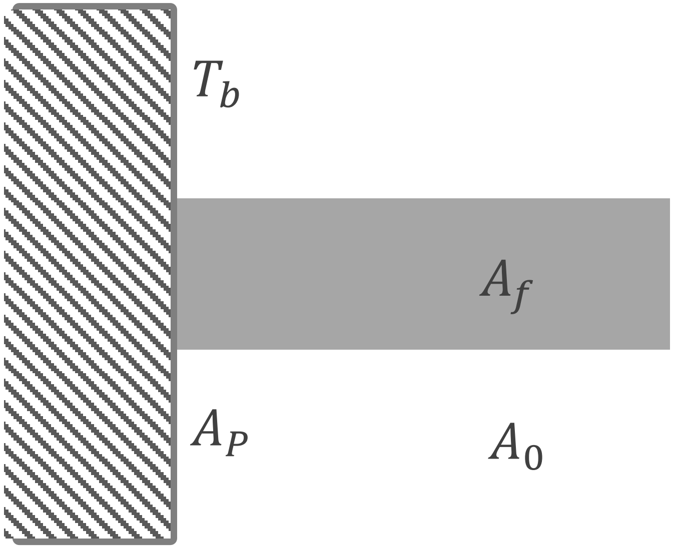
$$\eta_f = \frac{actual\ \dot{Q}_f}{\dot{Q}_f\ when\ fin\ is\ maintained\ at\ T_b}$$

3.23 Fin effectiveness

(For plate fin)

$$\eta_f = \frac{\tanh(ML)}{ML}, \qquad M = \left(\frac{h}{k_f \delta}\right)^{1/2}$$

3.23 Fin effectiveness



$A_0 = total\ surface\ area$

$A_f = fin\ surface\ area$

$A_p = primary\ surface\ area$

$\eta_0 = surface\ effectiveness$

3.23 Surface effectiveness

- Surface effectiveness

$$\eta_0 A_0 = (1)A_0 + \eta_f A_f = A_0 - A_f + \eta_f A_f$$

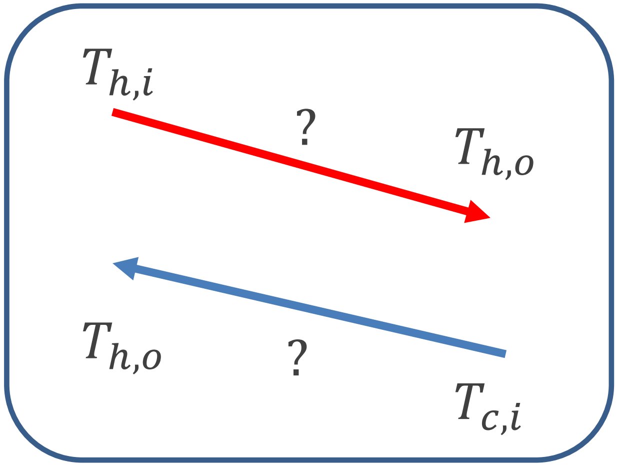
$$\eta_0 = 1 - (A_f/A_0)(1 - \eta_f)$$

3.23 Fin effectiveness

- Overall heat transfer coefficient

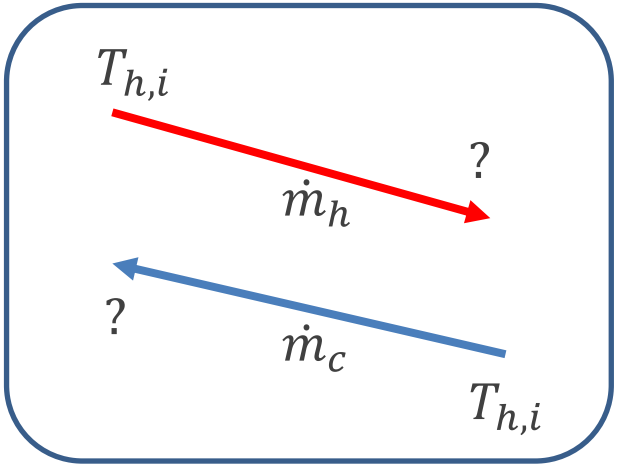
$$\dot{Q} = UA_0, \qquad \frac{1}{UA} = \frac{1}{A_o h_o} + \frac{1}{A_i h_i} + \frac{t}{kA} + R$$

3.24 Heat exchanger analysis method



LMTD method

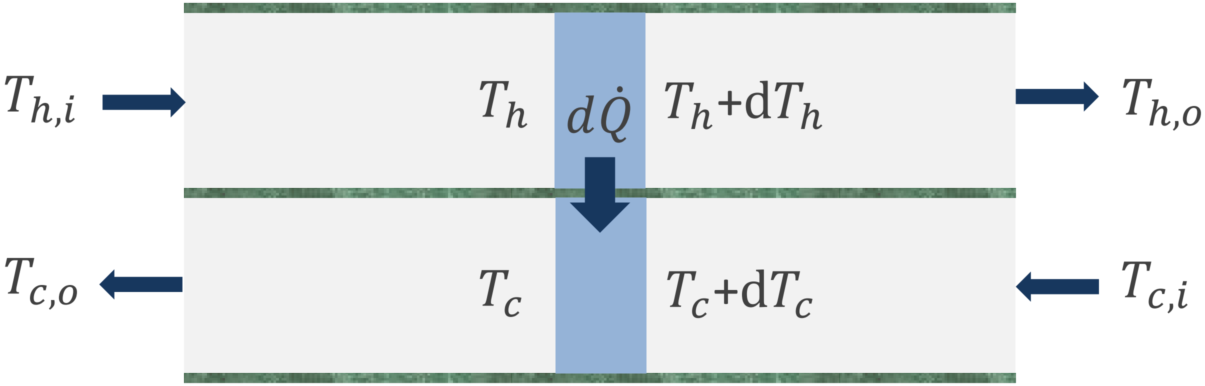
When inlet and outlet temperatures are known



e-NTU method

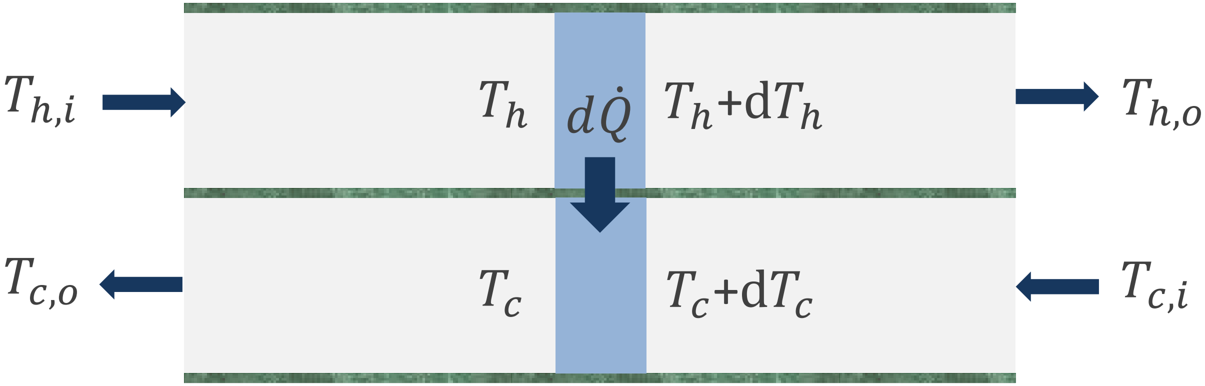
When inlet conditions and mass flows are known

3.24 LMTD method



$$\dot{Q} = C_h(T_{h,i} - T_{h,o}) = C_c(T_{c,o} - T_{c,i})$$

3.24 LMTD method



$$d\dot{Q} = -C_h dT_h = -C_c dT_c = U dA \Delta T$$

3.24 LMTD method

$$d(\Delta T) = dT_h - dT_c = -(\frac{UdA}{C_h}\Delta T - \frac{UdA}{C_c}\Delta T)$$

$$\frac{d(\Delta T)}{\Delta T} = -(\frac{1}{C_h} - \frac{1}{C_c})UdA$$

3.24 LMTD method

$$\rightarrow d(\Delta T) = dT_h - dT_c = -(\frac{UdA}{C_h}\Delta T - \frac{UdA}{C_c}\Delta T)$$

$$\rightarrow \frac{d(\Delta T)}{\Delta T} = -(\frac{1}{C_h} - \frac{1}{C_c})UdA$$

$$\rightarrow \ln\left(\frac{(\Delta T)_2}{(\Delta T)_1}\right) = \frac{(\Delta T)_2 - (\Delta T)_1}{\dot{Q}}UA$$

3.24 LMTD method

$$\dot{Q} = \frac{ln\left((\Delta T)_2/(\Delta T)_1\right)}{(\Delta T)_2 - (\Delta T)_1} UA$$

co-flow

$$(\Delta T)_1 = T_{h,i} - T_{c,i}$$
$$(\Delta T)_2 = T_{h,o} - T_{c,o}$$

counter flow

$$(\Delta T)_1 = T_{h,i} - T_{c,o}$$
$$(\Delta T)_2 = T_{h,o} - T_{c,i}$$

3.24 e-NTU method

$$\epsilon = \frac{q}{q_{max}} = \frac{C_c(T_{c,o} - T_{c,i})}{C_{min}(T_{h,i} - T_{c,i})} = \frac{C_h(T_{h,i} - T_{h,o})}{C_{min}(T_{h,i} - T_{c,i})}$$

$$NTU = \frac{UA}{C_{min}}$$

$C_{min} = C_C$ for most cryonic system

$$\epsilon = \frac{(T_{c,o} - T_{c,i})}{(T_{h,i} - T_{c,i})} \qquad NTU = \frac{UA}{C_C}$$

3.24 e-NTU method

$$\frac{(\Delta T)_2}{(\Delta T)_1} = \frac{UA}{C_C}\left(1 - \frac{C_C}{C_h}\right) \rightarrow \frac{T_{h,o} - T_{c,i}}{T_{h,i} - T_{c,o}} = \exp\left(\frac{UA}{C_{min}}\left(1 - \frac{C_{min}}{C_{max}}\right)\right)$$

$$\frac{T_{h,o} - T_{c,i}}{T_{h,i} - T_{c,o}} = \frac{T_{h,o} - T_{c,i}}{T_{h,i} - T_{c,o}} = \frac{T_{h,o} - T_{h,i} + T_{h,i} - T_{c,i}}{T_{h,i} - T_{c,i} + T_{c,i} - T_{c,o}} = \frac{1 - \frac{T_{h,i} - T_{h,o}}{T_{h,i} - T_{c,i}}}{1 - \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}}} = \frac{1 - C_R\epsilon}{1 - \epsilon} = \exp(NTU(1 - C_R))$$

for counter flow when $C_{min} = C_C$: $\epsilon = \dfrac{1 - \exp[-NTU(1 + C_r)]}{1 - C_r\exp[-NTU(1 - C_r)]}$

3.24 e-NTU method

Co-flow:

$$\epsilon = \frac{1 - \exp[-NTU(1 + C_r)]}{1 + C_r}$$

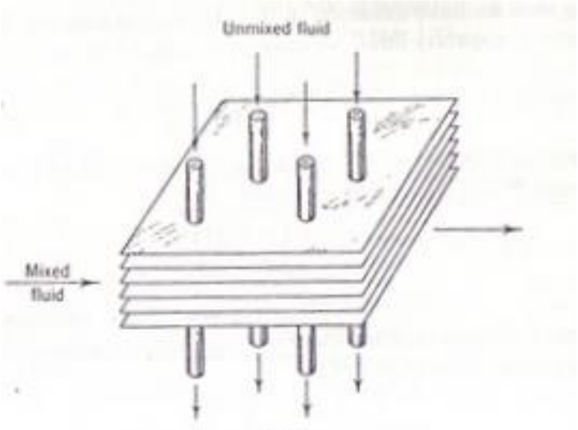
Counter-flow:

$$\epsilon = \frac{1 - \exp[-NTU(1 + C_r)]}{1 - C_r \exp[-NTU(1 - C_r)]}$$

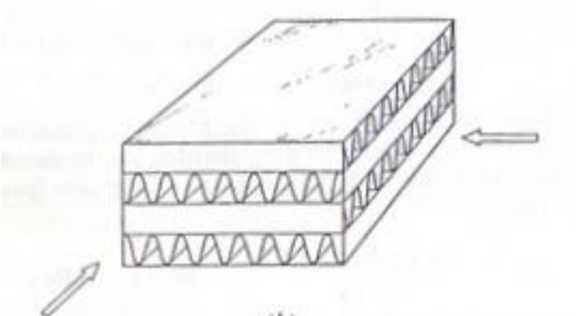
$$\epsilon = \frac{NTU}{1 + NTU} \qquad C_r = 1$$

$$\epsilon = 1 - \exp(-NTU) \quad C_r = 0$$

3.24 e-NTU method



(a)



(b)

both unmixed:

$$\epsilon = 1 - \exp\left[\left(\frac{1}{C_r}\right)(NTU)^{0.22}\{\exp[-C_r(NTU)^{0.78}] - 1\}\right]$$

C_{max} mixed:

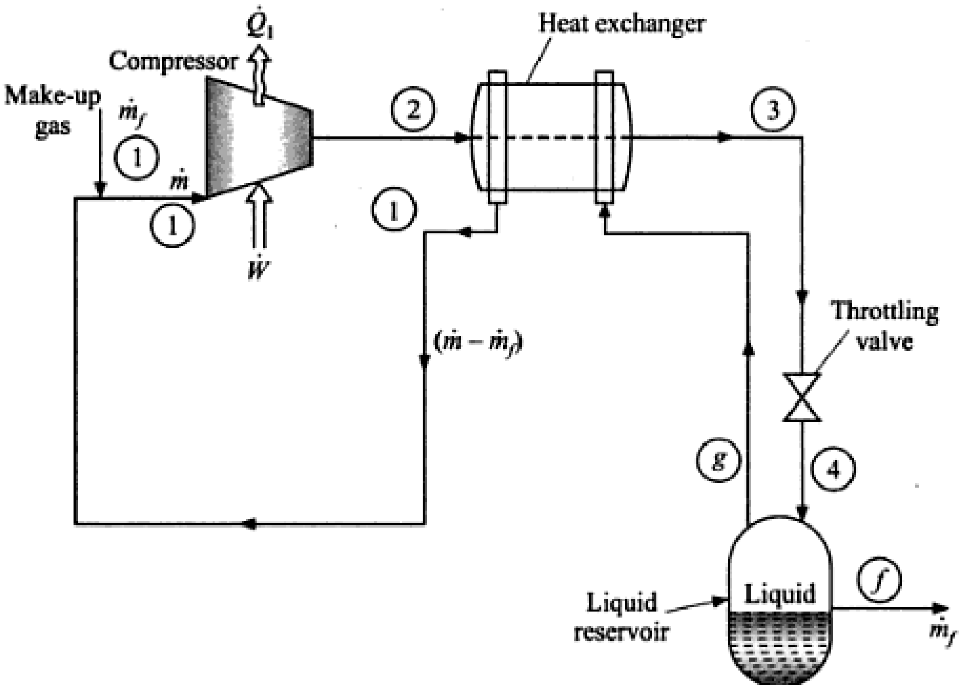
$$\epsilon = \left(\frac{1}{C_r}\right)(1 - \exp(-{C_r}^{-1}\{1 - \exp[-C_r(NTU)]\})$$

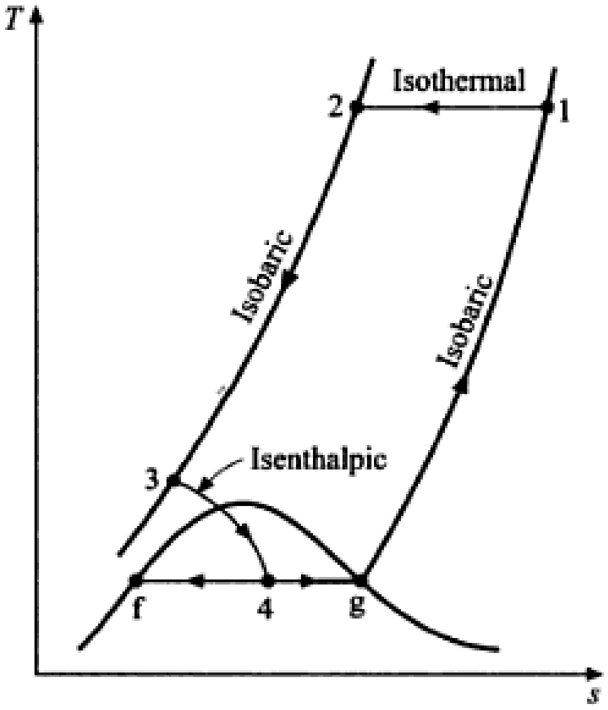
C_{in} mixed:

$$\epsilon = \ (1 - \exp(-{C_r}^{-1}\{1 - \exp[-C_r(NTU)]\})$$

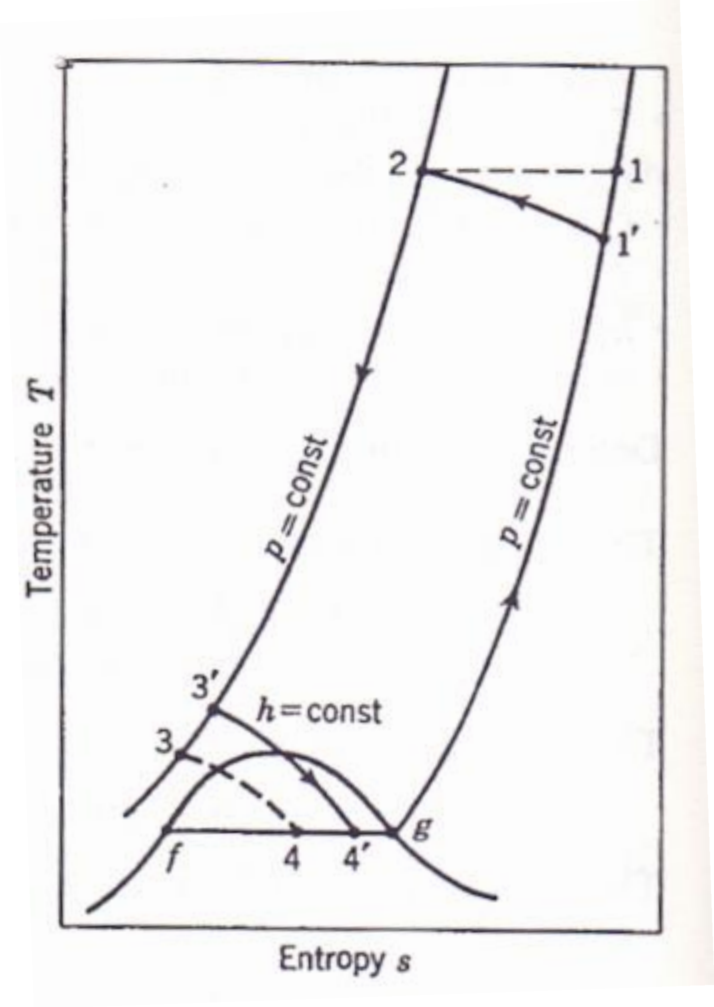
3.25 Effect of ε on system performance

- Simple Linde-Hampson System





3.25 Effect of ε on system performance



Linde-Hampson cycle with a heat exchanger with an effectiveness less than 100 percent. Points 1, 2, 3, and 4 are the state points for the ideal Linde-Hampson system. Point 1′, 2′, 3′, and 4′ are the state points for the system with the less effective heat exchanger.

3.25 Effect of ε on system performance

Mean specific heat

$$\bar{C}_{p,h} = \frac{h_2 - h_3}{T_2 - T_3} > \bar{C}_{p,c} = \frac{h_1 - h_g}{T_1 - T_g}$$

In Heat Exchanger

$$C_h = \dot{m}\bar{C}_{p,h} \quad > \quad C_c = \dot{m}(1-y)\bar{C}_{p,c}$$

$$C_{min} = C_c$$

HX Effectiveness

$$\varepsilon = \frac{C_c\left(T_1' - T_g\right)}{C_{min}\left(T_1 - T_g\right)} = \frac{h_1' - h_g}{h_1 - h_g}$$

3.25 Effect of ε on system performance

- **Liquid yield**

$$y = \frac{h_1' - h_2}{h_1' - h_f} = \frac{h_1 - h_2 + \left(h_1' - h_1\right)}{h_1 - h_f + \left(h_1' - h_1\right)} \qquad \left(\because h_1 - h_1' = (1-\varepsilon)\left(h_1 - h_g\right)\right)$$

$$= \frac{h_1 - h_2 + (1-\varepsilon)\left(h_1 - h_g\right)}{h_1 - h_f + (1-\varepsilon)\left(h_1 - h_g\right)} < \frac{h_1 - h_2}{h_1 - h_f}$$

➔ **liquid yield is decreased**

3.25 Effect of ε on system performance

The effect of heat-exchanger effectiveness on the liquid yield for a simple Linde-Hampson system using nitrogen as the working fluid. P_1 = 101.2 kPa (1 atm); p_2 = 20.27 MPa (200 atm); T_2 = 300 K (80°F).

3.25 Effect of ε on system performance

If y = 0

$$\varepsilon' = 1 - \frac{h_1 - h_2}{h_1 - h_g} = 0.869$$

HX is very critical in the liquefaction system

$$\Delta W = \dot{m}\left(h_1 - h_1'\right)$$

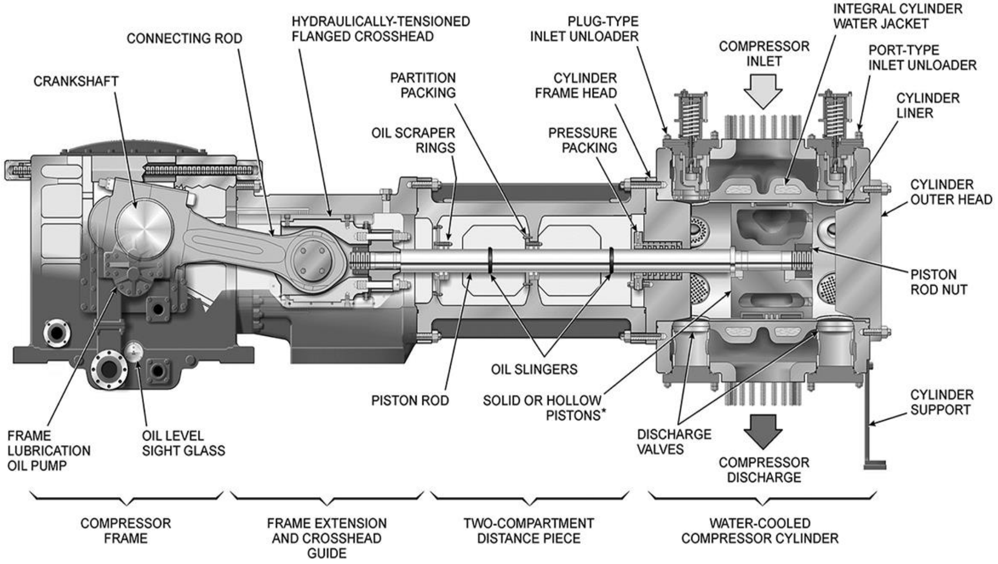
$$= \dot{m}\left(1-\varepsilon\right)\left(h_1 - h_g\right)$$ **: increased work**

3.26 Compressors and expanders

- There are two broad classes of compressors and expanders used in liquefaction systems

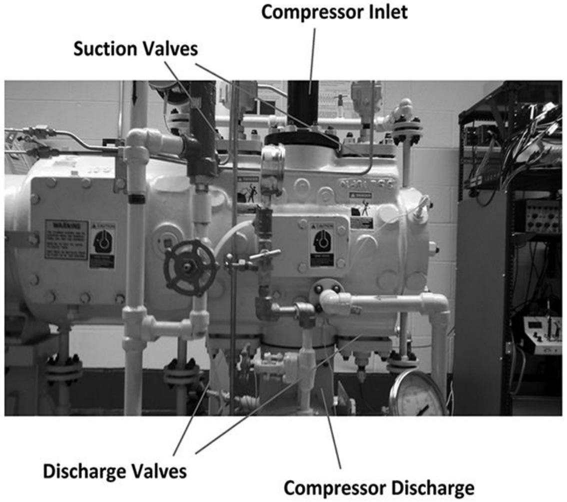
1. Reciprocating

Reciprocating Process Compressor Components



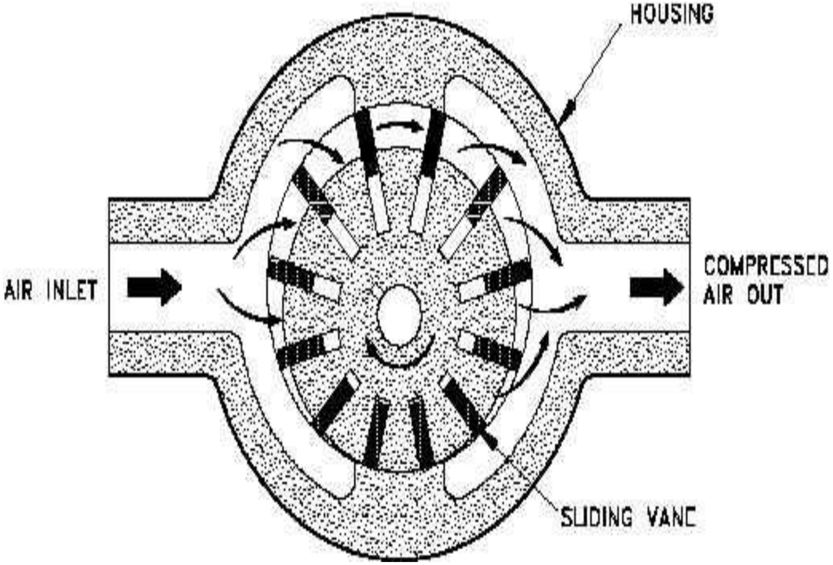
* *Illustration shows hollow nodular iron piston on top ha and hollow aluminum piston on bottom half. Solid nodular iron pistons and steel pistons, integral with piston rod, are also available.*

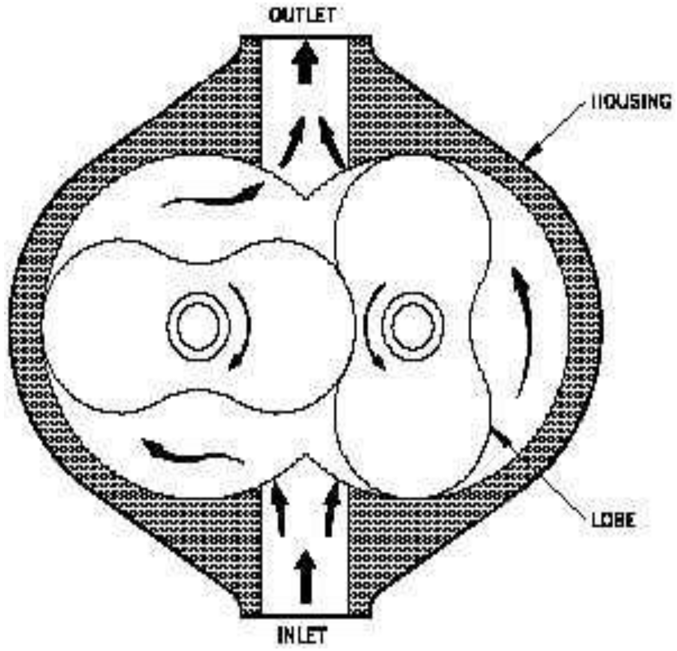
DRESSER-RAND



3.26 Compressors and expanders

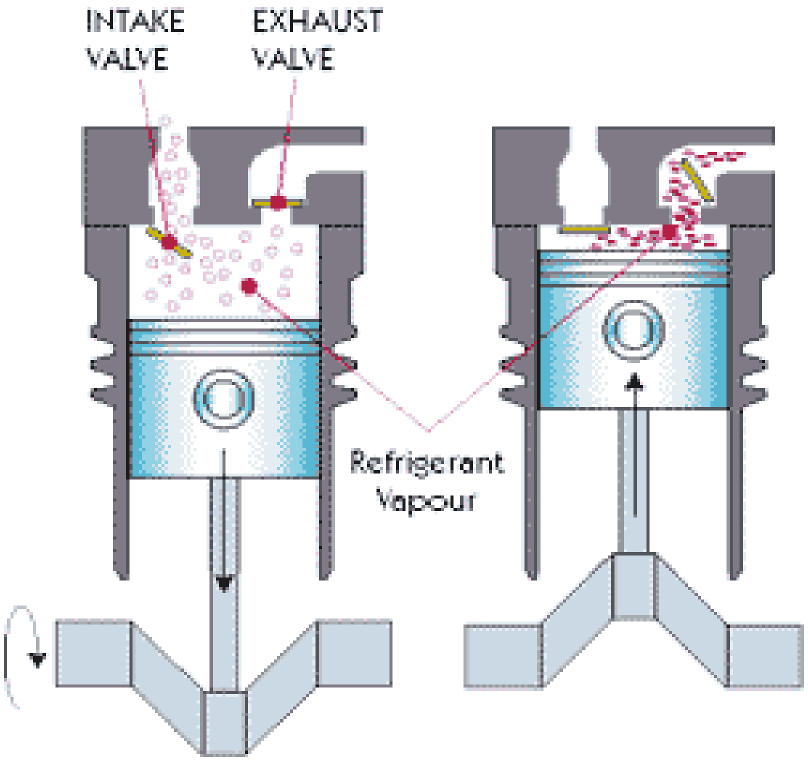
2. Rotary

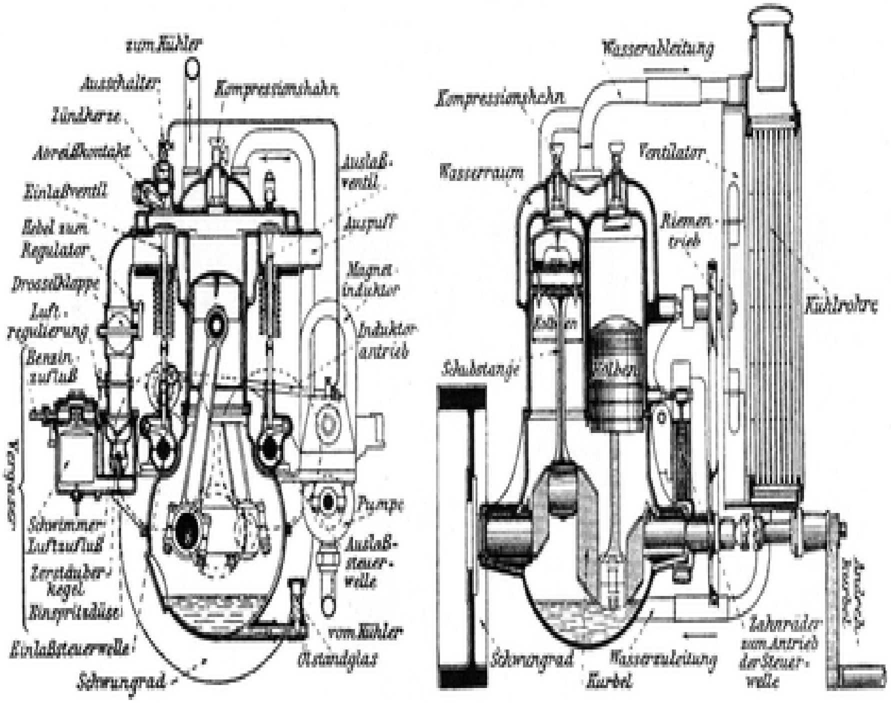




3.26 Compressors and expanders

1-1. Reciprocating Compressor-IC engine type



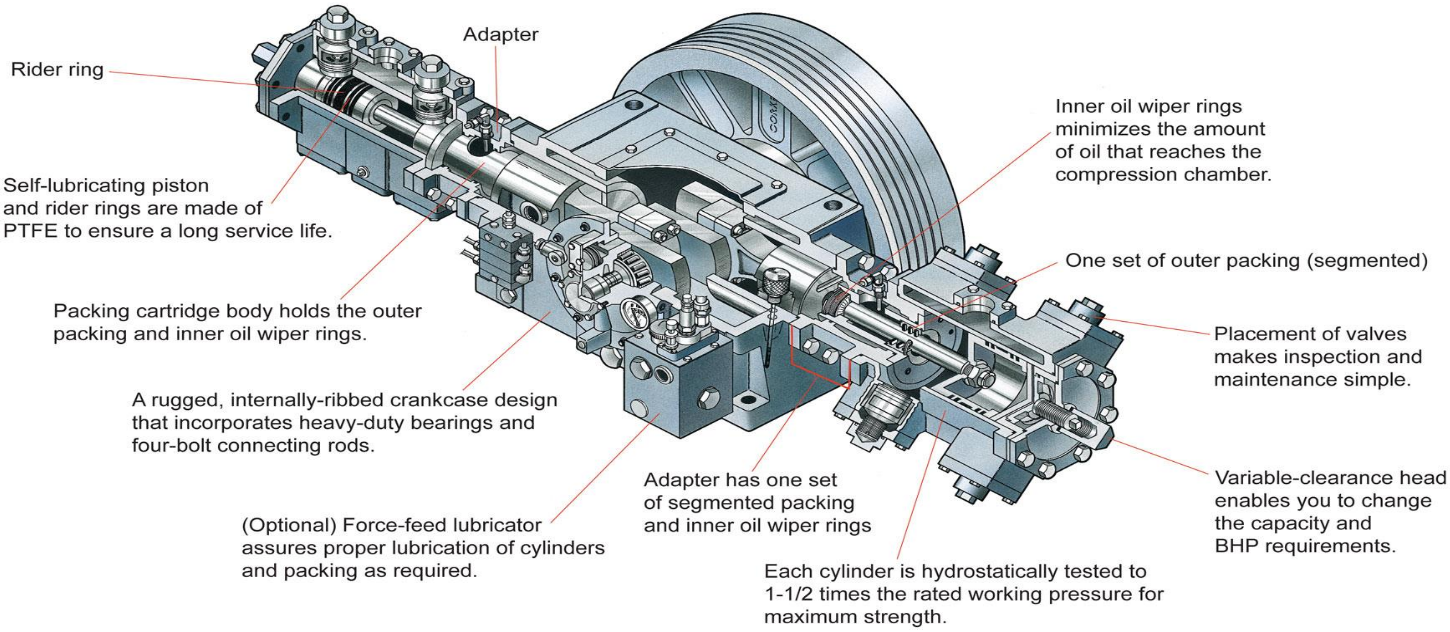


4. Zweizylindriger Benzinmotor.

3.26 Compressors and expanders

1-2. Reciprocating Compressor-steam engine type

Plain Style Horizontal Gas Compressor (Reciprocating)



Adapter

Rider ring

Self-lubricating piston and rider rings are made of PTFE to ensure a long service life.

Packing cartridge body holds the outer packing and inner oil wiper rings.

A rugged, internally-ribbed crankcase design that incorporates heavy-duty bearings and four-bolt connecting rods.

(Optional) Force-feed lubricator assures proper lubrication of cylinders and packing as required.

Adapter has one set of segmented packing and inner oil wiper rings

Each cylinder is hydrostatically tested to 1-1/2 times the rated working pressure for maximum strength.

Inner oil wiper rings minimizes the amount of oil that reaches the compression chamber.

One set of outer packing (segmented)

Placement of valves makes inspection and maintenance simple.

Variable-clearance head enables you to change the capacity and BHP requirements.

NOTE: A plain style compressor has one set of packing and no distance piece.

3.26 Compressors and expanders

1-2. Reciprocating Expander

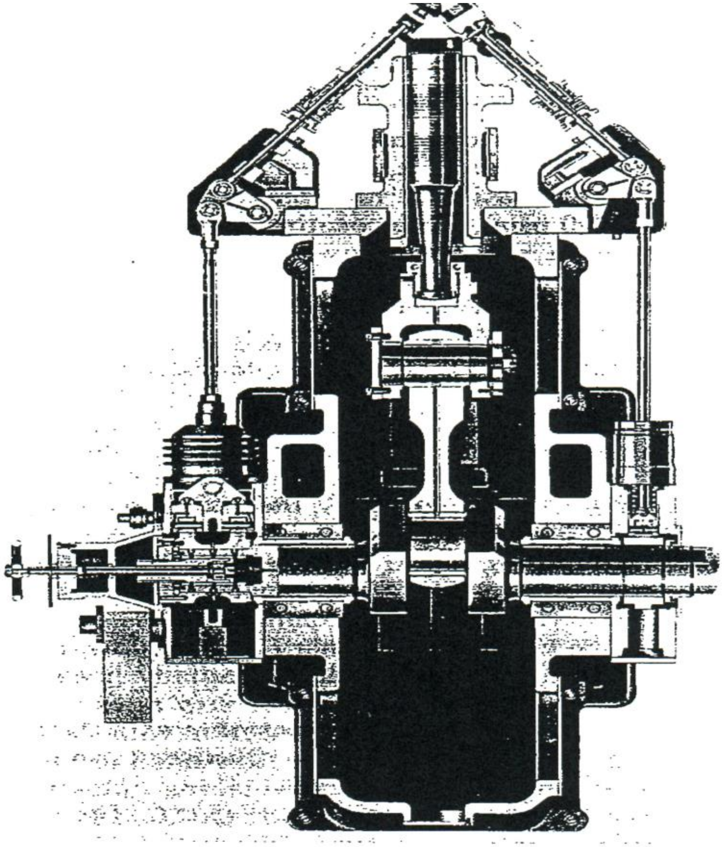
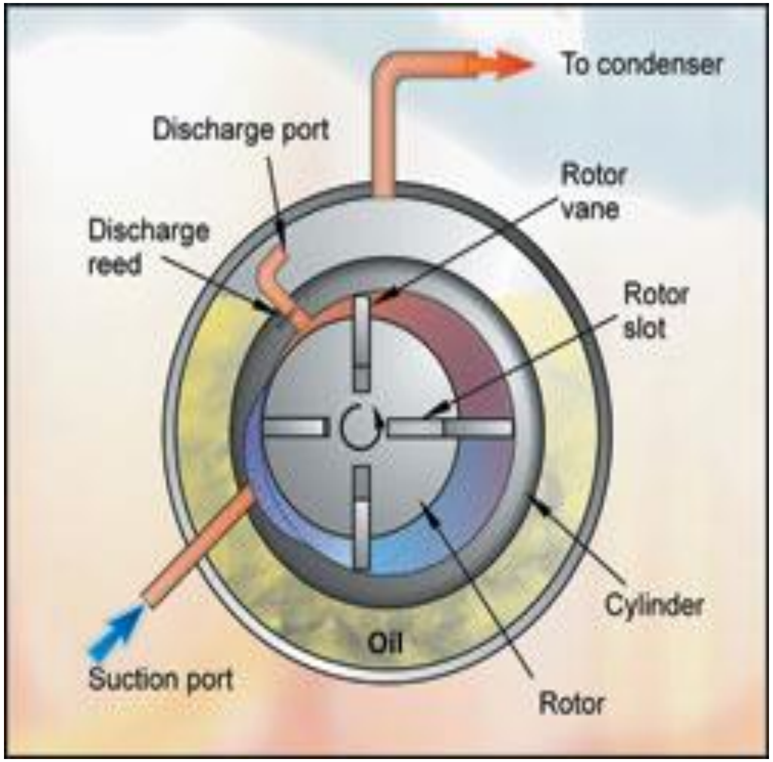
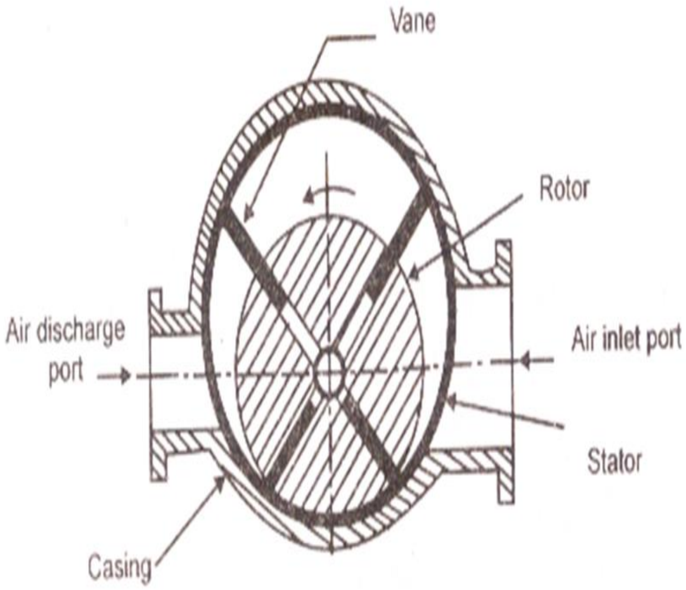


Fig. 3.43. Reciprocating expander (Worthington Corporation).

3.26 Compressors and expanders

2-1. Rotary Compressor





3.26 Compressors and expanders

2-2. Rotary Expander

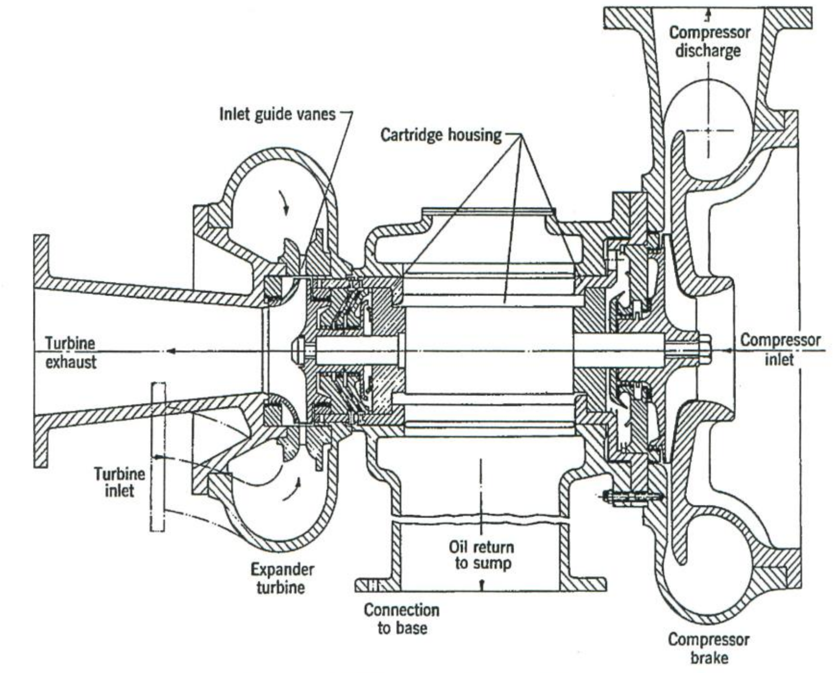
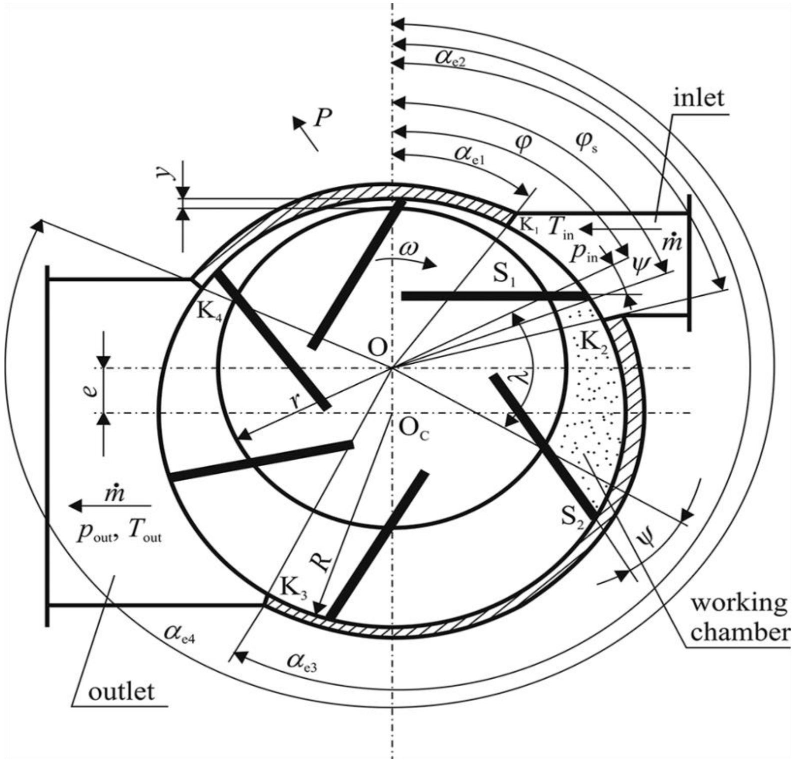


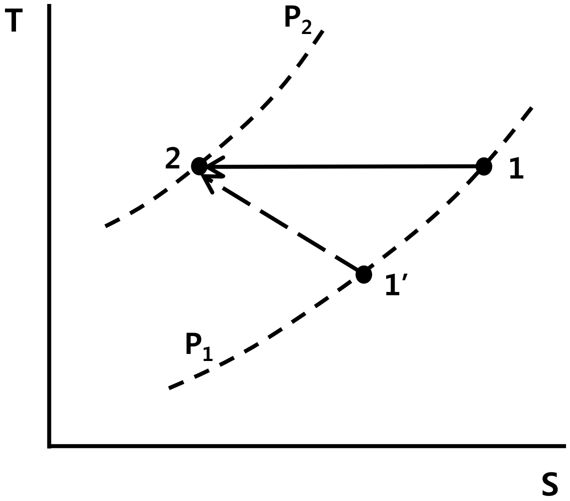
Fig. 3.44. Rotary expander (Worthington Corporation).



3.27 Losses for real machines

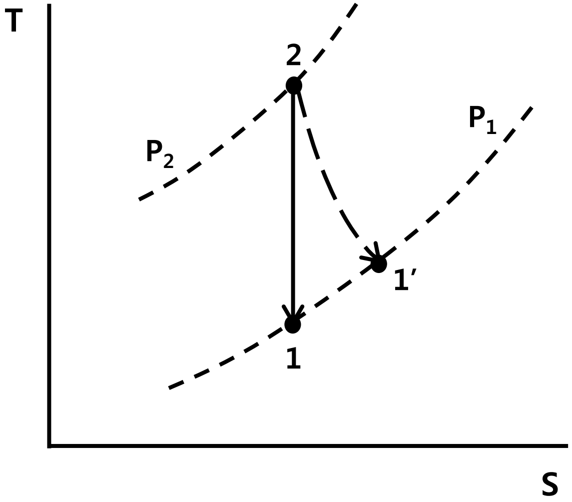
1. Isothermal efficiency of a compressor

$$\eta_{T=c} = \frac{\left(\dot{W}/\dot{m}\right)_{T=c}}{\left(\dot{W}/\dot{m}\right)_{\text{indicated}}}$$



2. Adiabatic efficiency of an expander

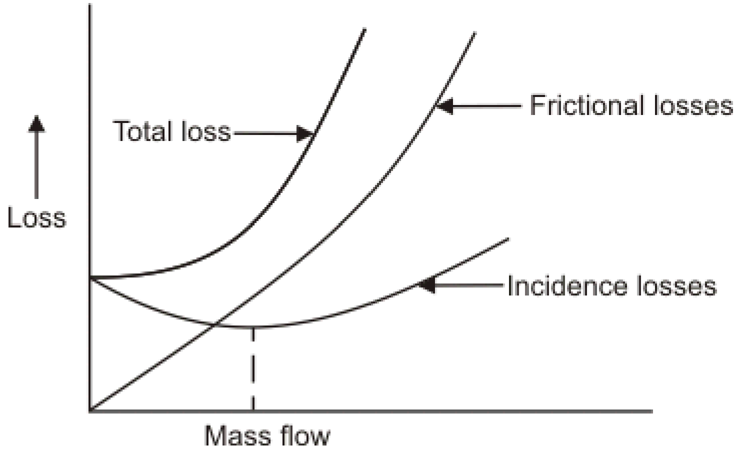
$$\eta_{ad} = \frac{\left(h_1 - h_2\right)_{\text{indicated}}}{\left(h_1 - h_2\right)_{ad}}$$



3.27 Losses for real machines

3. Mechanical efficiency of a compressor

$$\eta_{c,m} = \frac{\left(\dot{W}/\dot{m}\right)_{\text{indicated}}}{\left(\dot{W}/\dot{m}\right)_{\text{actual}}} = 1 - \frac{\dot{W}_{\text{friction}}}{\dot{W}_{\text{actual}}}$$



Dependence of various losses with mass flow in a centrifugal compressor

4. Mechanical efficiency of an expander

$$\eta_{e,m} = \frac{\left(\dot{W}/\dot{m}\right)_{\text{actual}}}{\left(h_1 - h_2\right)_{\text{actual}}} = \frac{\dot{W}_{\text{actual}}}{\dot{W}_{\text{actual}} + \dot{W}_{\text{friction}}}$$

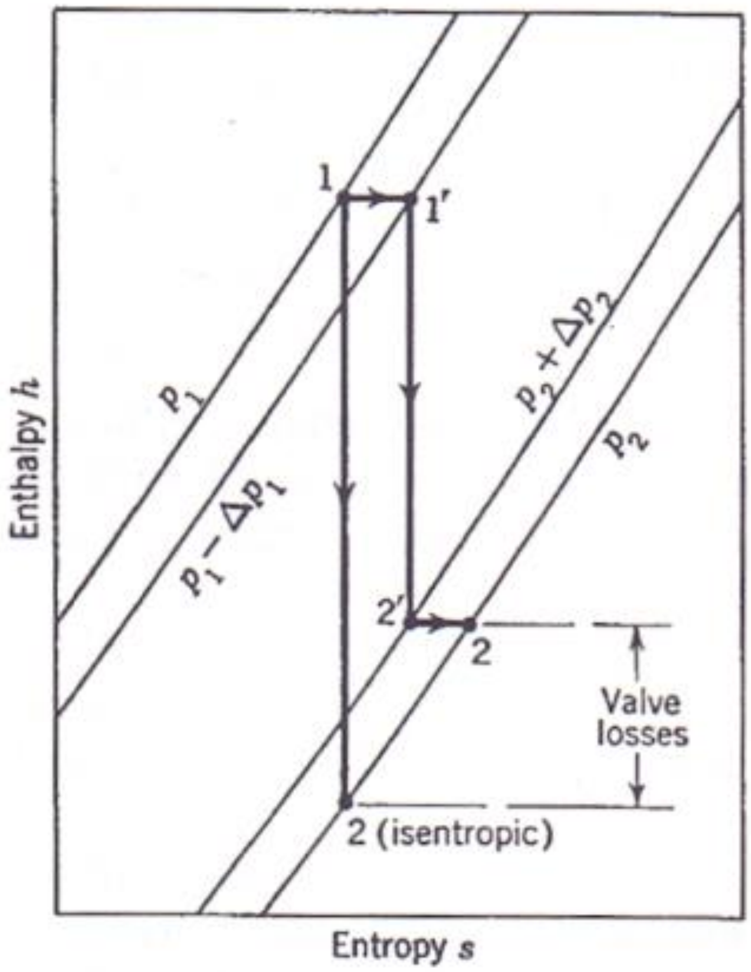
3.27 Losses for real machines

5. Overall efficiency

$\eta_o = \eta_{T=c}\eta_{c,m}$: compressor

$\eta_o = \eta_{ad}\eta_{e,m}$: expander

3.27 Losses for real machines



The effect of inlet and exhaust valve losses in an expander.

3.27 Losses for real machines

For **reciprocating** compressors and expanders, the following factors contribute to the efficiency being less than 100 percent:

1. Inlet-valve and outlet-valve losses. Pressure drops occur across the inlet and exhaust valves.

$$\frac{\Delta p_1}{p_1} = K_{L_1} \frac{\upsilon_1^2}{2g_c R T_1}$$

$$\frac{\Delta p_2}{p_2} = K_{L_2} \frac{\upsilon_2^2}{2g_c R T_2}$$

Δp : valve pressure drop

K_L : loss coefficient

υ : gas velocity through the valve

T : absolute temperature of the gas at the valve inlet

1 : inlet valve

2 : exhaust valve

3.27 Losses for real machines

- Work for adiabatic expansion
  (the valve losses were the only irreversible losses)

$$\frac{-\dot{W}}{\dot{m}}=\frac{\gamma RT_1}{\gamma-1}\left\{\left[\frac{p_2\left(1+\Delta p_2/p_2\right)}{p_1\left(1+\Delta p_1/p_1\right)}\right]^{(\gamma-1)/\gamma}-1\right\}$$

- Work for an isothermal compressor
  (the valve losses were the only irreversible losses)

$$\frac{-\dot{W}}{\dot{m}}=RT_1\ln\left[\frac{p_2\left(1+\Delta p_2/p_2\right)}{p_1\left(1+\Delta p_1/p_1\right)}\right]$$

3.27 Losses for real machines

2. Incomplete expansion. By shortening the cutoff, the friction losses are reduced at the expense of the small amount of work output at the end of the stroke.

3. Heat transfer.

$$\dot{Q} - \dot{W} = \dot{m}\left(h_2 - h_1\right)_{\text{actual}} \quad \text{or} \quad \left(h_1 - h_2\right)_{\text{actual}} = \left(\dot{W}/\dot{m}\right) - \left(\dot{Q}/\dot{m}\right)$$

$$\eta_{ad} = 1 - \frac{\dot{Q}/\dot{m}}{\left(h_1 - h_2\right)_{ad}}$$

4. Piston friction

3.27 Losses for real machines

For **rotary** compressors and expanders, the following factors affect the efficiency:

1. Inlet losses. (Same as reciprocating compressor)

2. Disk friction.

$$\frac{\dot{W}_{df}/\dot{m}}{\Delta h_{ad}} = 0.004\frac{D}{H_b}\left(\frac{U}{C_0}\right)^3$$

D : the blade-tip diameter (radial-flow unit) or the pitch-line or mean blade diameter (axial-flow unit)

H_b : the blade height

U : the blade-tip speed (radial-flow unit) or mean blade speed (axial-flow unit)

$$C_0 = \left(2g_c\Delta h_{ad}\right)^{1/2}$$

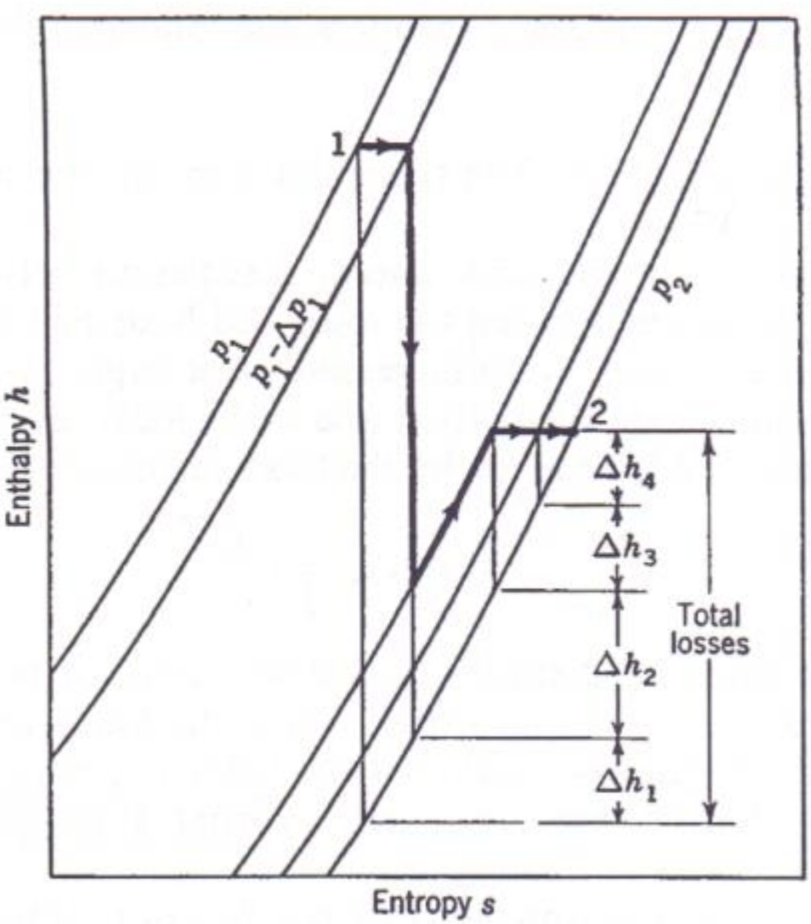
3.27 Losses for real machines

3. Impellor losses. A function of the blade angles (inlet and exit), the velocity of the fluid at the inlet and at the exit of the blade, and the type of unit (impulse, reaction, axial-flow, or radial-flow).

4. Leaving loss. The leaving loss is the loss in energy due to the kinetic energy of the outlet stream.

$$\Delta h_{exit} = \frac{\upsilon^2}{2g_c}$$

3.28 Effect of compressor and expander efficiency on system performance



- **Rotary-expander losses**

Δh_1 = inlet loss

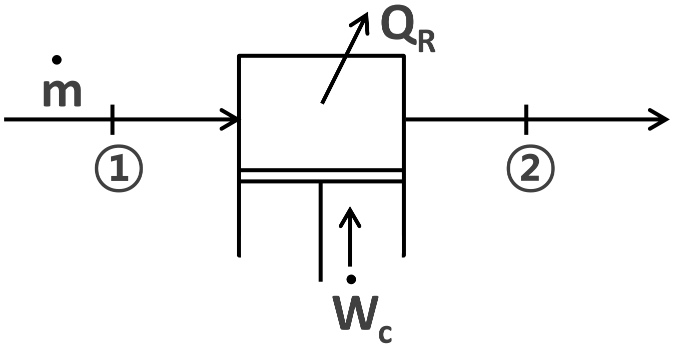
Δh_2 = loss due to heat in-leak to the gas in the cylinder and the frictional heating effect

Δh_3 = exhaust valve losses

Δh_4 = leaving loss associated with the kinetic energy of the leaving stream.

3.28 Effect of compressor and expander efficiency on system performance

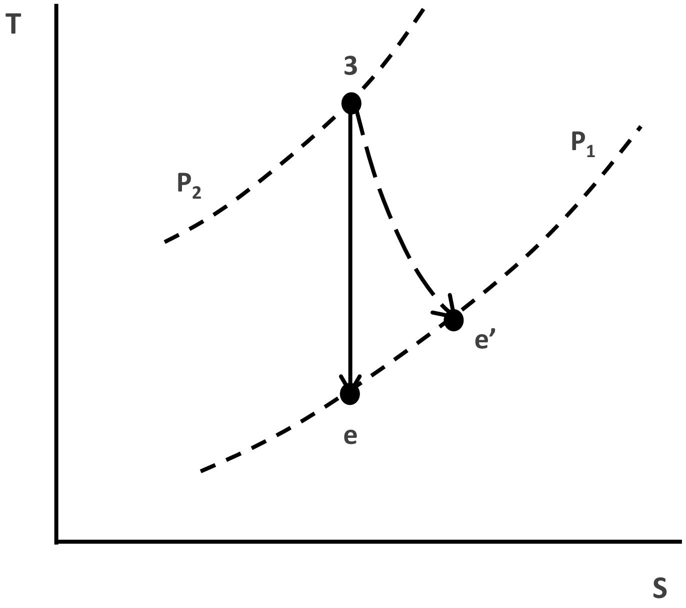
For compressor,



$$-\frac{\dot{W}_c}{\dot{m}} = \left[T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)\right] \times \frac{1}{\eta_{c,o}}$$

3.28 Effect of compressor and expander efficiency on system performance

For expander,



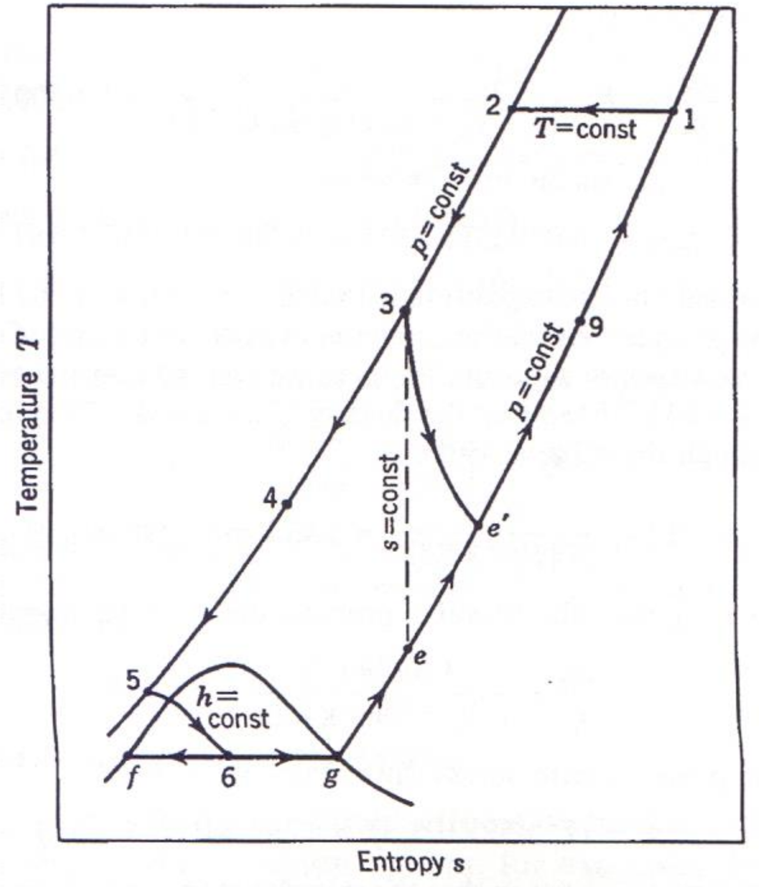
$$h_3 - h_e^{'} = \eta_{ad}\left(h_3 - h_e\right)$$

$$y = \frac{h_1 - h_2}{h_1 - h_f} + x\eta_{ad}\frac{h_3 - h_e}{h_1 - h_f}$$

$$-\frac{\dot{W}}{\dot{m}} = -\frac{\dot{W}_c}{\dot{m}} - x\left(h_3 - h_e\right)\times\eta_{e,o}$$

$$= \frac{T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)}{\eta_{c,o}} - x\eta_{e,o}\left(h_3 - h_e\right)$$

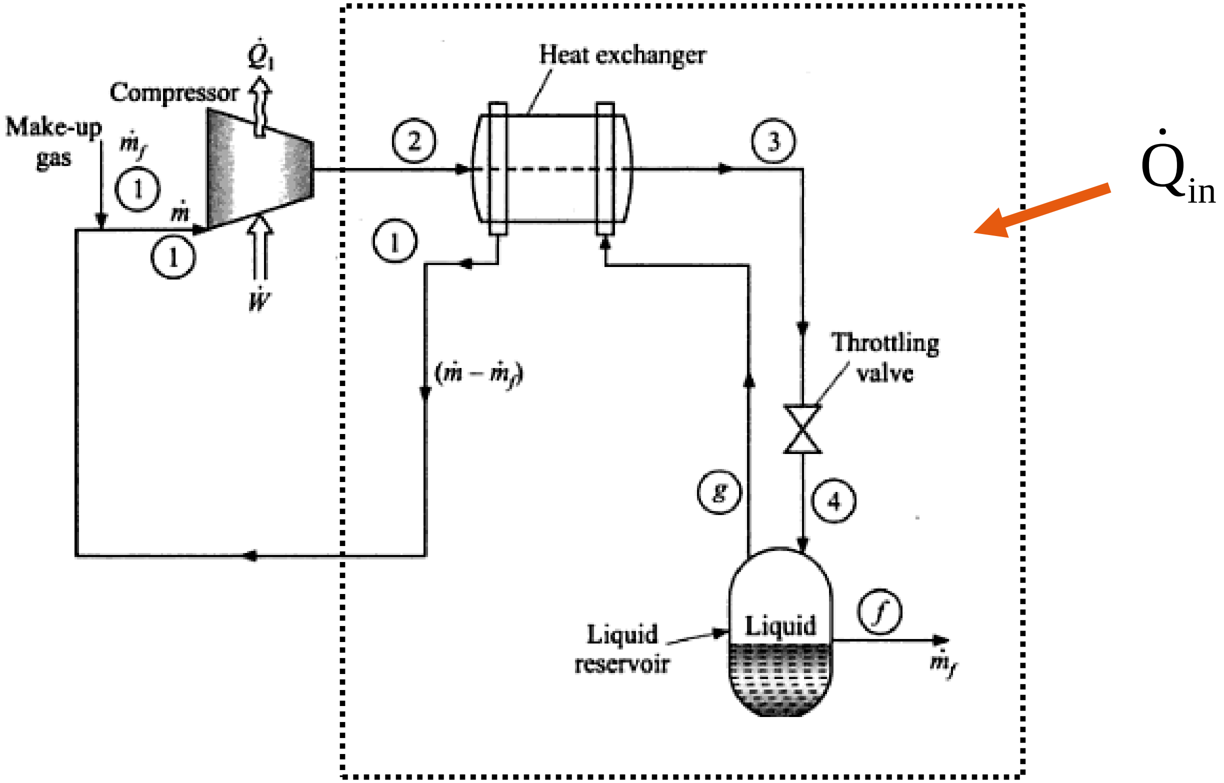
3.28 Effect of compressor and expander efficiency on system performance



Claude system with an expander having an adiabatic efficiency less than unity. The gas leaves the expander at point e’ in this case, instead of point e in the case of a reversible expander. (See Fig.3.18.)

3.29 Effect of heat transfer to the system

- **What happens when heat is coming into system?**



3.29 Effect of heat transfer to the system

$$\dot{Q}_{in} = \left(\dot{m} - \dot{m}_f\right) h_1 + \dot{m}_f h_f - \dot{m} h_2$$

$$= \dot{m}\left(1 - y\right) h_1 + \dot{m} y h_f - \dot{m} h_2$$

$$\Rightarrow \quad y = \frac{h_1 - h_2}{h_1 - h_f} - \frac{\boxed{\dot{Q}_{in}/\dot{m}}}{h_1 - h_f} \longrightarrow$$

The heat transfer rate per unit mass compressed is the important parameter!