Cryogenic Engineering

Chapter 5.

Cryogenic Refrigeration Systems

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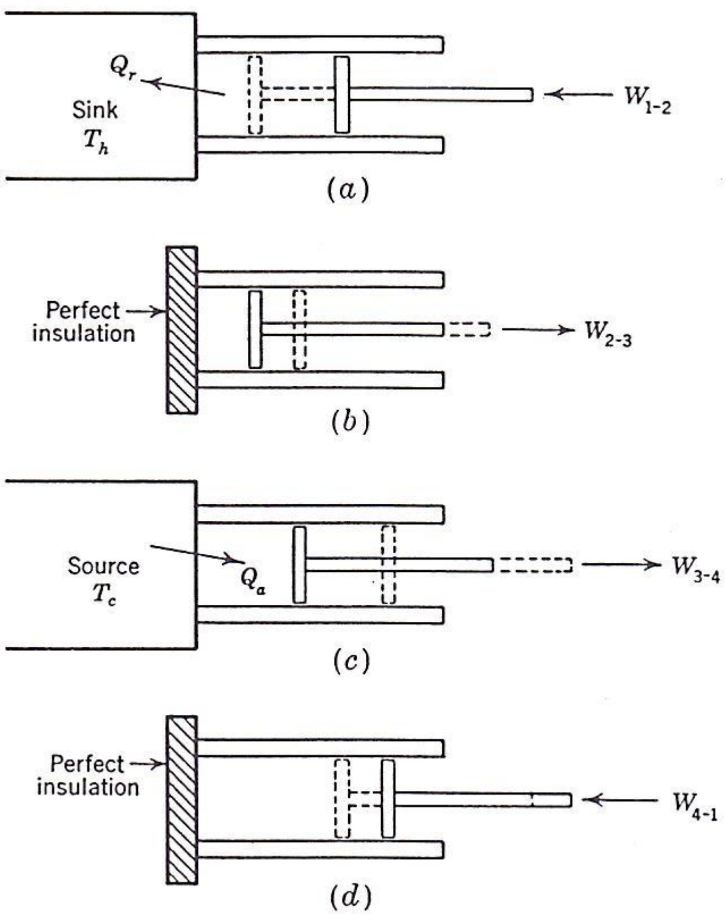
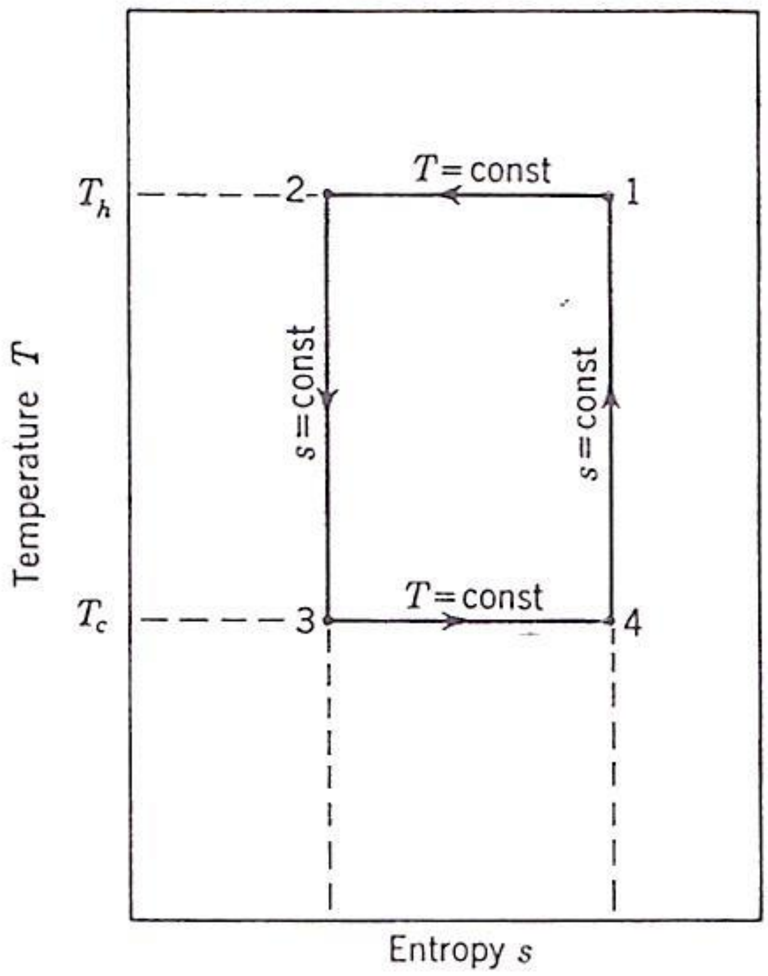
5.1 Thermodynamically ideal isothermal-source



Carnot Refrigeration

- (a) Reversible isothermal compression
- (b) Reversible adiabatic expansion
- (c) Reversible isothermal expansion with heat adsorption from the low temperature source
- (d) Reversible adiabatic compression

5.1 Thermodynamically ideal isothermal-source



Carnot Refrigeration
1-2 : Reversible isothermal compression with heat remover
2-3 : Reversible adiabatic expansion (isentropic)
3-4 : Reversible isothermal expansion with heat addition
4-1 : Reversible adiabatic compression (isentropic)

$$ds \equiv \left.\frac{\delta q}{T}\right)_{ideal}$$

5.1 Thermodynamically ideal isothermal-source

FOM is Figure of Merit

$$FOM = COP/COP_i$$

COP is the coefficient of performance of the actual system and COP_i is the coefficient of performance of the thermodynamically ideal system

5.1 Thermodynamically ideal isothermal-source

- **Coefficient of performance (COP)**

$$COP = \frac{Q_a}{W_{net}}$$

$$\begin{aligned} Q_{net} &= W_{net} \\ &= mT_h(s_2 - s_1) + mT_c(s_4 - s_3) \\ &= -(T_h - T_c)(s_1 - s_2) \end{aligned}$$

$$\oint \delta q = \oint du + \oint \delta \omega$$

$$\hookrightarrow 0$$

$$\therefore COP = \frac{T_c}{T_h - T_c}$$

5.1 Thermodynamically ideal isothermal-source

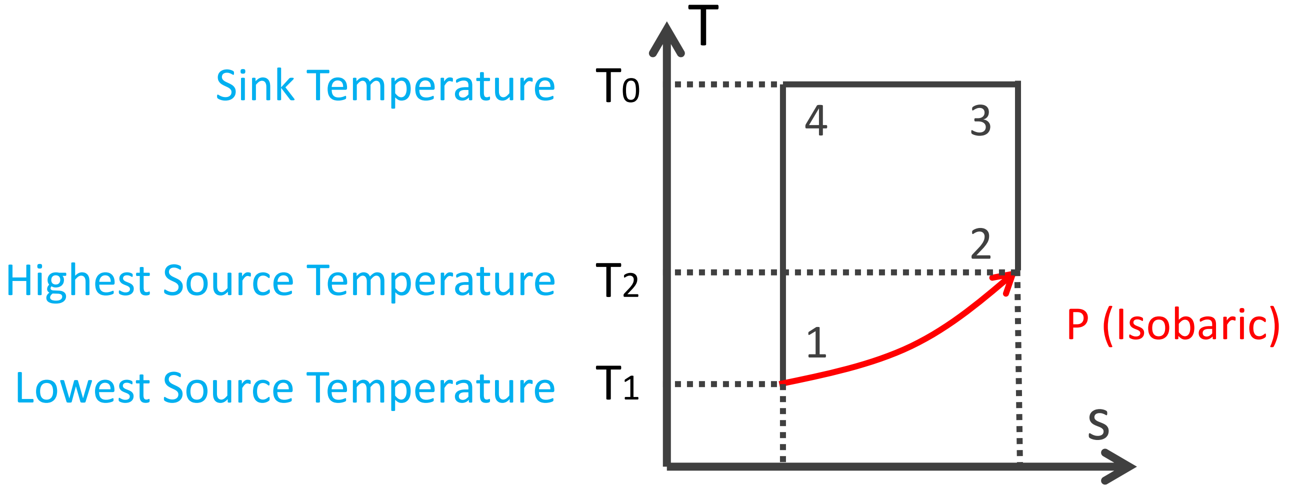
Source Temperature		
K	$COP_i = -Q_a/W_{net}$	$-W_{net}/Q_a$
111.7	0.5932	1.686
77.4	0.3477	2.876
20.3	0.07258	13.778
4.2	0.01420	70.43
1.0	0.003344	299.0
0.1	0.0003334	2999.0
0.01	0.0000333	29999.0

COP for a Carnot refrigerator operating between $300K$ and a low temperature T_c

5.2 The thermodynamically ideal isobaric-source system

It is for gas cooling. (not liquefaction!)

There is no phase change, so energy is absorbed at a varying temperature.



5.2 The thermodynamically ideal isobaric-source system

The energy rejected from the system is given by

$$Q_{rejected} = mT_0(s_4 - s_3)$$

The energy absorbed from the source is given by

$$Q_{absorbed} = \int_1^2 mTds = m\int_1^2 (dh - vdp) = m(h_2 - h_1)$$

From the First Law of Thermodynamics and the definition of the COP,

$$COP = -\frac{Q_a}{W_{net}} = -\frac{Q_a}{(Q_r + Q_a)} = \frac{h_2 - h_1}{T_0(s_2 - s_1) - (h_2 - h_1)}$$

5.2 The thermodynamically ideal isobaric-source system

For many cold-gas refrigerators, the pressures are sufficiently low that the working fluid may be assumed to behave as an ideal gas.

$$h_2 - h_1 = C_p(T_2 - T_1)$$

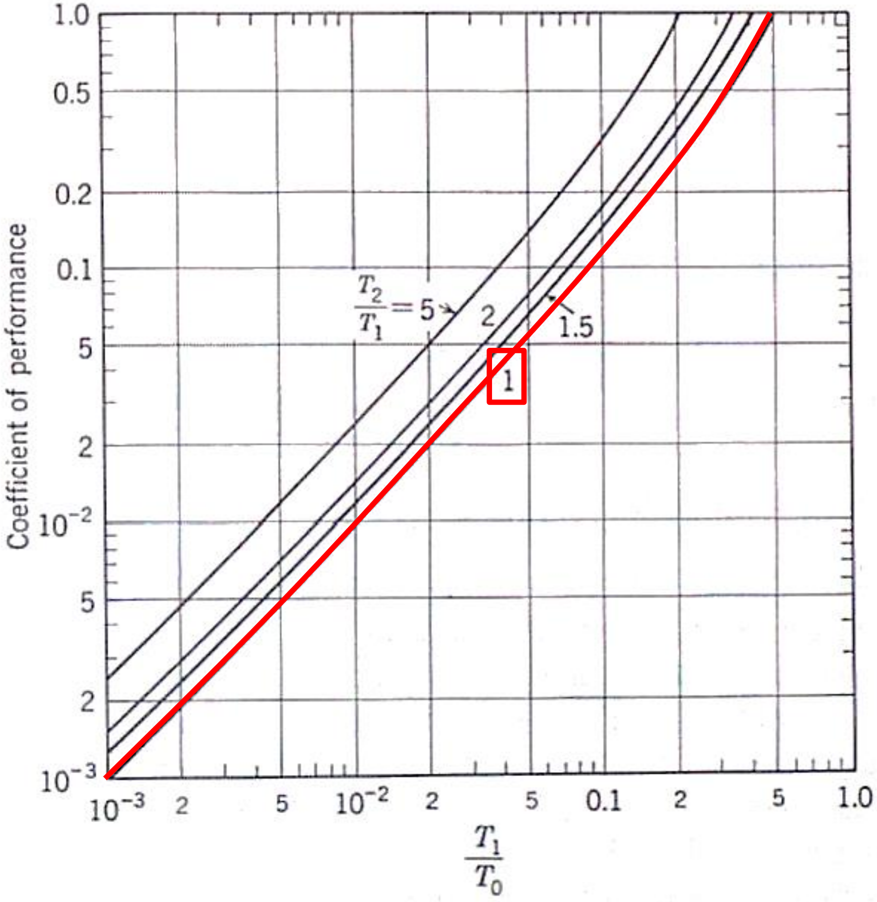
$$s_2 - s_1 = c_p ln\frac{T_2}{T_1} - R ln\frac{P_2}{P_1}$$

$$\therefore COP = \frac{\frac{T_2}{T_1} - 1}{\frac{T_0}{T} ln\frac{T_2}{T_1} - \frac{T_2}{T_1} + 1}$$

(COP is independent of the refrigerant)

5.2 The thermodynamically ideal isobaric-source system

The equation of COP is plotted in below figure.



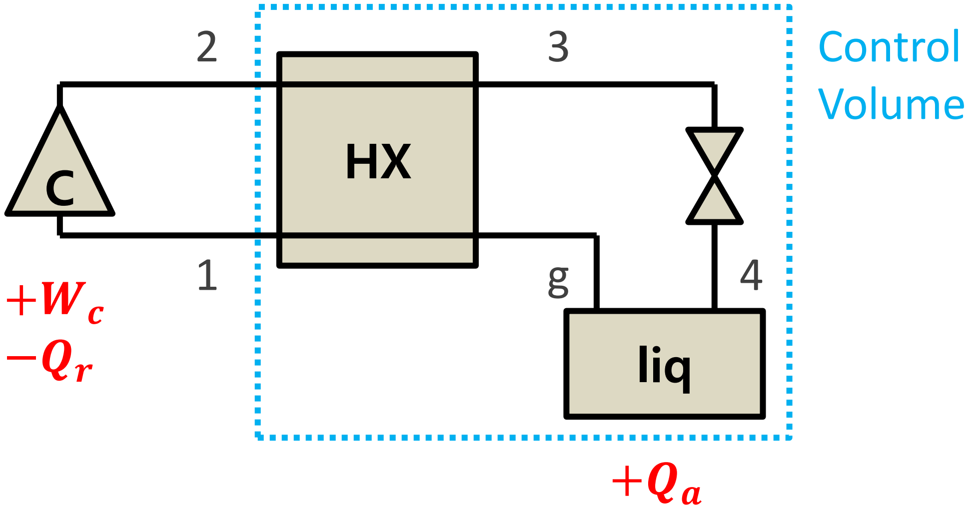
If $\frac{T_2}{T_1} \rightarrow 1$,

$COP \rightarrow COP_{carnot}$

5.3 Joule-Thomson refrigeration system

(Referring 3.5, Simple Linde-Hampson system)

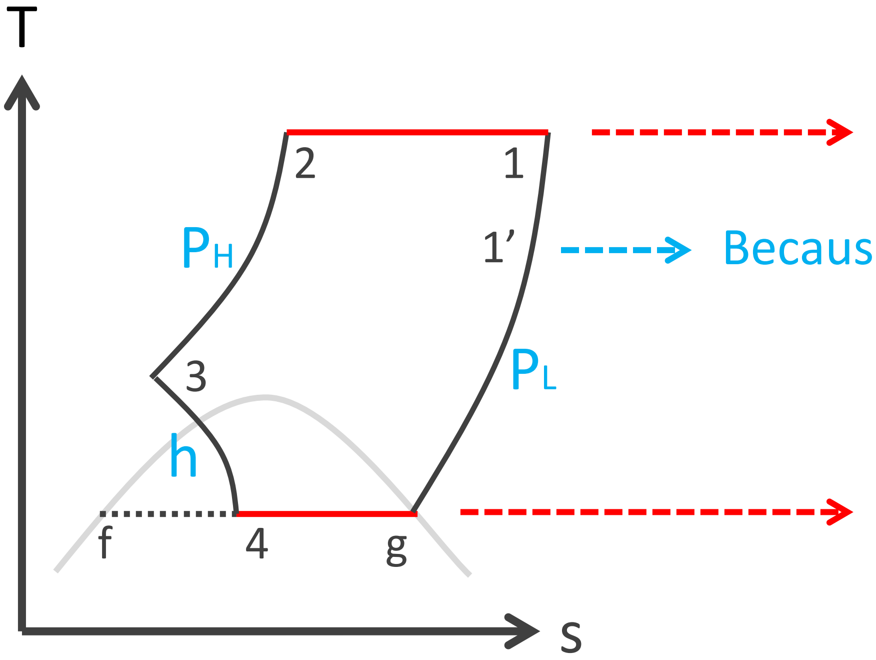
Instead of withdrawing the liquid, heat is absorbed to evaporate it.



At control volume, the total energy absorbed from the source is given by

$$\dot{Q}_a = \dot{m}(h_1' - h_2)$$

5.3 Joule-Thomson refrigeration system



Q_r Heating (Refrigerator outside)

Because ε is not 100%!

Q_a Cooling (Refrigerator inside)

The heat exchanger effectiveness $$\varepsilon = \frac{h_1' - h_g}{h_1 - h_g} = 1 - \frac{h_1 - h_1'}{h_1 - h_g}$$

5.3 Joule-Thomson refrigeration system

The total energy absorbed

$$\frac{\dot{Q}}{\dot{m}} = (h_1 - h_2) - (1 - \varepsilon)(h_1 - h_g)$$

$$\rightarrow \quad \frac{\dot{Q}}{\dot{m}} > 0$$ ($T_{warm.end} < T_{inversion}$ … not for Ne, He, H_2!)

$$\rightarrow \quad 1 - \frac{h_1 - h_2}{h_1 - h_g} < \varepsilon$$

Below this value, the refrigerator will not work!
(This limit is determined by setting the effect equal to zero)

$$\frac{\dot{Q}}{\dot{m}}$$

5.3 Joule-Thomson refrigeration system

The work requirement

$$-\frac{\dot{W}}{\dot{m}} = \frac{T_2(s_1 - s_2) - (h_1 - h_2)}{\eta_{c.o}}$$

$\eta_{c.o}$ = overall efficiency of the compressor (mechanical loss, friction …)

$$COP = \frac{-\dot{Q}_a}{\dot{W}} = \frac{\eta_{c.o}[(h_1 - h_2) - (1 - \varepsilon)\left(h_1 - h_g\right)]}{T_2(s_1 - s_2) - (h_1 - h_2)}$$

$$COP_i = \frac{T_c}{T_h - T_c} \quad \rightarrow \quad FOM = \frac{COP}{COP_i}$$

5.3 Joule-Thomson refrigeration system

The temperature in the evaporator can be regulated
by controlling the pressure by means of the expansion-valve setting.

Temperature	Pressure
65K	17.4kPa
115K	1.939MPa

Ex. When nitrogen is the working fluid

The temperature range is limited on the lower end
by the triple point and also
by the difficulty in maintaining low vacuum pressure with large flow rates

If the pressure were lowered below the triple-point,
N_2 snow would form and clogging valve.
The temperature is limited on the high end by the critical point.

5.3 Joule-Thomson refrigeration system

- **History of Joule-Thomson refrigerator (1)**

- 1970-1971, Stephens and Buller

Utilizing thermostatical expansion valve

The self regulating J.T. refrigerator has the advantage of rapid cooling because the initial gas flow rate is much larger than that for a fixed-orifice refrigerator

The problem of solid-contaminants in the gas stream is almost reduced

Reliable operation at 34.5~69MPa are made possible with the use of the self-regulating orifice.

5.3 Joule-Thomson refrigeration system

- **History of Joule-Thomson refrigerator (2)**

- 1981, Chan et al.

Using an adsorption compressor

Large quantities of gas are adsorbed in an adsorbent,
then the adsorbent is heated in a closed system to produce high P

The gas passes through a J.T. refrigerator and
is adsorbed in a second adsorbent chamber, which is cooled.

A minimum COP (0.22) was reported for N_2 system adsorbing at 77K

Compressor temperature : Cold(150K), Hot(470K)

5.4 Refrigerator optimization

- To design a refrigeration system

→ Need to determine the effectiveness and cost of primary components!

① Compressor costs

$$C_c = C_1 \dot{W}_c = C_1 \, (\dot{W}_C / \dot{m}) \dot{m}$$

$$C_1 = \mathit{Compressor\ cost\ per\ unit\ power\ requirement,}$$
$$\mathit{including\ both\ operating\ and\ capital\ costs}$$

② Exchanger costs

$$C_E = C_2 A$$

③ The sum of the compressor and heat exchanger costs

$$C_T = C_c + C_E$$

5.4 Refrigerator optimization

- The optimum condition → Derivative of the total cost equal to zero

$$\frac{dC_T}{di} = 0 = C_1\left(\frac{\dot{W}_c}{\dot{m}}\right)\frac{d\dot{m}}{di} + C_2\frac{dA}{di}$$

$$i = 1 - \varepsilon = heat\ exchanger\ ineffectiveness$$

And, we can determine $\frac{d\dot{m}}{di}, \frac{dA}{di}, \emptyset$

$$\frac{d\dot{m}}{di} = \frac{\dot{Q}_a}{(h_1 - h_g)(H - i)^2}$$

$$\frac{dA}{di} = -\frac{(\dot{m}c_c/U)}{i(1 - C_R + C_R i)}$$

$$\emptyset = \frac{C_2(h_1 - h_g)(\dot{m}c_c/U)}{C_1(\dot{W}_c/\dot{m})\dot{Q}_a}$$

5.4 Refrigerator optimization

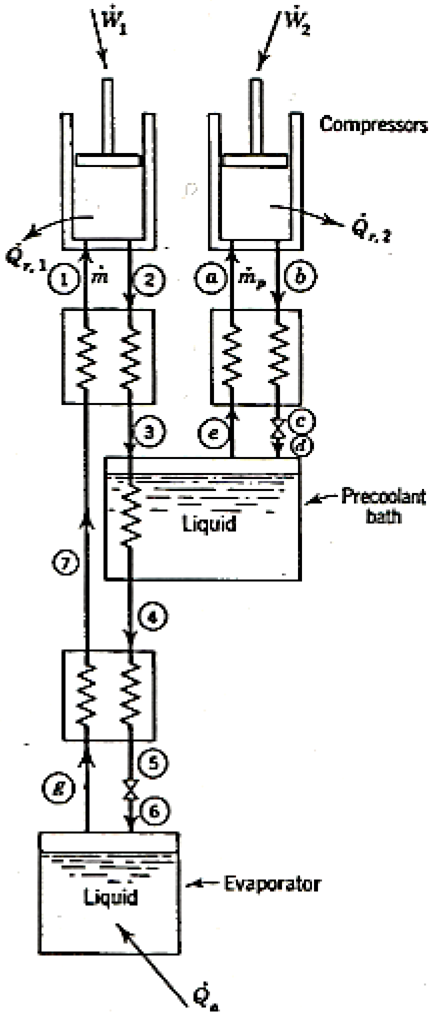
By using these equations, we can obtain the optimum ineffectiveness !

$$i_{opt} = \frac{C_R H^2}{2C_R H + 1 - C_R}$$

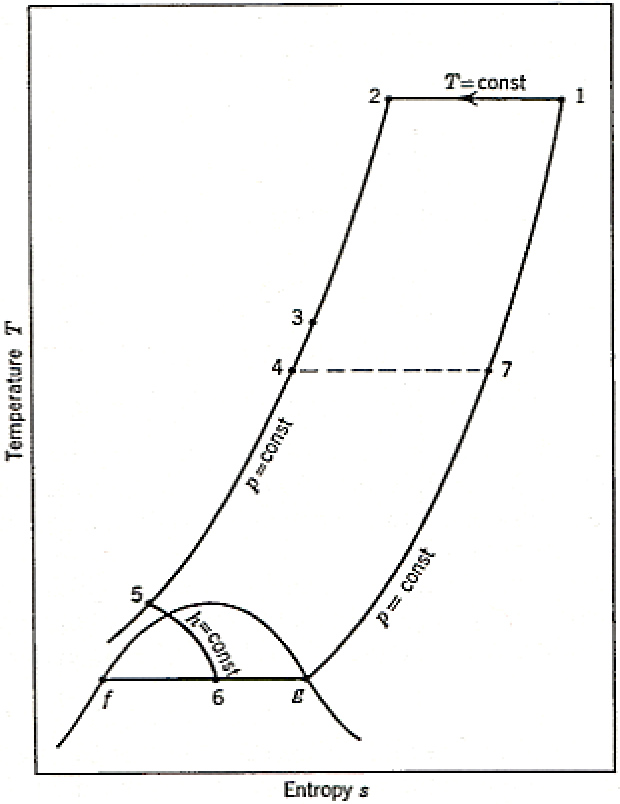
If, we are given the heat exchanger "free", that is, if $C_2 = 0$, we find that $\emptyset = 0$ and $B_2 = 0$.

So, $i_{opt} = 0$

5.5 Cascade or precooled Joule-Thomson refrigerators



<Precooled Linde-Hampson refrigerator>



<Thermodynamic cycle for the precooled Linde-Hampson refrigerator>

5.5 Cascade or precooled Joule-Thomson refrigerators

For temperatures lower than those obtainable with liquid nitrogen, the only available working fluids are neon, hydrogen, helium. For these fluids, precooled system is useful.

$$\dot{Q}_a = \dot{m}(h_1' - h_2) + \dot{m}_p(h_a' - h_b)$$

$$\dot{Q}_a/\dot{m} = (h_1' - h_2) + z(h_a' - h_b)$$
$$z = \dot{m}_p/\dot{m}$$

$$\varepsilon = \frac{h_1' - h_g}{h_1 - h_g}, \varepsilon_p = \frac{h_a' - h_e}{h_a - h_e}$$

$$\dot{Q}_a/\dot{m} = (h_1 - h_2) - (1 - \varepsilon)\left(h_1 - h_g\right) + z[(h_a - h_b) - \left(1 - \varepsilon_p\right)(h_a - h_e)]$$

5.5 Cascade or precooled Joule-Thomson refrigerators

Applying the First Law to the cold exchanger and the evaporator, we obtain

$$\dot{Q}_a/\dot{m} = h_7' - h_4$$

Introducing the effectiveness of the cold exchanger,

$$\varepsilon_c = \frac{h_7' - h_g}{h_7 - h_g}$$

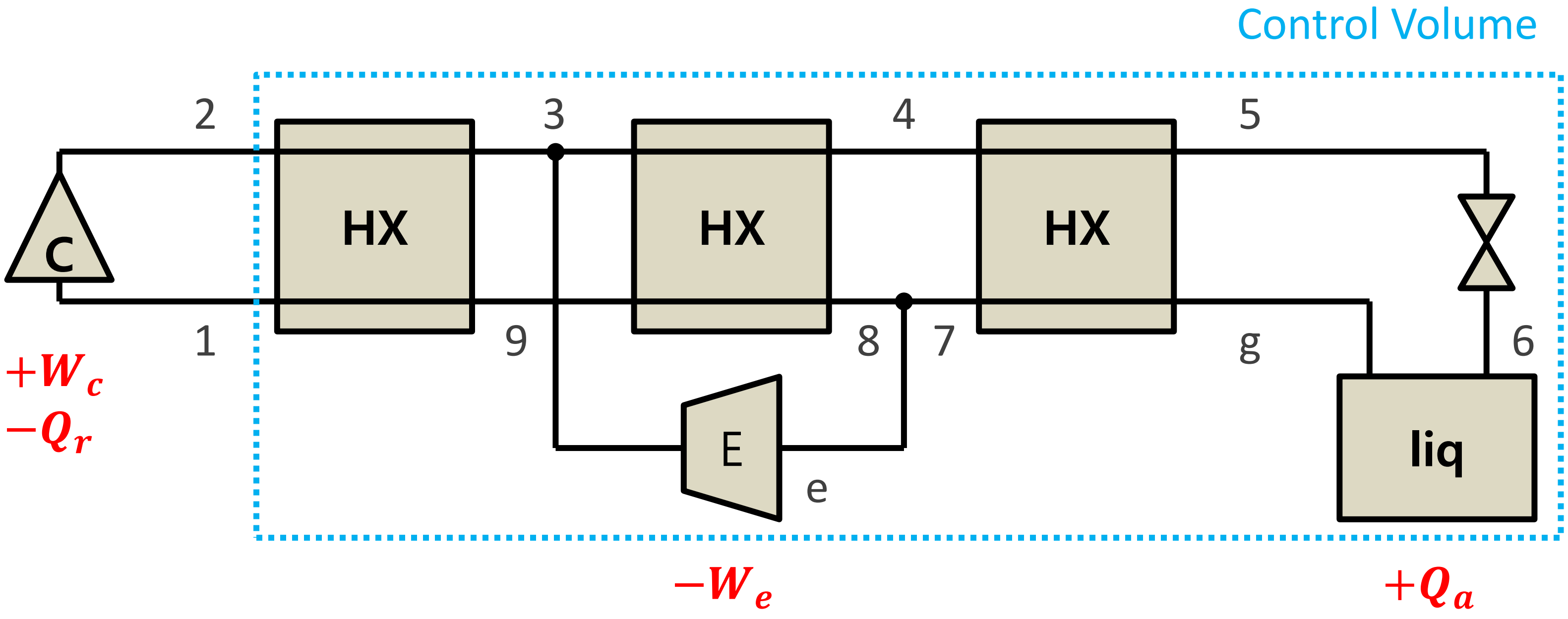
The refrigeration effect may be written as follows :

$$\dot{Q}_a/\dot{m} = (h_7 - h_4) - (1 - \varepsilon_c)(h_7 - h_g)$$

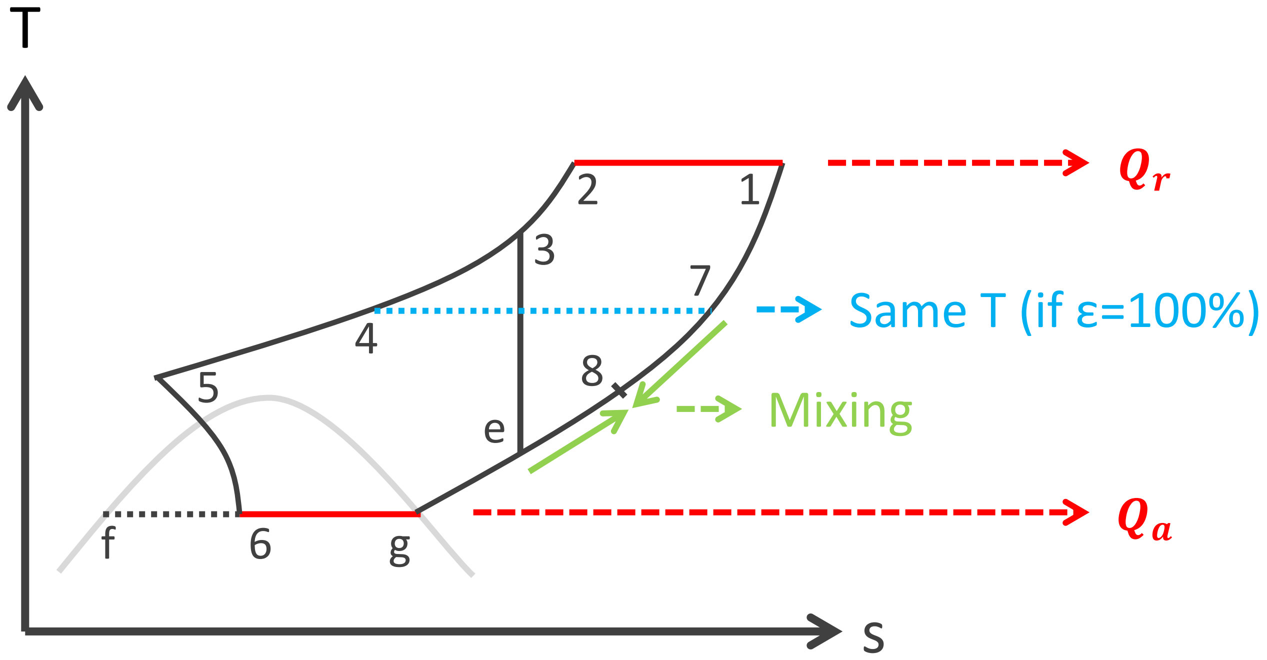
5.6 Expansion-engine refrigeration system

(Referring 3.9, Claude system(Collins system))

COP is very low, but it can make very low temperature



5.6 Expansion-engine refrigeration system



At control volume, the total energy absorbed from the source is given by

$$\frac{\dot{Q}_a}{\dot{m}} = (h_1' - h_2) + x(h_3 - h_e')$$

$$x = \frac{\dot{m}_e}{\dot{m}}$$

$$(h_3 - h_e') = \eta_{a.d}(h_3 - h_e)$$ ※ Adiabatic efficiency

5.6 Expansion-engine refrigeration system

The work requirement

$$-\frac{\dot{W}}{\dot{m}} = -\frac{\dot{W}_c}{\dot{m}} - x\frac{\dot{W}_e}{\dot{m}_e}$$

※ $\dot{m}h_1 + \dot{Q}_R = \dot{m}h_2 + \dot{W}_c$

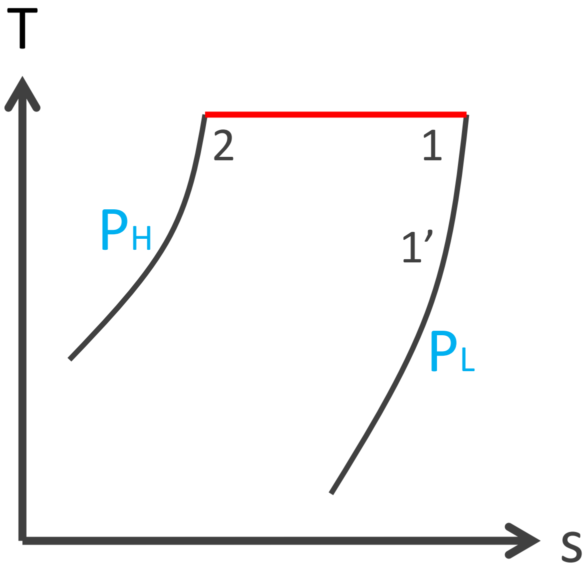
$$= \frac{[T_2(s_1 - s_2) - (h_1 - h_2)]}{\eta_{co}} - x\eta_{em}\eta_{ad}(h_3 - h_e)$$

※ overall efficiency of the compressor

※ mechanical efficiency of the expander

5.6 Expansion-engine refrigeration system

- **The heat exchanger effectiveness problem**



Case 1→2,

$$Q_r = T_1(s_2 - s_1) < 0$$

$$(h_1 + Q_r = h_2 + W_c),$$

$$W_c = T_{1(s_2 - s_1)} - (h_2 - h_1) < 0$$

Case 1’→2,

$$Q_r = \int_{1'}^{2} T ds$$ --> It can’t be caluculated!

$$W_c = \int_{1'}^{2} T ds - (h_2 - h_{1'})$$

5.6 Expansion-engine refrigeration system

- **History of Claude refrigerator (1)**

1966, Whitter
described the design utilizing two expansion engines
similar to the Collins liquefier

1968, Meier and Currie
described the performance used to maintain 4K
while providing 1 watt of refrigeration

5.6 Expansion-engine refrigeration system

- **History of Claude refrigerator (2)**

- Two significant modification

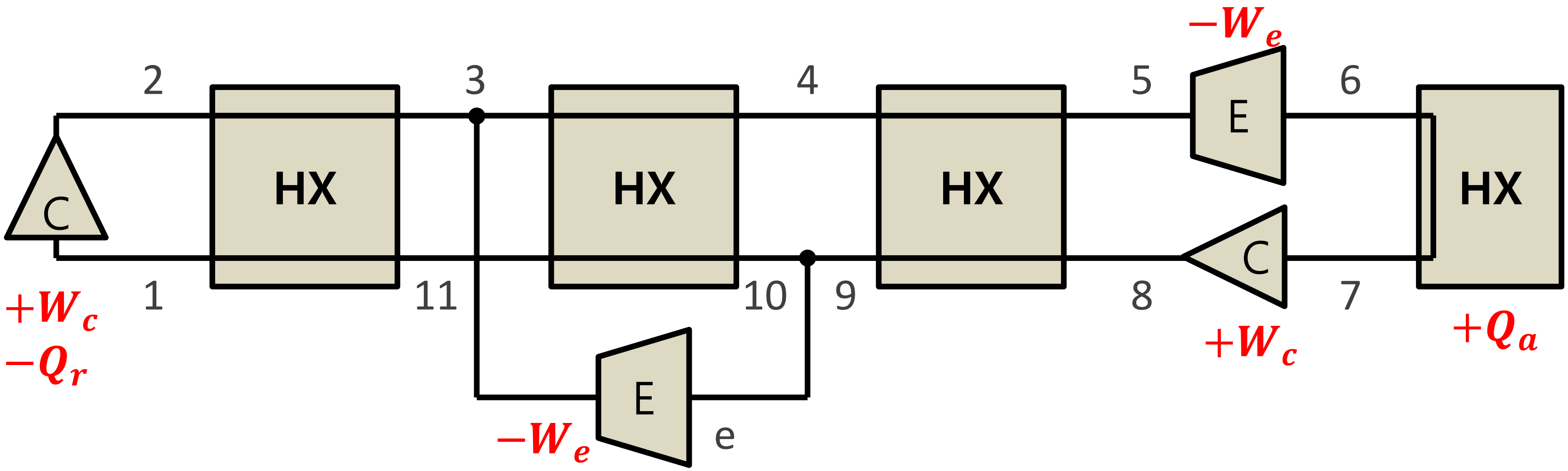
1971, Johnson et al.
using a “wet” expander
(operating in the two-phase region to replace the expansion valve)

1982, Minta and Smith
using a low temperature compressor

→ Figure at next page

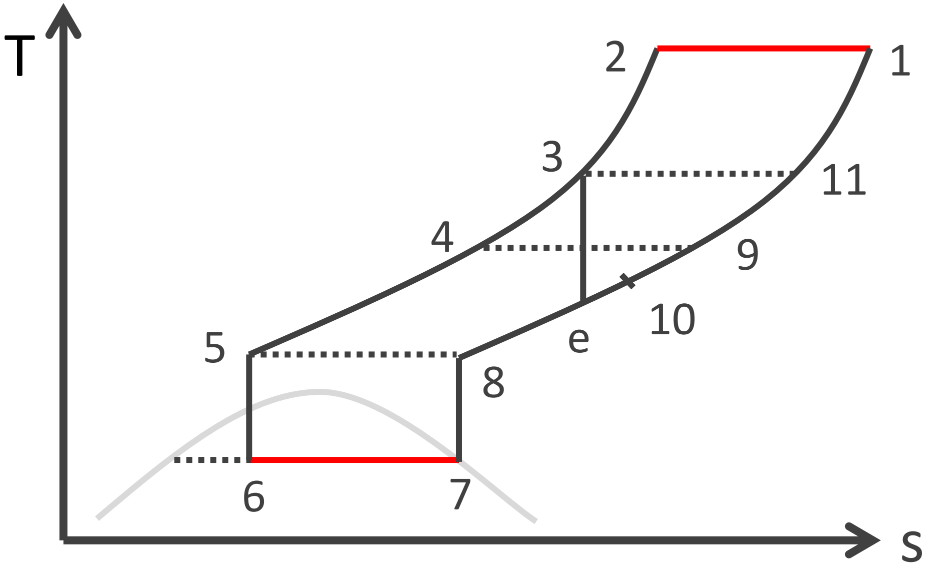
5.6 Expansion-engine refrigeration system

- Claude refrigerator with a wet expander and a saturated-vapor compressor



5.6 Expansion-engine refrigeration system

- **Claude refrigerator with a wet expander and a saturated-vapor compressor**



5.6 Expansion-engine refrigeration system

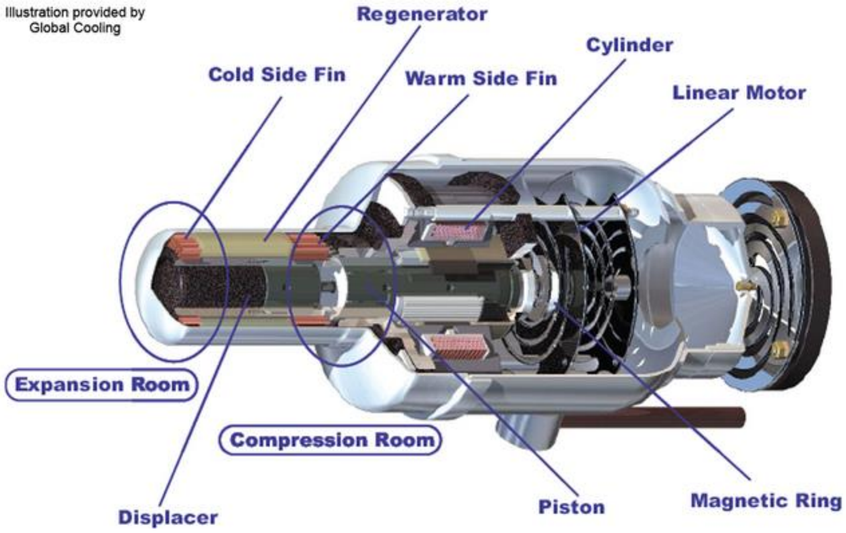
The wet(two-phase) expander is used primarily for helium system
because the thermal capacity of the compressed gas is
generally larger than the latent heat of the liquid phase

Unlike an air (or nitrogen) expansion engine
in which the engine efficiency is seriously affected
by the presence of liquid in the engine,
operation of the helium expander in the two-phase region
does not result in a serious deterioration of the engine performance.

This system required less heat exchanger surface
than the conventional Claude refrigerator
because the heat-transfer coefficients are higher
when the cold gas stream pressure is increased.

5.7 Philips refrigerator

- **Philips refrigerator?**

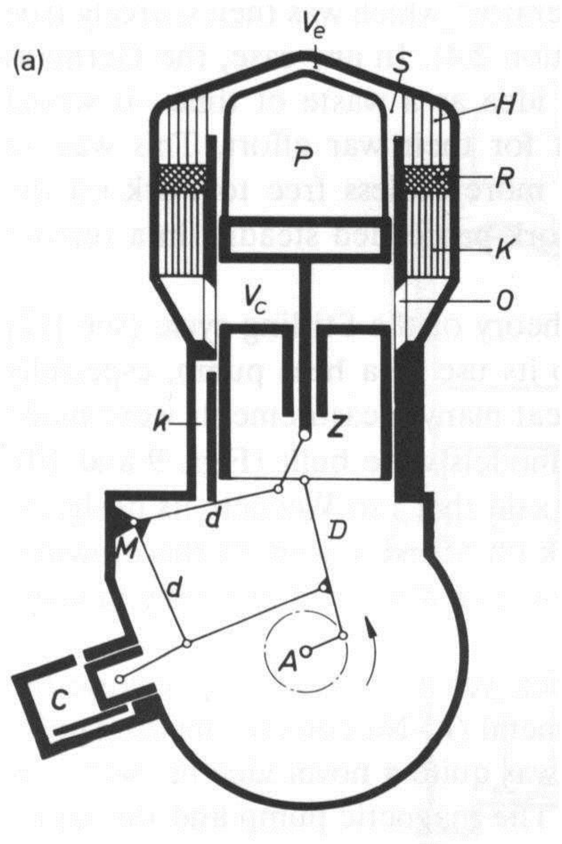


Philips refrigerator (www.apexinst.com/)

- An refrigeration device that operates on the reverse Stirling cycle, which was originally invented for use in a hot-air engine by a Scottish minister, Robert Stirling

5.7 Philips refrigerator

- **Stirling engine?**

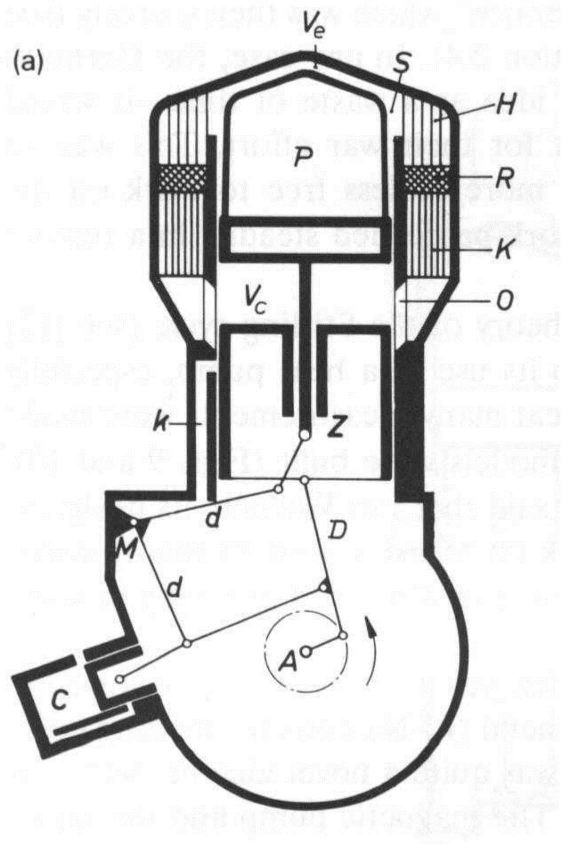


Stirling engine operates

- at high mean gas pressures (~200 bar)
- in an oil – free environment
- without valve gear

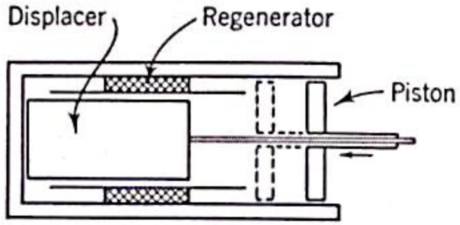
5.7 Philips refrigerator

- **Stirling engine?**

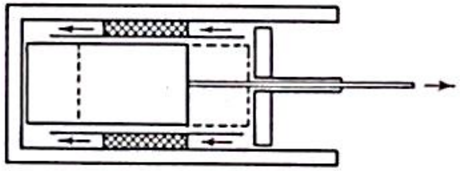


- Any source of thermal energy is possible
- Efficiency of Stirling engine is same as that of Carnot cycle in perfect regeneration
- It requires two different temperature levels

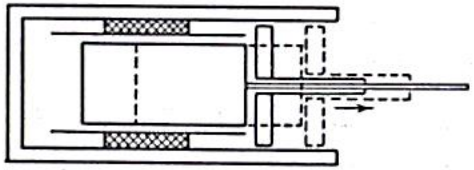
5.7 Philips refrigerator



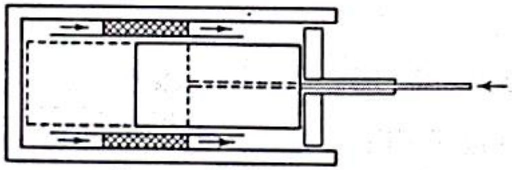
Process 1–2



Process 2–3



Process 3–4



Process 4–1

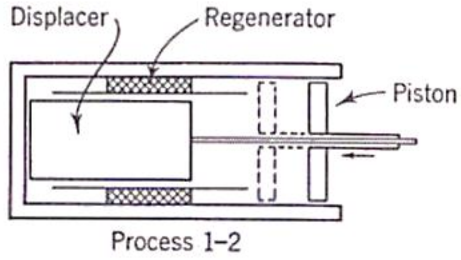
1 → 2 : Isothermal compression

2 → 3 : Const. volume cooling

3 → 4 : Isothermal expansion

4 → 1 : Const. volume heating

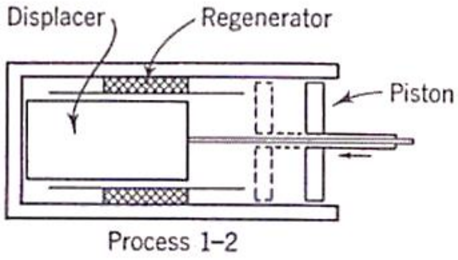
5.7 Philips refrigerator



Process 1 → 2 : Isothermal compression

The gas is compressed isothermally while rejecting heat to the high temperature sink (surroundings)

5.7 Philips refrigerator

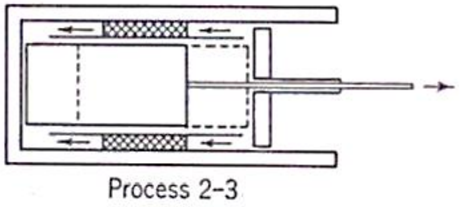


Process 1 → 2 : Isothermal compression

Heat rejected :

$$Q_r = mT_1(s_2 - s_1)$$

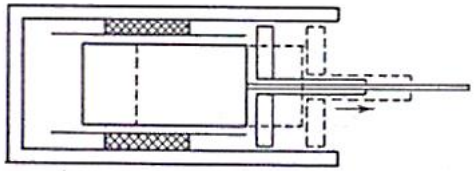
5.7 Philips refrigerator



Process 2 → 3 : Const. volume cooling

The gas is forced through the regenerator by the motion of the displacer. The gas is cooled at constant volume during this process. The energy removed from the gas is not transferred to the surroundings out is stored in the regenerator matrix

5.7 Philips refrigerator

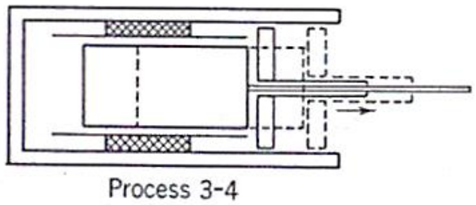


Process 3-4

Process 3 → 4 : Isothermal Expansion

The gas is expanded isothermally while absorbing heat rom from the low temperature source

5.7 Philips refrigerator

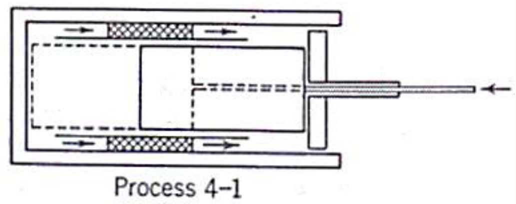

Process 3-4

Process 3 → 4 : Isothermal Expansion

Heat absorbed :

$$Q_a = mT_3(s_4 - s_3)$$

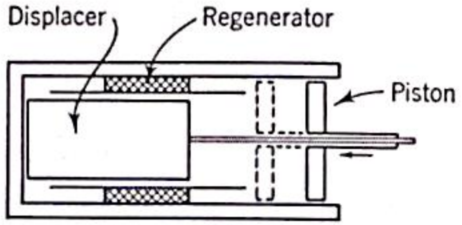
5.7 Philips refrigerator



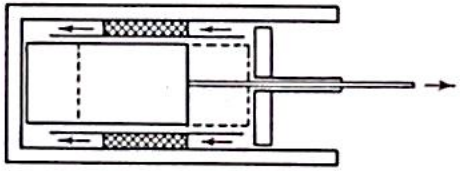
Process 4 → 1 : Const. volume heating

The cold gas is forced through the regenerator by the motion of the displacer; the gas is heated during this process. The energy stored during process 2-3 is transferred back to the gas. In the ideal case (no heat inleaks), heat is transferred to the refrigerator only during process 3-4, and heat is rejected from refrigerator only during process 1-2

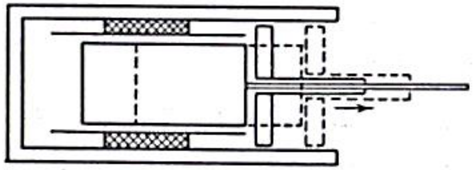
5.7 Philips refrigerator



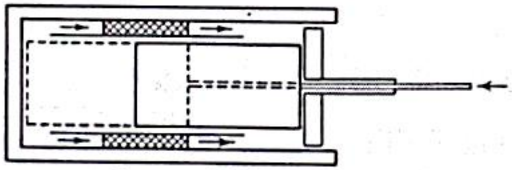
Process 1–2



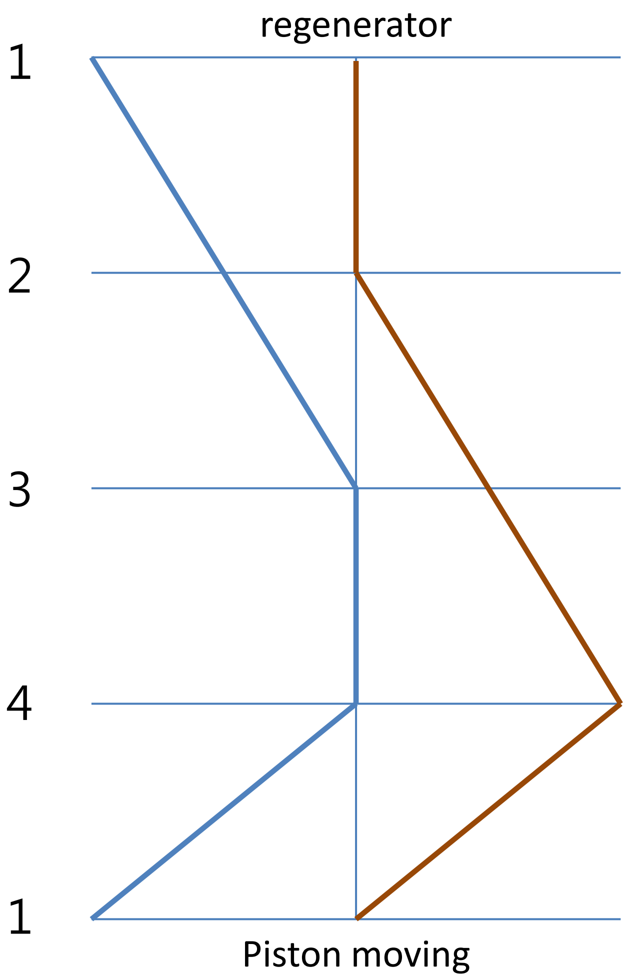
Process 2–3



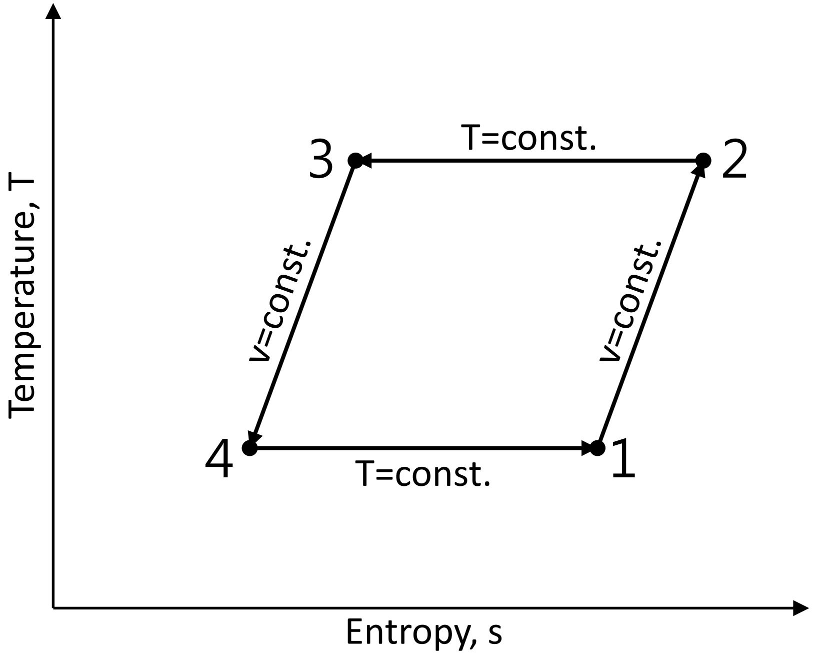
Process 3-4



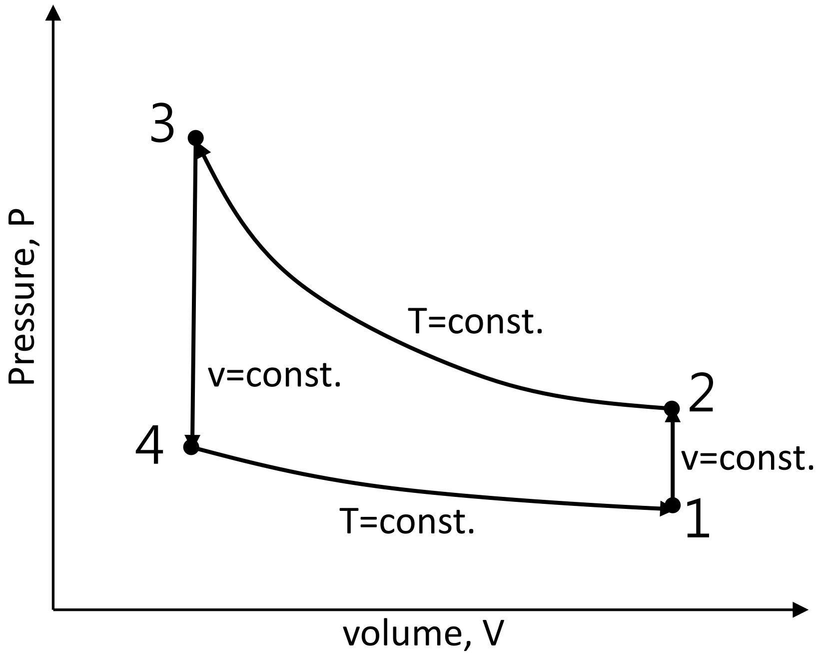
Process 4–1



5.7 Philips refrigerator



5.7 Philips refrigerator



5.7 Philips refrigerator

COP of the ideal Philips refrigerator

$$COP = \frac{-Q_a}{W_{net}} = \frac{-Q_a}{Q_a + Q_r} = \frac{1}{\frac{Q_r}{Q_a} - 1} = \frac{T_3}{\frac{T_1 s_1 - s_2}{(s_4 - s_3)} - T_3}$$

5.7 Philips refrigerator

If working fluid behaves as an ideal gas,

$$s_1 - s_2 = c_v \ln\left(\frac{T_1}{T_2}\right) + R\ ln\left(\frac{v_1}{v_2}\right) = R\ ln\left(\frac{v_1}{v_2}\right) = R\ ln\left(\frac{v_4}{v_3}\right)$$

$$= s_3 - s_4 \qquad (\because T_1 = T_4,\ T_3 = T_4,\ v_1 = v_4,\ v_3 = v_4)$$

5.7 Philips refrigerator

Finally, COP of an ideal Philips refrigerator with an deal gas as the refrigerant becomes

$$COP = \frac{T_3}{T_1 - T_3}$$

This is the same expression as that for the COP of a Carnot refrigerator

5.8 Importance of regenerator effectiveness for the Philips refrigerator

- The success of the Philips refrigerator

  → Large extent upon the effectiveness of the regenerator used in the system!

The actual energy absorbed from the low-temperature source becomes

$$Q_a = Q_{a,ideal} - \Delta Q$$

$$\varepsilon = \frac{Q_{actual}}{Q_{ideal}} = \frac{Q_{2-3,ideal} - \Delta Q}{Q_{2-3,ideal}}$$

$$\Delta Q = (1-\varepsilon)Q_{2-3,ideal} = (1-\varepsilon)mc_v(T_2 - T_3)$$

$Q_{2-3,ideal} = ideal\ heat\ transferred\ from\ the\ gas\ to\ the\ regenerator\ duting\ process\ 2-3$

$m = mass\ of\ gas\ flowing\ through\ the\ regenerator$

$c_v = specific\ heat\ of\ the\ gas\ flowing\ through\ the\ regenerator$

5.8 Importance of regenerator effectiveness for the Philips refrigerator

If the gas is ideal gas,

$$Q_{a,ideal} = mT_3(s_4 - s_3) = mRT_3\ln(v_4/v_3) = (\gamma - 1)mc_vT_3\ln(v_4/v_3)$$

$$\frac{\Delta Q}{Q_{a,ideal}} = \frac{1-\varepsilon}{\gamma - 1}\frac{T_2/T_3 - 1}{\ln(v_4/v_3)}$$

5.9. Vuilleumier refrigerator

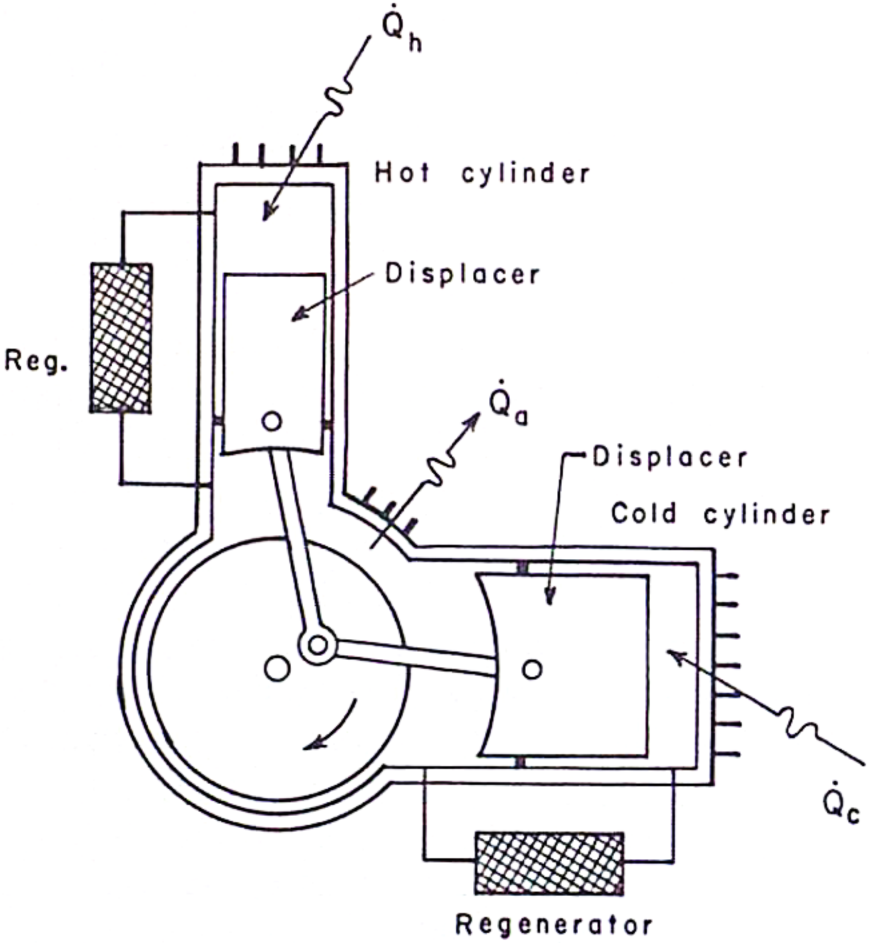


Fig. 5.16. Vuilleumier refrigerator schematic.

5.9. Vuilleumier refrigerator

- Vuilleumier Refrigerator (presented by Rudolph Vuilleumier in 1918)

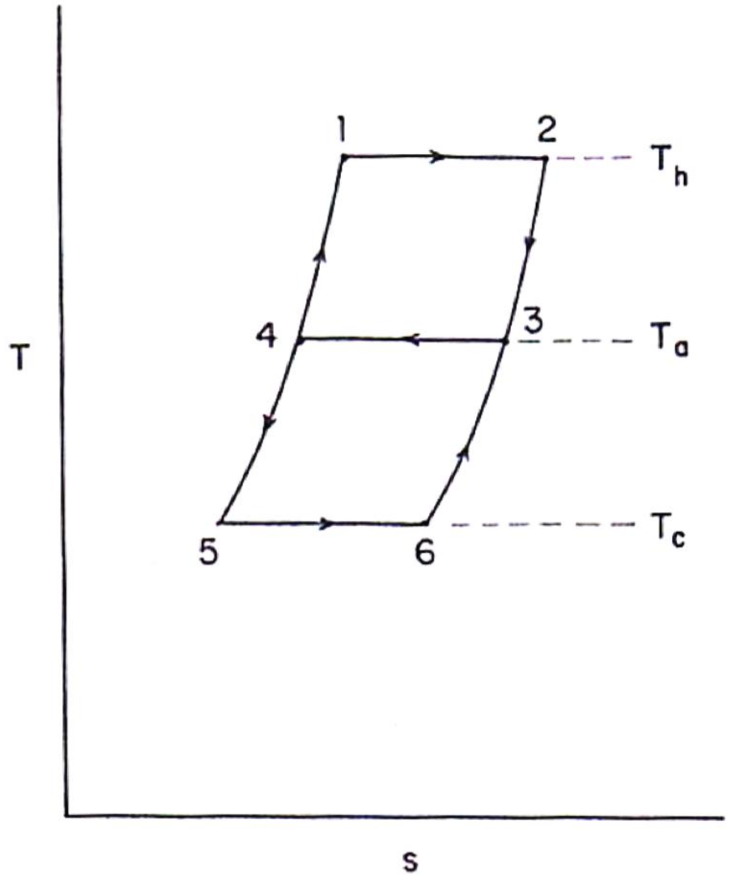
  (A variation of the VM device was also invented by Vannevar Bush (1938), and another version was patented by K. W. Taconis (1951).)

- Characteristics

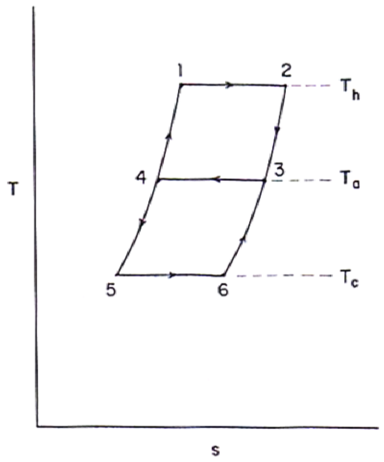
  - Thermal Compression
  - Constant Volume
  - No External Work

5.9. Vuilleumier refrigerator

Fig. 5.17. Thermodynamic cycle for the ideal Vuilleumier refrigerator.



5.9. Vuilleumier refrigerator



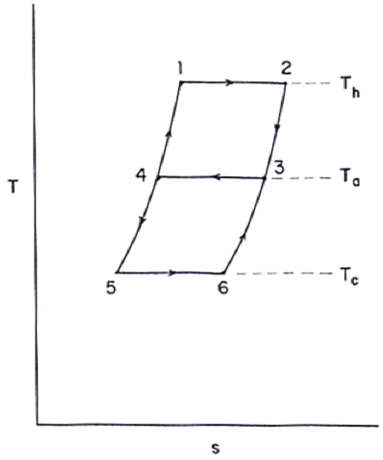
Process 1-2 : heat is added from a high-temperature source to the gas in the "hot" cylinder, displacer moves downward.

Process 4-1 : near-ambient temperature gas flows from the intermediate volume through the regenerator to the hot volume.

Process 2-3 : the displacer then moves upward and gas is displaced from the hot volume to the intermediate volume.

Process 3-4 : heat is rejected from the intermediate volume to maintain the temperature of the gas in the volume constant at T_a

5.9. Vuilleumier refrigerator



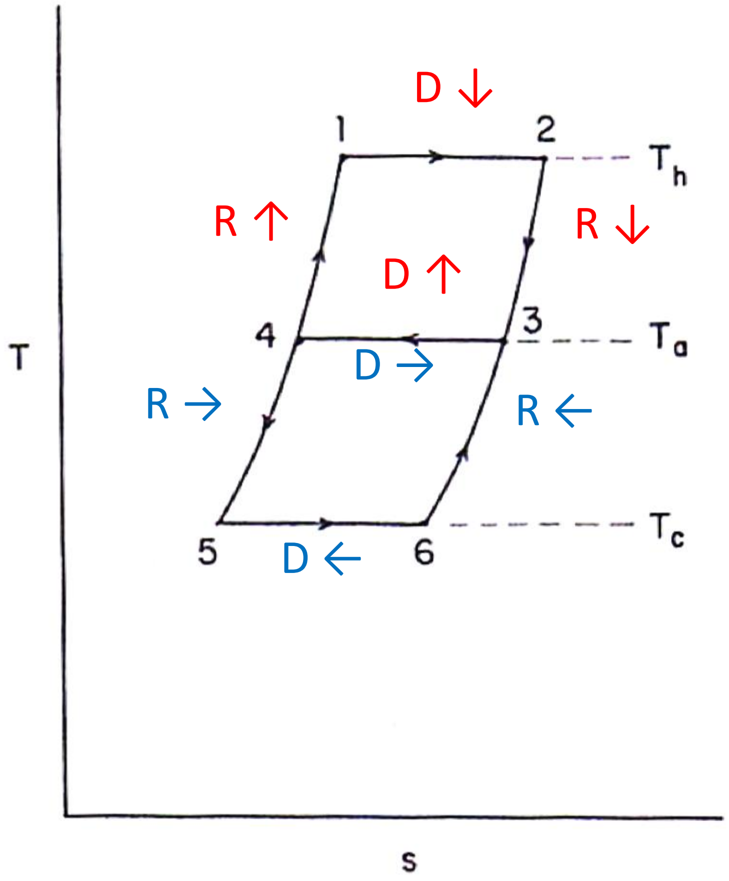
Process 5-6 : As the cold displacer is moved to the left, heat is absorbed by the gas in the cold volume from the low-temperature the same time, gas from the intermediate volume flows through the cold regenerator to the cold volume.

Process 4-5 : At the same time, gas from the intermediate volume flows through the cold regenerator to the cold volume.

Process 6-3 : the cold displacer then moves back to the right, and gas is displaced from the cold volume through the cold regenerator to the intermediate volume.

5.9. Vuilleumier refrigerator

Fig. 5.17. Thermodynamic cycle for the ideal Vuilleumier refrigerator.



R : Regenerator
D : Displacer

5.9. Vuilleumier refrigerator

$$Q_h = m_h T_h (s_2 - s_1) = m_h R T_h \ln\left(\frac{v_2}{v_1}\right)$$

$$Q_c = m_c T_c (s_6 - s_5) = m_c R T_c \ln\left(\frac{v_6}{v_5}\right)$$

$$Q_a = (m_h + m_c) T_a (s_4 - s_3) = -(m_h + m_c) R T_a \ln\left(\frac{v_3}{v_4}\right)$$

5.9. Vuilleumier refrigerator

$$v_2 = v_3 = v_6 \quad \text{and} \quad v_1 = v_4 = v_5$$

$$Q_h + Q_c + Q_a = 0$$

$$\rightarrow \quad m_h R T_h \ln\left(\frac{v_2}{v_1}\right) + m_c R T_c \ln\left(\frac{v_2}{v_1}\right) - (m_h + m_c) R T_a \ln\left(\frac{v_2}{v_1}\right) = 0$$

$$\rightarrow \quad \frac{m_c}{m_h} = \frac{T_h - T_a}{T_a - T_c}$$

5.9. Vuilleumier refrigerator

Definition of COP of this system

$$COP = \frac{Q_c}{Q_h} = \frac{m_c T_c}{m_h T_h}$$

$$\rightarrow \quad COP = \frac{T_c(T_h - T_a)}{T_h(T_a - T_c)}$$

5.9. Vuilleumier refrigerator

- Too Simplified

  - no harmonic motion
  - regenerator efficiency
  - other losses – shuttle heat transfer

- Advantage

  - heating only (solar, geothermal)
  - no driving engine

5.10. Solvay refrigerator

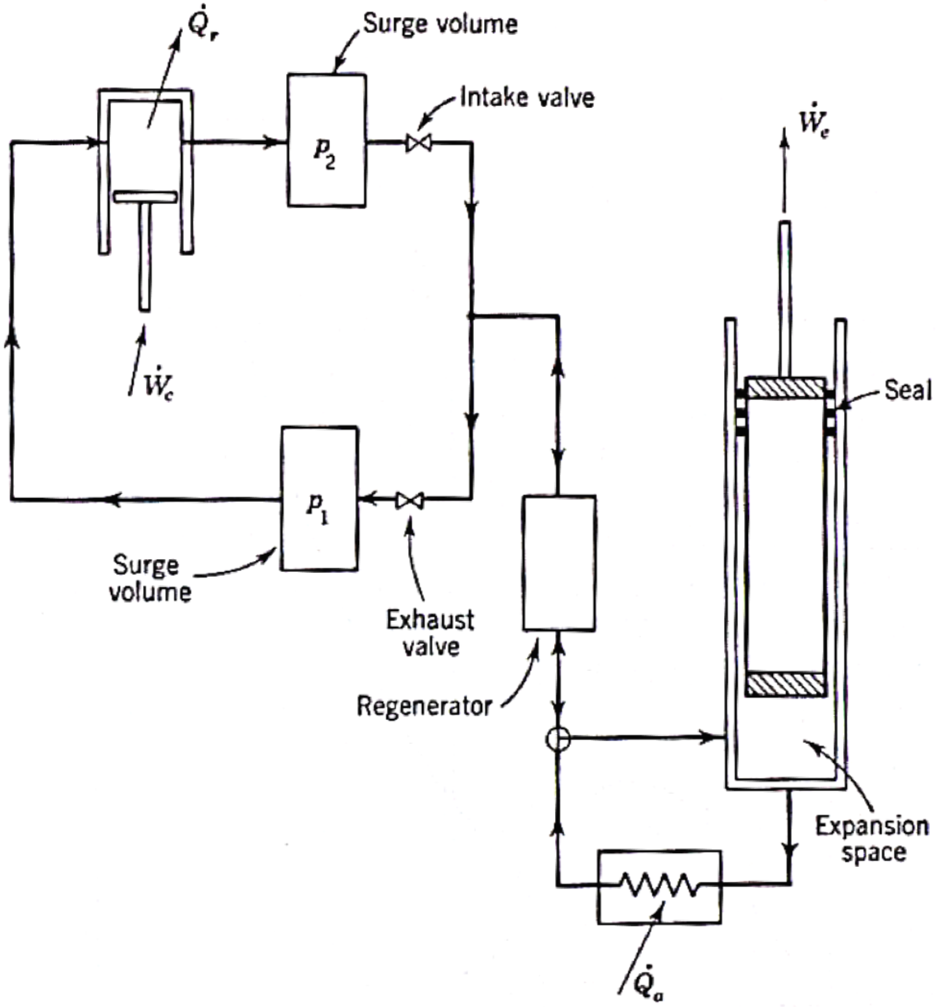


Fig. 5.18. Solvay refrigerator schematic.

5.10. Solvay refrigerator

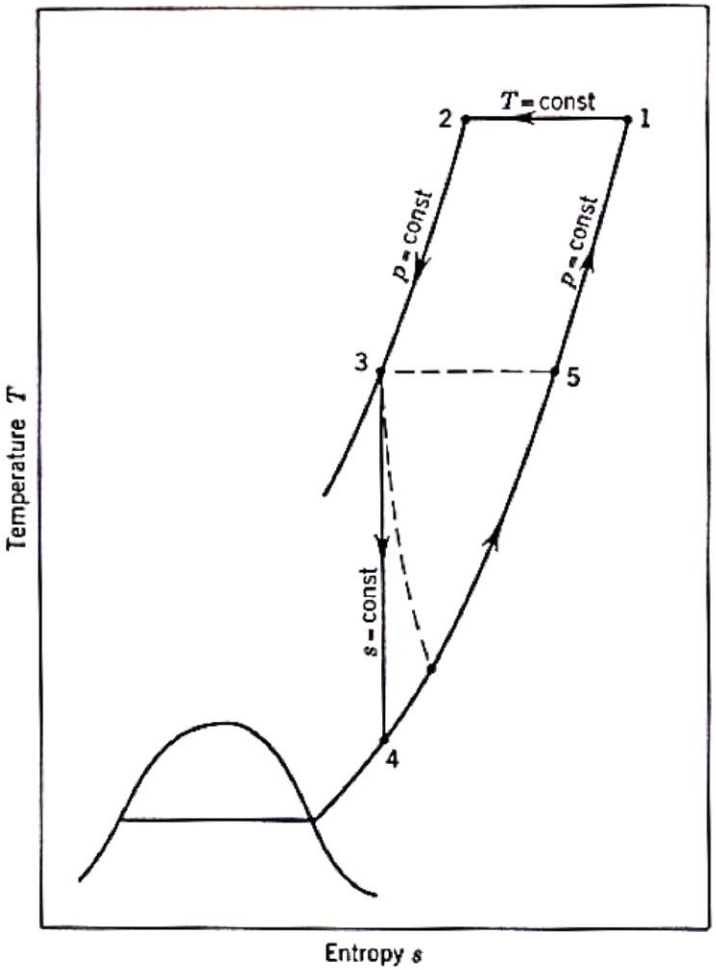
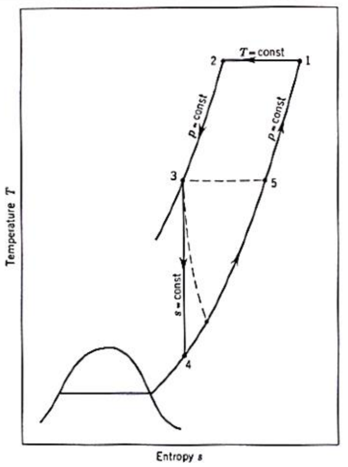


Fig. 5.19. Path traced out by a unit mass of gas on the T-s plane for the Solvay refrigerator.

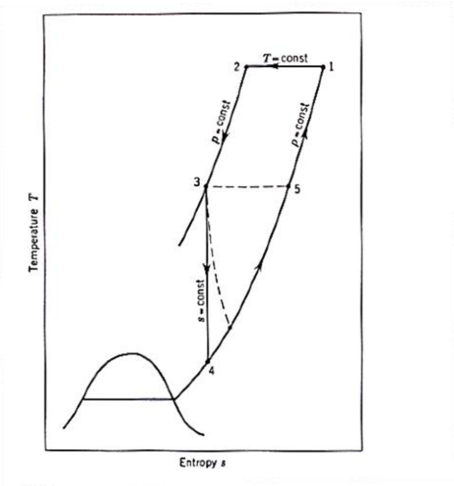
5.10. Solvay refrigerator



Process 1-2 : with the piston at the bottom of its stroke, the inlet valve is opened. The high-pressure gas flows into the regenerator, in which the gas is cooled, and the system pressure is increased from a low pressure P_1 to a higher pressure P_2

Process 2-3 : with the inlet valve still open, the piston is raised to draw a volume of gas into the cylinder. The gas has been cooled during its flow through the regenerator.

5.10. Solvay refrigerator



Process 3-4 : the inlet valve is closed, and the gas within the cylinder is expanded (isentropically in the ideal case) to the initial pressure P_1. As the gas expands, it does work on the piston, and energy is removed from the gas as work. The temperature of the gas therefore decreases.

Process 4-5 : the exhaust valve is opened, and the piston is lowered to force the cold gas out of the cylinder. During this process, the cold gas passes through a heat exchanger to remove heat from the region to be cooled.

Process 5-1 : the gas finally passes out through the regenerator, in which the cold gas is warmed back to room temperature.

5.10. Solvay refrigerator

	$\dot{W}_c/\dot{W}_e$	$\dot{Q}_r/\dot{Q}_a$	**Intake valve**	**Exhaust valve**	**Regenerator**	**Expansion space**
1-2	$\dot{W}_c$ (-) (consuming)	Release	Open	Close	No flow	Bottom
2-3			Open	Close	P_2 in (cooling)	Bottom
3-4	$\dot{W}_e$ (+)		Close	Close	No flow	Raise $P_2 \rightarrow P_1$
4-5		absorb	Close	Open		Down
5-1			Close	Open	P_1 in (heating)	

5.10. Solvay refrigerator

- Net work requirement

$$-\frac{\dot{W}_{net}}{\dot{m}} = \frac{T_2(s_1 - s_2) - (h_1 - h_2)}{\eta_{c,o}} - \eta_{e,m}\eta_{ad}(h_3 - h_4)$$

5.10. Solvay refrigerator

- Energy removed from low T source (α_a)

$$\eta_{ad} = \frac{h_3 - h_4'}{h_3 - h_4} = 1 - \frac{h_4' - h_4}{h_3 - h_4} \rightarrow \; h_4' - h_4 = -(1 - \eta_{ad})(h_3 - h_4)$$

$$\rightarrow \; \frac{Q_a}{m} = h_5 - h_4' = h_5 - h_4 - (1 - \eta_{ad})(h_3 - h_4)$$

5.11 Gifford– McMahon refrigerator (1960)

Gifford-McMahon refrigerator

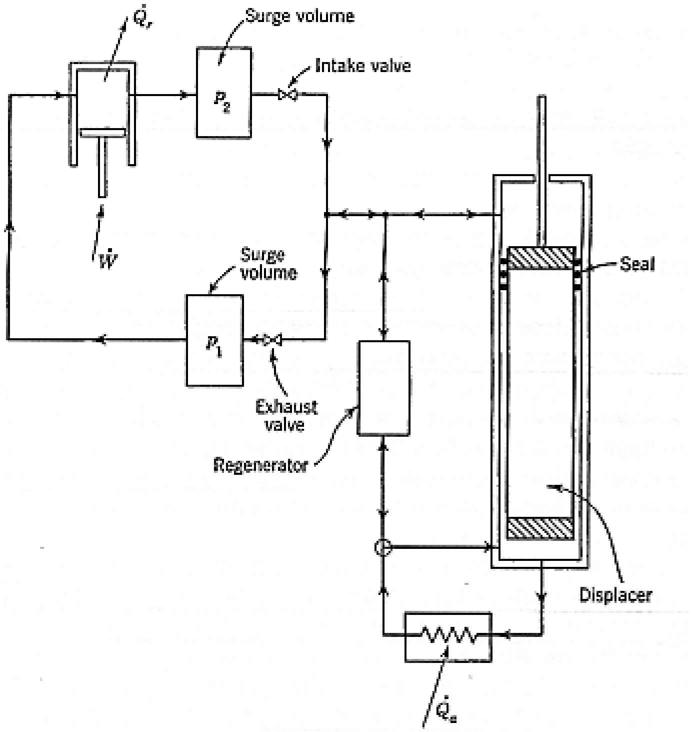
Gifford-McMahon (GM) coolers have found widespread application in many low-temperature systems; e.g., in MRI and cryopumps.

Gifford- McMahon refrigerator (www.cryomech.com)

5.11 Gifford– McMahon refrigerator (1960)

Gifford-McMahon refrigerator

This system consists of a compressor, a cylinder closed at both ends, a displacer within the cylinder, and a regenerator.

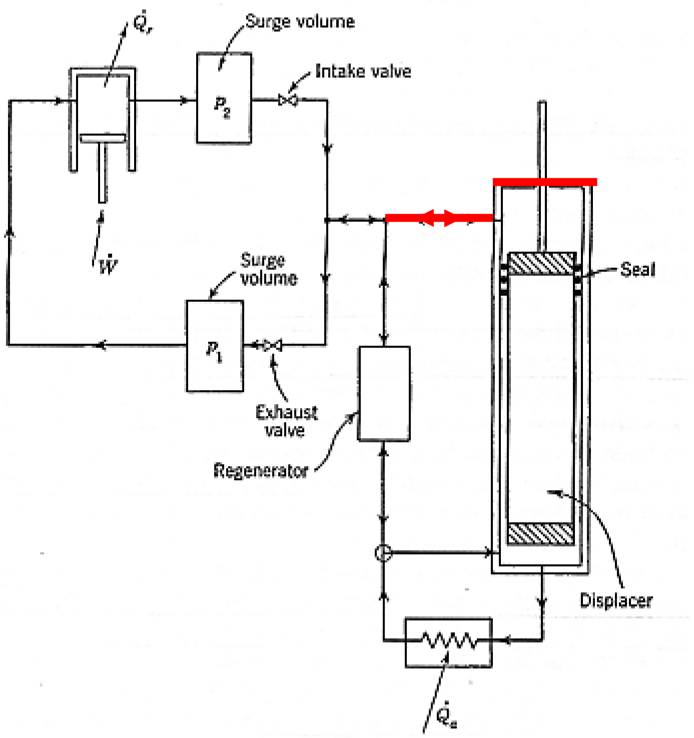


Gifford- McMahon refrigerator schematic

5.11 Gifford– McMahon refrigerator (1960)

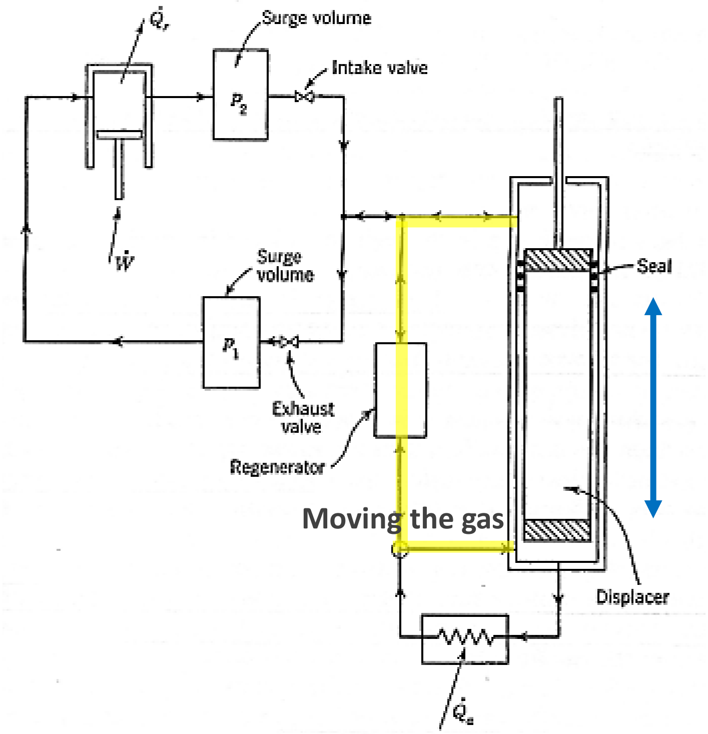
- What’s different from Solvay refrigerator?

- No work is transferred from the system during the expansion process.



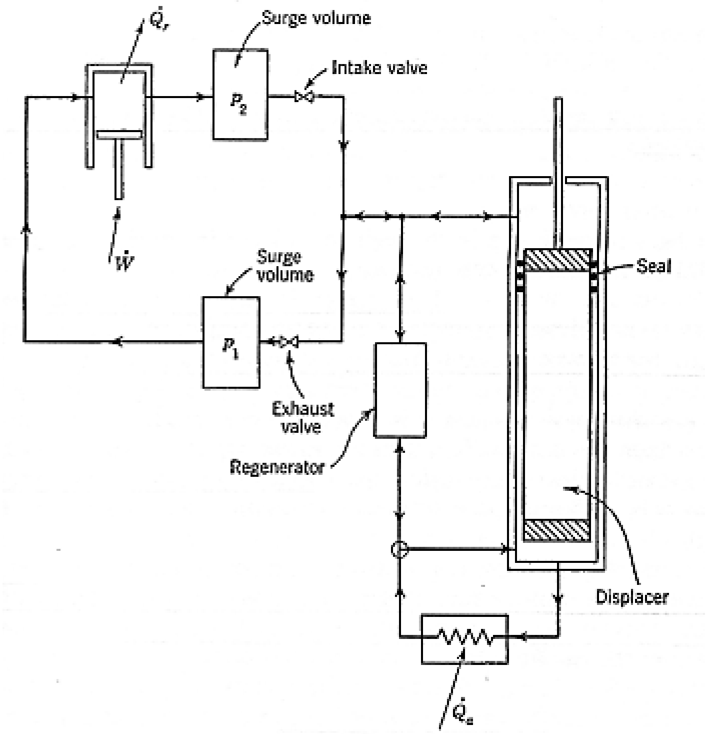
Gifford- McMahon refrigerator schematic

5.11 Gifford– McMahon refrigerator (1960)

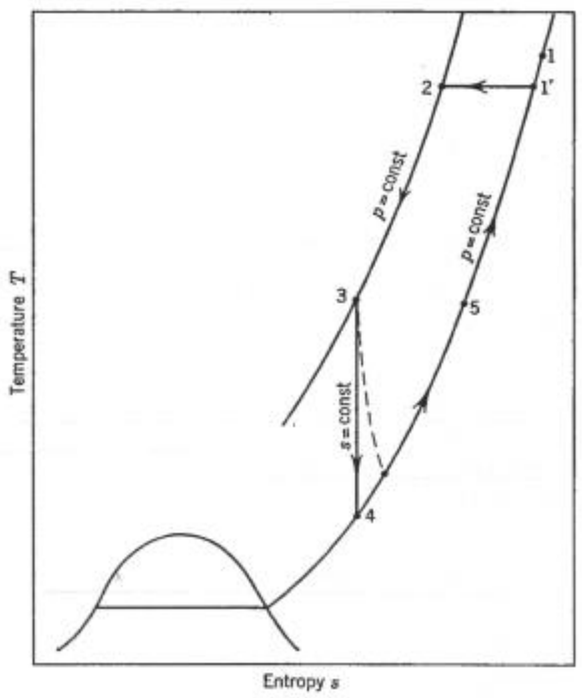


The displacer serves the purpose of moving the gas from one expansion space to another and would do zero net work in the ideal case of zero pressure drop in the regenerator.

5.11 Gifford– McMahon refrigerator (1960)

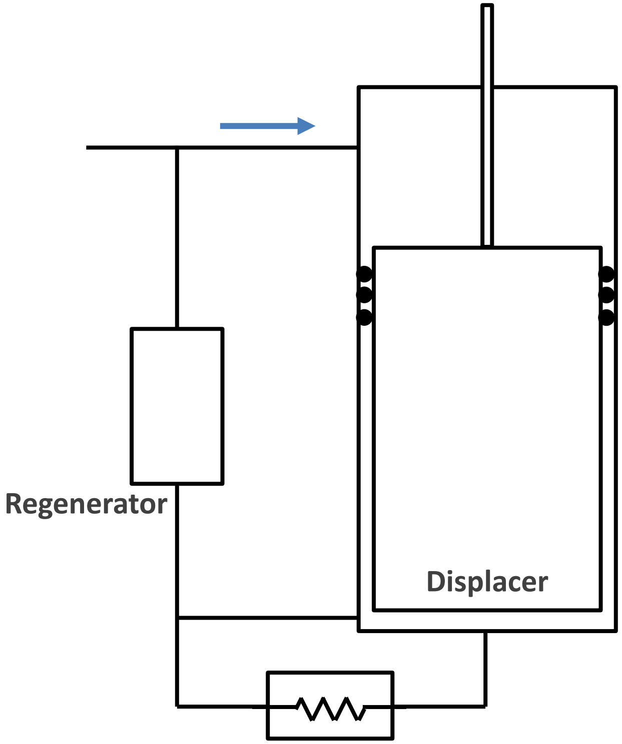


Gifford- McMahon refrigerator schematic



Path traced out by a unit mass of gas on the T-s plane for the Gifford-McMahon refrigerato

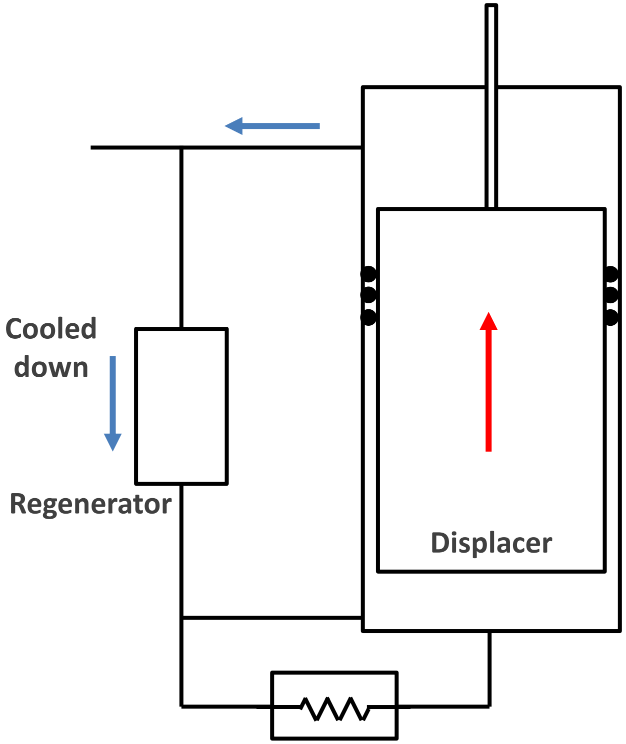
5.11 Gifford– McMahon refrigerator (1960)



Process 1 → 2

- The displacer at the bottom of the cylinder.
- The inlet valve is opened.
- The exhaust valve closed.
- The upper expansion space is increased from a low pressure P1 to a higher pressure P2.

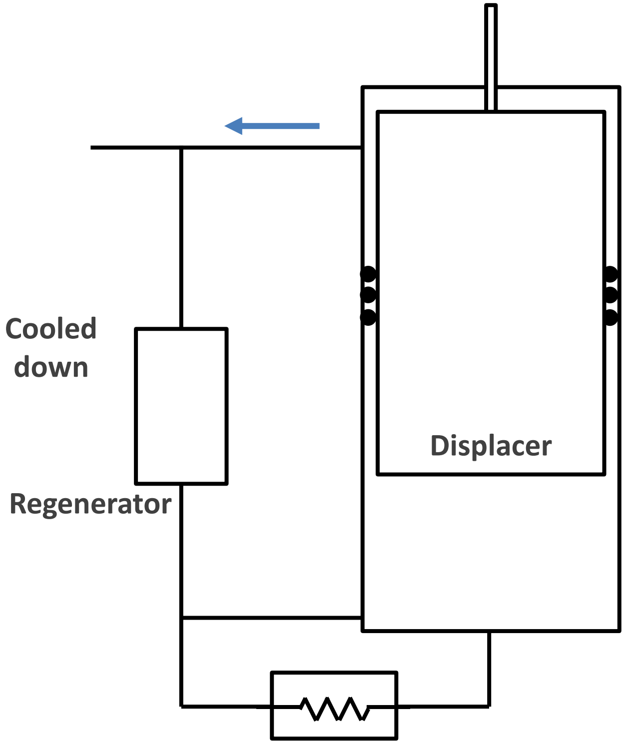
5.11 Gifford– McMahon refrigerator (1960)



Process 2 → 3

- The displacer is moved to the top of the cylinder
- The inlet valve is still opened.
- The exhaust valve closed.
- The gas that was in the upper expansion space down through the regenerator to the lower expansion space.
- The gas is cooled as it passes through the regenerator.

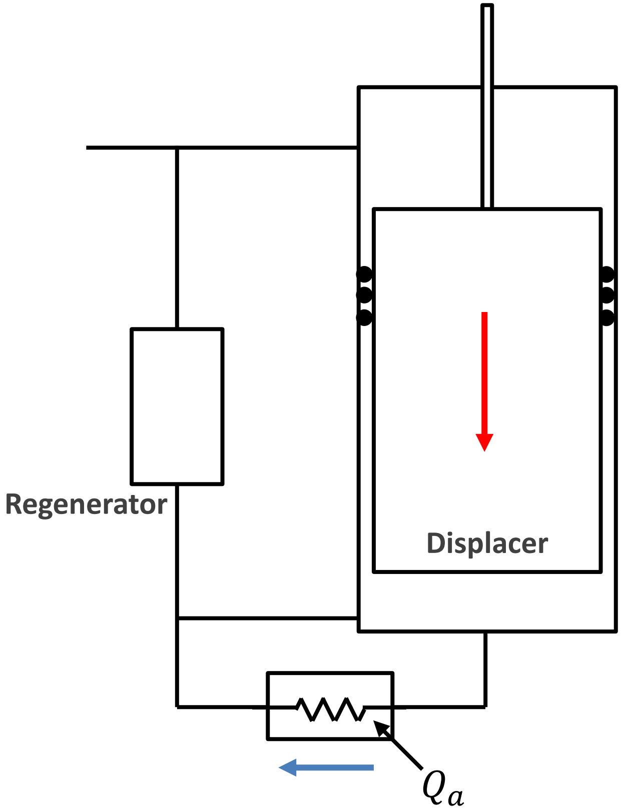
5.11 Gifford– McMahon refrigerator (1960)



Process 3 → 4

- The displacer at the top of the cylinder
- The inlet valve is closed.
- The exhaust valve is opened.
- The gas within the lower expansion space expands to the initial pressure P1.
- The gas within the lower expansion space drops to a low temperature.

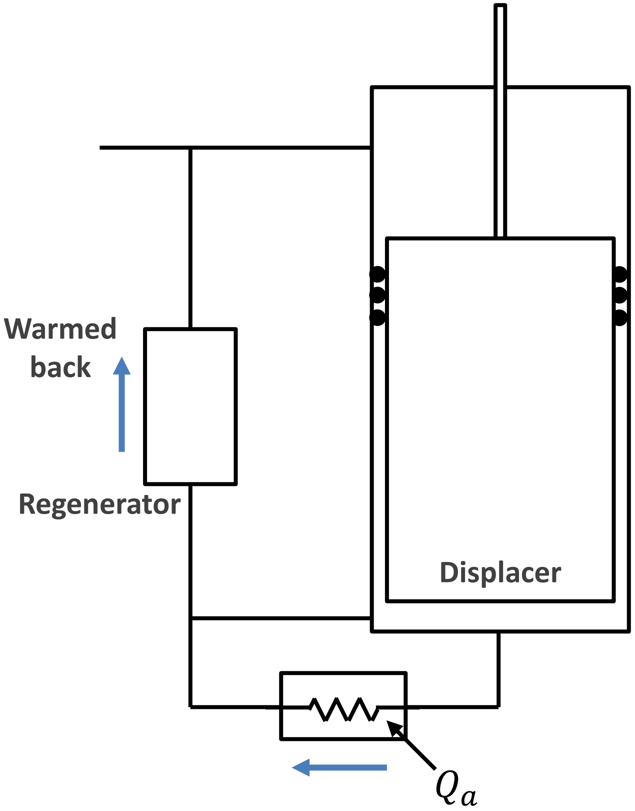
5.11 Gifford– McMahon refrigerator (1960)



Process 4 → 5

- The displacer is moved to the bottom of the cylinder
- The inlet valve is closed.
- The exhaust valve is opened.
- The cold gas flows through a heat exchanger in which heat is transferred.

5.11 Gifford– McMahon refrigerator (1960)



Process 4 → 5

- The cold gas flows through the regenerator, in which the bas is warmed back.

5.11 Gifford– McMahon refrigerator (1960)

Process	Compressor	Intake	Exhaust	Displacer	Upper expansion space	Lower expansion space	Regenerator
1-2	↑	O	C	Bottom	P1 -> P2	P1 -> P2	
2-3		O	C	↑	← (Gas flow direction)		Gas is cooled, V decreased -> Extra supply of hot gas
3-4		C	O	Top	P2 -> P1	P2 -> P1	
4-5		C	O	↓	→		Gas is warmed, V increased -> Gas is exhausted
5-1							

5.11 Gifford– McMahon refrigerator (1960)

- The net work requirement for this system

$$-\frac{\mathrm{W}}{\mathrm{m}}=\frac{[\mathrm{T}_1(\mathrm{s}_1-\mathrm{s}_2)-(\mathrm{h}_1-\mathrm{h}_2)]}{\eta_{c,o}}$$

5.11 Gifford– McMahon refrigerator (1960)

- The energy removed from the low-temperature source

$$-Q_a/\mathrm{m}=\left(\frac{m_e}{m}\right)\left(\mathrm{h}_5-\acute{h}_4\right)=\eta_{ad}\left(\frac{m_e}{m}\right)\left(\mathrm{h}_5-h_4\right)$$

m_e is the mass of gas within the lower expansion space
at the end of the expansion precess 3–4
m is the total mass of gas compressed

5.11 Gifford– McMahon refrigerator (1960)

Because the volume of the expansion space remains constant during the expansion process,

$$\frac{m_e}{m} = \rho'_4/\rho_3$$

5.11 Gifford– McMahon refrigerator (1960)

- Advantage of Solvay and Gifford-McMahon refrigerator

- The engine valves and displacer piston seals are at room temperature; therefore, low-temperature sealing problems are eliminated.

** Solvay

- COP of Solvay system is inherently higher than that of the Gifford-McMahon system.
- Expanding gas moves the displacer

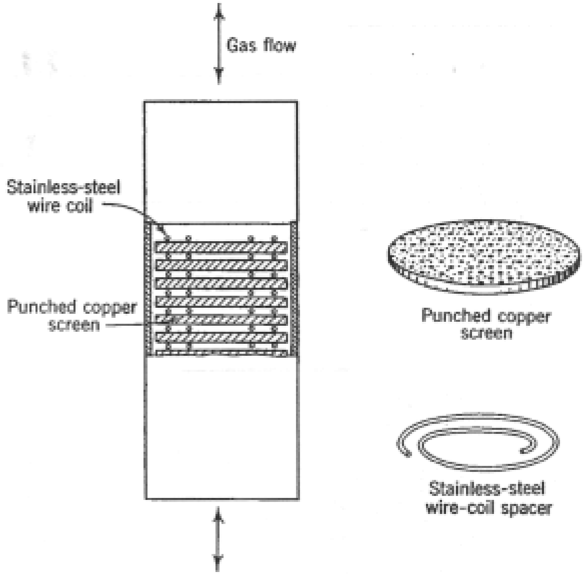
** Gifford-McMahon

- No leakage past the displacer
- Displace movement; simple and little force

5.11 Gifford– McMahon refrigerator (1960)

- Factors that contribute to a loss in performance of the Gifford-McMahon refrigerator

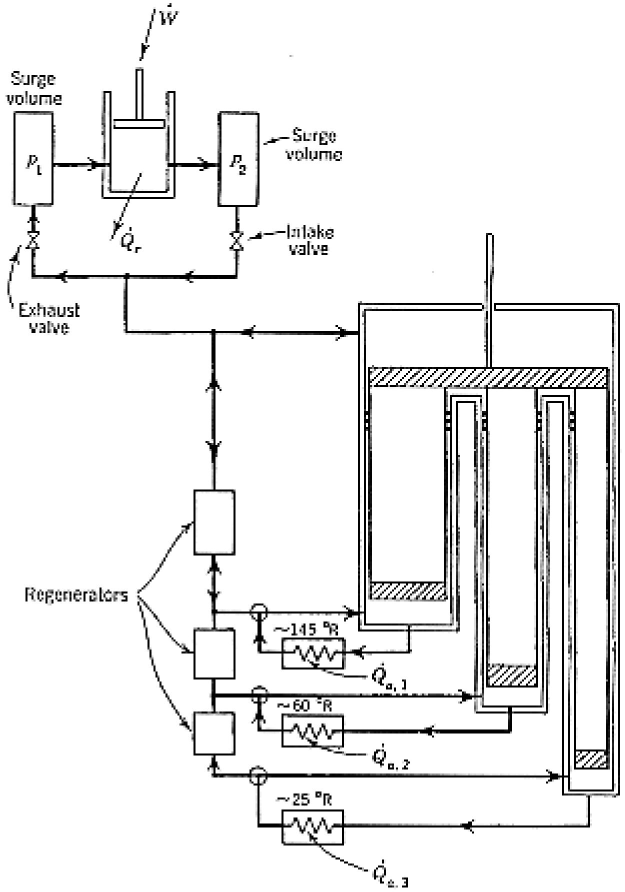
- Regenerator efficiency should be high > 98%
- Thermal conduction down the displacer and housing
- Shuttle heat transfer
- Finite volume within the regenerator



Regenerator schematic. The stainless-steel wire spacer is used to reduce the longitudinal conduction heat transfer within the matrix

5.11 Gifford– McMahon refrigerator (1960)

- Multistage Gifford-McMahon refrigerator



Three-stage Gifford-McMahon refrigerator

5.12 Regenerators

The regenerator is a critical component in refrigeration systems !

If we apply the First law to a differential element of gas flowing through the regenerator,

$$h(T_s - T)(A/L)dx - \dot{m}c_p \frac{\partial T}{\partial x} dx = \rho(V_g/L)c_p \frac{\partial T}{\partial \tau} dx$$

$h = heat - transfer\ coefficient\ between\ the\ flowing\ gas\ and\ solid\ material\ within\ the\ regenerator$

$T = gas\ temperature\ at\ location\ x\ and\ time\ \tau\ within\ regenerator$

$T_s = temperature\ of\ matrix\ at\ location\ x\ and\ time\ \tau$

$A = heat - transfer\ surface\ area\ of\ matrix$

$L = length\ of\ regenerator$

$\dot{m} = mass\ flow\ rate\ of\ gas\ through\ the\ regenerator$

$c_p = specific\ heat\ of\ the\ gas\ flowing\ through\ the\ regenerator$

$\rho = density\ of\ the\ gas$

$V_g = void\ volume\ or\ gas\ volume\ within\ the\ regenerator$

5.12 Regenerators

Rearrange the equation,

$$T - T_s = -\frac{\dot{m}c_pL}{hA}\frac{\partial T}{\partial x} - \frac{\rho c_p V_g}{hA}\frac{\partial T}{\partial \tau}$$

Apply the First law to a differential element of solid material within the regenerator,

$$h(T - T_s)\left(\frac{A}{L}\right)dx = (\frac{m_s}{L})c_s\frac{\partial T_s}{\partial \tau}dx$$

It can be written in the form

$$T - T_s = \frac{m_s c_s}{hA}\frac{\partial T_s}{\partial \tau}$$

5.12 Regenerators

Apply the dimensionless variables:

$$\xi = x/L$$

$$\eta = (\tau/P) - (\rho V_g/\dot{m}LP)x = [\tau - (x/L)\tau_d]/P$$

$$P = heating\ or\ cooling\ period\ for\ the\ regenerator$$

$$\tau_d = dwell\ time(fluid\ particle\ remain\ time\ in\ the\ regenerator$$

Use these relations and rearrange the equations,

$$T - T_s = -\frac{\dot{m}c_p}{hA}\frac{\partial T}{\partial \xi}$$

$$T - T_s = \frac{m_s c_s}{hAP}\frac{\partial T_s}{\partial \eta} = (\frac{\dot{m}c_p}{hA})(\frac{m_s c_s}{\dot{m}c_p P})\frac{\partial T_s}{\partial \eta}$$

5.12 Regenerators

- The effectiveness of a counterflow regenerator by Coppage and London(1953)

$$N_{tu,0} = \frac{1}{C_{min}}[\frac{1}{h_h A_h} + \frac{1}{h_c A_c}]^{-1}$$

$$C_R = C_{min}/C_{max} = (\dot{m}c_p)_{min}/(\dot{m}c_p)_{max}$$

Matrix capacity

$$C_m = m_s c_s / P_0 C_{min}$$

5.12 Regenerators

- The regenerator effectiveness is defined by

$$\varepsilon = \dot{Q}/C_{min}(T_{h1} - T_{c1})$$

$$\dot{Q} = average\ heat - transfer\ rate$$
$$T_{h1} = inlet\ temperature\ of\ the\ hot\ fluid\ steams$$
$$T_{c1} = inlet\ temperature\ of\ the\ cold\ fluid\ streams$$

The regenerator effectiveness

$$\varepsilon = \frac{1 - \exp(-X)}{1 - C_R \exp(-X)}$$

5.13 Magnetic cooling

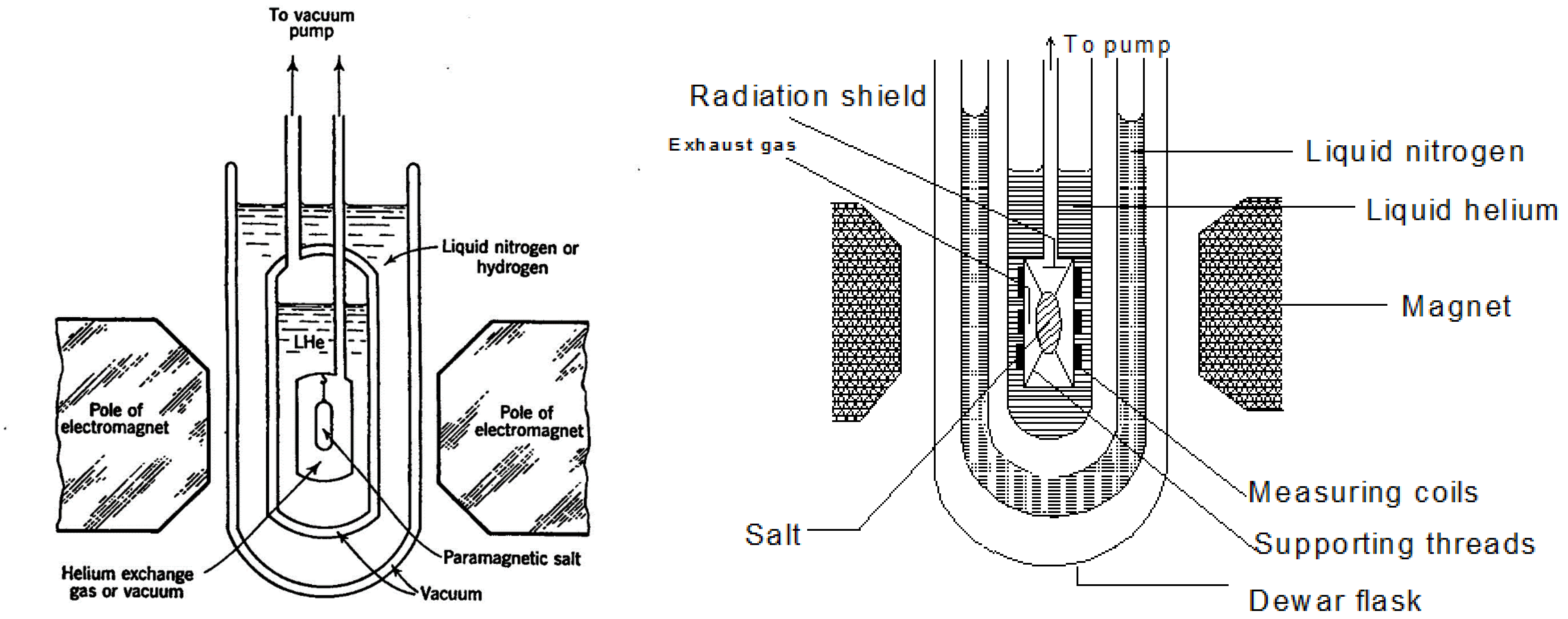


Fig. Apparatus for carrying out adiabatic demagnetization process

5.13 Magnetic cooling

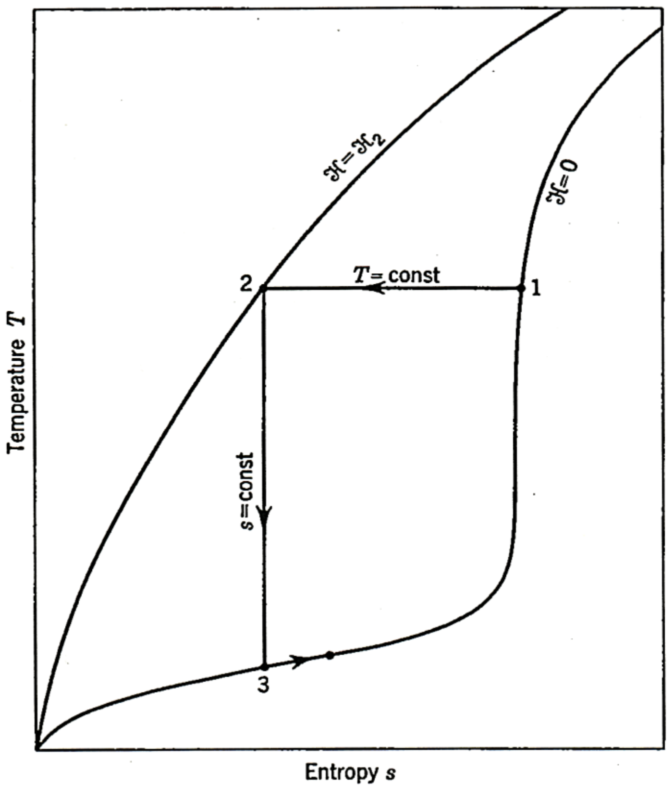


Fig. T-s diagram of the magnetic cooling

- Vapor compression cycle(Refrigerant)

1 – 2 : gas compression at constant T

– decrease s (increase order)

2 – 3 : reversible adiabatic expansion

– same s (no change in order)

– gas molecules farther apart

(random molecular velocity ↓)

5.13 Magnetic cooling

- Paramagnetic salt
  - – no magnetic field dipole : random even at low T
  - – with magnetic field at constant T

    dipole align (ordered, s ↓)

    heat of magnetization ↑
  - – if magnetic field is removed

    the same s,

    alignment should be disordered

    T should be lowered

    (Adiabatic, demagnetization)

5.14 Thermodynamics of magnetic cooling

The magnetic process may be analyzed thermodynamically

$$T\,ds = du - \mu_0 \boldsymbol{\mathcal{H}}\, d\zeta$$

μ_0=$4\pi \times 10^{-7}$T-m/A=permeability of free space in SI units
$\boldsymbol{\mathcal{H}}$= the magnetic field intensity
ζ=the magnetic moment per unit mass

5.14 Thermodynamics of magnetic cooling

For a pure substance,

$$Tds = dh - vdp = c_p dT - \mathrm{T}\left(\frac{\partial v}{\partial T}\right)_p dp$$

For a paramagnetic substance,

$$Tds = c_{\mathcal{H}} dT + \mu_0 T \left(\frac{\partial \zeta}{\partial T}\right)_{\mathcal{H}}$$

$c_{\mathcal{H}}$= the specific heat at constant magnetic field intensity
(analogous to c_p for a pure substance)

5.14 Thermodynamics of magnetic cooling

- Magnetocaloric coefficient
  (analogous to the isentropic expansion coefficient)

$$\mu_M = \left(\frac{\partial T}{\partial \boldsymbol{\mathcal{H}}}\right)_{\boldsymbol{\mathcal{H}}} = -\frac{\mu_0 T}{c_{\boldsymbol{\mathcal{H}}}}\left(\frac{\partial \zeta}{\partial T}\right)_{\boldsymbol{\mathcal{H}}}$$

- The Curie law
  : a sort of “ideal-gas law” for paramagnetic materials

$$\zeta = \frac{\boldsymbol{\mathcal{C}}\boldsymbol{\mathcal{H}}}{T}$$

$\boldsymbol{\mathcal{C}}$: Curie constant

5.14 Thermodynamics of magnetic cooling

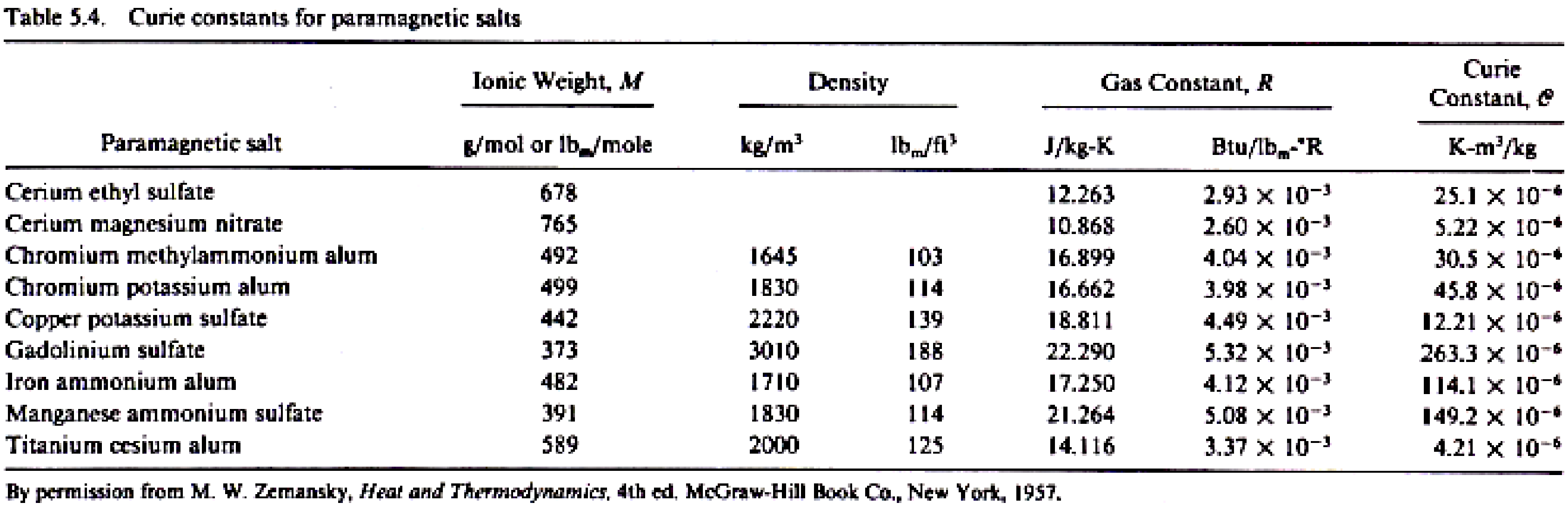


Table 5.4. Curie constants for paramagnetic salts

	Ionic Weight, M	Density		Gas Constant, R		Curie Constant, $\mathcal{C}$
Paramagnetic salt	g/mol or lb_m/mole	kg/m^3	lb_m/ft^3	J/kg-K	Btu/lb_m-°R	K-m^3/kg
Cerium ethyl sulfate	678			12.263	2.93×10^{-3}	25.1×10^{-6}
Cerium magnesium nitrate	765			10.868	2.60×10^{-3}	5.22×10^{-6}
Chromium methylammonium alum	492	1645	103	16.899	4.04×10^{-3}	30.5×10^{-6}
Chromium potassium alum	499	1830	114	16.662	3.98×10^{-3}	45.8×10^{-6}
Copper potassium sulfate	442	2220	139	18.811	4.49×10^{-3}	12.21×10^{-6}
Gadolinium sulfate	373	3010	188	22.290	5.32×10^{-3}	263.3×10^{-6}
Iron ammonium alum	482	1710	107	17.250	4.12×10^{-3}	114.1×10^{-6}
Manganese ammonium sulfate	391	1830	114	21.264	5.08×10^{-3}	149.2×10^{-6}
Titanium cesium alum	589	2000	125	14.116	3.37×10^{-3}	4.21×10^{-6}

By permission from M. W. Zemansky, *Heat and Thermodynamics*, 4th ed. McGraw-Hill Book Co., New York, 1957.

5.15 Magnetic moment and entropy of paramagnetic materials

- Magnetic moment of the paramagnetic material

$$\zeta = {}^{1}/_{2}\, n g \mu_B \, \mathrm{B}(\xi)$$

$B(\xi)$=Brilloui function

$B(\xi)=(2J+1)\coth[(2J+1)\xi]-\coth(\xi)$

$$where\ \ \xi = g\mu_B\mu_0\boldsymbol{\mathcal{H}}/2kT$$

5.15 Magnetic moment and entropy of paramagnetic materials

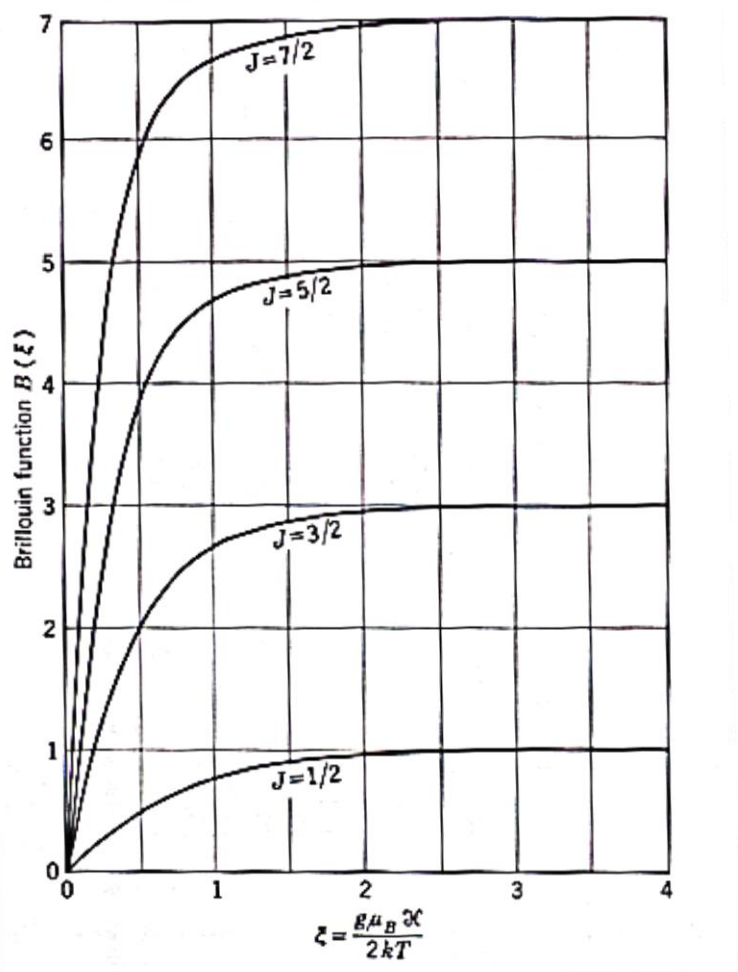
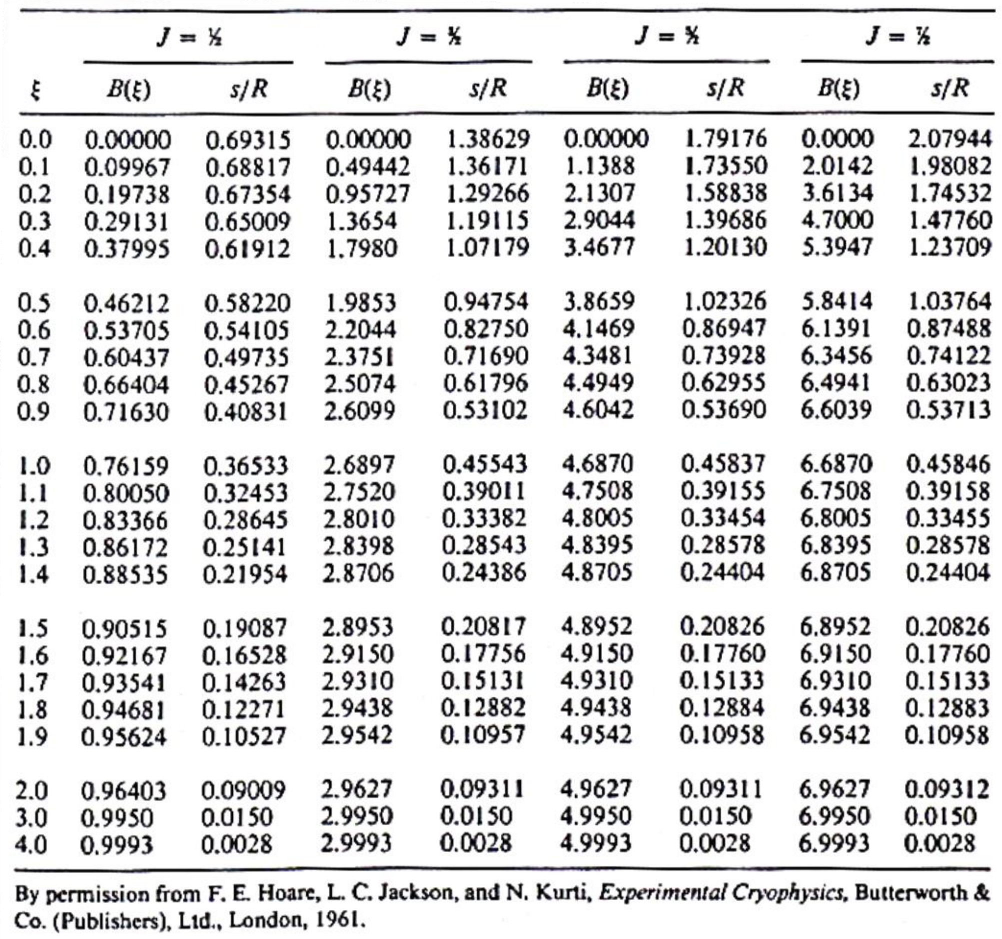


Fig. Brillounin function

5.15 Magnetic moment and entropy of paramagnetic materials

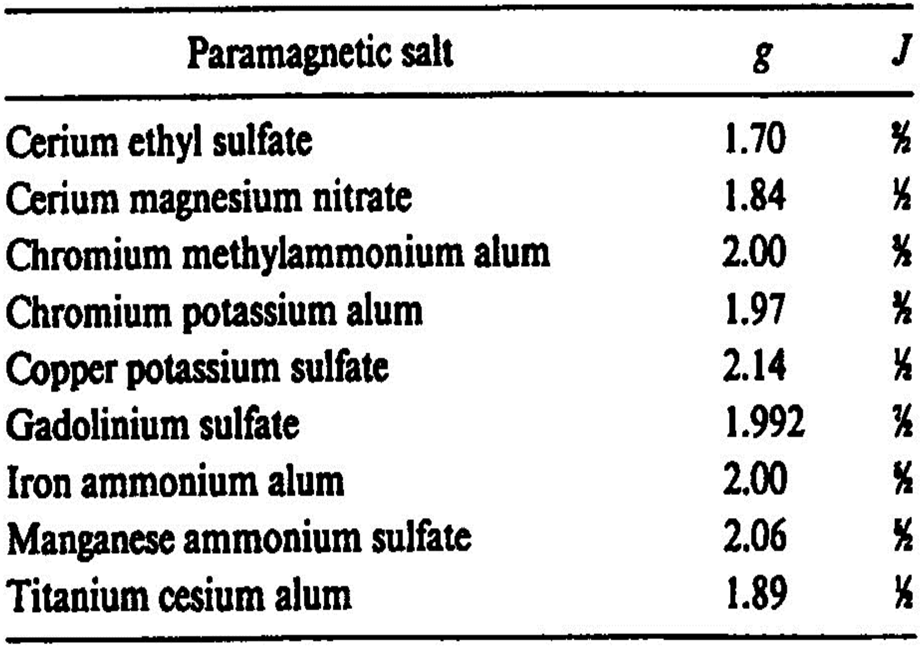


	$J = \frac{1}{2}$		$J = \frac{3}{2}$		$J = \frac{5}{2}$		$J = \frac{7}{2}$	
ξ	$B(\xi)$	s/R	$B(\xi)$	s/R	$B(\xi)$	s/R	$B(\xi)$	s/R
0.0	0.00000	0.69315	0.00000	1.38629	0.00000	1.79176	0.0000	2.07944
0.1	0.09967	0.68817	0.49442	1.36171	1.1388	1.73550	2.0142	1.98082
0.2	0.19738	0.67354	0.95727	1.29266	2.1307	1.58838	3.6134	1.74532
0.3	0.29131	0.65009	1.3654	1.19115	2.9044	1.39686	4.7000	1.47760
0.4	0.37995	0.61912	1.7980	1.07179	3.4677	1.20130	5.3947	1.23709
0.5	0.46212	0.58220	1.9853	0.94754	3.8659	1.02326	5.8414	1.03764
0.6	0.53705	0.54105	2.2044	0.82750	4.1469	0.86947	6.1391	0.87488
0.7	0.60437	0.49735	2.3751	0.71690	4.3481	0.73928	6.3456	0.74122
0.8	0.66404	0.45267	2.5074	0.61796	4.4949	0.62955	6.4941	0.63023
0.9	0.71630	0.40831	2.6099	0.53102	4.6042	0.53690	6.6039	0.53713
1.0	0.76159	0.36533	2.6897	0.45543	4.6870	0.45837	6.6870	0.45846
1.1	0.80050	0.32453	2.7520	0.39011	4.7508	0.39155	6.7508	0.39158
1.2	0.83366	0.28645	2.8010	0.33382	4.8005	0.33454	6.8005	0.33455
1.3	0.86172	0.25141	2.8398	0.28543	4.8395	0.28578	6.8395	0.28578
1.4	0.88535	0.21954	2.8706	0.24386	4.8705	0.24404	6.8705	0.24404
1.5	0.90515	0.19087	2.8953	0.20817	4.8952	0.20826	6.8952	0.20826
1.6	0.92167	0.16528	2.9150	0.17756	4.9150	0.17760	6.9150	0.17760
1.7	0.93541	0.14263	2.9310	0.15131	4.9310	0.15133	6.9310	0.15133
1.8	0.94681	0.12271	2.9438	0.12882	4.9438	0.12884	6.9438	0.12883
1.9	0.95624	0.10527	2.9542	0.10957	4.9542	0.10958	6.9542	0.10958
2.0	0.96403	0.09009	2.9627	0.09311	4.9627	0.09311	6.9627	0.09312
3.0	0.9950	0.0150	2.9950	0.0150	4.9950	0.0150	6.9950	0.0150
4.0	0.9993	0.0028	2.9993	0.0028	4.9993	0.0028	6.9993	0.0028

By permission from F. E. Hoare, L. C. Jackson, and N. Kurti, *Experimental Cryophysics*, Butterworth & Co. (Publishers), Ltd., London, 1961.

Table. Brillouin function and entropy for paramagnetic materials according to the Brillouin expression

5.15 Magnetic moment and entropy of paramagnetic materials



Paramagnetic salt	g	J
Cerium ethyl sulfate	1.70	$\frac{5}{2}$
Cerium magnesium nitrate	1.84	$\frac{1}{2}$
Chromium methylammonium alum	2.00	$\frac{3}{2}$
Chromium potassium alum	1.97	$\frac{3}{2}$
Copper potassium sulfate	2.14	$\frac{1}{2}$
Gadolinium sulfate	1.992	$\frac{7}{2}$
Iron ammonium alum	2.00	$\frac{5}{2}$
Manganese ammonium sulfate	2.06	$\frac{5}{2}$
Titanium cesium alum	1.89	$\frac{1}{2}$

Table. Atomic constants for
Paramagnetic materials:

g=Lande splitting factor;
J=total angular momentum
Quantum factor

5.16 Magnetic refrigeration system

- : Application of adiabatic demagnetization in maintaining T < 1.0 K

  Effective operation in zero gravity

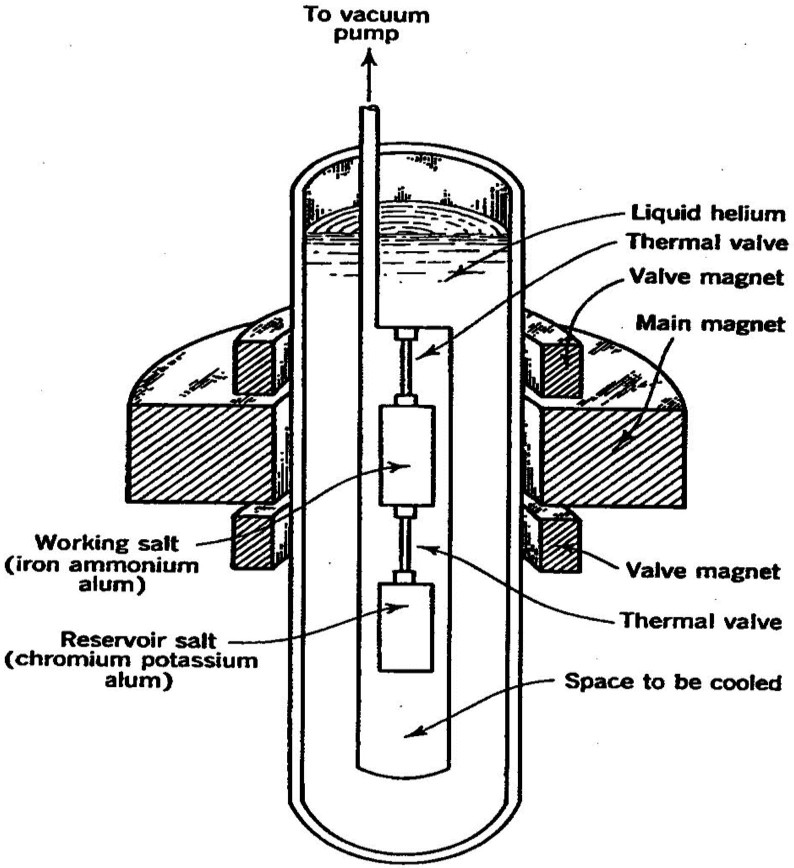


Fig. Magnetic refrigerator schematic

5.16 Magnetic refrigeration system

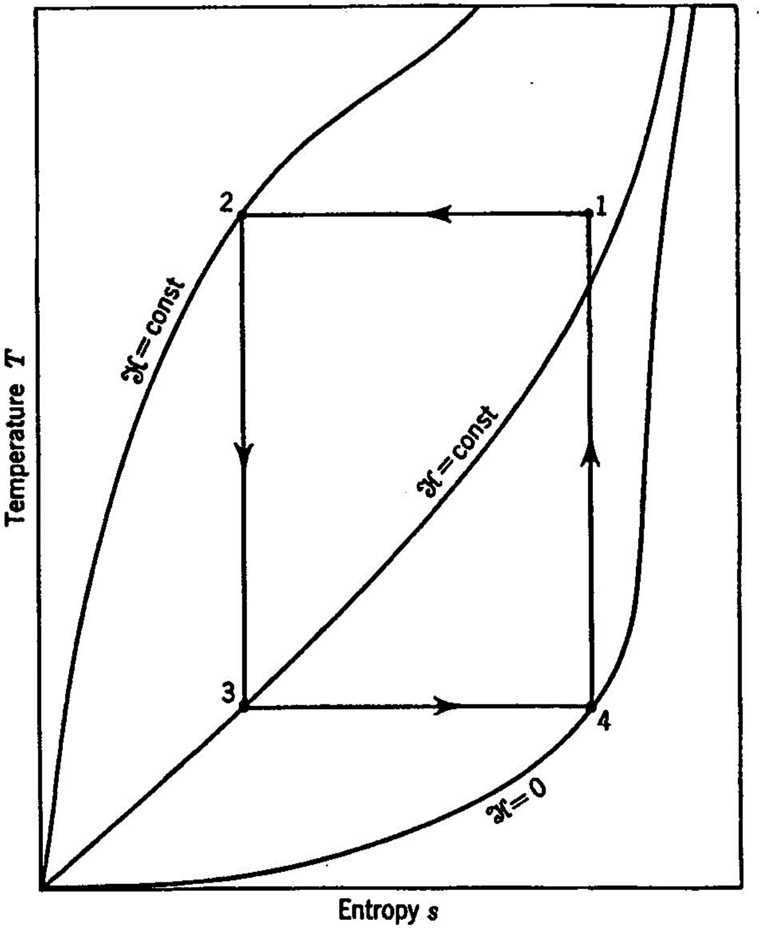


Fig. Thermodynamic cycle for the magnetic refrigerator

5.16 Magnetic refrigeration system

- The sequence of operations for the magnetic refrigerator

Process 1-2. The magnetic field is applied to the working salt while the Upper thermal valve is open and the lower thermal valve is closed. When The upper thermal valve is open, heat may be transferred from the working Salt to the liquid helium bath, thereby maintaining the salt temperature fairly constant. The thermal valve between the working salt and the reservoir salt is closed so that heat will not flow back into the low-temperature reservoir during this process.

Process 2-3. Both thermal valves are closed, and the magnetic field around the working salt is reduced adiabatically to some intermediate value. During this process, the temperature of the working salt decreases.

5.16 Magnetic refrigeration system

Process 3-4. the thermal valve between the working salt and the reservoir salt is opened, and the field around the working salt is reduced to zero while heat is absorbed isothermally by the working salt from the reservoir salt.

Process 4-1. Both thermal valves are closed, and the magnetic field around the working salt is adiabatically increased to its original value.

5.16 Magnetic refrigeration system

	$\mathcal{H}$	Upper thermal valve	Lower thermal valve
1 – 2	↑ applied to working salt (heat to the LHe bath)	open	closed
2 – 3	↓ (T of the working salt decreases)	closed	closed
3 – 4	↓ (to zero) (heat is absorbed isothermally)	closed	open
4 – 1	↑	closed	closed

5.16 Magnetic refrigeration system

- COP of the Magnetic refrigeration system

$$Q_a = mT_3(s_4 - s_3)$$

$$Q_r = mT_1(s_2 - s_1) = -mT_2(s_4 - s_3)$$

$$W_{net} = Q_a + Q_r = -m(T_1 - T_3)(s_4 - s_3)$$

$$COP = -\frac{Q_a}{W_{net}} = \frac{mT_3(s_4 - s_3)}{m(T_1 - T_3)(s_4 - s_3)} = \frac{T_3}{T_1 - T_3}$$

- COP for the ideal magnetic refrigerator is the same as that for a Carnot refrigerator.

5.16 Magnetic refrigeration system

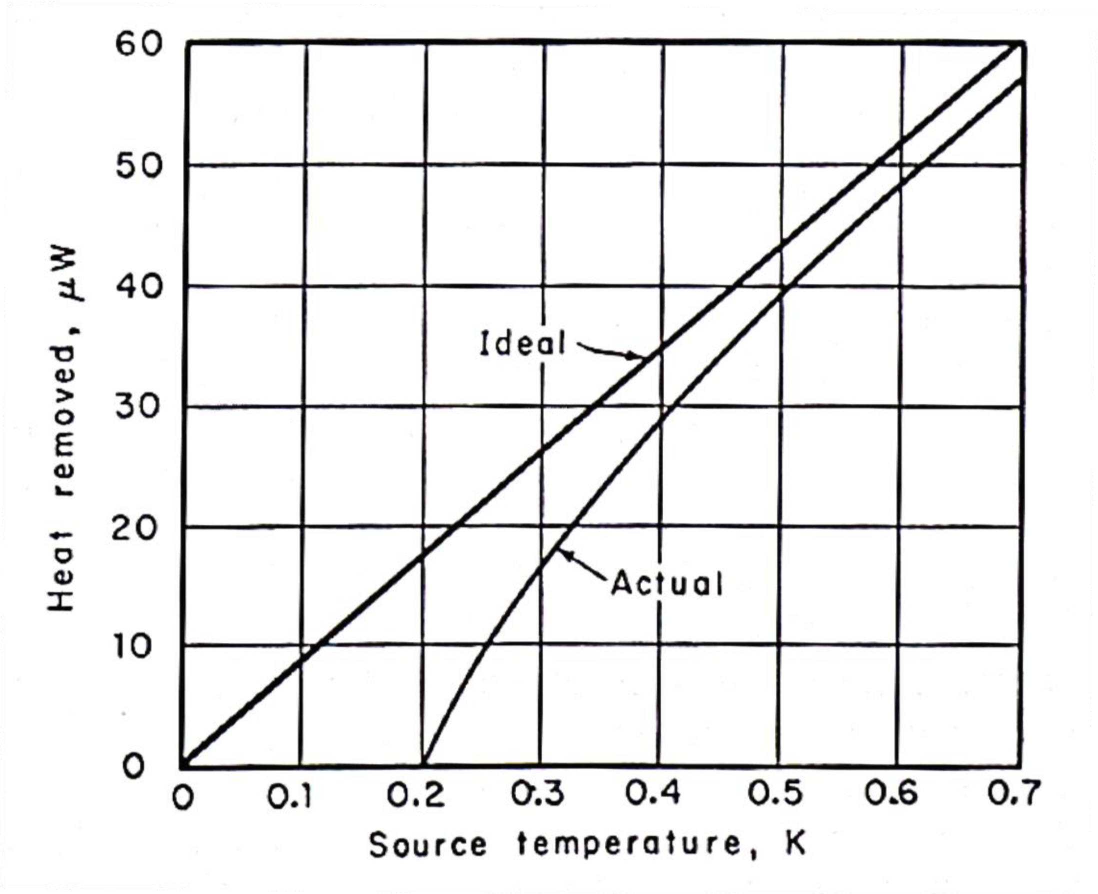


Fig. Actual and ideal performance of the magnetic refrigerator

5.16 Magnetic refrigeration system

- Irreversibility

- heat transfer from the ambient

- finite time rate of change of magnetic field : Entropy ↑ , Temperature ↑

5.17 Thermal valves

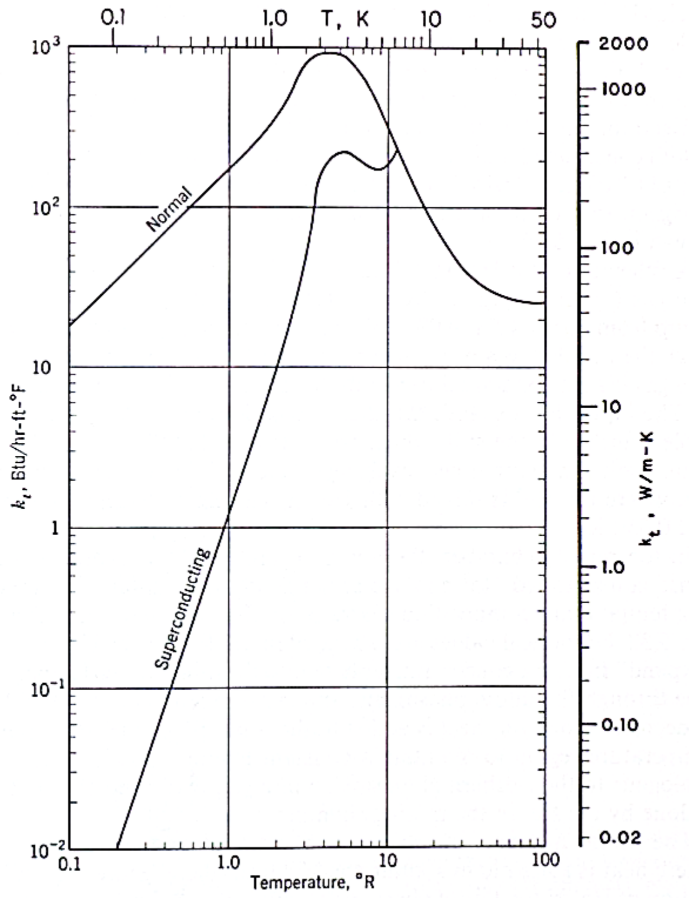


Fig. Thermal conductivity of lead
in the normal and superconducting states.

-k in the superconducting state ~ 1 (closed) at 0.5 K – no magnetic field

-k in the normal state ~ 200 (open) – with magnetic field

5.18 Dilution refrigerators

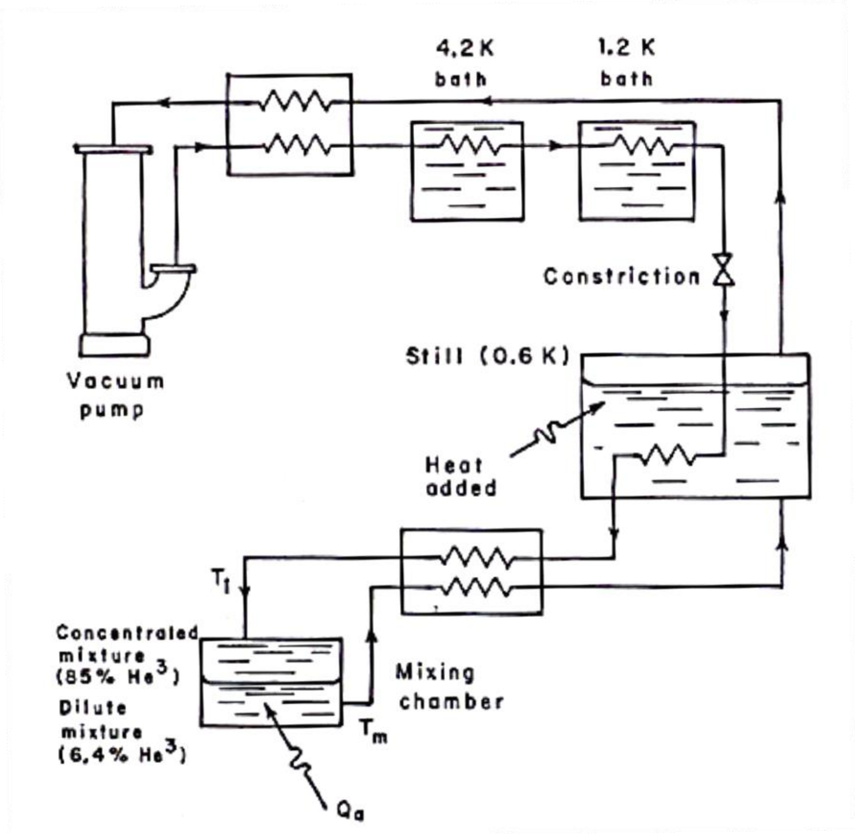
- Cooling could be achieved by means of dilution of He^3 by superfluid He^4 was first suggested by H.London(1951)

The refrigeration effect of the dilution refrigerator may be determined by application of the First law to the mixing chamber:

$$\dot{Q}_a = \dot{n}_3(h_m - h_i)$$

$$h_m = C_1 T_m^2, \qquad where\ C_1 = 94\ J/mol - K^2$$
$$h_i = C_2 T_i^2, \qquad where\ C_2 = 12\ J/mol - K^2$$

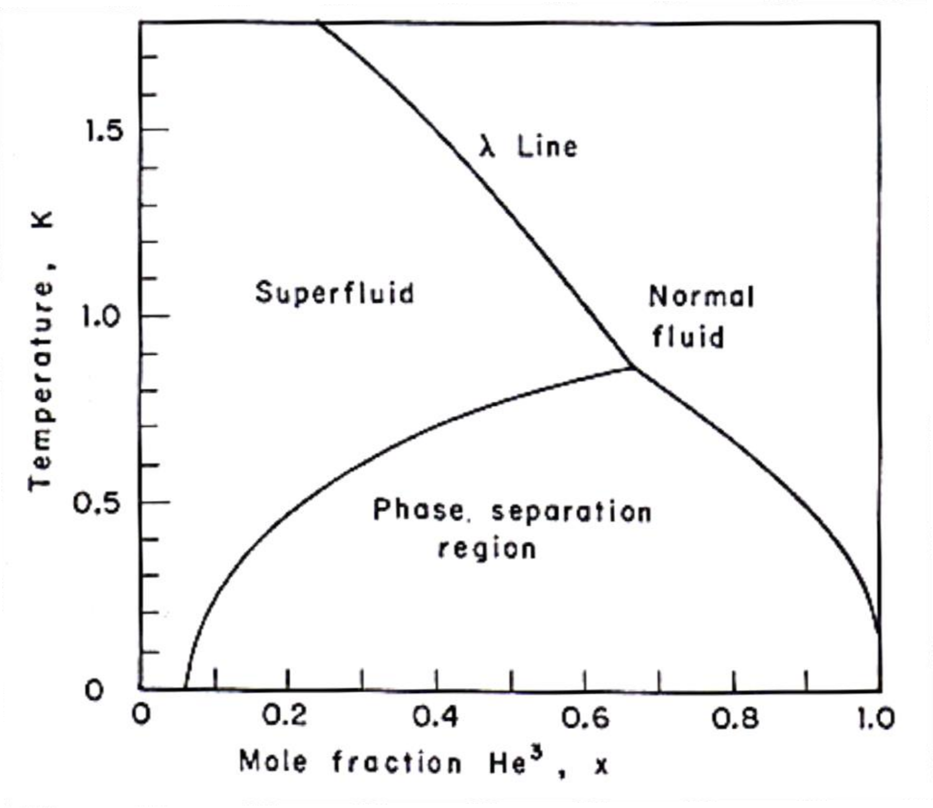
5.18 Dilution refrigerators



<$He^3 - He^4$ dilution refrigerator unit>
Oxford Instruments

<$He^3 - He^4$ dilution refrigerator schematic>

5.18 Dilution refrigerators



<Phase diagram for $He^3 - He^4$ mixtures>

5.19 Thermoacoustic Refrigeration (열음향냉동)

- **Thermoacoustic Refrigeration**

  Adiabatic compression and expansion of fluid(gas) by <u>standing wave</u>

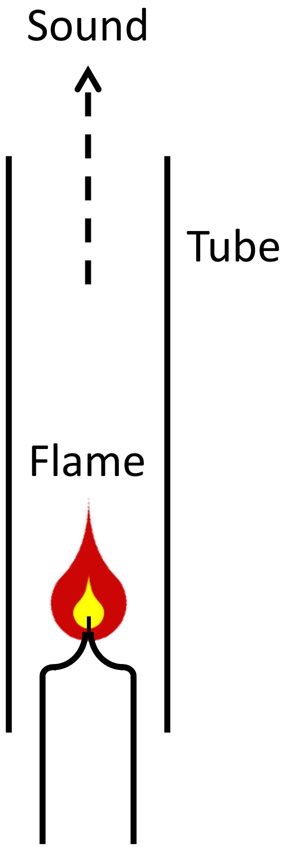
- Higgins (1777)

  Bryon Higgins was able to excited pipe oscillations in a large tube with two open ends.

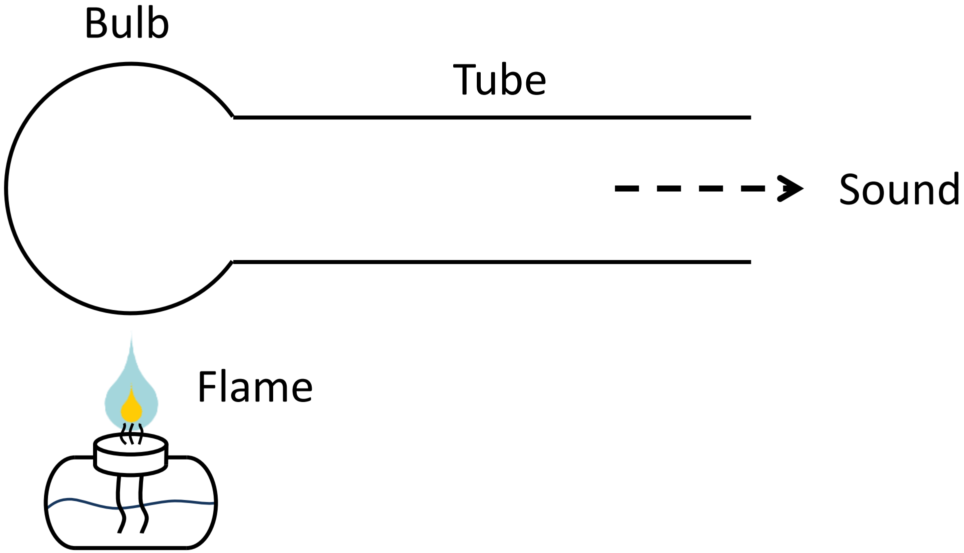
- Sondhauss (1850)

  Sondhauss experimented with a open-close tube.
  Heating the bulb at the close end produce sound.

5.19 Thermoacoustic Refrigeration



- Higgins (1777)



- Sondhauss (1850)

5.19 Thermoacoustic Refrigeration

- **Wheatly (1982) – Thermoacoustic refrigerator developed**

<Theory> Adiabatic compression & expansion

$$T\cancelto{0}{ds} = C_v dT + P dv$$

$$0 = C_v dT + \frac{RT}{v} dv \quad \text{(for ideal gas)}$$

$$0 = \frac{dT}{T} + (k-1)\frac{dv}{v}$$

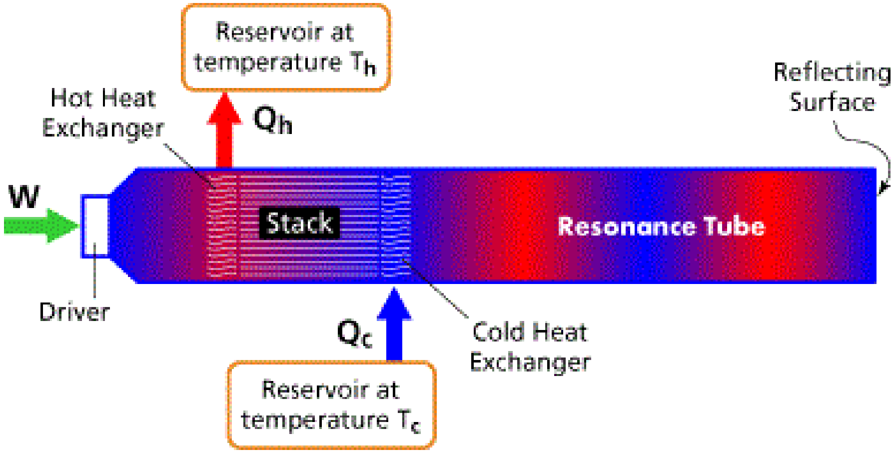
$$T v^{k-1} = c \quad \left(\text{where } \mathrm{k} = \frac{C_p}{C_v},\ \ C_p - C_v = R \ \rightarrow\ kC_v - C_v = R \ \rightarrow C_v = \frac{R}{k-1}\right)$$

$$P v^k = c$$

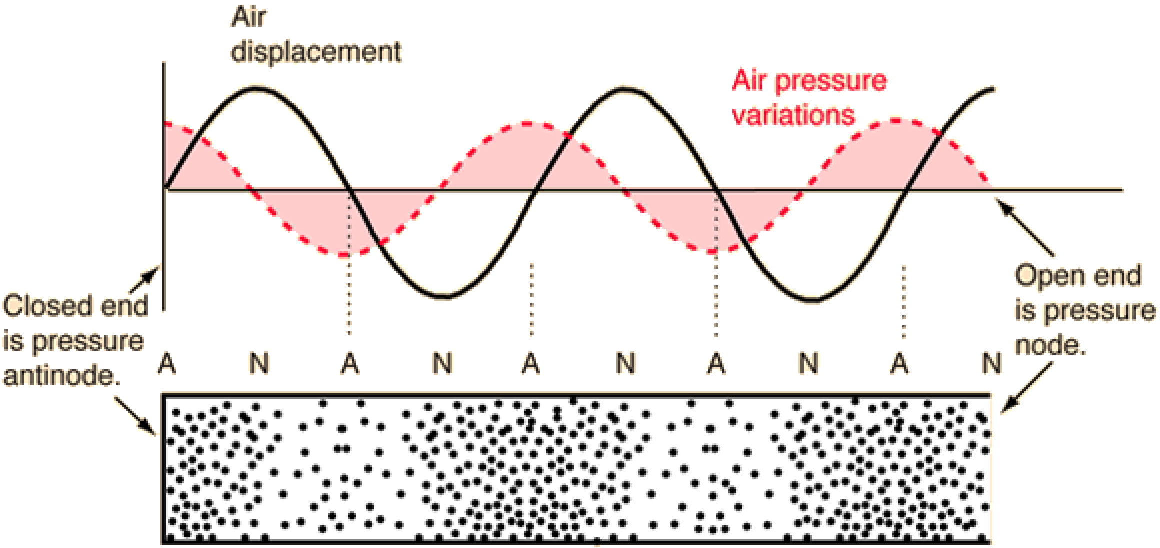
$$T\left(\frac{T}{P}\right)^{k-1} = \frac{T^k}{P^{k-1}} = c$$

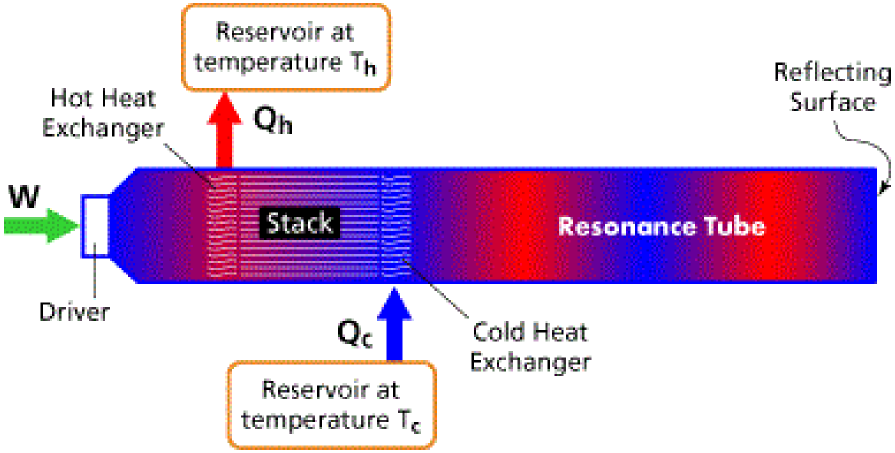
5.19 Thermoacoustic Refrigeration

- **Components**
  - Acoustic driver (Speaker)
  - Resonance tube (Resonator)
  - Stack of plates
  - Cold and hot heat exchanger



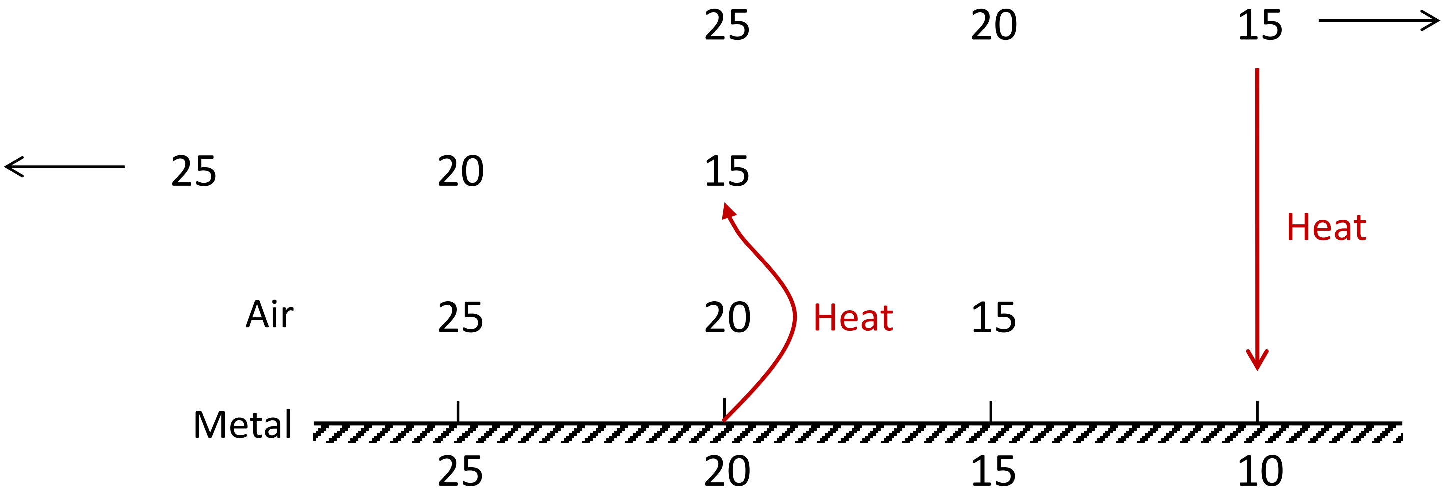
5.19 Thermoacoustic Refrigeration



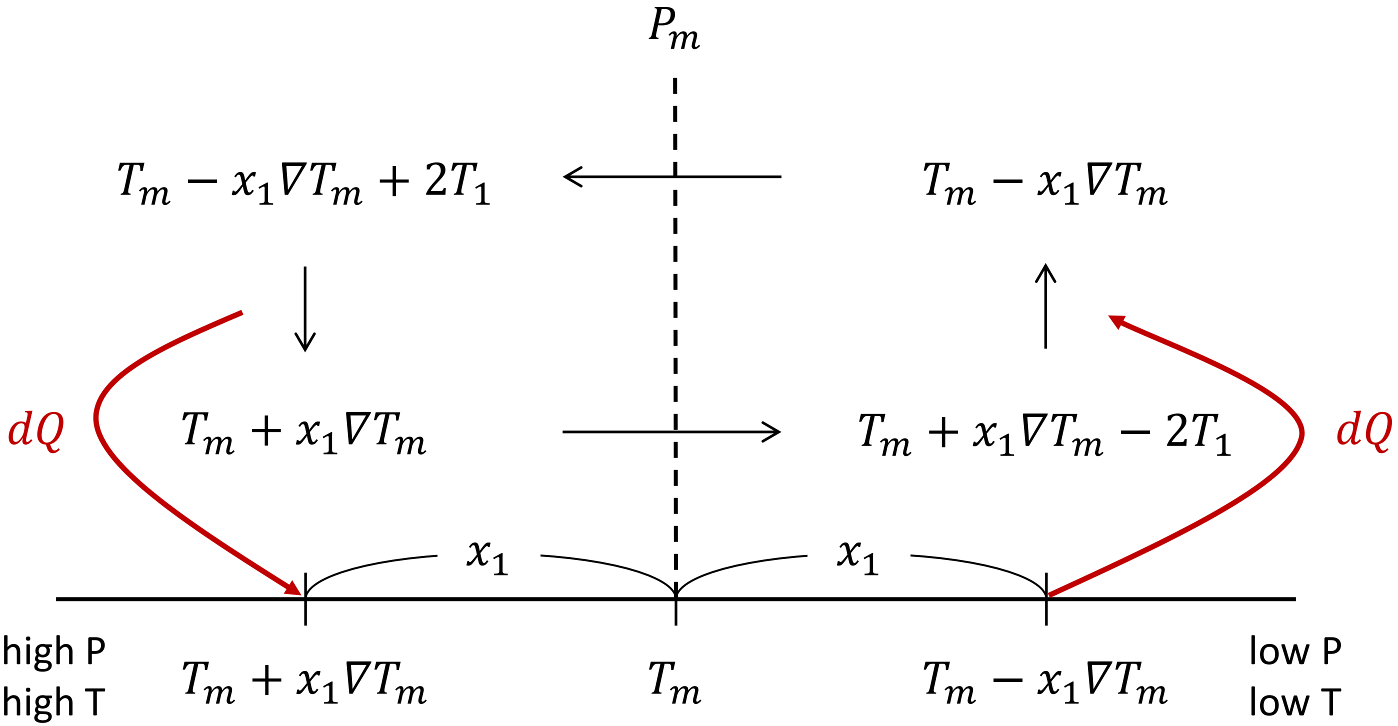


5.19 Thermoacoustic Refrigeration

- **Shuttle heat transfer**



5.19 Thermoacoustic Refrigeration



P_m

$T_m - x_1 \nabla T_m + 2T_1$

$T_m - x_1 \nabla T_m$

dQ

$T_m + x_1 \nabla T_m$

$T_m + x_1 \nabla T_m - 2T_1$

dQ

x_1

x_1

high P
high T

$T_m + x_1 \nabla T_m$

T_m

$T_m - x_1 \nabla T_m$

low P
low T

5.19 Thermoacoustic Refrigeration

(+) Advantages of thermoacoustic refrigeration

- Simplicity
- Reliability
- Low cost
- Minimal use of moving part
- No harmful chemicals

(-) Disadvantages of thermoacoustic refrigeration

- Capacity of thermoacoustic refrigerator very low ([W] scale)
- COP of thermoacoustic refrigerator low (< 0.1)