Cryogenic Engineering

Chapter 6.

Measurement Systems for Low Temperatures

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6.1. Theoretical plate calculations for columns

- **Temperature measurement from temperature property**

  ① Mercury in a capillary tube

  ② Pt (platinum) wire resistance

  ③ Ideal gas pressure

  ④ Thermoelectric EMF (voltage)

  ⑤ Equilibrium pressure of gas

  ⑥ Difference in thermal expansion

  ⑦ Speed of sound

  ⑧ Magnetic susceptibility

6.1. Theoretical plate calculations for columns

- **Temperature measurement from temperature property**

① Mercury in a capillary tube

< Mercury in a capillary tube >

Mercury in glass	-39 ^oC to +357 ^oC
Pressurised mercury in glass	-39 ^oC to +500 ^oC
Pressurised mercury in quartz	-39 ^oC to +800 ^oC
Alcohol in glass	-120 ^oC to +60 ^oC
Pentane in glass	-200 ^oC to +30 ^oC

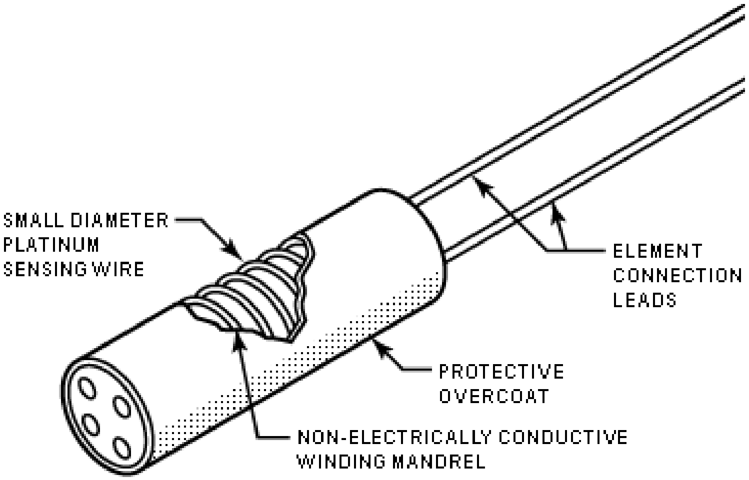
< The ranges of the most common liquid-in-glass thermometers >

6.1. Theoretical plate calculations for columns

- **Temperature measurement from temperature property**

② Pt (Platinum) wire resistance

< Platinum >



< wire-wound sensing element >

6.1. Theoretical plate calculations for columns

- **Temperature measurement from temperature property**

③ Ideal gas pressure

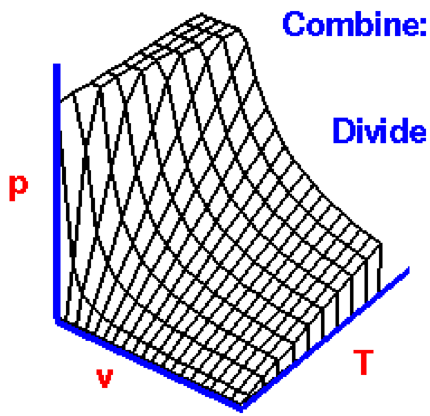
Properties

Density = ρ Pressure = p Temperature = T Volume = V Mass = M

Observations

Boyle: For a given mass, at constant temperature, the pressure times the volume is a constant. $pV = C_1$

Charles and Gay-Lussac: For a given mass, at constant pressure, the volume is directly proportional to the temperature. $V = C_2 T$



Combine: $pV/T = n\bar{R}$ $\bar{R} = 8.31$ J / mole / K (Universal)

$pV = n\bar{R}T$ n = number of moles

Divide by mass: $pv = \frac{n\bar{R}T}{M}$ Specific Volume = v

$v = \frac{\text{volume}}{\text{mass}} = \frac{1}{\rho}$

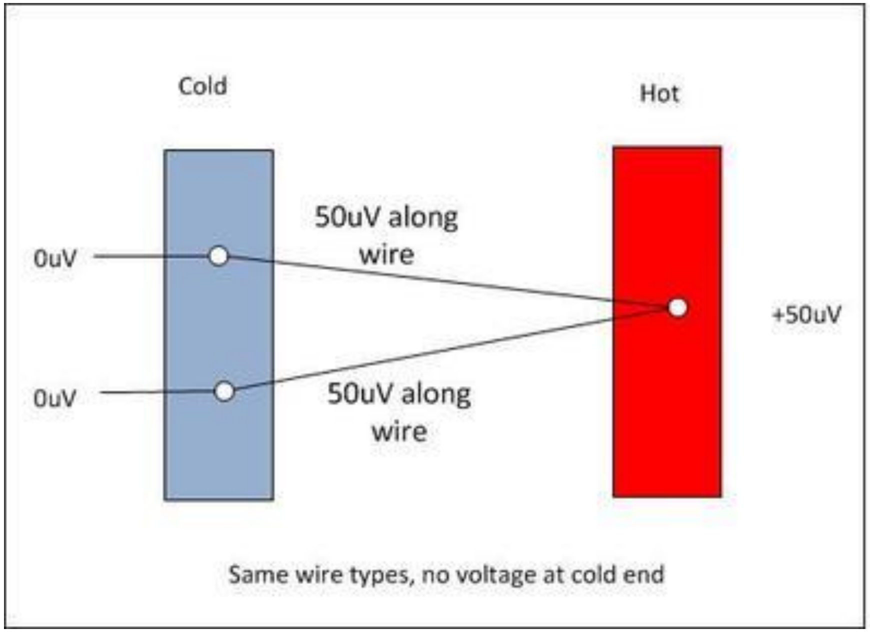
$pv = RT$ or $p = R\rho T$

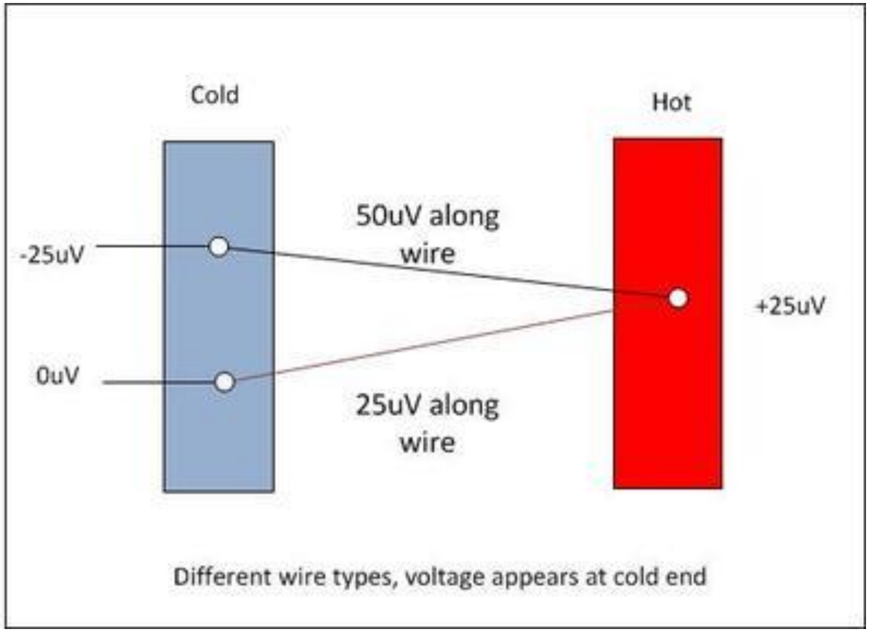
R = Constant value for each gas

= .286 kJ/ kg / K (for air)

- **Temperature measurement from temperature property**

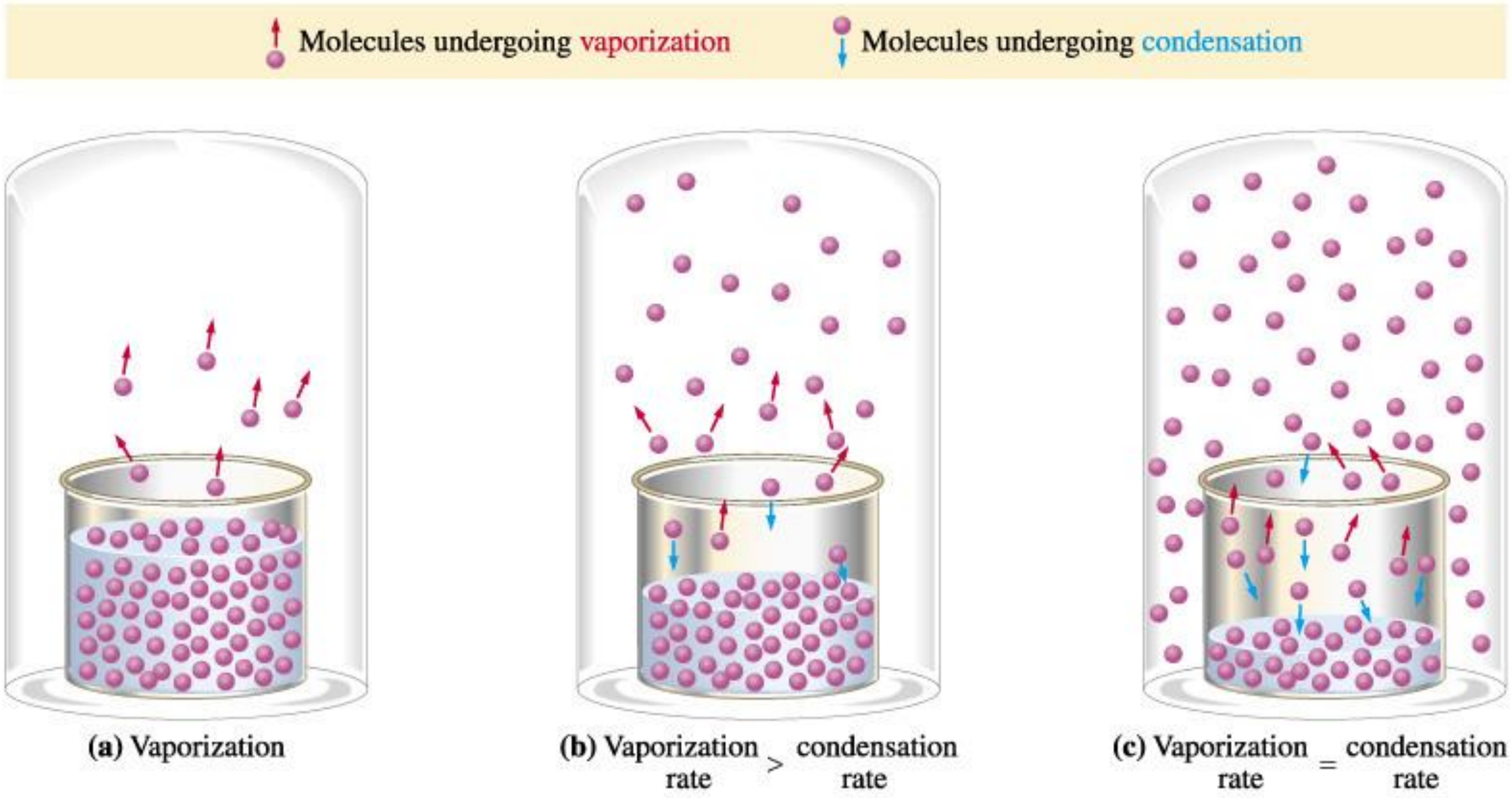
④ Thermoelectric EMF(Electro Motive Force)





▪ Temperature measurement from temperature property

⑤ Equilibrium pressure of gas



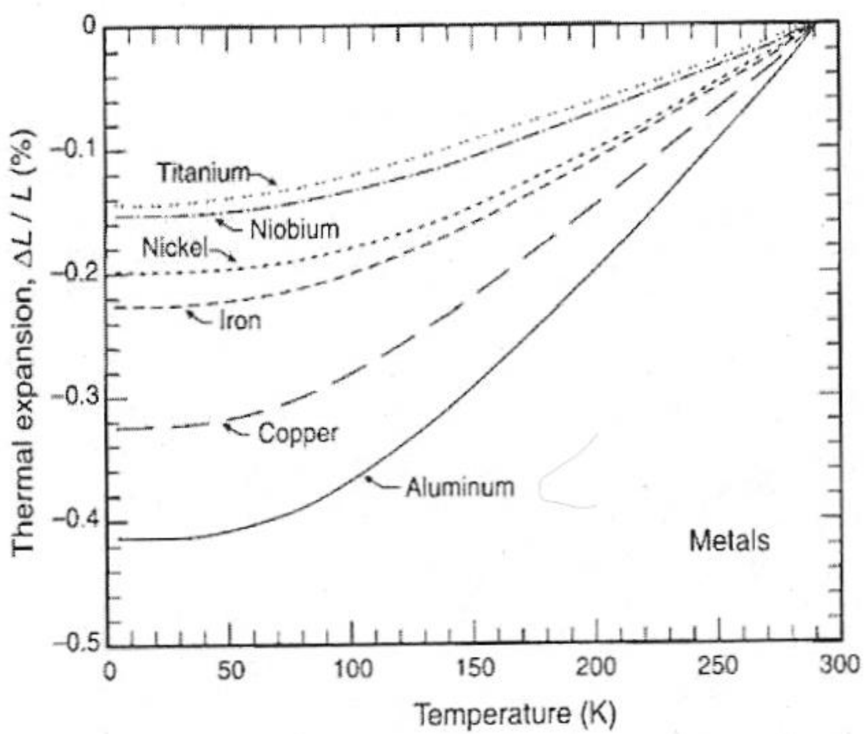
6.1. Theoretical plate calculations for columns

- **Temperature measurement from temperature property**

⑥ Difference in thermal expansion

Material	Fractional expansion per degree C x10^-6	Fractional expansion per degree F x10^-6
Glass, ordinary	9	5
Glass, pyrex	4	2.2
Quartz, fused	0.59	0.33
Aluminum	24	13
Brass	19	11
Copper	17	9.4
Iron	12	6.7
Steel	13	7.2
Platinum	9	5
Tungsten	4.3	2.4
Gold	14	7.8
Silver	18	10

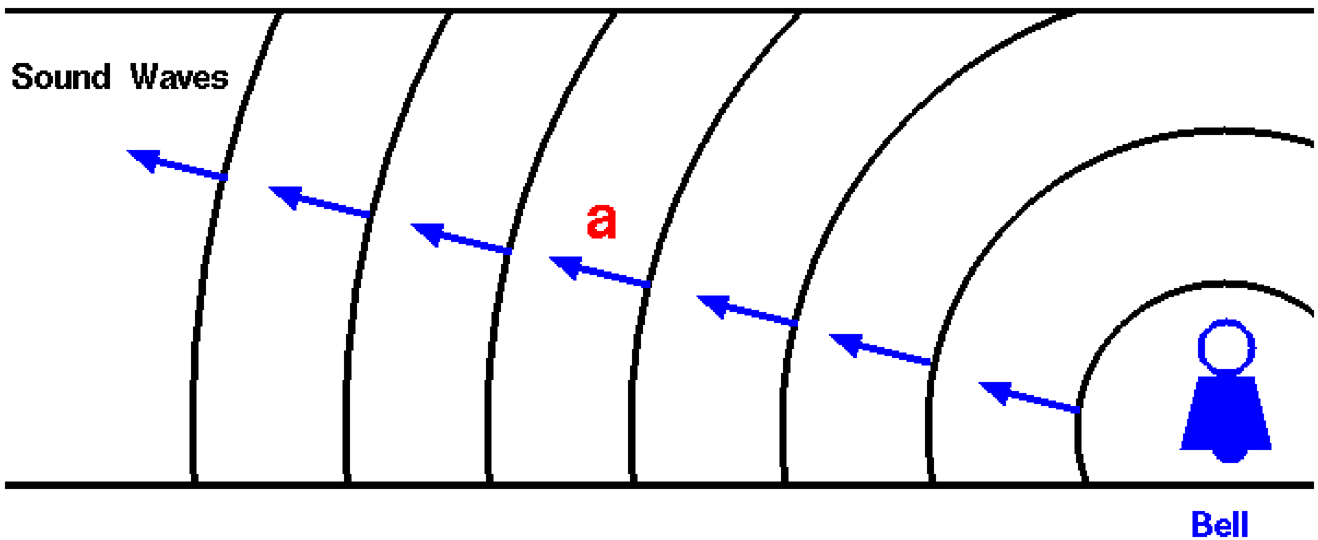
< Thermal Expansion Coefficients at 20 ℃ >



< Thermal linear expansion of common metals>

- **Temperature measurement from temperature property**

⑦ Speed of sound



Speed of sound (a) depends on the type of medium and the temperature of the medium.

a = sqrt (γ R T)

γ = ratio of specific heats (1.4 for air at STP)

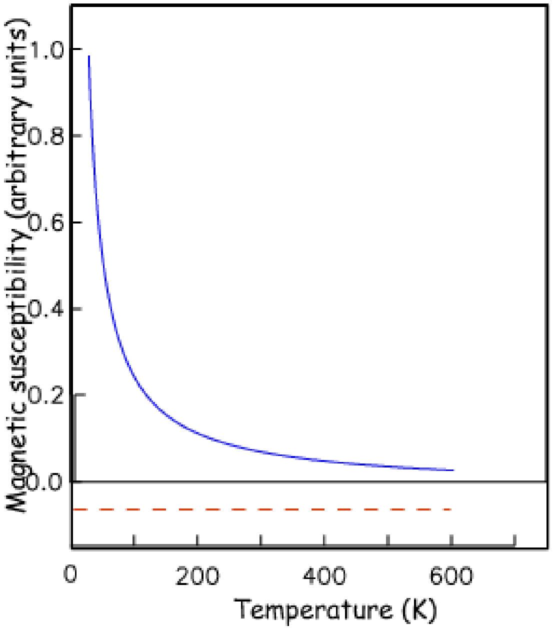
R = gas constant (286 $m^2 / s^2 / K$ for air)

T = absolute temperature (273.15 + °C)

▪ Temperature measurement from temperature property

⑧ Magnetic susceptibility

Magnetic susceptibility, quantitative measure of the extent to which a material may be magnetized in relation to a given applied magnetic field. The magnetic susceptibility of a material, commonly symbolized by χm, is equal to the ratio of the magnetization M within the material to the applied magnetic field strength H, or χm = M/H. This ratio, strictly speaking, is the volume susceptibility, because magnetization essentially involves a certain measure of magnetism (dipole moment) per unit volume.



< Schematic drawings of paramagnetic (solid line) and diamagnetic (dashed line) magnetic susceptibility as a function of temperature >

6.2. Temperature scales and fixed points

- **ITS-27 (International Temperature scale)**
  - The ITS-27 was based on six fixed points.
  - ITS-27 was defined only down to the NBP of oxygen or about - 190℃.

- **ITS-48 (1948)**
  - Adopted several changes in the international temperature scale
  - The same six fixed points were used.

- **ITS-48 (1960)**
  - Triple point of water became a standard fixed point instead of the freezing point of water

6.2. Temperature scales and fixed points

- **ITS-68**

Fixed Point	Temperature (K)
NMP of gold	1337.58
NMP of silver	1235.08
NMP of zinc	692.73
NMP of water	373.15
Standard-triple point of water	273.16
NMP of oxygen	90.188
TP of oxygen	54.361
NBP of neon	27.102
NBP of hydrogen	20.28
B.P. of hydrogen at 25 torr	17.042
TP of hydrogen	13.81

NMP: Normal Melting Point or freezing point
NBP: Normal Boiling Point
TP: Triple Point

6.2. Temperature scales and fixed points

- **ITS-90**

The International Temperature Scales 1990

Kelvin Scale: The triple point of water is set at 273.16

ITS 1990:	T_{90}-(K):	t_{90}-(°C):
triple point of H_2 at equilibrium	13.8033	-259.3467
triple point of Ne	24.5561	-248.5939
triple point of O_2	54.3584	-218.7916
triple point of Ar	83.8058	-189.3442
triple point of Hg	234.3156	-38.8344
triple point of water	273.16	0.01
equilibrium M of Ga	302.9146	29.7646
equilibrium F of In	429.7485	156.5985
equilibrium F of Sn	505.078	231.928
equilibrium F of Zn	692.677	419.527
equilibrium F of Al	933.473	660.323
equilibrium F of Ag	1234.93	961.78
equilibrium F of Au	1337.33	1064.18
equilibrium F of Cu	1357.77	1084.62

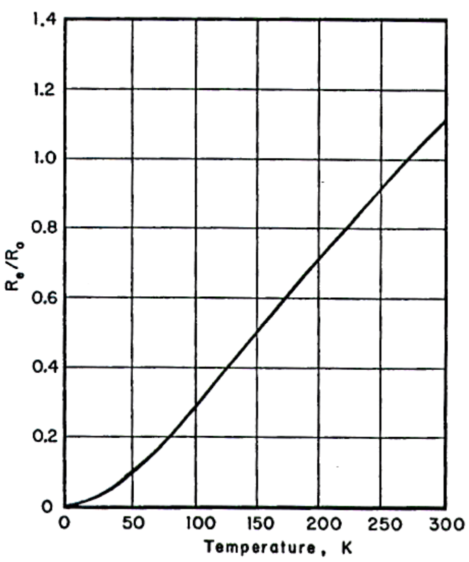
6.3. Metallic resistance thermometers

- **Callendar-van Dusen equation**

$$R_e / R_0 = 1 + At + Bt^2 + Ct^3(t - 100)$$

Typical values for platinum thermometers are

R_0=25Ω, A=3.946*10^{-3} ℃$^{-1}$, B= -1.108*10^{-6} ℃$^{-2}$, C= 3.33*10^{-12} ℃$^{-4}$



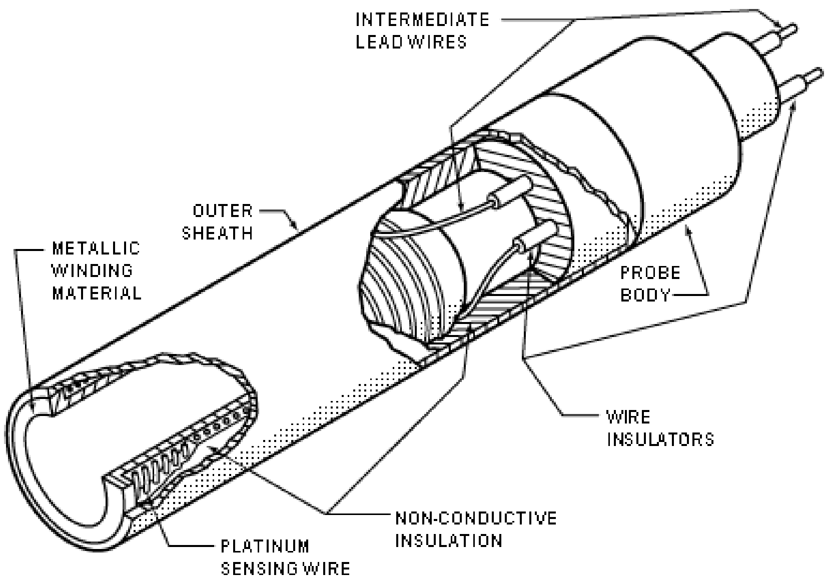
Choose materials for temperature range

- Platinum (Pt): below 630.74℃

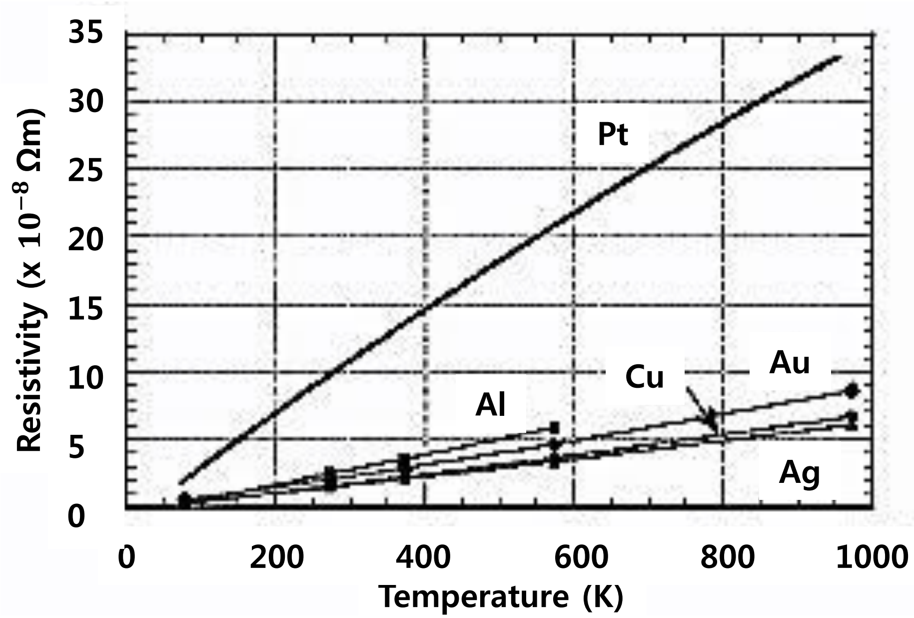
- Indium (In): low temperature range

< Reduced electric resistance ratio for platinum. R_0 is the electric at 0 ℃ >

- **Platinum resistance thermometer**



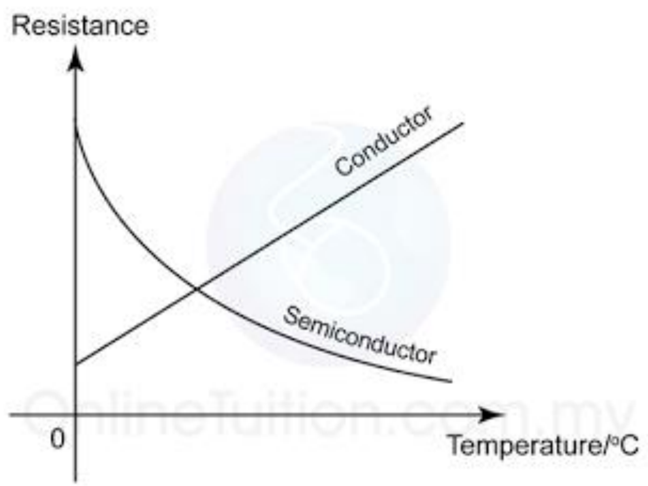
< The wire-wound sensing element >



< The resistivity of five metallic elements >

6.4. Semiconductor resistance thermometers

- **Electrical conductivity of semiconductors is temperature dependent**
  - High temperature : exponential proportion of the absolute temperature
  - Low temperature : due to the presence of impurities
- **Material of semiconductor thermometer**
  - Germanium : widely used
  - Carbon : for low temperature thermometer



< The resistivity change of a conductor and semiconductor against the temperature >

6.5. Thermocouples

One junction of thermocouple pair is placed at the point to be measured, the other junction is placed in a reference temperature region.

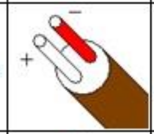
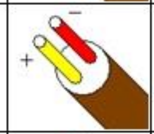
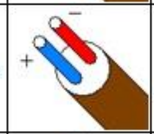
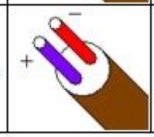
$$e = a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4$$

$$t = b_1 e + b_2 e^2 + b_3 e^3 + b_4 e^4$$

t : the difference in temperature

e : the thermocouple output E.M.F value

* Disadvantage : Output E.M.F is quite small

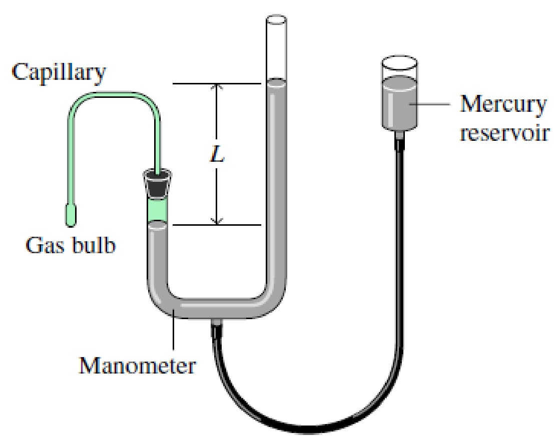
Type	Material		Color Code	Range (°C)	
Thermocouple Grade	Positive Wire	Negative Wire		Minimum	Maximum
J	Iron	Constantan		0	750
K	Chromel	Alumel		-200	1250
T	Copper	Constantan		-200	350
E	Chromel	Constantan		-200	900

6.6. Constant-volume gas thermometer

The constant-volume thermometer may be calibrated by measuring the gas pressure T_s at a standard temperature p_s.

From ideal gas law : $T = p(T_s / p_s)$

For accurate work : $$T = \frac{p(T_s / p_s)}{1 + (1 - p/p_s)(V_0/V)(T_s/T_0)} = K_1 p(T_s / p_s)$$



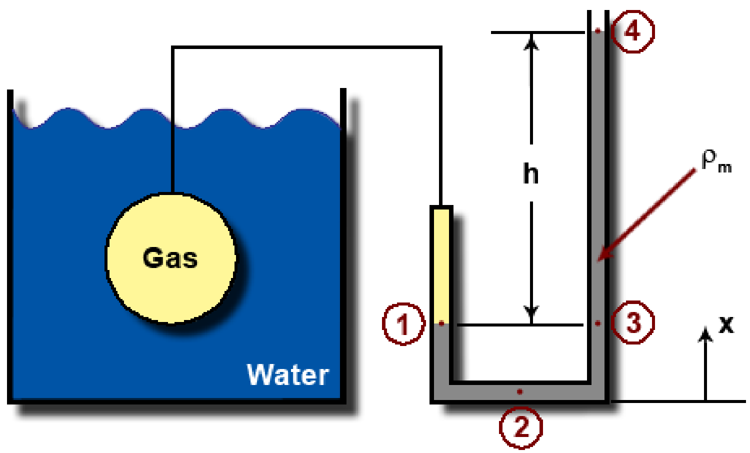
< Constant-volume gas thermometer >

6.7. Vapor-pressure thermometer

The vapor pressure(saturation pressure) is a definite function of the temperature of the liquid, so this property can be used in determining the temperature.

$$\ln(p/p_0) = C_1 - C_2/T - C_3 \ln(T/T_0) - C_4 T + C_5 T^2$$

- Advantage : Great sensitivity in the applicable temperature range
- Disadvantage :Limited range



< Vapor-pressure thermometer >

6.8. Magnetic thermometer

For temperature measurement below 1.0K.

From Curie law, define a magnetic temperature T^*

$$T^* = \frac{C}{\chi}\ ; C = The\ Curie\ constant, \chi = the\ magnetic\ susceptibility$$

Following are some correlations of magnetic temperature with the absolute temperature.

- Cerium magnesium nitrate, for $T \geq 0.004\text{K}$

$$T^* = T + (0.236 + 0.004137/T)(10^{-3})$$

- Chromic methylammonium alum, for $T \geq 0.070\text{K}$

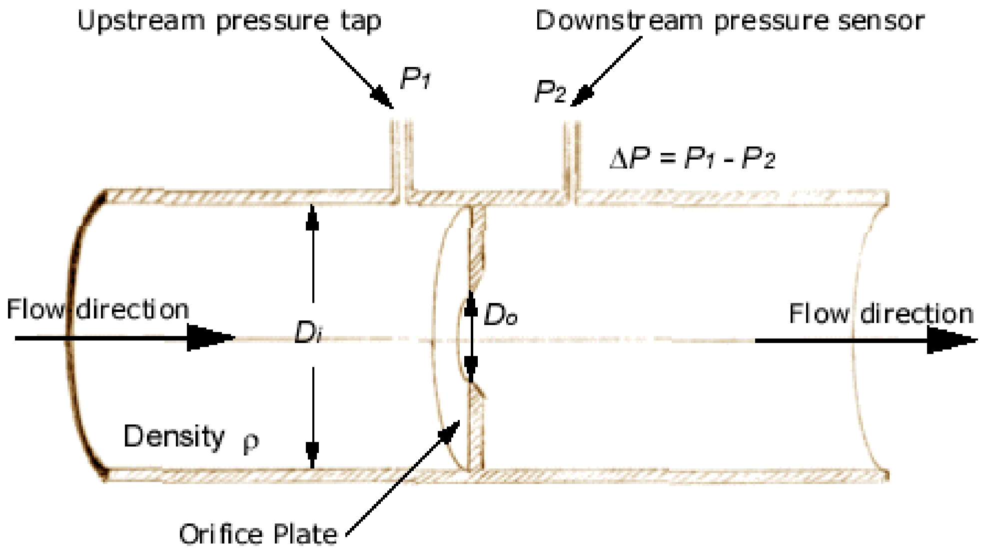
$$T^* = T + 0.00250 + 0.002422/T$$

- Chromium potassium alum, for $T \geq 0.10\text{K}$

$$T^* = T + 0.000862 + 0.002057/T$$

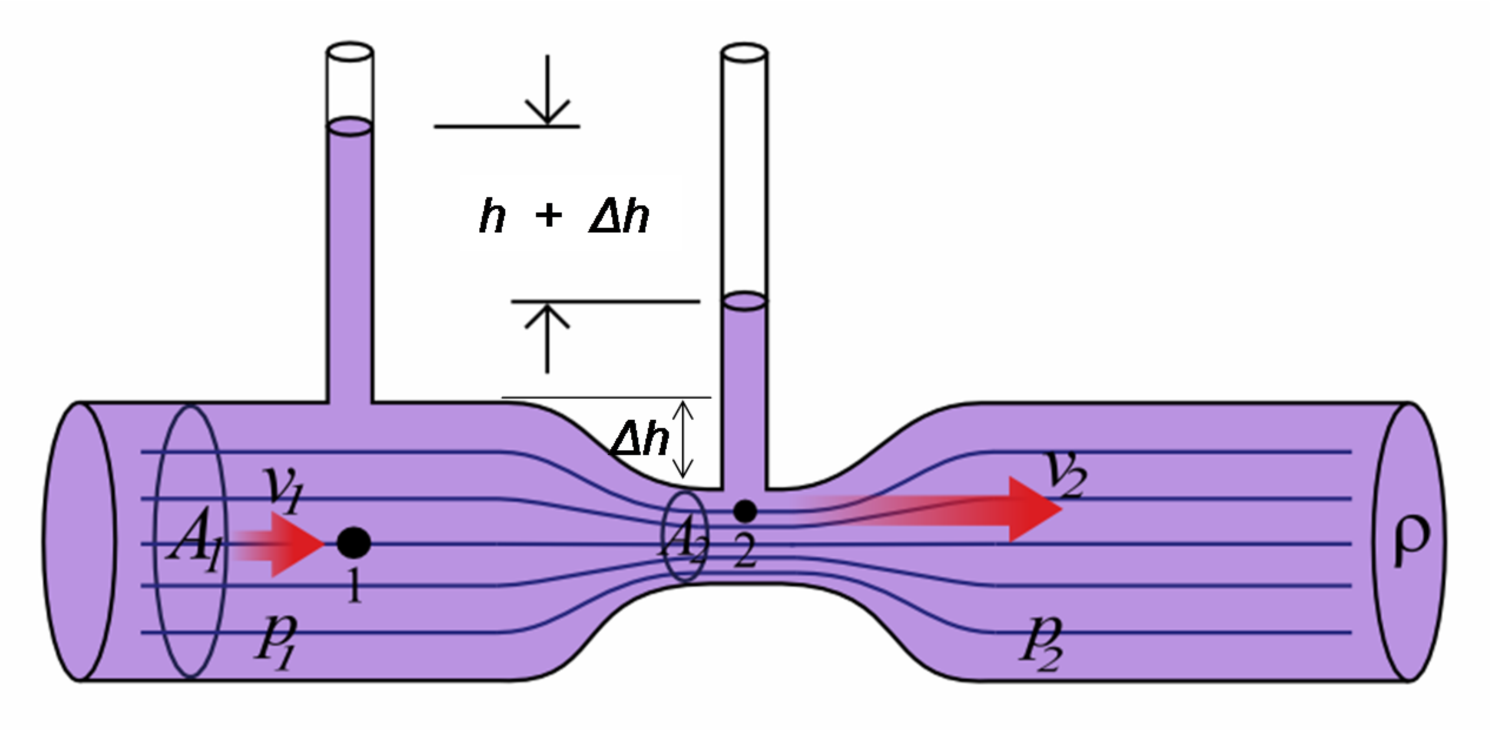
6.9. Orifice meters

A fluid passing though an orifice constriction will experience a drop in pressure across the orifice. This change can be used to measure the flowrate of the fluid. To calculate the flowrate of a fluid passing through an orifice plate, enter the parameters below.



$$\Delta p = \frac{1}{2}\,\rho\,Q^2\,\frac{1}{{A_0}^2}\left[1 - \left(\frac{A_o}{A_i}\right)^2\right]$$

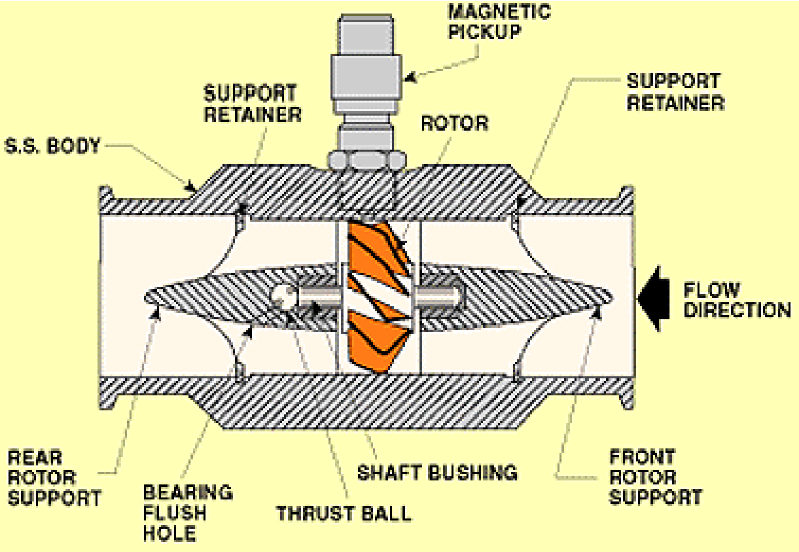
6.10. Venturi meter



$$P_1 - P_2 = \frac{\rho}{2}\left({v_2}^2 - {v_1}^2\right)$$

- **Advantages of turbine flow meter**

  - Simple, durable structure
  - Easy to install and maintain
  - Turbine meters are able to operate under a wide range of temperatures and pressures
  - Low pressure drop across the flow meter
  - Most effective in applications with steady, high-speed flows

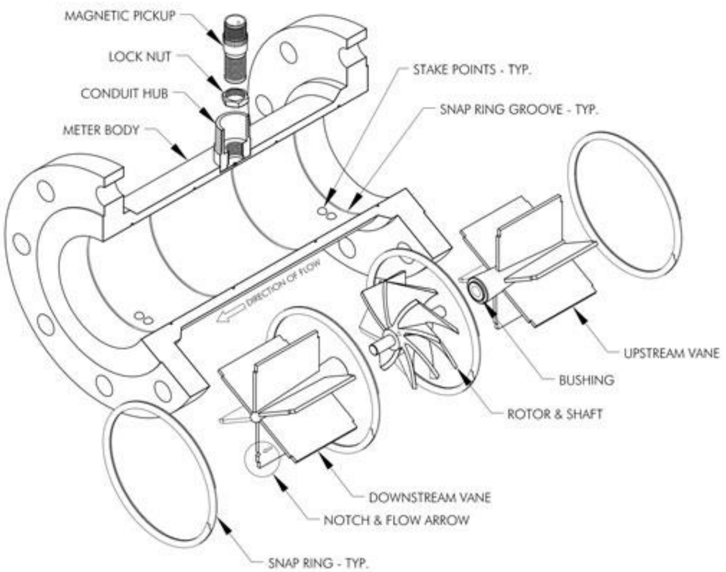


6.11. Turbine flowmeters

- **Disadvantages of turbine flow meter**

  - Require constant backpressure in order to avoid cavitation
  - Accuracy adversely affected by bubbles in liquids
  - Sensitive to changes in fluid viscosity

Turbine Meter Kits



6.12. Fluid-quality measurement

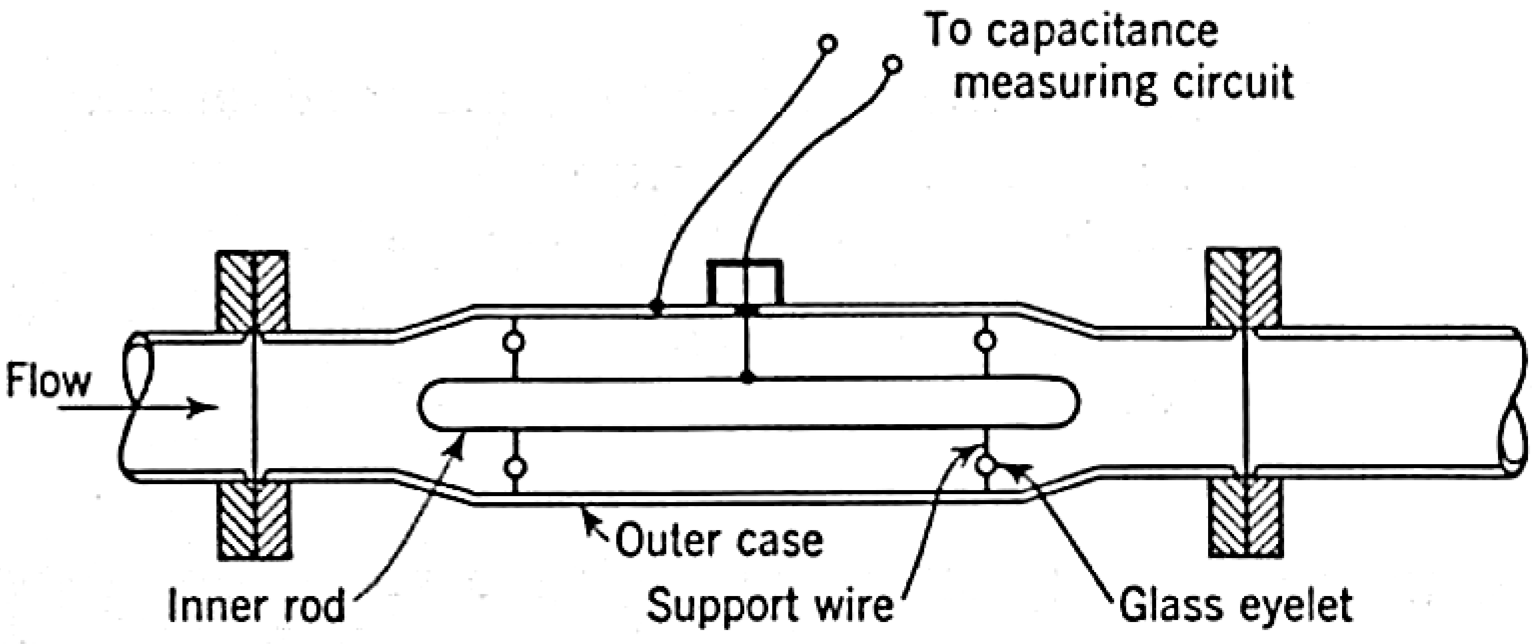


Fig. 6.12. Capacitance quality meter (Killian and Simpson 1960).

6.13. Hydrostatic gauges

- **Hydrostatic pressure is related to the liquid level L_f by**

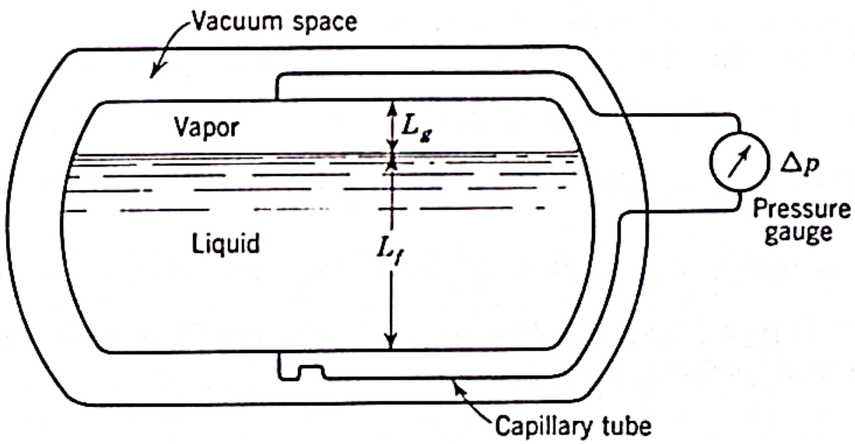


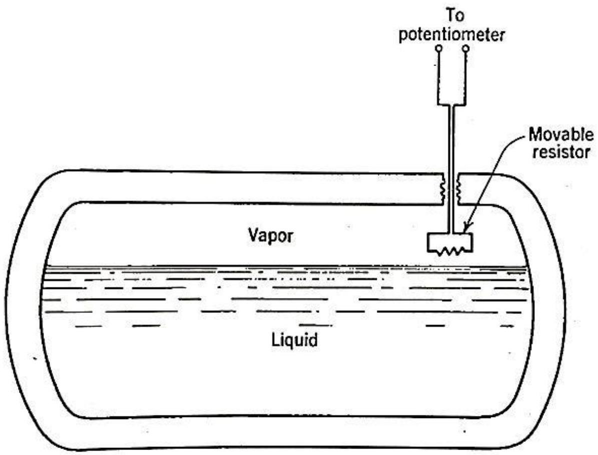
Fig. 6.13. Hydrostatic liquid-level gauge.

$$\Delta P = \frac{\rho_f L_f g}{g_c} + \frac{\rho_g L_g g}{g_c}$$

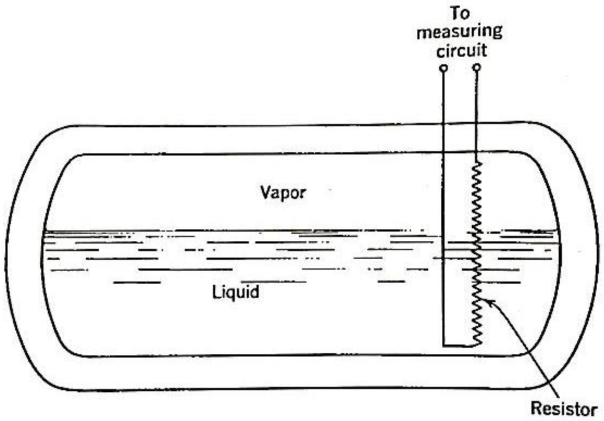
L_f = height of the liquid column
L_g= L-Lf = height of the vapor column
L = inside diameter of the vessel
ρ_f = liquid density
ρ_g = liquid density(saturated)
g = local acceleration due to gravity
g_c = conversion factor in Newton's Second Law of Motion

6.14. Electric-resistance gauges

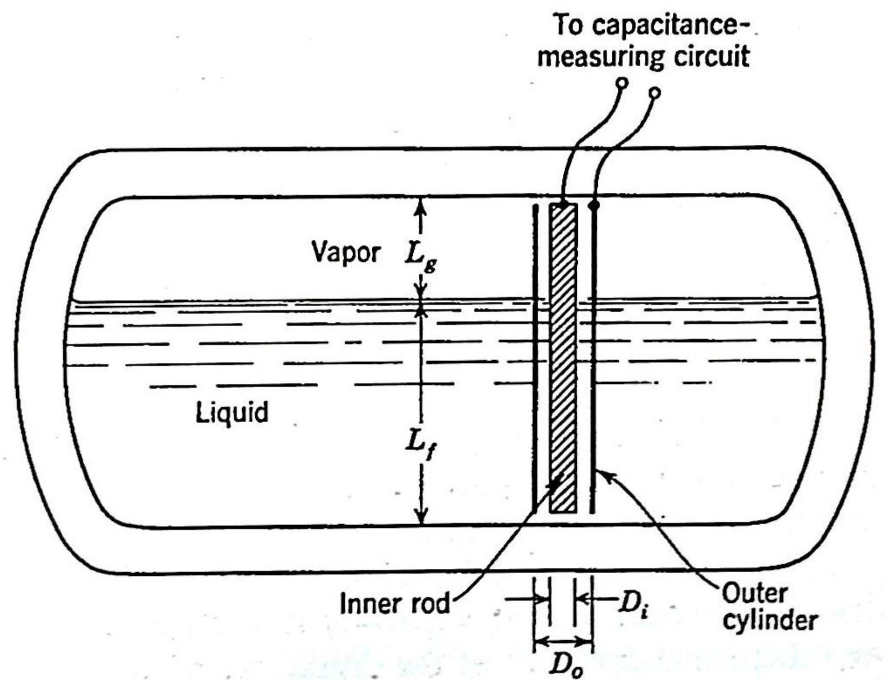
- **Movable electric resistance liquid-level gauge**



- **Fixed electric resistance liquid-level gauge**



6.15. Capacitance liquid-level probes



< Capacitance liquid-level gauge >

$$L_f = \frac{C\ ln\left(\frac{D_0}{D_i}\right)}{2\pi(\varepsilon_f - \varepsilon_g)\varepsilon_0} - \frac{\varepsilon_g L}{\varepsilon_f - \varepsilon_g}$$

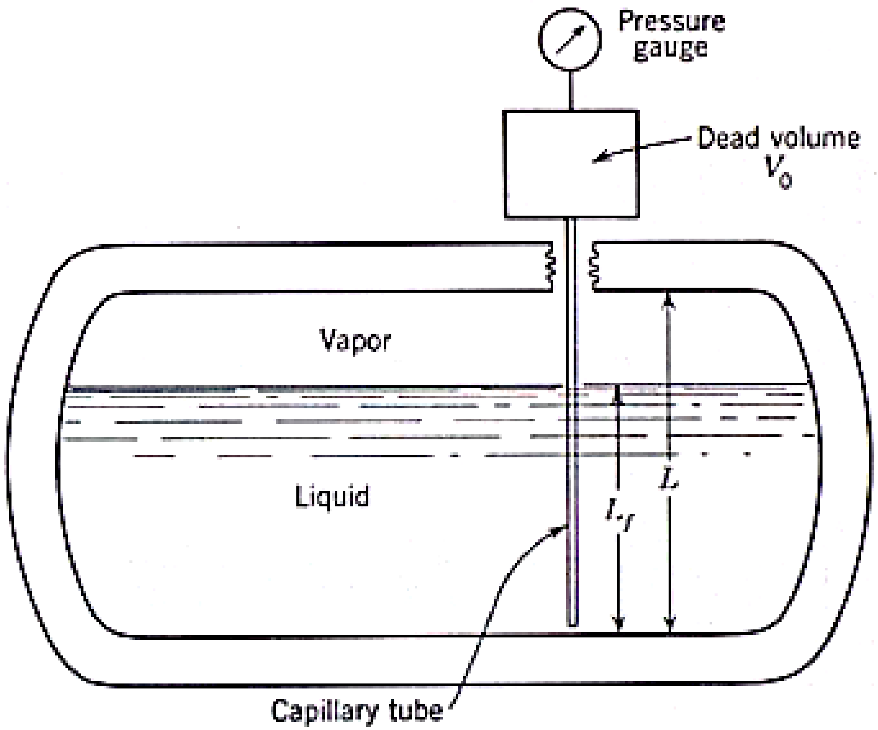
$\varepsilon_0 = 8.8542 * 10^{-12}\frac{\mathrm{F}}{\mathrm{M}} = \text{permittibity of free space}$

$\varepsilon_g = The\ dielectric\ constants\ for\ the\ vapor$

$\varepsilon_f = The\ dielectric\ constants\ for\ the\ fluid$

$C = The\ total\ capacitance\ for\ the\ gauge$

6.16. Thermodynamic liquid-level gauge



$$\frac{L_f}{L} = \frac{[(m - \rho_0 V_0)/A_c L] - \rho_g}{\rho_f - \rho_g}$$

< Thermodynamic liquid-level gauge >