Design Theories of Ship and Offshore Plant

Part II. Optimum Design

Ch. 1 Introduction to Optimum Design

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Contents

Ch. 1 Introduction to Optimum Design

1.1 Overview

Indeterminate and Determinate Problems (1/2)

Variables: x_1, x_2, x_3

Equation: $x_1 + x_2 + x_3 = 3$

- ✓ Number of variables: 3
- ✓ Number of equations: 1

Because the number of variables is larger than that of equations, this problem forms an indeterminate system.

Solution for the indeterminate problem:

We assume two unknown variables

↑ Number of variables (3) – Number of equations (1)

Example) assume that $x_1 = 1, x_2 = 0$

➡ $x_3 = 2$

Equation of straight line

$y = a_0 + a_1 x$ Where, a_0, a_1 are given.

- ✓ Number of variables: 2 x, y
- ✓ Number of equations: 1

☞ We can get the value of y by assuming x.

Finding intersection point (x^*, y^*) of two straight lines

$y = a_0 + a_1 x$ Where, a_0, a_1, b_0, b_1 are given.

$y = b_0 + b_1 x$

- ✓ Number of variables: 2 x, y
- ✓ Number of equations: 2

Because the number of variables is equals to that of equations, this problem forms an determinate system.

Indeterminate and Determinate Problems (2/2)

Determinate problem

Variables: x_1, x_2, x_3

Equations: $f_1(x_1, x_2, x_3) = 0$

$f_2(x_1, x_2, x_3) = 0$

$f_3(x_1, x_2, x_3) = 0$

If f_1, f_2, and f_3 are linearly independent, then

- ✓ Number of variables: 3
- ✓ Number of equations: 3

Since the number of equations is equal to that of variables, this problem can be solved.

What happens if $2 \times f_3 = f_2$?

Then f_2 and f_3 are linearly dependent.

Since the number of equations, which are linearly independent, is less than that of variables, this problem forms an indeterminate system.

Indeterminate problem

Variables: x_1, x_2, x_3

Equations: $f_1(x_1, x_2, x_3) = 0$

$f_2(x_1, x_2, x_3) = 0$

$f_3(x_1, x_2, x_3) = 0$

If f_1 and f_2 are only linearly independent, then

- ✓ Number of variables: 3
- ✓ Number of equations: 2

Since the number of equations is less than that of variables, one equation should be added to solve this problem.

Added Equation	Solution
$f_4^1 = 0$	(x_1^1, x_2^1, x_3^1)
$f_4^2 = 0$	(x_1^2, x_2^2, x_3^2)
⋮	

We can obtain many sets of solutions by assuming different equations.

➡ Indeterminate problem

We need a certain criteria to determine the proper solution. By adding the criteria, this problem can be formulated as an optimization problem.

Example of a Design Problem

Esthetic* Design of a Dress

Find (Design variables)
- Size, material, color, etc.

Constraints
- There are some requirements, but it can be difficult to formulate them.
- By using the sense of a designer, the requirements are satisfied.

Objective function (Criteria to determine the proper design variables)
- There are many design alternatives.
- Among them, we should select the best one. How?
- Criteria: Preference, cost, etc.
- But it can be also difficult to formulate the objective function.

➡ **Indeterminate, optimization problem**

Mathematical Model for Determination of Optimal Principal Dimensions of a Ship - Summary ("Conceptual Ship Design Equation")

Find (Design variables) $L,\ B,\ D,\ C_B$ (length, breadth, depth, block coefficient)

Given (Owner's requirements) $DWT,\ CC_{req},\ T_{\max}(=T),\ V$ (deadweight, Required cargo hold capacity, maximum draft, ship speed)

Physical constraint

→ Displacement - Weight equilibrium (Weight equation) – Equality constraint

$$L\cdot B\cdot T\cdot C_B\cdot \rho_{sw}\cdot C_\alpha = DWT_{given} + LWT(L,B,D,C_B)$$
$$= DWT_{given} + C_s\cdot L^{1.6}(B+D) + C_o\cdot L\cdot B$$
$$+ C_{power}\cdot (L\cdot B\cdot T\cdot C_B)^{2/3}\cdot V^3 \quad \cdots(2.3)$$

Economical constraints (Owner's requirements)

→ Required cargo hold capacity (Volume equation) - Equality constraint

$$CC_{req} = C_{CH}\cdot L\cdot B\cdot D \cdots(3.1)$$

- DFOC (Daily Fuel Oil Consumption)
  : It is related with the resistance and propulsion.
- Delivery date
  : It is related with the shipbuilding process.

Regulatory constraint

→ Freeboard regulation (ICLL 1966) - Inequality constraint

$$D \ge T + C_{FB}\cdot D \cdots(4)$$

$$Min.Roll\ Period : e.g.,$$
$$T_R \ge 12\ \sec ....(6)$$

Stability regulation (MARPOL, SOLAS, ICLL)

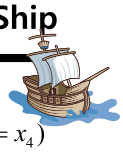
$$GM \ge GM_{Required} \cdots(5)$$
$$GZ \ge GZ_{Required}$$

Objective function (Criteria to determine the proper principal dimensions)

$$Building\ Cost = C_{PS}\cdot C_s\cdot L^{1.6}(B+D) + C_{PO}\cdot C_o\cdot L\cdot B + C_{PM}\cdot C_{power}\cdot (L\cdot B\cdot T\cdot C_B)^{2/3}\cdot V^3$$

4 variables (L, B, D, C_B), 2 equality constraints ((2.3), (3.1)), 3 inequality constraints ((4), (5), (6))
➡ Optimization problem

Determination of Optimal Principal Dimensions of a Ship



Optimization Problem

➡ Minimize/maximize an objective function with constraints on design variables

Find (Design variables)

x_1, x_2, x_3, x_4

Equality constraint

$h(x_1, x_2, x_3, x_4) = 0$

Inequality constraint

$g(x_1, x_2, x_3, x_4) \le 0$

Objective function

$f(x_1, x_2, x_3, x_4)$

Engineering Design of Ship (Simplified)

Find (Design variables) $L(=x_1), B(=x_2), D(=x_3), C_B(=x_4)$

length, breadth, depth, block coefficient

Weight equation

$$L \cdot B \cdot T \cdot C_B \cdot \rho_{sw} \cdot C_\alpha = DWT_{given} + LWT(L, B, D, C_B)$$

$$x_1 \cdot x_2 \cdot x_4 \cdot C_1 = C_2 + h'(x_1, x_2, x_3, x_4)$$

$$x_1 \cdot x_2 \cdot x_4 \cdot C_1 - C_2 - h'(x_1, x_2, x_3, x_4) = h(x_1, x_2, x_3, x_4) = 0$$

Objective function (Criteria to determine the proper principal dimensions)

$$Building\ Cost = C_{PS} \cdot C_s \cdot L^{1.6}(B + D) + C_{PO} \cdot C_o \cdot L \cdot B + C_{PM} \cdot C_{ma} \cdot NMCR$$

$$f(x_1, x_2, x_3, x_4) = C_3 \cdot x_1^{1.6} \cdot (x_2 + x_3) + C_4 \cdot x_1 \cdot x_2 + C_5$$

- $T, C_\alpha, \rho_{sw}, DWT_{given}, C_{PS}, C_s, C_{PO}, C_o, C_{PM}, C_{ma}, NMCR$ *are* **Given**

Characteristics of the constraints

- ✓ **Physical constraints are usually formulated as equality constraints.**
  (Example of ship design: Weight equation)
- ✓ **Economical constraints, regulatory constraints, and constraints related with politics and culture are formulated as inequality constraints.**
  (Example of ship design: Required cargo hold capacity (Volume equation), Freeboard regulation (ICLL 1966))

Classification of Optimization Problems and Methods - Summary

<table>
<tr><th rowspan="2"></th><th colspan="2">Unconstrained optimization problem</th><th colspan="3">Constrained optimization problem</th></tr>
<tr><th>Linear</th><th>Nonlinear</th><th>Linear</th><th colspan="2">Nonlinear</th></tr>
<tr><td>Objective function (example)</td><td>Minimize $f(\mathbf{x})$
$f(\mathbf{x}) = x_1 + 2x_2$</td><td>Minimize $f(\mathbf{x})$
$f(\mathbf{x}) = x_1^2 + x_2^2 - 3x_1x_2$</td><td>Minimize $f(\mathbf{x})$
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$f(\mathbf{x}) = x_1^2 + x_2^2 - 3x_1x_2$</td></tr>
<tr><td>Constraints (example)</td><td>None</td><td>None</td><td>$h(\mathbf{x}) = x_1 + 5x_2 = 0$
$g(\mathbf{x}) = -x_1 \le 0$</td><td>$h(\mathbf{x}) = x_1 + 5x_2 = 0$
$g(\mathbf{x}) = -x_1 \le 0$</td><td>$g_1(\mathbf{x}) = \frac{1}{6}x_1^2 + \frac{1}{6}x_2^2 - 1.0 \le 0$
$g_2(\mathbf{x}) = -x_1 \le 0$</td></tr>
<tr><td rowspan="3">Optimization methods for continuous value</td><td colspan="2" rowspan="3">① Gradient method
- Steepest descent method
- Conjugate gradient method
- Newton method
- Davidon-Fletcher-Powell (DFP) method
- Broyden-Fletcher-Goldfarb-Shanno (BFGS) method

② Enumerative method
- Hooke & Jeeves method
- Nelder & Mead method
- Golden section search method</td><td>Linear Programming (LP) method is usually used.</td><td colspan="2">Penalty function method: Converting the constrained optimization problem to the unconstrained optimization problem by using the penalty function, the problem can be solved using unconstrained optimization method.</td></tr>
<tr><td>Simplex Method
(Linear Programming)</td><td rowspan="2">Quadratic programming (QP) method</td><td>Sequential Linear Programming (SLP) method
First, linearize the nonlinear problem and then obtain the solution to this linear approximation problem using the linear programming method. And then, repeat the linearization.</td></tr>
<tr><td></td><td>Sequential Quadratic Programming (SQP) method
First, approximate a quadratic objective function and linear constraints, find the search direction and then obtain the solution to this quadratic programming problem in this direction. And then, repeat the approximation.</td></tr>
<tr><td>Optimization methods for discrete value</td><td colspan="5">Integer programming: ① Cut algorithm ② Enumeration algorithm ③ Constructive algorithm</td></tr>
<tr><td>Metaheuristic optimization</td><td colspan="5">Genetic algorithm (GA), Ant algorithm, Simulated annealing, etc.</td></tr>
</table>

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1.2 Problem Statement of Optimum Design

General Formulation of an Optimization Problem

Minimize **Objective Function**

$$f(\mathbf{x})$$

Subject to **Constraints**

$$g_j(\mathbf{x}) \le 0, j = 1, \cdots, m$$

: Inequality Constraint

$$h_k(\mathbf{x}) = 0, k = 1, \cdots, p$$

: Equality Constraint

$$\mathbf{x}_l \le \mathbf{x} \le \mathbf{x}_u$$

: Upper and Lower Limits of Design Variables

Where $\mathbf{x} = (x_1, x_2, \cdots, x_n)$ **Design Variables**

Components of an Optimization Problem (1/3)

☑ **Design variable**

- **A set of variables that describe the system such as size and position, etc.**
- **It is also called 'Free variable' or 'Independent variable'.**
- **Cf. Dependent Variable**
  : A variable that is dependent on the design variable (independent variable)

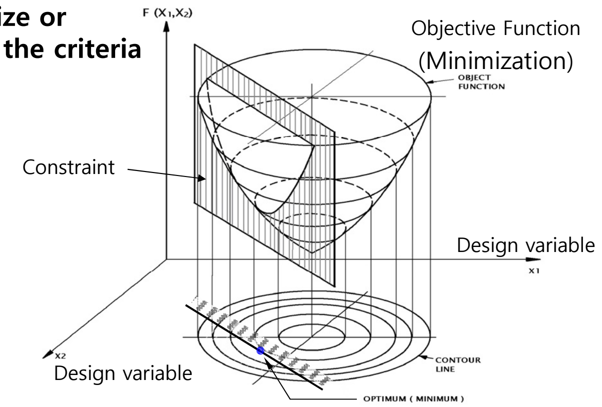
☑ **Constraint**

- **A certain set of specified requirements and restrictions placed on a design**
- **It is a function of the design variables.**
- **Inequality Constraint ('≤' or '≥'), Equality Constraint ('=')**

Components of an Optimization Problem (2/3)

☑ **Objective function**

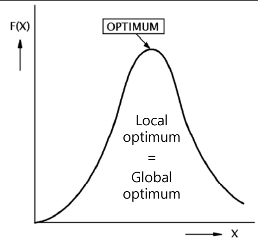
- **A criteria to compare the different design and determine the proper design such as cost, profit, weight, etc.**
- **It is a function of the design variables.**
- **To minimize or maximize the criteria**



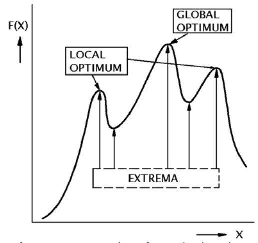
Components of an Optimization Problem (3/3)

Determination of the optimal design considering the objective function (maximization) and constraints

Local optimum: The best near or in narrow region
Global optimum: The best in whole region

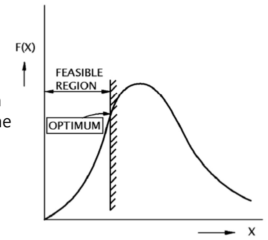


(a) Unconstrained optimization, Unimodal case

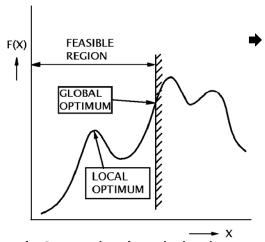


(b) Unconstrained optimization, Multimodal case

The region satisfying the constraint



(c) Constrained optimization, Unimodal case



(d) Constrained optimization, Multimodal case

➡ **Optimal design can be changed according to the constraints.**

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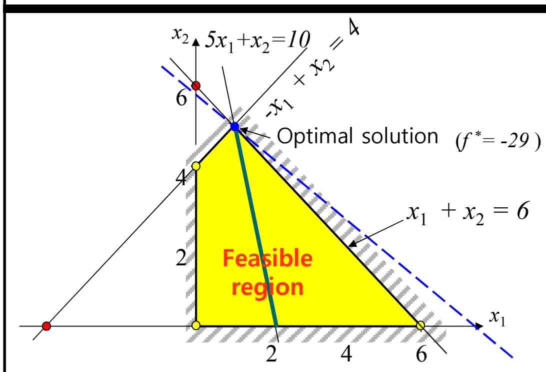
Example of the Formulation of an Optimization Problem



Minimize $f=-4x_1-5x_2$		*Minimize* $f(\mathbf{x})$ — **Objective Function**
Subject to $-x_1+x_2\le 4$, $x_1+x_2\le 6$	⇨ $-x_1+x_2-4\le 0$, $x_1+x_2-6\le 0$	*Subject to* $g_j(\mathbf{x})\le 0, j=1,\cdots,m$ — **Constraints** : Inequality constraint
$5x_1+x_2=10$	⇨ $5x_1+x_2-10=0$	$h_k(\mathbf{x})=0, k=1,\cdots,p$: Equality constraint
$0\le x_1,x_2$		$\mathbf{x}_l\le\mathbf{x}\le\mathbf{x}_u$: Constraint about limits of design variables

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1.3 Classification of Optimization Problems

Classification of Optimization Problems (1/4)

☑ Existence of constraints

- **Unconstrained optimization problem**
  - Minimize the objective function $f(\mathrm{x})$ without any constraints on the design variables x.

Minimize	$f(\mathrm{x})$

- **Constrained optimization problem**
  - Minimize the objective function $f(\mathrm{x})$ with some constraints on the design variables x.

Minimize	$f(\mathrm{x})$
Subject to	$h(\mathrm{x})=0$
	$g(\mathrm{x})\leq 0$

Classification of Optimization Problems (2/4)

☑ Number of objective functions

- **Single-objective optimization problem**

Minimize	$f(\mathrm{x})$
Subject to	$h(\mathrm{x})=0$
	$g(\mathrm{x})\leq 0$

- **Multi-objective optimization problem**
  - Weighting Method, Constraint Method, etc.

Minimize	$f_1(\mathrm{x}), f_2(\mathrm{x}), f_3(\mathrm{x})$
Subject to	$h(\mathrm{x})=0$
	$g(\mathrm{x})\leq 0$

Classification of Optimization Problems (3/4)

☑ Linearity of objective function and constraints

- **Linear optimization problem**
  - The objective function ($f(\mathbf{x})$) and constraints ($h(\mathbf{x})$, $g(\mathbf{x})$) are linear functions of the design variables $\mathbf{x}$.

Minimize
$$f(\mathbf{x}) = x_1 + 2x_2$$
Subject to
$$h(\mathbf{x}) = x_1 + 5x_2 = 0$$
$$g(\mathbf{x}) = -x_1 \leq 0$$

- **Nonlinear optimization problem**
  - The objective function ($f(\mathbf{x})$) or constraints ($h(\mathbf{x})$, $g(\mathbf{x})$) are nonlinear functions of the design variables $\mathbf{x}$.

Minimize
$$f(\mathbf{x}) = x_1^2 + x_2^2 - 3x_1x_2$$
Subject to
$$h(\mathbf{x}) = x_1 + 5x_2 = 0$$
$$g(\mathbf{x}) = -x_1 \leq 0$$

Minimize
$$f(\mathbf{x}) = x_1^2 + x_2^2 - 3x_1x_2$$
Subject to
$$g_1(\mathbf{x}) = \frac{1}{6}x_1^2 + \frac{1}{6}x_2^2 - 1.0 \leq 0$$
$$g_2(\mathbf{x}) = -x_1 \leq 0$$

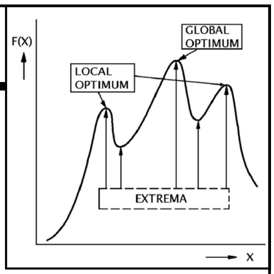
Classification of Optimization Problems (4/4)

☑ Type of design variables

- Continuous optimization problem
  - Design variables are continuous in the optimization problem.
- Discrete optimization problem
  - Design variables are discrete in the optimization problem.
  - It is also called a 'combinatorial optimization problem'.
  - Example) Integer programming problem

1.4 Classification of Optimization Methods

Classification of Optimization Method



☑ Global Optimization Method

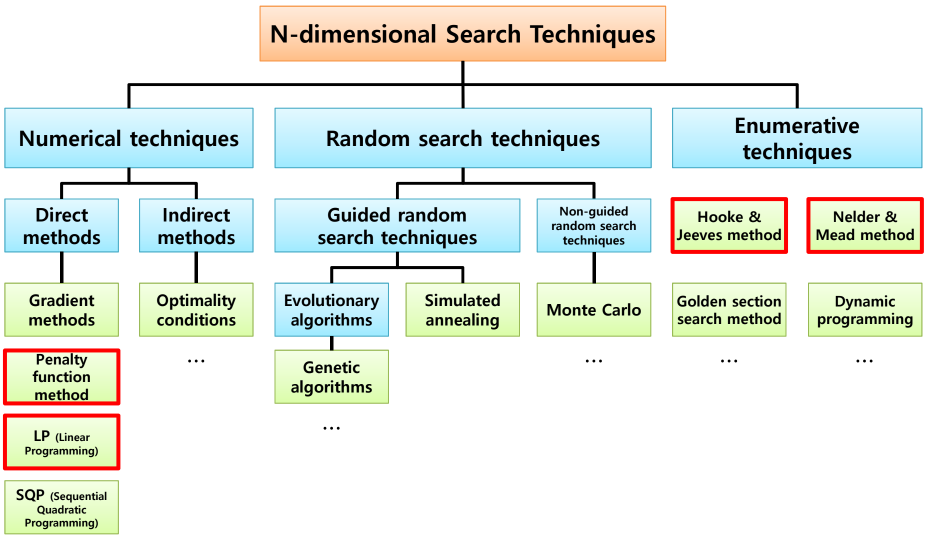
- **Advantage**
  - It is useful for solving the global optimization problem which has many local optima.
- **Disadvantage**
  - It needs many iterations (much time) to obtain the optimum.
- **Genetic Algorithms (GA), Simulated Annealing, etc.**

☑ Local Optimization Method

- **Advantage**
  - It needs relatively few iterations (less time) to obtain the optimum.
- **Disadvantage**
  - It is only able to find the local optimum which is near to the starting point.
- **Sequential Quadratic Programming (SQP), Method of Feasible Directions (MFD), Multi-Start Optimization Method, etc.**

* Local optimum: The best near or in narrow region
* Global optimum: The best in whole region

Classes of Search Techniques (1/4)



Classes of Search Techniques (2/4)

☑ Numerical techniques (Classical or calculus based techniques)

- Use deterministic approach to find best solution. That is, Use a set of necessary and sufficient conditions to be satisfied by the solutions of an optimization problem.
- Require knowledge of gradients or higher order derivatives.
- Indirect methods
  - Search for local extremes by solving the usually nonlinear set of equations resulting from setting the gradient of the objective function to zero.
  - The search for possible solutions (function peaks) starts by restricting itself to points with zero slope in all directions.
  - Methods using optimality conditions (e.g., Kuhn-Tucker condition)
- Direct methods
  - Seek extremes by hopping around the search space and assessing the gradient of the new point, which guides the search.
  - This is simply the notion of hill climbing, which finds the best local point by climbing the steepest permissible gradient.
  - These techniques can be used only on a restricted set of well behaved functions.
  - Gradient methods, Penalty function method, LP, SQP, etc.

Classes of Search Techniques (3/4)

☑ Guided random search techniques (Stochastic techniques)

- Based on enumerative, stochastic techniques but use additional information to guide the search.
- Two major subclasses are simulated annealing and evolutionary algorithms that both can be seen as evolutionary processes.
- Evolutionary algorithms
  - Use natural selection principles.
  - This form of search evolves throughout generations, improving the features of potential solutions by means of biological inspired operations.
  - Genetic algorithms, Evolutionary Strategies (ES), etc.
- Simulated annealing
  - Uses a thermodynamic evolution process to search minimum energy states.

Classes of Search Techniques (4/4)

☑ Enumerative techniques

- Search every point related to the function's domain space (finite or discretized), one point at a time.
- At each point, all possible solutions are generated and tested to find optimum.
- They are very simple to implement but usually require significant computation.
- These techniques are not suitable for applications with large domain spaces.
- Dynamic programming, Hooke and Jeeves method, Nelder and Mead method, golden section method, etc.