Design Theories of Ship and Offshore Plant

Part II. Optimum Design

Ch. 1 Introduction to Optimum Design

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1.1 Overview
### Indeterminate and Determinate Problems (1/2)

**Variables:** \(x_1, x_2, x_3\)  
**Equation:** \(x_1 + x_2 + x_3 = 3\)

- Number of variables: 3  
- Number of equations: 1

Because the number of variables is larger than that of equations, this problem forms an indeterminate system.

**Solution for the indeterminate problem:**

We assume two unknown variables

Number of variables (3) – Number of equations (1)  

\(x_3 = 2\)

Finding intersection point \((x', y')\) of two straight lines

**Equation of straight line**

\[y' = a_0 + a_1 x\]  
Where \(a_0, a_1\) are given.

- Number of variables: 2  
- Number of equations: 1

We can get the value of \(y\) by assuming \(x\).

### Indeterminate and Determinate Problems (2/2)

**Determinate problem**

**Variables:** \(x_1, x_2, x_3\)  
**Equations:**

\[f_1(x_1, x_2, x_3) = 0\]
\[f_2(x_1, x_2, x_3) = 0\]
\[f_3(x_1, x_2, x_3) = 0\]

- Number of variables: 3  
- Number of equations: 3

Since the number of equations is equal to that of variables, this problem can be solved.

**Indeterminate problem**

**Variables:** \(x_1, x_2, x_3\)  
**Equations:**

\[f_1(x_1, x_2, x_3) = 0\]
\[f_2(x_1, x_2, x_3) = 0\]
\[f_3(x_1, x_2, x_3) = 0\]

- Number of variables: 3  
- Number of equations: 2

Since the number of equations is less than that of variables, one equation should be added to solve this problem.

We can obtain many sets of solutions by assuming different equations.

We need a certain criteria to determine the proper solution. By adding the criteria, this problem can be formulated as an optimization problem.
Example of a Design Problem

Esthetic* Design of a Dress

Find (Design variables)
- Size, material, color, etc.

Constraints
- There are some requirements, but it can be difficult to formulate them.
- By using the sense of a designer, the requirements are satisfied.

Objective function (Criteria to determine the proper design variables)
- There are many design alternatives.
- Among them, we should select the best one. How?
- Criteria: Preference, cost, etc.
- But it can be also difficult to formulate the objective function.

Mathematical Model for Determination of Optimal Principal Dimensions of a Ship
- Summary ("Conceptual Ship Design Equation")

<table>
<thead>
<tr>
<th>Find (Design variables)</th>
<th>Given (Owner’s requirements)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L, B, D, C_b</td>
<td>DWT, CC req, T_max (= T), V</td>
</tr>
</tbody>
</table>

Physical constraint

Displacement - Weight equilibrium (Weight equation) - Equality constraint

\[ L \cdot B \cdot T \cdot C_b \cdot \rho = DWT_{\text{given}} + LWT(L, B, D, C_b) \]

\[ DWT_{\text{given}} = C_1 \cdot L^2 \cdot (B + D) + C_2 \cdot L \cdot B + C_3 \left( L \cdot B \cdot T \cdot C_b \right)^{2/3} \cdot V^{3/2} \]

Economical constraints (Owner’s requirements)

Required cargo hold capacity (Volume equation) - Equality constraint

\[ CC_{\text{req}} = C_{\text{Ch}} \cdot L \cdot B \cdot D \]

Regulatory constraint

Freeboard regulation (ICLL 1966) - Inequality constraint

\[ D \geq T + C_{\text{Fg}} \cdot D \]

Stability regulation (MARPOL, SOLAS, ICLL)

\[ GM \geq GM_{\text{Required}} \]

Objective function (Criteria to determine the proper principal dimensions)

\[ GZ \geq GZ_{\text{req}} \]

Building Cost = \[ C_{\text{Fg}} \cdot C_{\text{Ch}} \cdot L^2 \cdot (B + D) + C_{\text{Fg}} \cdot C_{\text{Ch}} \cdot L \cdot B + C_{\text{Fg}} \cdot C_{\text{Power}} \cdot (L \cdot B \cdot T \cdot C_b)^{2/3} \cdot V^{3/2} \]

4 variables (L, B, D, C_b), 2 equality constraints ((2.3), (3.1)), 3 inequality constraints ((4), (5), (6))

Optimization problem
## Determination of Optimal Principal Dimensions of a Ship

### Classification of Optimization Problems and Methods

- **Unconstrained optimization problem**
  - Linear
  - Nonlinear
- **Constrained optimization problem**
  - Linear
  - Nonlinear

<table>
<thead>
<tr>
<th>Objective function (example)</th>
<th>Minimize $f(x)$</th>
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<th>Minimize $f(x)$</th>
<th>Minimize $f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear programming (LP) method&lt;br&gt;(Linear Programming)&lt;br&gt;</td>
<td>$f(x) = x_1 + 2x_2$</td>
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</tr>
<tr>
<td>Nonlinear programming (QP) method&lt;br&gt;Penalty function method&lt;br&gt;</td>
<td>$f(x) = x_1^2 + x_2^2 - 3x_3 x_4$</td>
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</tbody>
</table>

**Optimization methods for continuous value**


**Optimization methods for discrete value**

1. Cut algorithm<br>2. Enumeration algorithm<br>3. Constructive algorithm

**Metaheuristic optimization**

- **Design Theories of Ship and Offshore Plant, September 2016, Myung-Il Roh**

- **Summary**

- **Classification of Optimization Problems and Methods**

- **Design of Ship and Offshore Plant**

- **Optimization Problem**

- **Engineering Design of Ship (Simplified)**

<table>
<thead>
<tr>
<th>Find (Design variables)</th>
<th>$L(x_1), B(x_2), D(x_3), C_B(x_4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Weight equation</strong></td>
<td>$L \cdot B \cdot T \cdot C_B \cdot D_B \cdot C_p = DWT_{exp} + LWT(L, B, D, C_B)$</td>
</tr>
<tr>
<td><strong>Constraints</strong></td>
<td>$h(x_1, x_2, x_3, x_4) = 0$</td>
</tr>
<tr>
<td><strong>Inequality constraint</strong></td>
<td>$g(x_1, x_2, x_3, x_4) \leq 0$</td>
</tr>
<tr>
<td><strong>Objective function</strong></td>
<td>$f(x_1, x_2, x_3, x_4)$</td>
</tr>
</tbody>
</table>

- **Characteristics of the constraints**

  - Physical constraints are usually formulated as equality constraints.
  - Economical constraints, regulatory constraints, and constraints related with politics and culture are formulated as inequality constraints.
  - Economical constraints, regulatory constraints, and constraints related with politics and culture are formulated as inequality constraints.

- **Example of ship design: Required cargo hold capacity (Volume equation), Freeboard regulation (ICLL 1966)**
1.2 Problem Statement of Optimum Design

General Formulation of an Optimization Problem

Minimize

\[ f(x) \]

Subject to

\[ g_j(x) \leq 0, \ j = 1, \ldots, m \]

: Inequality Constraint

\[ h_k(x) = 0, \ k = 1, \ldots, p \]

: Equality Constraint

\[ x_l \leq x \leq x_u \]

: Upper and Lower Limits of Design Variables

Where \( x = (x_1, x_2, \ldots, x_n) \)
Components of an Optimization Problem (1/3)

- **Design variable**
  - A set of variables that describe the system such as size and position, etc.
  - It is also called ‘Free variable’ or ‘Independent variable’.
  - Cf. Dependent Variable
    : A variable that is dependent on the design variable (independent variable)

- **Constraint**
  - A certain set of specified requirements and restrictions placed on a design
  - It is a function of the design variables.
  - Inequality Constraint (‘≤’ or ‘≥’), Equality Constraint (‘=’)

Components of an Optimization Problem (2/3)

- **Objective function**
  - A criteria to compare the different design and determine the proper design such as cost, profit, weight, etc.
  - It is a function of the design variables.
  - To minimize or maximize the criteria

![Diagram of Objective Function (Minimization)]
Components of an Optimization Problem (3/3)

**Determination of the optimal design considering the objective function (maximization) and constraints**

- Local optimum: The best near or in narrow region
- Global optimum: The best in whole region

(a) Unconstrained optimization, Unimodal case
(b) Unconstrained optimization, Multimodal case
(c) Constrained optimization, Unimodal case
(d) Constrained optimization, Multimodal case

The region satisfying the constraint

Optimal design can be changed according to the constraints.

Example of the Formulation of an Optimization Problem

Minimize: \( f = -4x_1 - 5x_2 \)
Subject to:
- \( -x_1 + x_2 \leq 4 \)
- \( x_1 + x_2 \leq 6 \)
- \( x_1 + x_2 - 4 \leq 0 \)
- \( x_1 + x_2 - 6 \leq 0 \)

Minimize: \( f(x) \)
Subject to:
- \( g_k(x) \leq 0, f = 1, \ldots, m \) : Inequality constraint
- \( h_k(x) = 0, k = 1, \ldots, p \) : Equality constraint
- \( x_i \leq x \leq x_i \) : Constraint about limits of design variables

Optimal solution: \( f^* = -29 \)

Feasible region
1.3 Classification of Optimization Problems

Classification of Optimization Problems (1/4)

☑ Existence of constraints

- Unconstrained optimization problem
  - Minimize the objective function \( f(x) \) without any constraints on the design variables \( x \).

\[
\begin{align*}
\text{Minimize} & \quad f(x) \\
\end{align*}
\]

- Constrained optimization problem
  - Minimize the objective function \( f(x) \) with some constraints on the design variables \( x \).

\[
\begin{align*}
\text{Minimize} & \quad f(x) \\
\text{Subject to} & \quad h(x) = 0 \\
& \quad g(x) \leq 0
\end{align*}
\]
Classification of Optimization Problems (2/4)

- Number of objective functions
  - Single-objective optimization problem
    \[
    \text{Minimize} \quad f(x) \\
    \text{Subject to} \quad h(x) = 0 \\
    g(x) \leq 0
    \]

  - Multi-objective optimization problem
    - Weighting Method, Constraint Method, etc.
    \[
    \begin{align*}
    \text{Minimize} & \quad f_1(x), f_2(x), f_3(x) \\
    \text{Subject to} & \quad h(x) = 0 \\
    & \quad g(x) \leq 0
    \end{align*}
    \]

Classification of Optimization Problems (3/4)

- Linearity of objective function and constraints
  - Linear optimization problem
    - The objective function \( f(x) \) and constraints \( h(x), g(x) \) are linear functions of the design variables \( x \).
    \[
    \begin{align*}
    \text{Minimize} & \quad f(x) = x_1 + 2x_2 \\
    \text{Subject to} & \quad h(x) = x_1 + 5x_2 = 0 \\
    & \quad g(x) = -x_1 \leq 0
    \end{align*}
    \]

  - Nonlinear optimization problem
    - The objective function \( f(x) \) or constraints \( h(x), g(x) \) are nonlinear functions of the design variables \( x \).
    \[
    \begin{align*}
    \text{Minimize} & \quad f(x) = x_1^2 + x_2^2 - 3x_1x_2 \\
    \text{Subject to} & \quad h(x) = x_1 + 5x_2 = 0 \\
    & \quad g(x) = -x_1 \leq 0
    \end{align*}
    \]
    \[
    \begin{align*}
    \text{Minimize} & \quad f(x) = x_1^2 + x_2^2 - 3x_1x_2 \\
    \text{Subject to} & \quad g_1(x) = \frac{1}{6}x_1^2 + \frac{1}{6}x_2^2 - 1.0 \leq 0 \\
    & \quad g_2(x) = -x_1 \leq 0
    \end{align*}
    \]

Classification of Optimization Problems (4/4)

- Type of design variables
  - Continuous optimization problem
    - Design variables are continuous in the optimization problem.
  - Discrete optimization problem
    - Design variables are discrete in the optimization problem.
    - It is also called a ‘combinatorial optimization problem’.
    - Example) Integer programming problem

1.4 Classification of Optimization Methods
Classification of Optimization Method

- **Global Optimization Method**
  - **Advantage**
    - It is useful for solving the global optimization problem which has many local optima.
  - **Disadvantage**
    - It needs many iterations (much time) to obtain the optimum.
    - Genetic Algorithms (GA), Simulated Annealing, etc.

- **Local Optimization Method**
  - **Advantage**
    - It needs relatively few iterations (less time) to obtain the optimum.
  - **Disadvantage**
    - It is only able to find the local optimum which is near to the starting point.
    - Sequential Quadratic Programming (SQP), Method of Feasible Directions (MFD), Multi-Start Optimization Method, etc.

Classes of Search Techniques (1/4)

- **N-dimensional Search Techniques**
- **Numerical techniques**
  - Direct methods
  - Indirect methods
    - Penalty function method
    - LP (Linear Programming)
    - SQP (Sequential Quadratic Programming)
- **Random search techniques**
  - Guided random search techniques
    - Evolutionary algorithms
  - Optimality conditions
    - Hooke & Jeeves method
    - Nelder & Mead method
- **Enumerative techniques**
  - Non-guided random search techniques
    - Genetic algorithms
    - Simulated annealing
    - Monte Carlo
    - Golden section search method
    - Dynamic programming
  - LP (Linear Programming)
  - SQP (Sequential Quadratic Programming)

* Local optimum: The best near or in narrow region
* Global optimum: The best in whole region
Classes of Search Techniques (2/4)

- **Numerical techniques (Classical or calculus based techniques)**
  - Use deterministic approach to find best solution. That is, Use a set of necessary and sufficient conditions to be satisfied by the solutions of an optimization problem.
  - Require knowledge of gradients or higher order derivatives.
  - **Indirect methods**
    - Search for local extremes by solving the usually nonlinear set of equations resulting from setting the gradient of the objective function to zero.
    - The search for possible solutions (function peaks) starts by restricting itself to points with zero slope in all directions.
    - Methods using optimality conditions (e.g., Kuhn-Tucker condition)
  - **Direct methods**
    - Seek extremes by hopping around the search space and assessing the gradient of the new point, which guides the search.
    - This is simply the notion of hill climbing, which finds the best local point by climbing the steepest permissible gradient.
    - These techniques can be used only on a restricted set of well behaved functions.
    - Gradient methods, Penalty function method, LP, SQP, etc.

Classes of Search Techniques (3/4)

- **Guided random search techniques (Stochastic techniques)**
  - Based on enumerative, stochastic techniques but use additional information to guide the search.
  - Two major subclasses are simulated annealing and evolutionary algorithms that both can be seen as evolutionary processes.
  - Evolutionary algorithms
    - Use natural selection principles.
    - This form of search evolves throughout generations, improving the features of potential solutions by means of biological inspired operations.
    - Genetic algorithms, Evolutionary Strategies (ES), etc.
  - Simulated annealing
    - Uses a thermodynamic evolution process to search minimum energy states.
Classes of Search Techniques (4/4)

Enumerative techniques

- Search every point related to the function’s domain space (finite or discretized), one point at a time.
- At each point, all possible solutions are generated and tested to find optimum.
- They are very simple to implement but usually require significant computation.
- These techniques are not suitable for applications with large domain spaces.
- Dynamic programming, Hooke and Jeeves method, Nelder and Mead method, golden section method, etc.