Design Theories of Ship and Offshore Plant

Part I. Ship Design

Ch. 5 Naval Architectural Calculation

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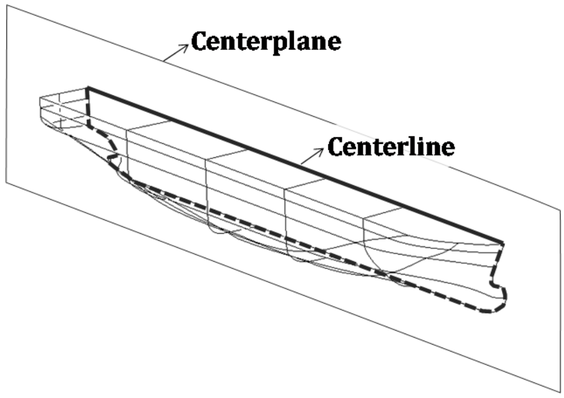
Contents

Ch. 5 Naval Architectural Calculation

5.1 Static Equilibrium

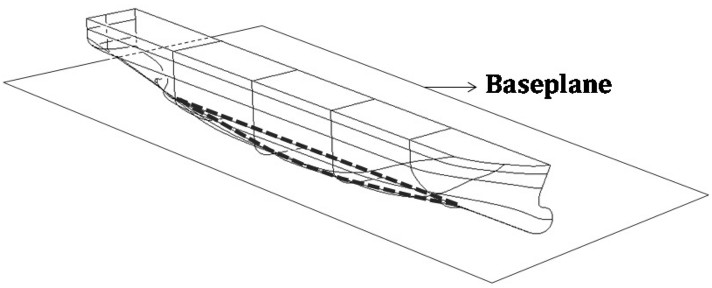
Center Plane

Before defining the coordinate system of a ship, we first introduce three planes, which are all standing perpendicular to each other.



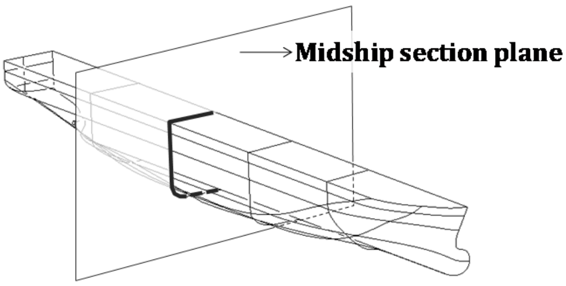
Generally, a ship is **symmetrical** about starboard and port.
The first plane is the vertical longitudinal plane of symmetry, or **center plane**.

Base Plane



The second plane is the horizontal plane, containing the bottom of the ship, which is called **base plane**.

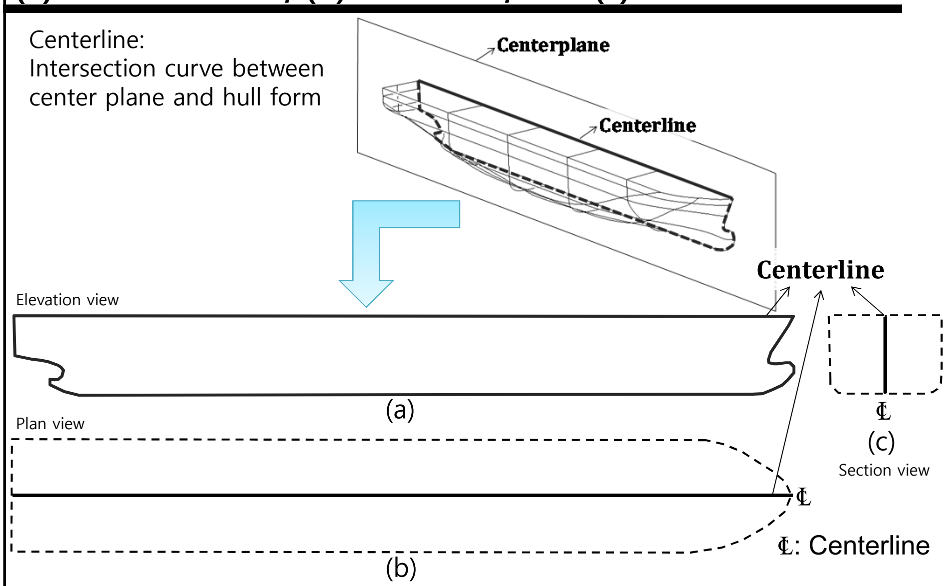
Midship Section Plane



The third plane is the vertical transverse plane through the midship, which is called **midship section plane**.

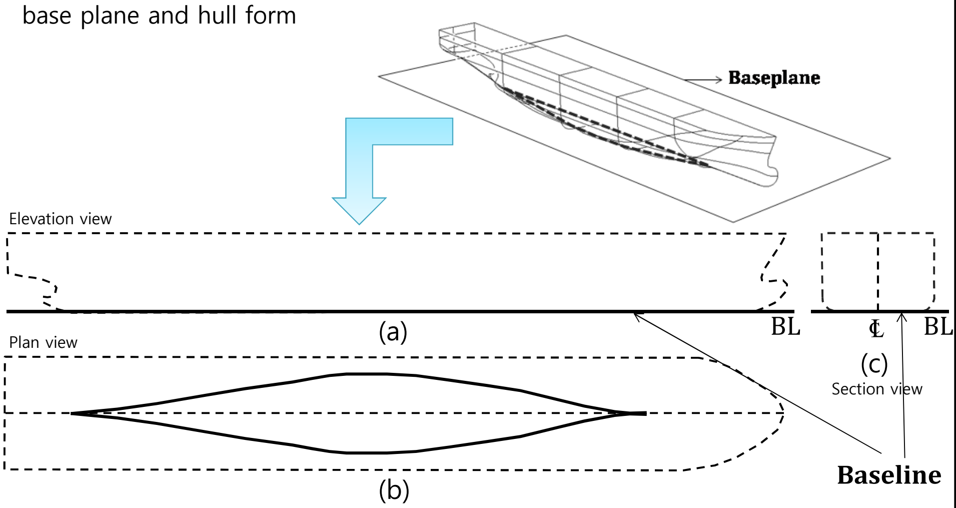
Centerline in (a) Elevation view, (b) Plan view, and (c) Section view

Centerline:
Intersection curve between center plane and hull form

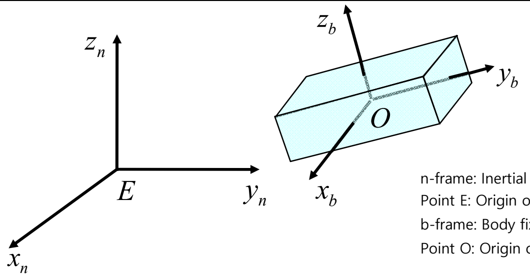


Baseline in (a) Elevation view, (b) Plan view, and (c) Section view

Baseline:
Intersection curve between base plane and hull form



System of Coordinates



n-frame: Inertial frame $x_n\,y_n\,z_n$ or $x\,y\,z$
Point E: Origin of the inertial frame(n-frame)
b-frame: Body fixed frame $x_b\,y_b\,z_b$ or $x'\,y'\,z'$
Point O: Origin of the body fixed frame(b-frame)

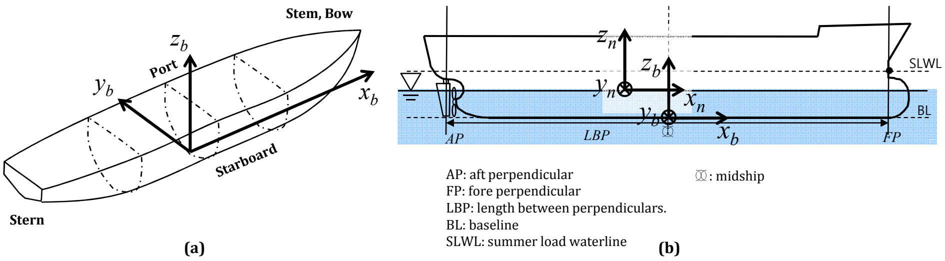
1) ***Body fixed coordinate system***
The right handed coordinate system with the axis called x_b(or x'), y_b(or y'), and z_b(or z') is **fixed to the object**. This coordinate system is called ***body fixed coordinate system*** or ***body fixed reference frame (b-frame)***.

2) ***Space fixed coordinate system***
The right handed coordinate system with the axis called x_n(or x), y_n(or y) and z_n(or z) is **fixed to the space**. This coordinate system is called ***space fixed coordinate system*** or ***space fixed reference frame*** or ***inertial frame (n-frame)***.

In general, a change in the position and orientation of the object is described with respect to the inertial frame. Moreover Newton's 2^{nd} law is only valid for the inertial frame.

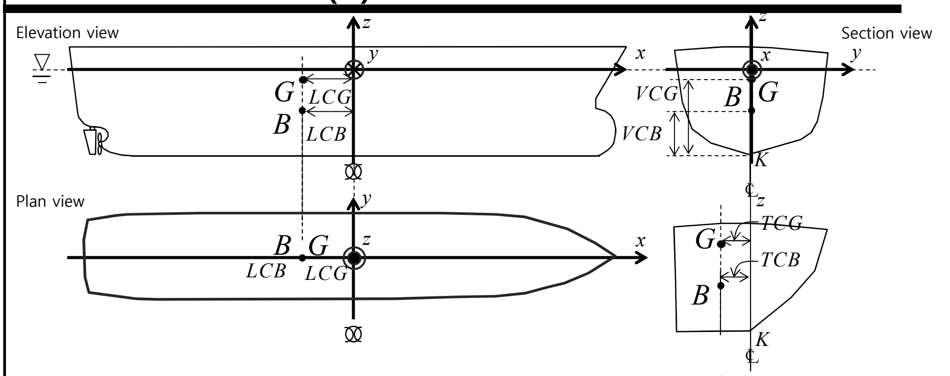
System of Coordinates for a Ship

Body fixed coordinate system (b-frame): Body fixed frame $x_b\, y_b\, z_b$ or $x'\, y'\, z'$
Space fixed coordinate system (n-frame): Inertial frame $x_n\, y_n\, z_n$ or $x\, y\, z$



Center of Buoyancy (B) and Center of Mass (G)

K: keel
LCB: longitudinal center of buoyancy
VCB: vertical center of buoyancy
TCB: transverse center of buoyancy
LCG: longitudinal center of gravity
VCG: vertical center of gravity
TCG: transverse center of gravity



※ In the case that the shape of a ship is asymmetrical with respect to the centerline.

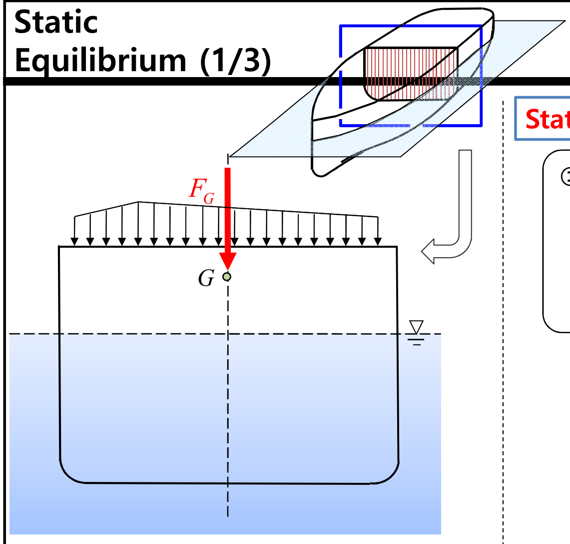
Center of buoyancy (B)

It is the point at which all the vertically upward forces of support (buoyant force) can be considered to act.
It is equal to the center of volume of the submerged volume of the ship. Also, It is equal to the first moment of the submerged volume of the ship about particular axis divided by the total buoyant force (displacement).

Center of mass or Center of gravity (G)

It is the point at which all the vertically downward forces of weight of the ship (gravitational force) can be considered to act.
It is equal to the first moment of the weight of the ship about particular axis divided by the total weight of the ship.

Static Equilibrium (1/3)



m: mass of ship
a: acceleration of ship

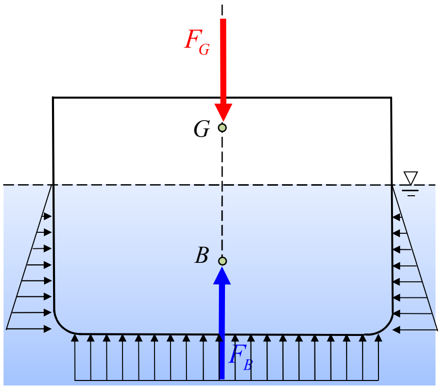
G: Center of mass
F_G: Gravitational force of ship

Static Equilibrium

① **Newton's 2nd law**

$$ma = \sum F$$
$$= -F_G$$

Static Equilibrium (2/3)



B: Center of buoyancy at upright position(center of volume of the submerged volume of the ship)
F_B: Buoyant force acting on ship

Static Equilibrium

① **Newton's 2nd law**

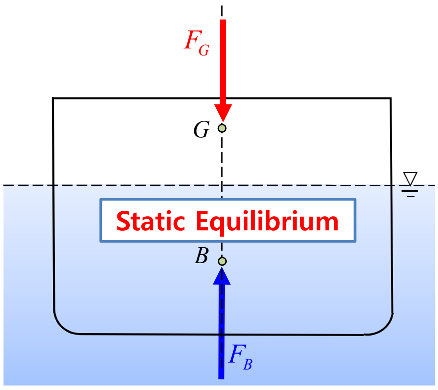
$$ma = \sum F$$
$$= -F_G + F_B$$

for the ship to be in static equilibrium

$$0 = \sum F \quad ,(\because a = 0)$$

$$\therefore F_G = F_B$$

Static Equilibrium (3/3)



τ: Moment
I: Mass moment of inertia
ω: Angular velocity

Static Equilibrium

① **Newton's 2nd law**

$$ma = \sum F$$
$$= -F_G + F_B$$

for the ship to be in static equilibrium

$$0 = \sum F \quad ,(\because a = 0)$$
$$\therefore F_G = F_B$$

② **Euler equation**

$$I\dot{\omega} = \sum \tau$$

for the ship to be in static equilibrium

$$0 = \sum \tau \quad ,(\because \dot{\omega} = 0)$$

When the buoyant force (F_B) lies on the **same line** of action as the gravitational force (F_G), total summation of the moment becomes 0.

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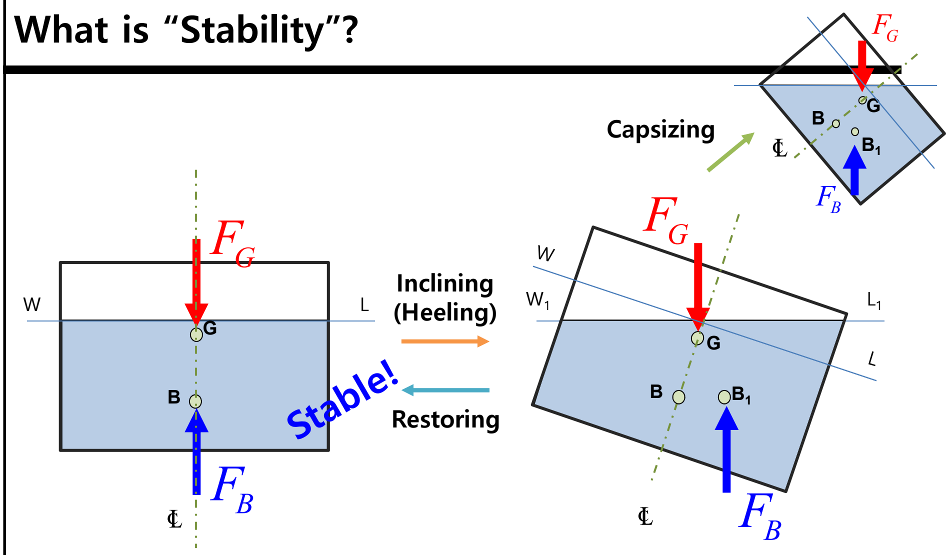
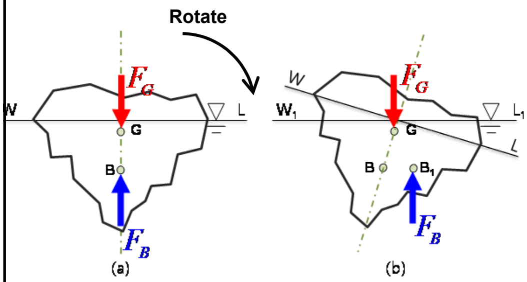
What is "Stability"?



Stability = Stable + Ability

Stability of a Floating Object

- You have a torque on this object relative to any point that you choose. It does not matter where you pick a point.
- The torque will only be zero when the buoyant force and the gravitational force are on one line. Then the torque becomes zero.



Static Equilibrium

① Newton's 2nd law

$$ma = \sum F$$
$$= -F_G + F_B$$

for the ship to be in static equilibrium

$$0 = \sum F \quad ,(\because a = 0)$$
$$\therefore F_G = F_B$$

② Euler equation

$$I\dot{\omega} = \sum \tau$$

for the ship to be in static equilibrium

$$0 = \sum \tau \quad ,(\because \dot{\omega} = 0)$$

When the buoyant force (F_B) lies on the **same line** of action as the gravitational force (F_G), total summation of the moment becomes 0.

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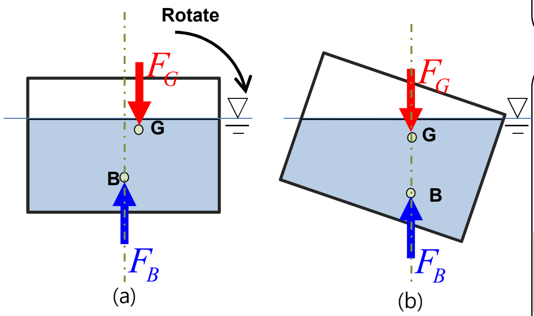
Stability of a Ship

- You have a torque on this object relative to any point that you choose. It does not matter where you pick a point.
- The torque will only be zero when the buoyant force and the gravitational force are on one line. Then the torque becomes zero.



Static Equilibrium

Static Equilibrium

① Newton's 2nd law

$$ma = \sum F$$
$$= -F_G + F_B$$

for the ship to be in static equilibrium

$$0 = \sum F \quad ,(\because a = 0)$$
$$\therefore F_G = F_B$$

② Euler equation

$$I\dot{\omega} = \sum \tau$$

for the ship to be in static equilibrium

$$0 = \sum \tau \quad ,(\because \dot{\omega} = 0)$$

When the buoyant force (F_B) lies on the **same line** of action as the gravitational force (F_G), total summation of the moment becomes 0.

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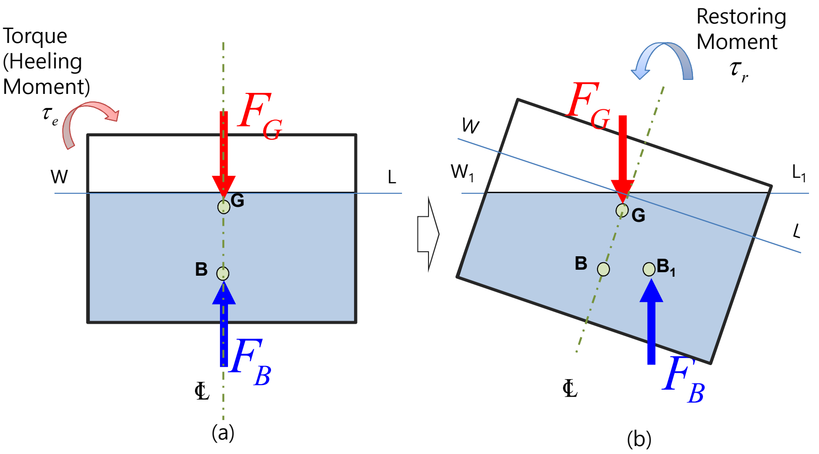
Interaction of Weight and Buoyancy of a Floating Body (1/2)

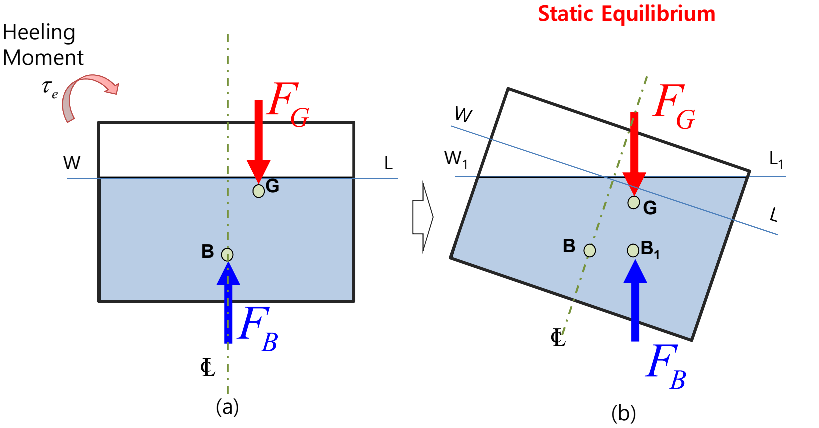


Euler equation: $I\dot{\omega} = \sum \tau \Rightarrow \dot{\omega} \neq 0$

Interaction of weight and buoyancy resulting in intermediate state

Interaction of Weight and Buoyancy of a Floating Body (2/2)

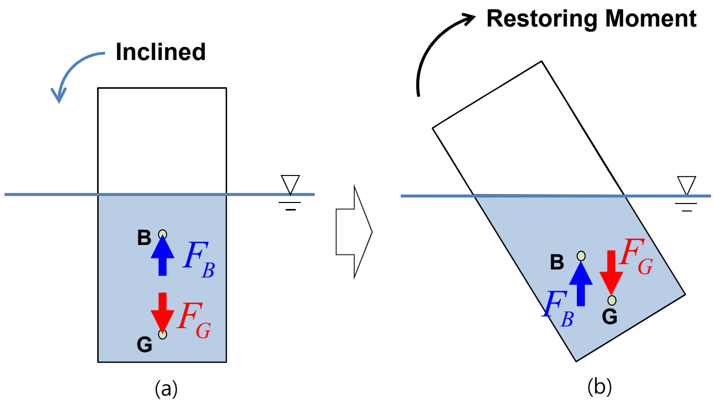
Static Equilibrium



Euler equation: $I\dot{\omega} = \sum \tau \Rightarrow \dot{\omega} = 0$

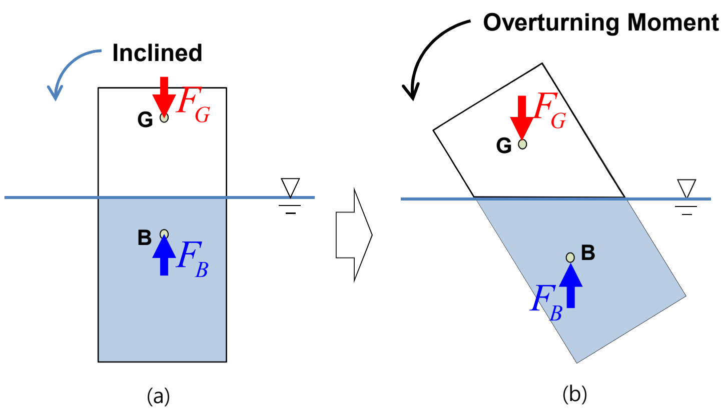
Interaction of weight and buoyancy resulting in static equilibrium state

Stability of a Floating Body (1/2)



Floating body in stable state

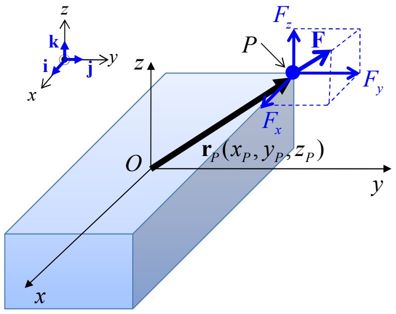
Stability of a Floating Body (2/2)



Floating body in unstable state

Transverse, Longitudinal, and Yaw Moment

Question) **If the force F is applied on the point of rectangle object, what is the moment?**



$$\mathbf{M} = \mathbf{r}_P \times \mathbf{F}$$

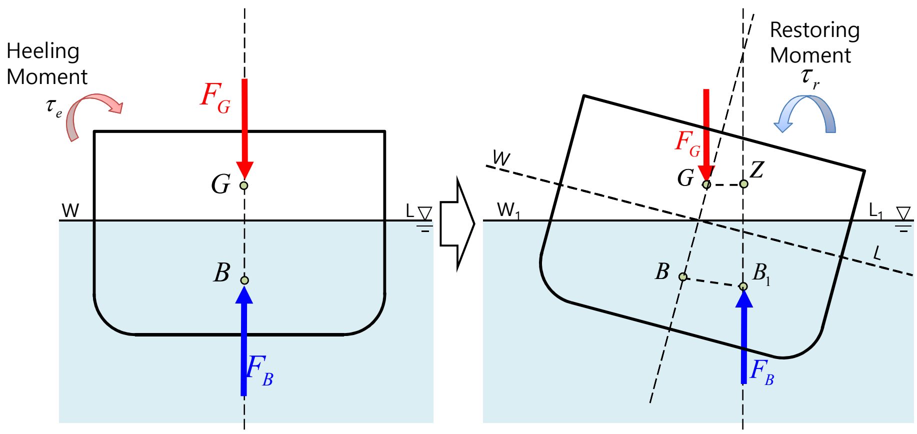
$$= \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_P & y_P & z_P \\ F_x & F_y & F_z \end{bmatrix} = \mathbf{i}\underbrace{(y_P \cdot F_z - z_P \cdot F_y)}_{M_x} + \mathbf{j}\underbrace{(-x_P \cdot F_z + z_P \cdot F_x)}_{M_y} + \mathbf{k}\underbrace{(x_P \cdot F_y - y_P \cdot F_x)}_{M_z}$$

Transverse moment (M_x), Longitudinal moment (M_y), Yaw moment (M_z)

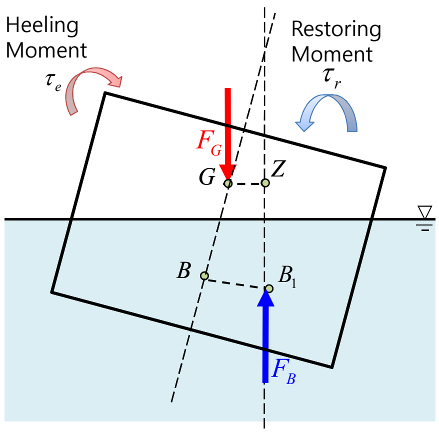
The x-component of the moment, i.e., the bracket term of unit vector **i,** indicates the **transverse moment,** which is the moment caused by the force F acting on the point P **about x axis**. Whereas the y-component, the term of unit vector **j,** indicates the **longitudinal moment about y axis**, and the z-component, the last term **k**, represents the **yaw moment about z axis**.

5.2 Restoring Moment and Restoring Arm

Restoring Moment Acting on an Inclined Ship



Restoring Arm (GZ, Righting Arm)



- The value of the restoring moment is found by multiplying the buoyant force of the ship (displacement), F_B, by the perpendicular distance from G to the line of action of F_B.
- It is customary to label as *Z* the point of intersection of the line of action of F_B and the parallel line to the waterline through *G* to it.
- This distance *GZ* is known as the '**restoring arm**' or '**righting arm**'.

- **Transverse Restoring Moment**

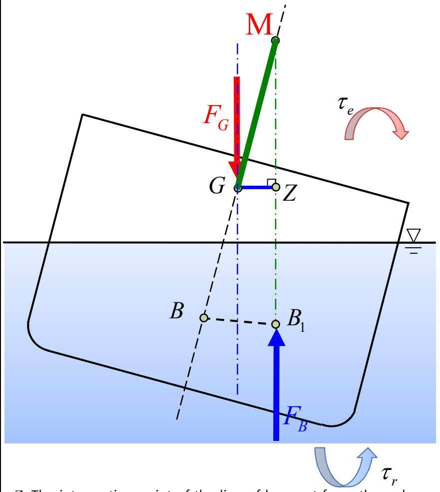
$$\tau_{restoring} = F_B \cdot GZ$$

G: Center of mass K: Keel
B: Center of buoyancy at upright position
B_1: Changed center of buoyancy
F_G: Weight of ship F_B: Buoyant force acting on ship

Metacenter (M)

• Restoring Moment

$\tau_{restoring} = F_B \cdot GZ$



Z: The intersection point of the line of buoyant force through B_1 with the transverse line through G

Definition of M (Metacenter)

• The intersection point of the vertical line through the center of buoyancy at **previous** **position** (*B*) with the vertical line through the center of buoyancy at **new** **position** (*B_1*) **after inclination**

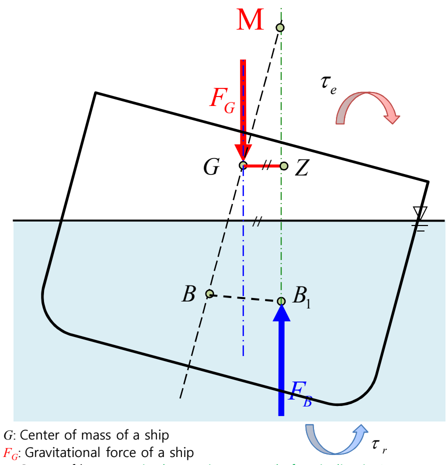
• The term **meta** was selected as a prefix for center because its Greek meaning implies **movement**. The **metacenter** therefore is a **moving center**.

• GM ➡ Metacentric height

• From the figure, GZ can be obtained with assumption that *M* does not change within a small angle of inclination (about 7° to 10°), as below.

$$GZ \approx GM \cdot \sin\phi$$

Restoring Moment at Large Angle of Inclination (1/3)



G: Center of mass of a ship
F_G: Gravitational force of a ship
B: Center of buoyancy in the previous state (before inclination)
F_B: Buoyant force acting on a ship
B_1: New position of center of buoyancy after the ship has been inclined
Z: The intersection point of a vertical line through the new position of the center of buoyancy(B_1) with the transversely parallel line to a waterline through the center of mass(G)

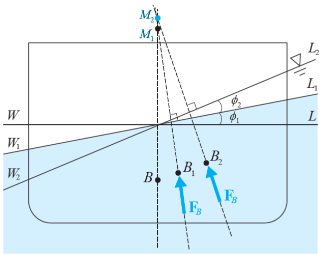
$$GZ \approx GM \cdot \sin\phi$$

For a small angle of inclination (about 7° to 10°)

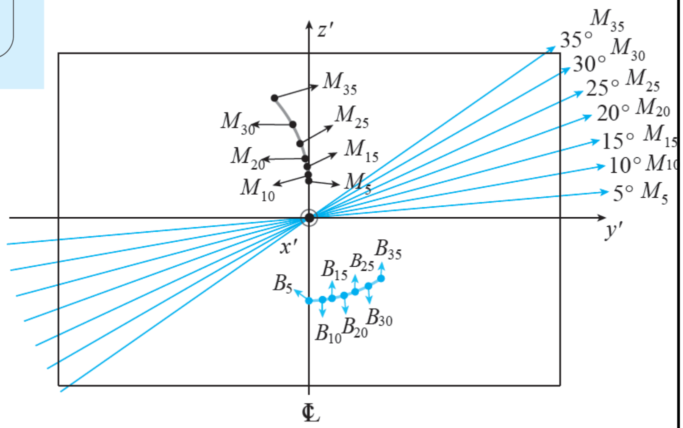
• The use of metacentric height (*GM*) as the restoring arm is not valid for a ship at a large angle of inclination.

To determine the restoring arm "GZ", it is necessary to know the positions of the center of mass (*G*) and the new position of the center of buoyancy (*B_1*).

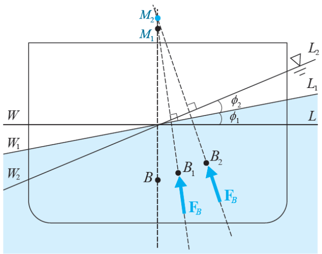
Restoring Moment at Large Angle of Inclination (2/3)



M: The intersection point of the vertical line through the center of buoyancy at **previous position** (B_{i-1}) with the vertical line through the center of buoyancy at **present position** (B_i) **after inclination**

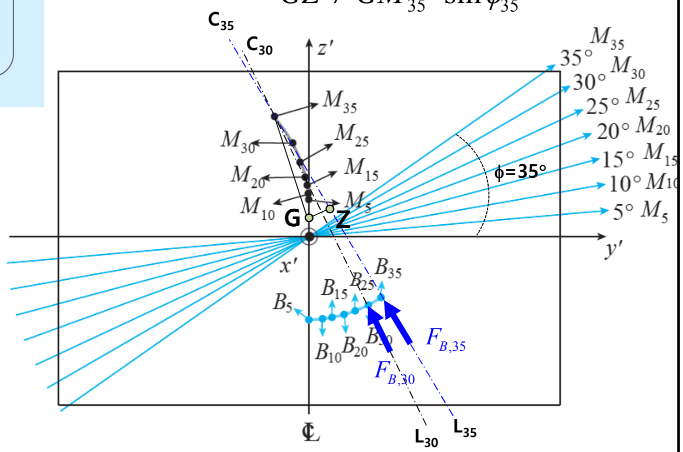


Restoring Moment at Large Angle of Inclination (3/3)



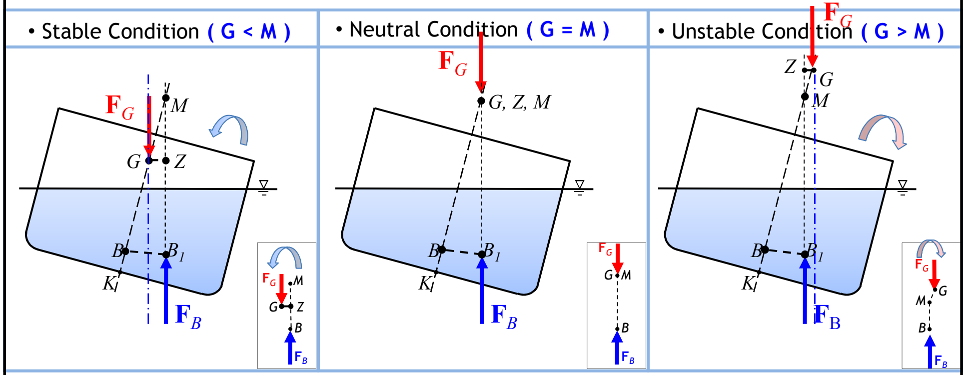
M: The intersection point of the vertical line through the center of buoyancy at **previous position** (B_{i-1}) with the vertical line through the center of buoyancy at **present position** (B_i) **after inclination**

$$GZ \neq GM_{35} \cdot \sin \phi_{35}$$



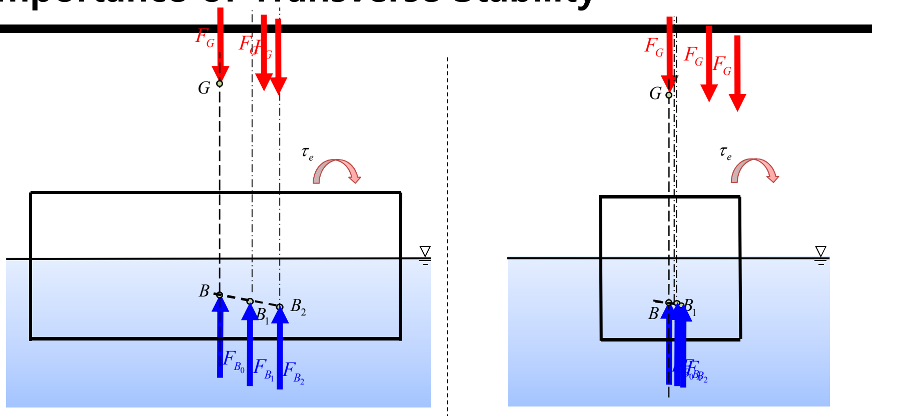
Stability of a Ship According to Relative Position between "G", "B", and "M" at Small Angle of Inclination

- **Righting (Restoring) Moment:** Moment to return the ship to the upright floating position
- **Stable / Neutral / Unstable Condition:** Relative height of G with respect to M is one measure of stability.



G: Center of mass   *K*: Keel
B: Center of buoyancy at upright position B_1: Changed center of buoyancy
F_G: Weight of ship F_B: Buoyant force acting on ship
Z: The intersection of the line of buoyant force through B_1 with the transverse line through G
M: The intersection of the line of buoyant force through B_1 with the centerline of the ship

Importance of Transverse Stability



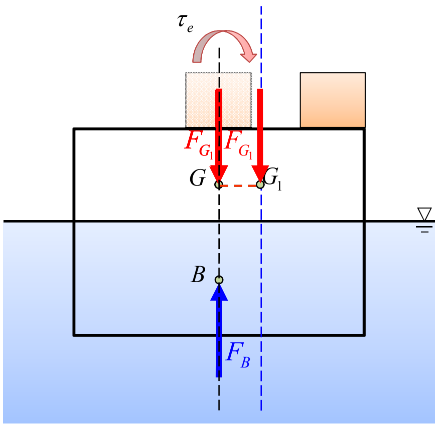
The ship is inclined further from it.
The ship is in static equilibrium state.

The ship is inclined further from it.
Because of the limit of the breadth, "B" can not move further. the ship will capsize.

As the ship is inclined, the position of the center of buoyancy "B" is changed.
Also the **position of the center of mass "G" relative to inertial frame is changed**.

One of the most important factors of stability is the breadth.
So, we usually consider that transverse stability is more important than longitudinal stability.

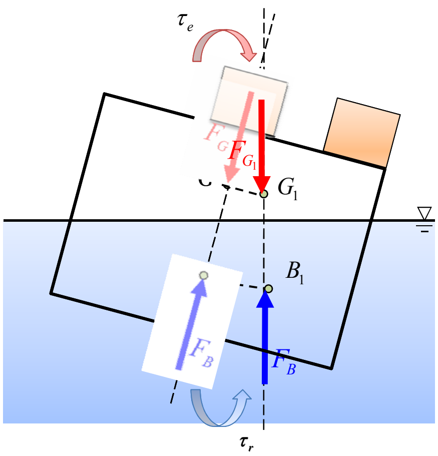
Summary of Static Stability of a Ship (1/3)



G: Center of mass of a ship
G_1: New position of center of mass after the object on the deck moves to the right side
F_G: Gravitational force of a ship
B: Center of buoyancy at initial position
F_B: Buoyant force acting on a ship
B_1: New position of center of buoyancy after the ship has been inclined
Z: The intersection of a line of buoyant force(F_B) through the new position of the center of buoyancy (B_1) with the transversely parallel line to the waterline through the center of mass of a ship(G)

- When an object on the deck moves to the right side of a ship, the total center of mass of the ship moves to the point G_1, off the centerline.
- Because the buoyant force and the gravitational force are not on one line, the forces induces a moment to incline the ship.
  - * We have a moment on this object relative to any point that we choose. It does not matter where we pick a point.

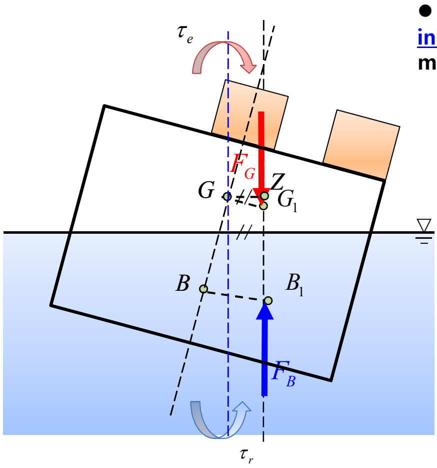
Summary of Static Stability of a Ship (2/3)



G: Center of mass of a ship
G_1: New position of center of mass after the object on the deck moves to the right side
F_G: Gravitational force of a ship
B: Center of buoyancy at initial position
F_B: Buoyant force acting on a ship
B_1: New position of center of buoyancy after the ship has been inclined
Z: The intersection of a line of buoyant force(F_B) through the new position of the center of buoyancy (B_1) with the transversely parallel line to the waterline through the center of mass of a ship(G)

- The total moment will only be zero when the buoyant force and the gravitational force are on one line. If the moment becomes zero, the ship is in static equilibrium state.

Summary of Static Stability of a Ship (3/3)



G: Center of mass of a ship
G_1: New position of center of mass after the object on the deck moves to the right side
F_G: Gravitational force of a ship
B: Center of buoyancy at initial position
F_B: Buoyant force acting on a ship
B_1: New position of center of buoyancy after the ship has been inclined
Z: The intersection of a line of buoyant force(F_B) through the new position of the center of buoyancy (B_1) with the transversely parallel line to the waterline through the center of mass of a ship(G)

● When the object on the deck returns to the initial position in the centerline, the center of mass of the ship returns to the initial point G.

- ● Then, because the buoyant force and the gravitational force are not on one line, the forces induces a restoring moment to return the ship to the initial position.

※ Naval architects refer to the restoring moment as "righting moment".

- ● The moment arm of the buoyant force and gravitational force about G is expressed by GZ, where Z is defined as the intersection point of the line of buoyant force(F_B) through the new position of the center of buoyancy(B_1) with the transversely parallel line to the waterline through the center of mass of the ship (G).

• Transverse Righting Moment

$$\tau_{righting} = F_B \cdot \underline{GZ}$$

- ● By the restoring moment, the ship returns to the initial position.

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