Electromagnetics:
Normal Incidence at Multiple Dielectric Interfaces
(8-9)

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Normal Incidence at Multiple Dielectric Interfaces (1)

For medium 1:

$$\mathbf{E}_1 = \mathbf{a}_x (E_{i0} e^{-j\beta_1 z} + E_{r0} e^{+j\beta_1 z})$$

$$\mathbf{H}_1 = \mathbf{a}_y \frac{1}{\eta_1} (E_{i0} e^{-j\beta_1 z} - E_{r0} e^{+j\beta_1 z})$$

For medium 2:

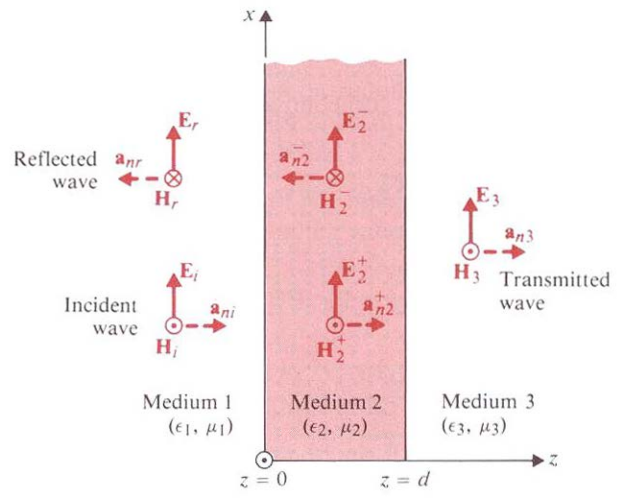
$$\mathbf{E}_2 = \mathbf{a}_x (E_2^+ e^{-j\beta_2 z} + E_2^- e^{+j\beta_2 z})$$

$$\mathbf{H}_2 = \mathbf{a}_y \frac{1}{\eta_2} (E_2^+ e^{-j\beta_2 z} - E_2^- e^{+j\beta_2 z})$$

For medium 3:

$$\mathbf{E}_3 = \mathbf{a}_x E_3^+ e^{-j\beta_3 z}$$

$$\mathbf{H}_3 = \mathbf{a}_y \frac{1}{\eta_3} E_3^+ e^{-j\beta_3 z}$$



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

Boundary conditions:

At z = 0:

$$\mathbf{E}_1(0) = \mathbf{E}_2(0)$$

$$\mathbf{H}_1(0) = \mathbf{H}_2(0)$$

At z = d:

$$\mathbf{E}_2(d) = \mathbf{E}_3(d)$$

$$\mathbf{H}_2(d) = \mathbf{H}_3(d)$$

Normal Incidence at Multiple Dielectric Interfaces (2)

Boundary conditions:

At z = 0:

$$\rightarrow \begin{pmatrix} E_1(0) \\ H_1(0) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \frac{1}{\eta_1} & -\frac{1}{\eta_1} \end{pmatrix} \begin{pmatrix} E_{i0} \\ E_{r0} \end{pmatrix} \qquad \rightarrow \begin{pmatrix} E_2(0) \\ H_2(0) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \frac{1}{\eta_2} & -\frac{1}{\eta_2} \end{pmatrix} \begin{pmatrix} E_2^+ \\ E_2^- \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 1 \\ \frac{1}{\eta_1} & -\frac{1}{\eta_1} \end{pmatrix} \begin{pmatrix} E_{i0} \\ E_{r0} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \frac{1}{\eta_2} & -\frac{1}{\eta_2} \end{pmatrix} \begin{pmatrix} E_2^+ \\ E_2^- \end{pmatrix}$$

At z = d:

$$\rightarrow \begin{pmatrix} E_2(d) \\ H_2(d) \end{pmatrix} = \begin{pmatrix} e^{-j\beta_2 d} & e^{+j\beta_2 d} \\ \frac{1}{\eta_2} e^{-j\beta_2 d} & -\frac{1}{\eta_2} e^{+j\beta_2 d} \end{pmatrix} \begin{pmatrix} E_2^+ \\ E_2^- \end{pmatrix} \qquad \rightarrow \begin{pmatrix} E_3(d) \\ H_3(d) \end{pmatrix} = \begin{pmatrix} e^{-j\beta_3 d} \\ \frac{1}{\eta_3} e^{-j\beta_3 d} \end{pmatrix} E_3^+$$

$$\rightarrow \begin{pmatrix} e^{-j\beta_2 d} & e^{+j\beta_2 d} \\ \frac{1}{\eta_2} e^{-j\beta_2 d} & -\frac{1}{\eta_2} e^{+j\beta_2 d} \end{pmatrix} \begin{pmatrix} E_2^+ \\ E_2^- \end{pmatrix} = \begin{pmatrix} e^{-j\beta_3 d} \\ \frac{1}{\eta_3} e^{-j\beta_3 d} \end{pmatrix} E_3^+$$

For medium 1:

$$\mathbf{E}_1 = \mathbf{a}_x (E_{i0} e^{-j\beta_1 z} + E_{r0} e^{+j\beta_1 z})$$

$$\mathbf{H}_1 = \mathbf{a}_y \frac{1}{\eta_1} (E_{i0} e^{-j\beta_1 z} - E_{r0} e^{+j\beta_1 z})$$

For medium 2:

$$\mathbf{E}_2 = \mathbf{a}_x (E_2^+ e^{-j\beta_2 z} + E_2^- e^{+j\beta_2 z})$$

$$\mathbf{H}_2 = \mathbf{a}_y \frac{1}{\eta_2} (E_2^+ e^{-j\beta_2 z} - E_2^- e^{+j\beta_2 z})$$

For medium 3:

$$\mathbf{E}_3 = \mathbf{a}_x E_3^+ e^{-j\beta_3 z}$$

$$\mathbf{H}_3 = \mathbf{a}_y \frac{1}{\eta_3} E_3^+ e^{-j\beta_3 z}$$

Normal Incidence at Multiple Dielectric Interfaces (3)

Boundary conditions:

$$\rightarrow \begin{pmatrix} E_2^+ \\ E_2^- \end{pmatrix} = \begin{pmatrix} e^{-j\beta_2 d} & e^{+j\beta_2 d} \\ \frac{1}{\eta_2} e^{-j\beta_2 d} & -\frac{1}{\eta_2} e^{+j\beta_2 d} \end{pmatrix}^{-1} \begin{pmatrix} e^{-j\beta_3 d} \\ \frac{1}{\eta_3} e^{-j\beta_3 d} \end{pmatrix} E_3^+$$

$$= -\frac{\eta_2}{2} \begin{pmatrix} -\frac{1}{\eta_2} e^{+j\beta_2 d} & -e^{+j\beta_2 d} \\ -\frac{1}{\eta_2} e^{-j\beta_2 d} & e^{-j\beta_2 d} \end{pmatrix} \begin{pmatrix} e^{-j\beta_3 d} \\ \frac{1}{\eta_3} e^{-j\beta_3 d} \end{pmatrix} E_3^+$$

$$= \begin{pmatrix} \frac{1}{2} e^{+j\beta_2 d} & \frac{\eta_2}{2} e^{+j\beta_2 d} \\ \frac{1}{2} e^{-j\beta_2 d} & -\frac{\eta_2}{2} e^{-j\beta_2 d} \end{pmatrix} \begin{pmatrix} e^{-j\beta_3 d} \\ \frac{1}{\eta_3} e^{-j\beta_3 d} \end{pmatrix} E_3^+$$

Recall the B.C. at z = 0:

$$\rightarrow \boxed{\begin{pmatrix} 1 & 1 \\ \frac{1}{\eta_1} & -\frac{1}{\eta_1} \end{pmatrix} \begin{pmatrix} E_{i0} \\ E_{r0} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \frac{1}{\eta_2} & -\frac{1}{\eta_2} \end{pmatrix} \begin{pmatrix} E_2^+ \\ E_2^- \end{pmatrix}}$$

Normal Incidence at Multiple Dielectric Interfaces (3)

Boundary conditions:

$$\begin{pmatrix} E_2^+ \\ E_2^- \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} 1 & 1 \\ \frac{1}{\eta_1} & -\frac{1}{\eta_1} \end{pmatrix} \begin{pmatrix} E_{i0} \\ E_{r0} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \frac{1}{\eta_2} & -\frac{1}{\eta_2} \end{pmatrix} \begin{pmatrix} \frac{1}{2} e^{+j\beta_2 d} & \frac{\eta_2}{2} e^{+j\beta_2 d} \\ \frac{1}{2} e^{-j\beta_2 d} & -\frac{\eta_2}{2} e^{-j\beta_2 d} \end{pmatrix} \begin{pmatrix} 1 \\ \frac{1}{\eta_3} \end{pmatrix} E_3^+ e^{-j\beta_3 d}$$

$$= \begin{pmatrix} 1 & 1 \\ \frac{1}{\eta_2} & -\frac{1}{\eta_2} \end{pmatrix} \begin{pmatrix} \frac{1}{2} e^{+j\beta_2 d} + \frac{\eta_2}{2\eta_3} e^{+j\beta_2 d} \\ \frac{1}{2} e^{-j\beta_2 d} - \frac{\eta_2}{2\eta_3} e^{-j\beta_2 d} \end{pmatrix} E_3^+ e^{-j\beta_3 d}$$

$$= \begin{pmatrix} \frac{1}{2} e^{+j\beta_2 d} + \frac{\eta_2}{2\eta_3} e^{+j\beta_2 d} + \frac{1}{2} e^{-j\beta_2 d} - \frac{\eta_2}{2\eta_3} e^{-j\beta_2 d} \\ \frac{1}{\eta_2}[\frac{1}{2} e^{+j\beta_2 d} + \frac{\eta_2}{2\eta_3} e^{+j\beta_2 d} - \frac{1}{2} e^{-j\beta_2 d} + \frac{\eta_2}{2\eta_3} e^{-j\beta_2 d}] \end{pmatrix} E_3^+ e^{-j\beta_3 d}$$

$$= \begin{pmatrix} \cos\beta_2 d + j\frac{\eta_2}{\eta_3}\sin\beta_2 d \\ \frac{1}{\eta_2}[j\sin\beta_2 d + \frac{\eta_2}{\eta_3}\cos\beta_2 d] \end{pmatrix} E_3^+ e^{-j\beta_3 d}$$

Normal Incidence at Multiple Dielectric Interfaces (4)

Boundary conditions:

$$\begin{pmatrix} 1 & 1 \\ \dfrac{1}{\eta_1} & -\dfrac{1}{\eta_1} \end{pmatrix}\begin{pmatrix} E_{i0} \\ E_{r0} \end{pmatrix} = \begin{pmatrix} \cos\beta_2 d + j\dfrac{\eta_2}{\eta_3}\sin\beta_2 d \\ \dfrac{1}{\eta_2}[j\sin\beta_2 d + \dfrac{\eta_2}{\eta_3}\cos\beta_2 d] \end{pmatrix} E_3^+ e^{-j\beta_3 d}$$

$$= \begin{pmatrix} 1 \\ 1\Big/\eta_2 \dfrac{\eta_3\cos\beta_2 d + j\eta_2\sin\beta_2 d}{\eta_2\cos\beta_2 d + j\eta_3\sin\beta_2 d} \end{pmatrix} E_3^+ e^{-j\beta_3 d}(\cos\beta_2 d + j\frac{\eta_2}{\eta_3}\sin\beta_2 d)$$

$$= \begin{pmatrix} 1 \\ 1\Big/\underbrace{\eta_2 \dfrac{\eta_3\cos\beta_2 d + j\eta_2\sin\beta_2 d}{\eta_2\cos\beta_2 d + j\eta_3\sin\beta_2 d}}_{\to \eta_2'} \end{pmatrix} E_3'^+$$

$$\to \Gamma_0 = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2' - \eta_1}{\eta_2' + \eta_1}$$

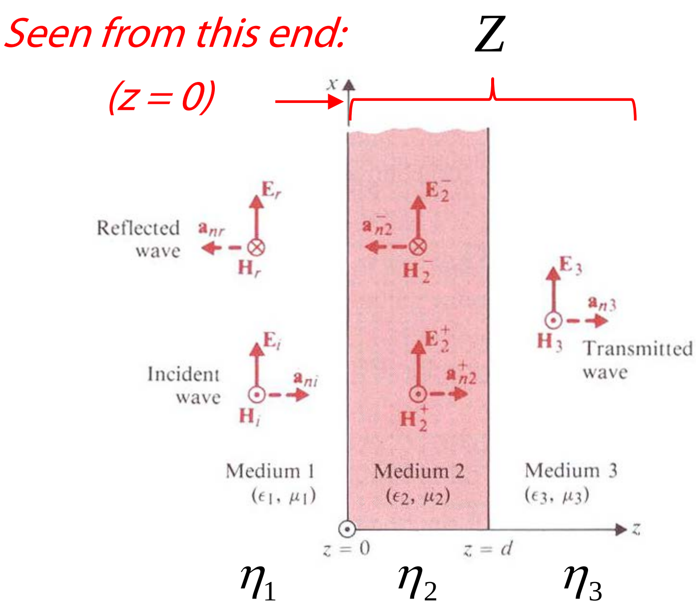
Recall:

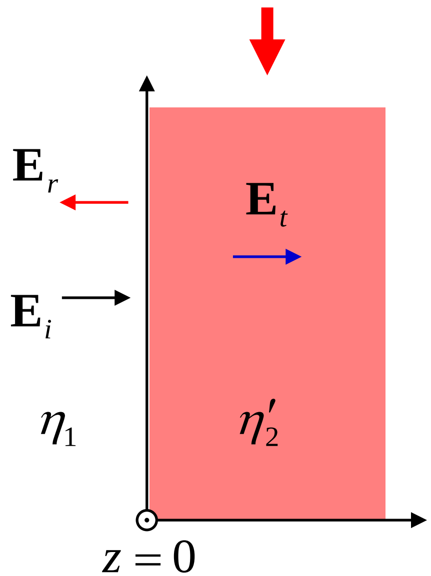
$$\begin{pmatrix} 1 & 1 \\ \dfrac{1}{\eta_1} & -\dfrac{1}{\eta_1} \end{pmatrix}\begin{pmatrix} E_{i0} \\ E_{r0} \end{pmatrix} = \begin{pmatrix} 1 \\ \dfrac{1}{\eta_2} \end{pmatrix} E_{t0}$$

$$\to \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$ ← *Reflection coefficient*

← *B.C. for the normal incidence at a plane dielectric boundary*

Impedance Transformation with Multiple Dielectrics





Wave impedance of the total field:

$$\eta_2' = \eta_2 \frac{\eta_3 \cos \beta_2 d + j\eta_2 \sin \beta_2 d}{\eta_2 \cos \beta_2 d + j\eta_3 \sin \beta_2 d} = \left. \frac{\text{Total } E_x}{\text{Total } H_y} \right|_{z=0} \equiv Z(0)$$

Note that the wave impedance is a function of η_2, η_3, & d.

We can transform η_3 to η_2' with a variety of choices of η_2 & d.

$\eta_2' \to \eta_1$ $\to$ *No reflection* $\to$ *Impedance matched*