Electromagnetics:
Introduction to Waveguides
General Wave Behaviors along Uniform Guiding Structure
(10-1, 10-2)

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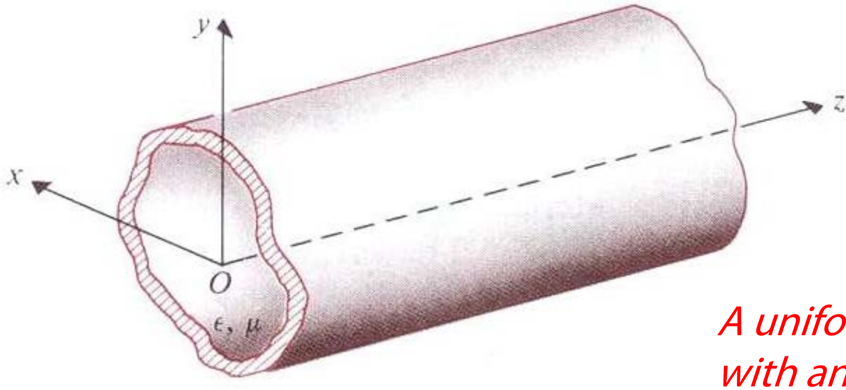
Waveguides



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

A uniform waveguide (waveguiding structure) with an arbitrary cross section.

Notation to use hereafter:

$$\gamma = \alpha + j\beta \quad \rightarrow \text{Propagation constant}$$

$$\mathbf{E}(x, y, z; t) = \mathrm{Re}[\mathbf{E}^0(x, y)e^{(j\omega t - \gamma z)}]$$

Helmholtz equations for source-free media:

$$k^2 = \omega^2 \mu\varepsilon \rightarrow \quad \nabla^2\mathbf{E} + k^2\mathbf{E} = 0 \quad \rightarrow \nabla^2_{xy}\mathbf{E} + (\gamma^2 + k^2)\mathbf{E} = 0$$

$$\nabla^2\mathbf{H} + k^2\mathbf{H} = 0 \quad \rightarrow \nabla^2_{xy}\mathbf{H} + (\gamma^2 + k^2)\mathbf{H} = 0$$

$$\rightarrow \nabla^2\mathbf{E} = (\nabla^2_{xy} + \nabla^2_z)\mathbf{E} = (\nabla^2_{xy} + \frac{\partial^2}{\partial z^2})\mathbf{E}$$

$$= \nabla^2_{xy}\mathbf{E} + \gamma^2\mathbf{E}$$

Interrelationship among the E and H Fields

From: $\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \rightarrow \frac{\partial E_z^0}{\partial y} + \gamma E_y^0 = -j\omega\mu H_x^0$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \rightarrow -\gamma E_x^0 - \frac{\partial E_z^0}{\partial x} = -j\omega\mu H_y^0$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \rightarrow \frac{\partial E_y^0}{\partial x} - \frac{\partial E_x^0}{\partial y} = -j\omega\mu H_z^0$$

From: $\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E}$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \rightarrow \frac{\partial H_z^0}{\partial y} + \gamma H_y^0 = j\omega\varepsilon E_x^0$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \rightarrow -\gamma H_x^0 - \frac{\partial H_z^0}{\partial x} = j\omega\varepsilon E_y^0$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \rightarrow \frac{\partial H_y^0}{\partial x} - \frac{\partial H_x^0}{\partial y} = j\omega\varepsilon E_z^0$$

Transverse field components represented by E_z^0 & H_z^0

$$H_x^0 = -\frac{1}{h^2}\left(\gamma\frac{\partial H_z^0}{\partial x} - j\omega\varepsilon\frac{\partial E_z^0}{\partial y}\right)$$

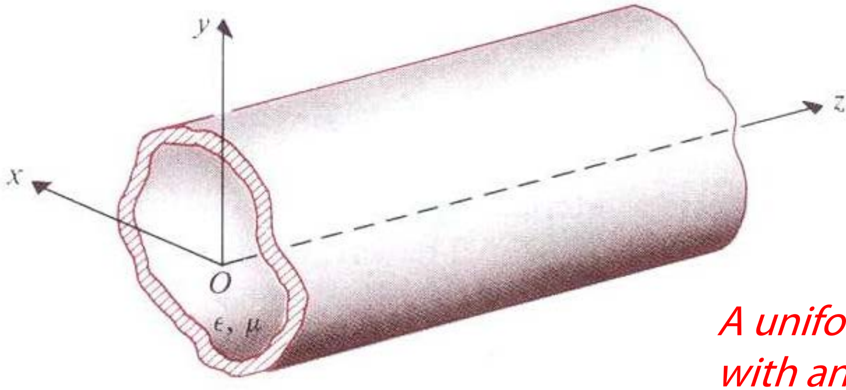
$$H_y^0 = -\frac{1}{h^2}\left(\gamma\frac{\partial H_z^0}{\partial y} + j\omega\varepsilon\frac{\partial E_z^0}{\partial x}\right)$$

$$E_x^0 = -\frac{1}{h^2}\left(\gamma\frac{\partial E_z^0}{\partial x} + j\omega\mu\frac{\partial H_z^0}{\partial y}\right)$$

$$E_y^0 = -\frac{1}{h^2}\left(\gamma\frac{\partial E_z^0}{\partial y} - j\omega\mu\frac{\partial H_z^0}{\partial x}\right)$$

$$\leftarrow h^2 = \gamma^2 + k^2$$

Classification of Waveguide Modes



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

A uniform waveguide (waveguiding structure) with an arbitrary cross section.

- Transverse electromagnetic (TEM) waves: $E_z = H_z = 0$
- Transverse magnetic (TM) waves: $H_z = 0$
- Transverse electric (TE) waves: $E_z = 0$
- Hybrid (HE or EH) waves: $E_z \neq 0 \,\&\, H_z \neq 0$

  HE: $H_z > E_z$ ← *TE-like*

  EH: $E_z > H_z$ ← *TM-like*

Transverse Electromagnetic (TEM) Waves

In case: $E_z = H_z = 0 \rightarrow What\ if\ h^2 = 0?$

Can TEM waves exist in a single-conductor hollow (or dielectric-filled) waveguide of any shape?

$$H_x^0 = -\frac{1}{h^2}\left(\gamma\frac{\partial H_z^0}{\partial x} - j\omega\varepsilon\frac{\partial E_z^0}{\partial y}\right)$$

$$H_y^0 = -\frac{1}{h^2}\left(\gamma\frac{\partial H_z^0}{\partial y} + j\omega\varepsilon\frac{\partial E_z^0}{\partial x}\right)$$

$$E_x^0 = -\frac{1}{h^2}\left(\gamma\frac{\partial E_z^0}{\partial x} + j\omega\mu\frac{\partial H_z^0}{\partial y}\right)$$

$$E_y^0 = -\frac{1}{h^2}\left(\gamma\frac{\partial E_z^0}{\partial y} - j\omega\mu\frac{\partial H_z^0}{\partial x}\right)$$

$$\rightarrow h^2 = \gamma_{TEM}^2 + k^2 = 0$$

$$\rightarrow \gamma_{TEM} = jk = j\omega\sqrt{\mu\varepsilon}$$

Recall:

$$\rightarrow \frac{\partial E_z^0}{\partial y} + \gamma E_y^0 = -j\omega\mu H_x^0$$

$$\rightarrow -\gamma E_x^0 - \frac{\partial E_z^0}{\partial x} = -j\omega\mu H_y^0$$

$$\rightarrow \frac{\partial E_y^0}{\partial x} - \frac{\partial E_x^0}{\partial y} = -j\omega\mu H_z^0$$

$$\rightarrow \frac{\partial H_z^0}{\partial y} + \gamma H_y^0 = j\omega\varepsilon E_x^0$$

$$\rightarrow -\gamma H_x^0 - \frac{\partial H_z^0}{\partial x} = j\omega\varepsilon E_y^0$$

$$\rightarrow \frac{\partial H_y^0}{\partial x} - \frac{\partial H_x^0}{\partial y} = j\omega\varepsilon E_z^0$$

Phase velocity: $u_{p,TEM} = \frac{\omega}{\beta} = \frac{\omega}{k} = \frac{1}{\sqrt{\mu\varepsilon}}$ ← No explicit frequency dependence!

Wave impedance: $Z_{TEM} = \frac{E_x^0}{H_y^0} = \frac{j\omega\mu}{\gamma_{TEM}} = -\frac{E_y^0}{H_x^0} = \frac{\gamma_{TEM}}{j\omega\varepsilon} = \sqrt{\frac{\mu}{\varepsilon}} = \eta$

$$\rightarrow \mathbf{H} = \frac{1}{Z_{TEM}}\mathbf{a}_z \times \mathbf{E}$$

Transverse Magnetic (TM) Waves (1)

For $H_z = 0$: $\rightarrow \nabla_{xy}^2 \mathbf{E} + (\gamma^2 + k^2)\mathbf{E} = 0 \quad \rightarrow \nabla_{xy}^2 E_z^0 + (\gamma^2 + k^2) E_z^0 = 0$

$$\rightarrow \nabla_{xy}^2 E_z^0 + h^2 E_z^0 = 0$$ ← *Characteristic eq.*

$$H_x^0 = -\frac{1}{h^2}\left(\gamma \frac{\partial H_z^0}{\partial x} - j\omega\varepsilon \frac{\partial E_z^0}{\partial y}\right)$$

$$H_y^0 = -\frac{1}{h^2}\left(\gamma \frac{\partial H_z^0}{\partial y} + j\omega\varepsilon \frac{\partial E_z^0}{\partial x}\right)$$

$$E_x^0 = -\frac{1}{h^2}\left(\gamma \frac{\partial E_z^0}{\partial x} + j\omega\mu \frac{\partial H_z^0}{\partial y}\right)$$

$$E_y^0 = -\frac{1}{h^2}\left(\gamma \frac{\partial E_z^0}{\partial y} - j\omega\mu \frac{\partial H_z^0}{\partial x}\right)$$

$$\rightarrow \mathbf{E}_{T,TM}^0 = \mathbf{a}_x E_x^0 + \mathbf{a}_y E_y^0 = -\frac{\gamma}{h^2} \nabla_T E_z^0$$

Recall:

$$\rightarrow \frac{\partial H_z^0}{\partial y} + \gamma H_y^0 = j\omega\varepsilon E_x^0$$

$$\rightarrow -\gamma H_x^0 - \frac{\partial H_z^0}{\partial x} = j\omega\varepsilon E_y^0$$

$$\rightarrow \frac{\partial H_y^0}{\partial x} - \frac{\partial H_x^0}{\partial y} = j\omega\varepsilon E_z^0$$

Wave impedance: $Z_{TM} = \frac{E_x^0}{H_y^0} = -\frac{E_y^0}{H_x^0} = \frac{\gamma}{j\omega\varepsilon} \quad \rightarrow \mathbf{H} = \frac{1}{Z_{TM}} \mathbf{a}_z \times \mathbf{E}$

Characteristic values or eigenvalues: $h^2 = \gamma^2 + k^2$

$$\rightarrow \gamma = \sqrt{h^2 - k^2} = \sqrt{h^2 - \omega^2 \mu\varepsilon}$$

Transverse Magnetic (TM) Waves (2)

Propagation constant: $\gamma = \sqrt{h^2 - k^2} = \sqrt{h^2 - \omega^2 \mu\varepsilon}$

$$\rightarrow \gamma = 0 \;\rightarrow f_c = \frac{h}{2\pi\sqrt{\mu\varepsilon}}$$ $\rightarrow$ *Cutoff frequency*

$$\rightarrow \boxed{\gamma = h\sqrt{1-(f/f_c)^2}}$$

What if: $(f/f_c)^2 < 1$?

For $(f/f_c)^2 > 1$: $\rightarrow \gamma = j\beta = jk\sqrt{1-(h/k)^2} = jk\sqrt{1-(f_c/f)^2}$

$\rightarrow$ *Propagating mode*

Wavelength in the guide: $\lambda_g = \dfrac{2\pi}{\beta} = \dfrac{2\pi}{k\sqrt{1-(f_c/f)^2}} = \dfrac{\lambda}{\sqrt{1-(f_c/f)^2}} > \lambda$

Phase velocity: $u_p = \dfrac{\omega}{\beta} = \dfrac{\omega}{k\sqrt{1-(f_c/f)^2}} = \dfrac{u}{\sqrt{1-(f_c/f)^2}} > u$

Group velocity: $u_g = \dfrac{1}{d\beta/d\omega} = u\sqrt{1-(f_c/f)^2} < u$

Why?

Wave impedance: $\boxed{Z_{TM} = \dfrac{\gamma}{j\omega\varepsilon} = \dfrac{jk\sqrt{1-(f_c/f)^2}}{j\omega\varepsilon} = \eta\sqrt{1-(f_c/f)^2} < \eta}$

Transverse Electric (TE) Waves

For $E_z = 0$: $\rightarrow \nabla_{xy}^2 \mathbf{H} + (\gamma^2 + k^2)\mathbf{H} = 0 \ \rightarrow \nabla_{xy}^2 H_z^0 + (\gamma^2 + k^2) H_z^0 = 0$

$\rightarrow \nabla_{xy}^2 H_z^0 + h^2 H_z^0 = 0$ ← *Characteristic eq.*

$$H_x^0 = -\frac{1}{h^2}\left(\gamma \frac{\partial H_z^0}{\partial x} - j\omega\varepsilon \frac{\partial E_z^0}{\partial y}\right)$$

$$H_y^0 = -\frac{1}{h^2}\left(\gamma \frac{\partial H_z^0}{\partial y} + j\omega\varepsilon \frac{\partial E_z^0}{\partial x}\right)$$

$$E_x^0 = -\frac{1}{h^2}\left(\gamma \frac{\partial E_z^0}{\partial x} + j\omega\mu \frac{\partial H_z^0}{\partial y}\right)$$

$$E_y^0 = -\frac{1}{h^2}\left(\gamma \frac{\partial E_z^0}{\partial y} - j\omega\mu \frac{\partial H_z^0}{\partial x}\right)$$

$$\rightarrow \mathbf{H}_{T,TM}^0 = \mathbf{a}_x H_x^0 + \mathbf{a}_y H_y^0 = -\frac{\gamma}{h^2}\nabla_T H_z^0$$

Recall:

$$\rightarrow \frac{\partial E_z^0}{\partial y} + \gamma E_y^0 = -j\omega\mu H_x^0$$

$$\rightarrow -\gamma E_x^0 - \frac{\partial E_z^0}{\partial x} = -j\omega\mu H_y^0$$

$$\rightarrow \frac{\partial E_y^0}{\partial x} - \frac{\partial E_x^0}{\partial y} = -j\omega\mu H_z^0$$

Wave impedance: $Z_{TE} = \frac{E_x^0}{H_y^0} = -\frac{E_y^0}{H_x^0} = \frac{j\omega\mu}{\gamma} \ \rightarrow \mathbf{E} = -Z_{TE}(\mathbf{a}_z \times \mathbf{H})$

For $(f/f_c)^2 > 1$: $$Z_{TE} = \frac{j\omega\mu}{jk\sqrt{1-(f_c/f)^2}} = \frac{\eta}{\sqrt{1-(f_c/f)^2}} > \eta$$

Why?