

# Electromagnetics:

## Optical Fibers

Yoonchan Jeong

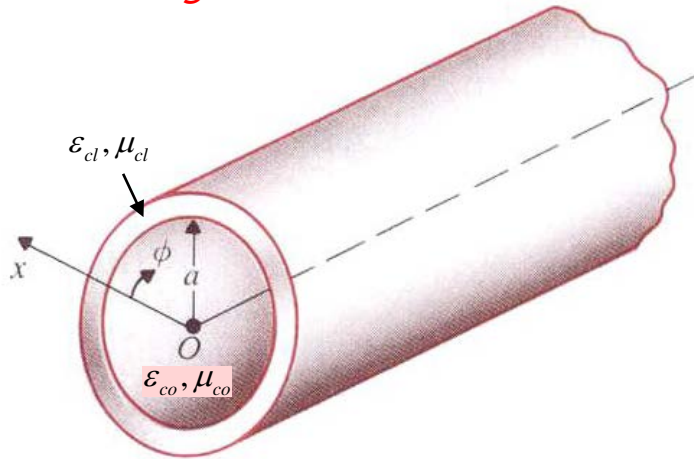
School of Electrical Engineering, Seoul National University

Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953

Email: [yunchan@snu.ac.kr](mailto:yunchan@snu.ac.kr)

# Circular Dielectric Waveguides: Optical Fibers

*A circular waveguide with a dielectric cladding*



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

$$\mu_{co} \epsilon_{co} > \mu_{cl} \epsilon_{cl}$$

Let:  $\gamma = j\beta$

$$k^2 = \omega^2 \mu \epsilon = \omega^2 \begin{cases} \mu_{co} \epsilon_{co}, & r \leq a \\ \mu_{cl} \epsilon_{cl}, & r > a \end{cases}$$

$$\rightarrow h^2 = k_{co}^2 - \beta^2$$

$$\rightarrow q^2 = \beta^2 - k_{cl}^2$$

Let:  $E_{co,z}(r, \phi, z) = E_{co,z}^0(r, \phi) e^{-\gamma z}$   
 $H_{co,z}(r, \phi, z) = H_{co,z}^0(r, \phi) e^{-\gamma z}$  → Core

$E_{cl,z}(r, \phi, z) = E_{cl,z}^0(r, \phi) e^{-\gamma z}$   
 $H_{cl,z}(r, \phi, z) = H_{cl,z}^0(r, \phi) e^{-\gamma z}$  → Cladding



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Recall:

$$R(r) = \begin{cases} C J_n(hr) + D N_n(hr) & \text{for } h^2 > 0 \\ C I_n(hr) + D K_n(hr) & \text{for } h^2 < 0 \end{cases}$$

$\rightarrow E_{co,z}^0, H_{co,z}^0 \propto J_n(hr) \cos n\phi$   
 $\rightarrow E_{cl,z}^0, H_{cl,z}^0 \propto K_n(hr) \cos n\phi$

# TM Modes in Circular Dielectric Waveguides

Unless  $n = 0$ , the B.C. cannot be satisfied only with  $E_z^0$ !

$$\leftarrow Z_{TM} = \gamma / j\omega\epsilon$$

Recall:

$$\mathbf{E}_T^0 = -\frac{\gamma}{h^2} \nabla_T E_z^0 \quad \leftarrow \nabla_T E_z^0 = \left( \mathbf{a}_r \frac{\partial}{\partial r} + \mathbf{a}_\phi \frac{\partial}{r \partial \phi} \right) E_z^0 \quad \rightarrow \mathbf{H} = \frac{1}{Z_{TM}} \mathbf{a}_z \times \mathbf{E}$$

Core:

$E_{co,r}^0 = -\frac{j\beta}{h^2} h A_n J'_n(hr) \cos n\phi$	$H_{co,r}^0 = \frac{\omega\epsilon_{co}}{\beta} (-1) E_{co,\phi}^0 = -\frac{j\omega\epsilon_{co} n}{h^2 r} A_n J_n(hr) \sin n\phi$
$E_{co,\phi}^0 = -\frac{j\beta}{h^2} \frac{(-n)}{r} A_n J_n(hr) \sin n\phi$	$H_{co,\phi}^0 = \frac{\omega\epsilon_{co}}{\beta} E_{co,r}^0 = -\frac{j\omega\epsilon_{co}}{h} A_n J'_n(hr) \cos n\phi$
$E_{co,z}^0 = A_n J_n(hr) \cos n\phi$	$H_{co,z}^0 = 0$

Cladding:

$E_{cl,r}^0 = \frac{j\beta}{q^2} q C_n K'_n(qr) \cos n\phi$	$H_{cl,r}^0 = \frac{\omega\epsilon_{cl}}{\beta} (-1) E_{cl,\phi}^0 = \frac{j\omega\epsilon_{cl} n}{q^2 r} C_n K_n(qr) \sin n\phi$
$E_{cl,\phi}^0 = \frac{j\beta}{q^2} \frac{(-n)}{r} C_n K_n(qr) \sin n\phi$	$H_{cl,\phi}^0 = \frac{\omega\epsilon_{cl}}{\beta} E_{cl,r}^0 = \frac{j\omega\epsilon_{cl}}{q} C_n K'_n(qr) \cos n\phi$
$E_{cl,z}^0 = C_n K_n(qr) \cos n\phi$	$H_{cl,z}^0 = 0$

B.C.:

$E_{co,z}^0(a) = E_{cl,z}^0(a)$	$H_{co,z}^0(a) = H_{cl,z}^0(a)$
$E_{co,\phi}^0(a) = E_{cl,\phi}^0(a)$	$H_{co,\phi}^0(a) = H_{cl,\phi}^0(a)$

# TE Modes in Circular Dielectric Waveguides

Unless  $n = 0$ , the B.C. cannot be satisfied only with  $H_z^0$ !

$$\leftarrow Z_{TE} = j\omega\mu / \gamma$$

Recall:

$$\mathbf{H}_T^0 = -\frac{\gamma}{h^2} \nabla_T H_z^0 \leftarrow \nabla_T H_z^0 = \left( \mathbf{a}_r \frac{\partial}{\partial r} + \mathbf{a}_\phi \frac{\partial}{r \partial \phi} \right) H_z^0 \rightarrow \mathbf{E} = -Z_{TE} \mathbf{a}_z \times \mathbf{H}$$

Core:

$H_{co,r}^0 = -\frac{j\beta}{h^2} h B_n J'_n(hr) \cos n\phi$	$E_{co,r}^0 = -\frac{\omega\mu_{co}}{\beta} (-1) H_{co,\phi}^0 = \frac{j\omega\mu_{co} n}{h^2 r} B_n J_n(hr) \sin n\phi$
$H_{co,\phi}^0 = -\frac{j\beta}{h^2} \frac{(-n)}{r} B_n J_n(hr) \sin n\phi$	$E_{co,\phi}^0 = -\frac{\omega\mu_{co}}{\beta} H_{co,r}^0 = \frac{j\omega\mu_{co}}{h} B_n J'_n(hr) \cos n\phi$
$H_{co,z}^0 = B_n J_n(hr) \cos n\phi$	$E_{co,z}^0 = 0$

Cladding:

$H_{cl,r}^0 = \frac{j\beta}{q^2} q D_n K'_n(qr) \cos n\phi$	$E_{cl,r}^0 = -\frac{\omega\mu_{cl}}{\beta} (-1) H_{cl,\phi}^0 = -\frac{j\omega\mu_{cl} n}{q^2 r} D_n K_n(qr) \sin n\phi$
$H_{cl,\phi}^0 = \frac{j\beta}{q^2} \frac{(-n)}{r} D_n K_n(qr) \sin n\phi$	$E_{cl,\phi}^0 = -\frac{\omega\mu_{cl}}{\beta} H_{cl,r}^0 = -\frac{j\omega\mu_{cl}}{q} D_n K'_n(qr) \cos n\phi$
$H_{cl,z}^0 = D_n K_n(qr) \cos n\phi$	$E_{cl,z}^0 = 0$

B.C.:

$H_{co,z}^0(a) = H_{cl,z}^0(a)$	$E_{co,z}^0(a) = E_{cl,z}^0(a)$
$H_{co,\phi}^0(a) = H_{cl,\phi}^0(a)$	$E_{co,\phi}^0(a) = E_{cl,\phi}^0(a)$

# Hybrid Modes in Circular Dielectric Waveguides (1)

TM

TE

TM

TE

Core:

$E_{co,r}^0 = -\frac{j\beta}{h^2} h A_n J_n'(hr) \cos n\phi$ $E_{co,\phi}^0 = -\frac{j\beta}{h^2} \frac{(-n)}{r} A_n J_n(hr) \sin n\phi$ $E_{co,z}^0 = A_n J_n(hr) \cos n\phi$	$E_{co,r}^0 = \frac{j\omega\mu_{co}n}{h^2 r} B_n J_n(hr) \sin(n\phi + \phi_0)$ $E_{co,\phi}^0 = \frac{j\omega\mu_{co}}{h} B_n J_n'(hr) \cos(n\phi + \phi_0)$ $E_{co,z}^0 = 0$	$H_{co,r}^0 = -\frac{j\omega\epsilon_{co}n}{h^2 r} A_n J_n(hr) \sin n\phi$ $H_{co,\phi}^0 = -\frac{j\omega\epsilon_{co}}{h} A_n J_n'(hr) \cos n\phi$ $H_{co,z}^0 = 0$	$H_{co,r}^0 = -\frac{j\beta}{h^2} h B_n J_n'(hr) \cos(n\phi + \phi_0)$ $H_{co,\phi}^0 = -\frac{j\beta}{h^2} \frac{(-n)}{r} B_n J_n(hr) \sin(n\phi + \phi_0)$ $H_{co,z}^0 = B_n J_n(hr) \cos(n\phi + \phi_0)$
$\rightarrow E_{co,r}^0 = -\frac{j\beta}{h^2} [h A_n J_n'(hr) \cos n\phi - \frac{\omega\mu_{co}n}{\beta r} B_n J_n(hr) \sin(n\phi + \phi_0)]$ $\rightarrow E_{co,\phi}^0 = -\frac{j\beta}{h^2} [\frac{(-n)}{r} A_n J_n(hr) \sin n\phi - \frac{\omega\mu_{co}}{\beta} B_n h J_n'(hr) \cos(n\phi + \phi_0)]$ $\rightarrow E_{co,z}^0 = A_n J_n(hr) \cos n\phi$	$\rightarrow H_{co,r}^0 = -\frac{j\beta}{h^2} [\frac{\omega\epsilon_{co}n}{\beta r} A_n J_n(hr) \sin n\phi + h B_n J_n'(hr) \cos(n\phi + \phi_0)]$ $\rightarrow H_{co,\phi}^0 = -\frac{j\beta}{h^2} [\frac{\omega\epsilon_{co}}{\beta} h A_n J_n'(hr) \cos n\phi - \frac{n}{r} B_n J_n(hr) \sin(n\phi + \phi_0)]$ $\rightarrow H_{co,z}^0 = B_n J_n(hr) \cos(n\phi + \phi_0)$		

Cladding:

$E_{cl,r}^0 = \frac{j\beta}{q^2} q C_n K_n'(qr) \cos n\phi$ $E_{cl,\phi}^0 = \frac{j\beta}{q^2} \frac{(-n)}{r} C_n K_n(qr) \sin n\phi$ $E_{cl,z}^0 = C_n K_n(qr) \cos n\phi$	$E_{cl,r}^0 = -\frac{j\omega\mu_{cl}n}{q^2 r} D_n K_n(qr) \sin(n\phi + \phi_0)$ $E_{cl,\phi}^0 = -\frac{j\omega\mu_{cl}}{q} D_n K_n'(qr) \cos(n\phi + \phi_0)$ $E_{cl,z}^0 = 0$	$H_{cl,r}^0 = \frac{j\omega\epsilon_{cl}n}{q^2 r} C_n K_n(qr) \sin n\phi$ $H_{cl,\phi}^0 = \frac{j\omega\epsilon_{cl}}{q} C_n K_n'(qr) \cos n\phi$ $H_{cl,z}^0 = 0$	$H_{cl,r}^0 = \frac{j\beta}{q^2} q D_n K_n'(qr) \cos(n\phi + \phi_0)$ $H_{cl,\phi}^0 = \frac{j\beta}{q^2} \frac{(-n)}{r} D_n K_n(qr) \sin(n\phi + \phi_0)$ $H_{cl,z}^0 = D_n K_n(qr) \cos(n\phi + \phi_0)$
$\rightarrow E_{cl,r}^0 = \frac{j\beta}{q^2} [q C_n K_n'(qr) \cos n\phi - \frac{\omega\mu_{cl}n}{\beta r} D_n K_n(qr) \sin(n\phi + \phi_0)]$ $\rightarrow E_{cl,\phi}^0 = \frac{j\beta}{q^2} [\frac{(-n)}{r} C_n K_n(qr) \sin n\phi - \frac{\omega\mu_{cl}}{\beta} q D_n K_n'(qr) \cos(n\phi + \phi_0)]$ $\rightarrow E_{cl,z}^0 = C_n K_n(qr) \cos n\phi$	$\rightarrow H_{cl,r}^0 = \frac{j\beta}{q^2} [\frac{\omega\epsilon_{cl}n}{\beta r} C_n K_n(qr) \sin n\phi + q D_n K_n'(qr) \cos(n\phi + \phi_0)]$ $\rightarrow H_{cl,\phi}^0 = \frac{j\beta}{q^2} [\frac{\omega\epsilon_{cl}}{\beta} q C_n K_n'(qr) \cos n\phi + \frac{(-n)}{r} D_n K_n(qr) \sin(n\phi + \phi_0)]$ $\rightarrow H_{cl,z}^0 = D_n K_n(qr) \cos(n\phi + \phi_0)$		

B.C.:

$$E_{co,z}^0(a) = E_{cl,z}^0(a)$$

$$E_{co,\phi}^0(a) = E_{cl,\phi}^0(a)$$

$$H_{co,z}^0(a) = H_{cl,z}^0(a)$$

$$H_{co,\phi}^0(a) = H_{cl,\phi}^0(a)$$

# Hybrid Modes in Circular Dielectric Waveguides (2)

## Boundary condition:

$$E_{co,z}^0(a) = E_{cl,z}^0(a) \rightarrow A_n J_n(ha) \cos n\phi = C_n K_n(qa) \cos n\phi \rightarrow A_n = C_n \frac{K_n(qa)}{J_n(ha)}$$

$$E_{co,\phi}^0(a) = E_{cl,\phi}^0(a) \rightarrow -\frac{j\beta}{h^2} \left[ \frac{(-n)}{a} A_n J_n(ha) \sin n\phi - \frac{\omega\mu_{co}}{\beta} B_n h J_n'(ha) \cos(n\phi + \phi_0) \right] = \frac{j\beta}{q^2} \left[ \frac{(-n)}{a} C_n K_n(qa) \sin n\phi - \frac{\omega\mu_{cl}}{\beta} q D_n K_n'(qa) \cos(n\phi + \phi_0) \right]$$

$$H_{co,z}^0(a) = H_{cl,z}^0(a) \rightarrow B_n J_n(ha) \cos(n\phi + \phi_0) = D_n K_n(qa) \cos(n\phi + \phi_0) \rightarrow D_n = B_n \frac{J_n(ha)}{K_n(qa)}$$

$$H_{co,\phi}^0(a) = H_{cl,\phi}^0(a) \rightarrow -\frac{j\beta}{h^2} \left[ \frac{\omega\epsilon_{co}}{\beta} h A_n J_n'(ha) \cos n\phi - \frac{n}{a} B_n J_n(ha) \sin(n\phi + \phi_0) \right] = \frac{j\beta}{q^2} \left[ \frac{\omega\epsilon_{cl}}{\beta} q C_n K_n'(qa) \cos n\phi + \frac{(-n)}{a} D_n K_n(qa) \sin(n\phi + \phi_0) \right]$$

$$\rightarrow A_n \frac{n}{h^2 a} J_n(ha) \sin n\phi + B_n \frac{\omega\mu_{co}}{h\beta} J_n'(ha) \cos(n\phi + \phi_0) + \frac{n}{q^2 a} C_n K_n(qa) \sin n\phi + D_n \frac{\omega\mu_{cl}}{q\beta} K_n'(qa) \cos(n\phi + \phi_0) = 0$$

$$\rightarrow A_n \frac{\omega\epsilon_{co}}{h\beta} J_n'(ha) \cos n\phi - B_n \frac{n}{h^2 a} J_n(ha) \sin(n\phi + \phi_0) + C_n \frac{\omega\epsilon_{cl}}{q\beta} K_n'(qa) \cos n\phi - D_n \frac{n}{q^2 a} K_n(qa) \sin(n\phi + \phi_0) = 0$$

$$\rightarrow B_n \left[ \frac{\omega\mu_{co}}{h\beta} J_n'(ha) + \frac{\omega\mu_{cl}}{q\beta} \frac{K_n'(qa)}{K_n(qa)} J_n(ha) \right] \cos(n\phi + \phi_0) + C_n \left[ \frac{n}{q^2 a} + \frac{n}{h^2 a} \right] K_n(qa) \sin n\phi = 0$$

$$\rightarrow -B_n \left[ \frac{n}{h^2 a} + \frac{n}{q^2 a} \right] J_n(ha) \sin(n\phi + \phi_0) + C_n \left[ \frac{\omega\epsilon_{co}}{h\beta} \frac{J_n'(ha)}{J_n(ha)} K_n(qa) + \frac{\omega\epsilon_{cl}}{q\beta} K_n'(qa) \right] \cos n\phi = 0$$

$$\rightarrow \text{For a non-trivial solution to exist:} \quad \rightarrow \phi_0 = \pm \frac{\pi}{2}$$

$$\rightarrow \left[ \frac{\omega\mu_{co}}{h\beta} J_n'(ha) + \frac{\omega\mu_{cl}}{q\beta} \frac{K_n'(qa)}{K_n(qa)} J_n(ha) \right] \left[ \frac{\omega\epsilon_{co}}{h\beta} \frac{J_n'(ha)}{J_n(ha)} K_n(qa) + \frac{\omega\epsilon_{cl}}{q\beta} K_n'(qa) \right] \cos n\phi \cos(n\phi + \phi_0) + \left[ \frac{n}{h^2 a} + \frac{n}{q^2 a} \right]^2 J_n(ha) K_n(qa) \sin n\phi \sin(n\phi + \phi_0) = 0$$

Transcendental equation for modes:

$$\rightarrow \left[ \frac{\mu_{co}}{ha} \frac{J_n'(ha)}{J_n(ha)} + \frac{\mu_{cl}}{qa} \frac{K_n'(qa)}{K_n(qa)} \right] \left[ \frac{\epsilon_{co}}{ha} \frac{J_n'(ha)}{J_n(ha)} + \frac{\epsilon_{cl}}{qa} \frac{K_n'(qa)}{K_n(qa)} \right] - n^2 \left[ \left( \frac{1}{ha} \right)^2 + \left( \frac{1}{qa} \right)^2 \right] \left( \frac{\beta}{\omega} \right)^2 = 0$$

# Characteristics of Hybrid Modes (1)

Transcendental equation for modes:

$$\left[ \frac{\mu_{co}}{ha} \frac{J'_n(ha)}{J_n(ha)} + \frac{\mu_{cl}}{qa} \frac{K'_n(qa)}{K_n(qa)} \right] \left[ \frac{\epsilon_{co}}{ha} \frac{J'_n(ha)}{J_n(ha)} + \frac{\epsilon_{cl}}{qa} \frac{K'_n(qa)}{K_n(qa)} \right] = n^2 \left[ \left( \frac{1}{ha} \right)^2 + \left( \frac{1}{qa} \right)^2 \right]^2 \left( \frac{\beta}{\omega} \right)^2 = 0$$

*For non-magnetic media:*  $\rightarrow \mu_{co} = \mu_{cl} = \mu_0$

$$\rightarrow \left[ \frac{J'_n(ha)}{haJ_n(ha)} + \frac{K'_n(qa)}{qaK_n(qa)} \right] \left[ \frac{n_{co}^2 J'_n(ha)}{haJ_n(ha)} + \frac{n_{cl}^2 K'_n(qa)}{qaK_n(qa)} \right] = n^2 \left[ \left( \frac{1}{ha} \right)^2 + \left( \frac{1}{qa} \right)^2 \right]^2 \left( \frac{\beta}{k_0} \right)^2 = 0$$

**Solution:**

$$\rightarrow \frac{J'_n(ha)}{haJ_n(ha)} = -\frac{n_{co}^2 + n_{cl}^2}{2n_{co}^2} \frac{K'_n(qa)}{qaK_n(qa)} \pm \left\{ \left( \frac{n_{co}^2 - n_{cl}^2}{2n_{co}^2} \right)^2 \left( \frac{K'_n(qa)}{qaK_n(qa)} \right)^2 + \frac{n^2}{n_{co}^2} \left( \frac{\beta}{k_0} \right)^2 \left[ \frac{1}{(ha)^2} + \frac{1}{(qa)^2} \right]^2 \right\}^{1/2}$$

$$\leftarrow \begin{cases} J'_n(x) = -J'_{n+1}(x) + \frac{n}{x} J_n(x) \\ J'_n(x) = J'_{n-1}(x) - \frac{n}{x} J_n(x) \end{cases}$$

*EH modes:*  $\rightarrow \frac{J_{n+1}(ha)}{haJ_n(ha)} = \frac{n_{co}^2 + n_{cl}^2}{2n_{co}^2} \frac{K'_n(qa)}{qaK_n(qa)} + \frac{n}{(ha)^2} - \left[ \left( \frac{n_{co}^2 - n_{cl}^2}{2n_{co}^2} \right)^2 \left( \frac{K'_n(qa)}{qaK_n(qa)} \right)^2 + \frac{n^2}{n_{co}^2} \left( \frac{\beta}{k_0} \right)^2 \left( \frac{1}{h^2 a^2} + \frac{1}{q^2 a^2} \right)^2 \right]^{1/2}$

*HE modes:*  $\rightarrow \frac{J_{n-1}(ha)}{haJ_n(ha)} = -\frac{n_{co}^2 + n_{cl}^2}{2n_{co}^2} \frac{K'_n(qa)}{qaK_n(qa)} + \frac{n}{(ha)^2} - \left[ \left( \frac{n_{co}^2 - n_{cl}^2}{2n_{co}^2} \right)^2 \left( \frac{K'_n(qa)}{qaK_n(qa)} \right)^2 + \frac{n^2}{n_{co}^2} \left( \frac{\beta}{k_0} \right)^2 \left( \frac{1}{h^2 a^2} + \frac{1}{q^2 a^2} \right)^2 \right]^{1/2}$

# Characteristics of Hybrid Modes (2)

***EH modes:***  $\rightarrow \frac{J_{n+1}(ha)}{haJ_n(ha)} = \frac{n_{co}^2 + n_{cl}^2}{2n_{co}^2} \frac{K'_n(qa)}{qaK_n(qa)} + \frac{n}{(ha)^2} - \left[ \left( \frac{n_{co}^2 - n_{cl}^2}{2n_{co}^2} \right)^2 \left( \frac{K'_n(qa)}{qaK_n(qa)} \right)^2 + \frac{n^2}{n_{co}^2} \left( \frac{\beta}{k_0} \right)^2 \left( \frac{1}{h^2 a^2} + \frac{1}{q^2 a^2} \right)^2 \right]^{1/2}$   
 $E_z \uparrow H_z \downarrow \rightarrow$  ***TM-like***

***HE modes:***  $\rightarrow \frac{J_{n-1}(ha)}{haJ_n(ha)} = -\frac{n_{co}^2 + n_{cl}^2}{2n_{co}^2} \frac{K'_n(qa)}{qaK_n(qa)} + \frac{n}{(ha)^2} - \left[ \left( \frac{n_{co}^2 - n_{cl}^2}{2n_{co}^2} \right)^2 \left( \frac{K'_n(qa)}{qaK_n(qa)} \right)^2 + \frac{n^2}{n_{co}^2} \left( \frac{\beta}{k_0} \right)^2 \left( \frac{1}{h^2 a^2} + \frac{1}{q^2 a^2} \right)^2 \right]^{1/2}$   
 $E_z \downarrow H_z \uparrow \rightarrow$  ***TE-like***

***For n = 0:***

***TM modes:***  $\rightarrow \frac{J_1(ha)}{haJ_0(ha)} = \frac{n_{cl}^2}{n_{co}^2} \frac{K'_0(qa)}{qaK_0(qa)} = -\frac{n_{cl}^2}{n_{co}^2} \frac{K_1(qa)}{qaK_0(qa)}$

***TE modes:***  $\rightarrow \frac{J_1(ha)}{haJ_0(ha)} = \frac{K'_0(qa)}{qaK_0(qa)} = -\frac{K_1(qa)}{qaK_0(qa)}$

***To be solved numerically***



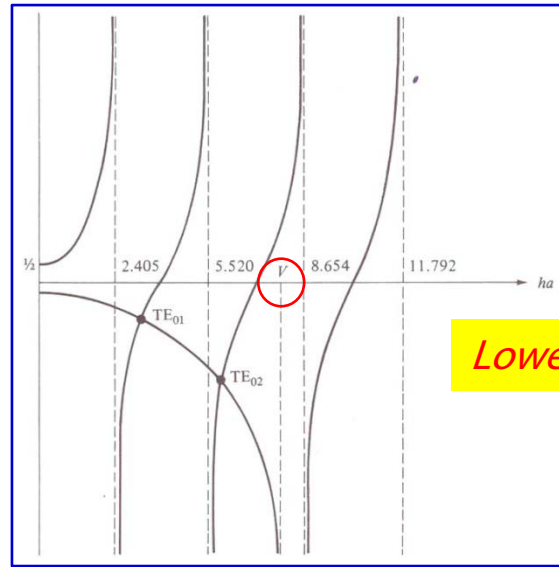
# Cutoff Conditions

$$TM_{0m}: \rightarrow \frac{J_1(ha)}{haJ_0(ha)} = -\frac{n_{cl}^2}{n_{co}^2} \frac{K_1(qa)}{qaK_0(qa)}$$

$$TE_{0m}: \rightarrow \frac{J_1(ha)}{haJ_0(ha)} = -\frac{K_1(qa)}{qaK_0(qa)}$$

$$\text{If } qa \rightarrow 0, ha \rightarrow \frac{2\pi a}{\lambda_0} \sqrt{n_{co}^2 - n_{cl}^2} \equiv V$$

$V < 2.405 \rightarrow$  *Single-mode!*



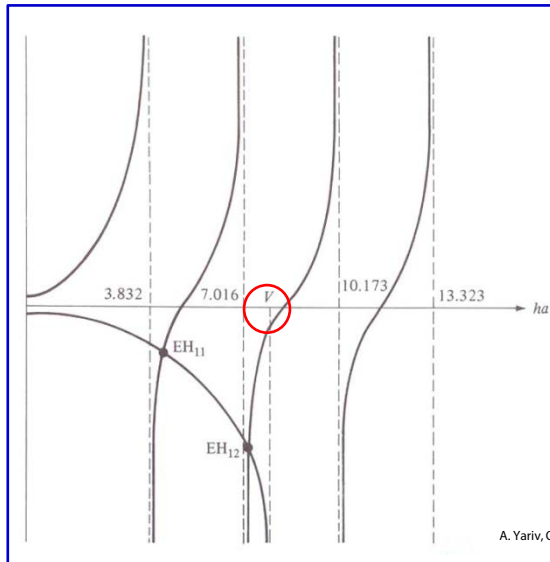
*Graphical method*

A. Yariv, Optical Electronics, 4th ed., HBJ, 1991.

$$EH_{1m}: \rightarrow \frac{J_2(ha)}{haJ_1(ha)} = \frac{n_{co}^2 + n_{cl}^2}{2n_{co}^2} \frac{K_1'(qa)}{qaK_1(qa)} + \frac{1}{(ha)^2} - \left[ \left( \frac{n_{co}^2 - n_{cl}^2}{2n_{co}^2} \right)^2 \left( \frac{K_1'(qa)}{qaK_1(qa)} \right)^2 + \frac{1}{n_{co}^2} \left( \frac{\beta}{k_0} \right)^2 \left( \frac{1}{h^2 a^2} + \frac{1}{q^2 a^2} \right)^2 \right]^{1/2}$$

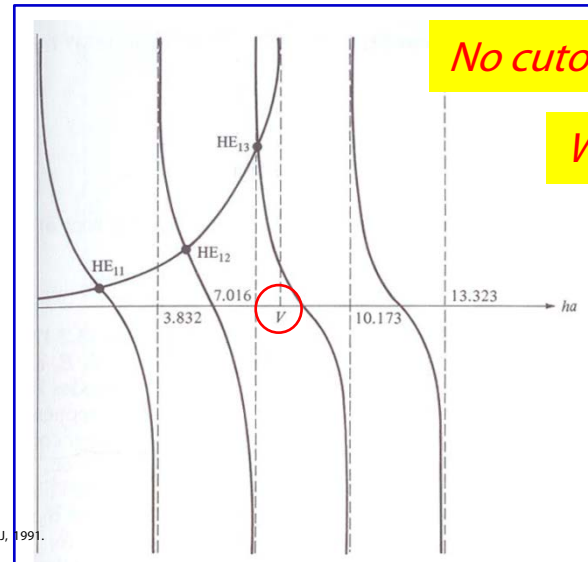
$$HE_{1m}: \rightarrow \frac{J_0(ha)}{haJ_1(ha)} = -\frac{n_{co}^2 + n_{cl}^2}{2n_{co}^2} \frac{K_1'(qa)}{qaK_1(qa)} + \frac{1}{(ha)^2} - \left[ \left( \frac{n_{co}^2 - n_{cl}^2}{2n_{co}^2} \right)^2 \left( \frac{K_1'(qa)}{qaK_1(qa)} \right)^2 + \frac{1}{n_{co}^2} \left( \frac{\beta}{k_0} \right)^2 \left( \frac{1}{h^2 a^2} + \frac{1}{q^2 a^2} \right)^2 \right]^{1/2}$$

$EH_{1m}:$



A. Yariv, Optical Electronics, 4th ed., HBJ, 1991.

$HE_{1m}:$



*Why?*