Electromagnetics:
Wave Characteristics on Finite Transmission Lines
(9-4)

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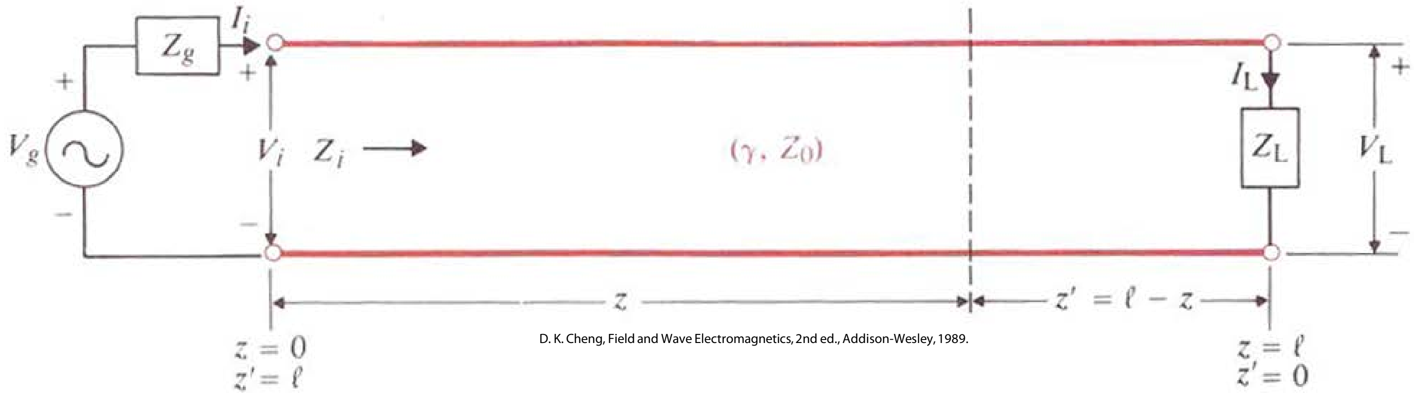
Finite Transmission Lines (1)



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

General solution:

$$V(z) = V^+(z) + V^-(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

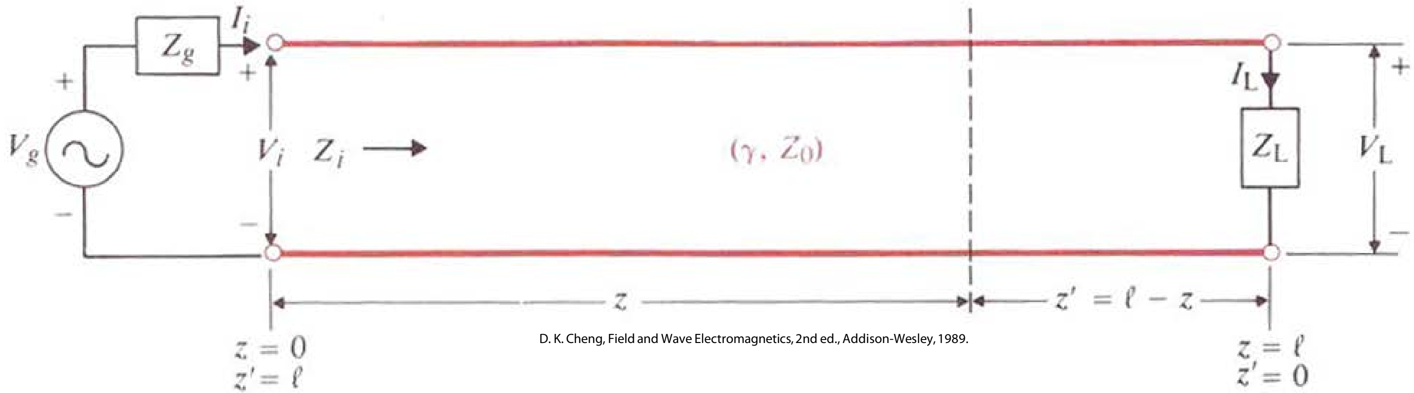
$$I(z) = I^+(z) + I^-(z) = I_0^+ e^{-\gamma z} + I_0^- e^{+\gamma z}$$

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-}$$ ← *Characteristic impedance*

Load impedance:

$$\left(\frac{V}{I}\right)_{z=l} = \frac{V_L}{I_L} = Z_L$$

Finite Transmission Lines (2)



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$$\rightarrow V_L = V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l} \qquad \rightarrow V_0^+ = \frac{1}{2}\left(V_L + I_L Z_0\right)e^{\gamma l}$$

$$\rightarrow I_L = \frac{V_0^+}{Z_0} e^{-\gamma l} - \frac{V_0^-}{Z_0} e^{\gamma l} \qquad \rightarrow V_0^- = \frac{1}{2}\left(V_L - I_L Z_0\right)e^{-\gamma l}$$

$$\rightarrow V(z) = \frac{I_L}{2}\left[(Z_L + Z_0)e^{\gamma(l-z)} + (Z_L - Z_0)e^{-\gamma(l-z)}\right]$$

$$\rightarrow I(z) = \frac{I_L}{2Z_0}\left[(Z_L + Z_0)e^{\gamma(l-z)} - (Z_L - Z_0)e^{-\gamma(l-z)}\right]$$

$l - z \rightarrow z'$

$$\rightarrow V(z') = \frac{I_L}{2}\left[(Z_L + Z_0)e^{\gamma z'} + (Z_L - Z_0)e^{-\gamma z'}\right]$$

$$\rightarrow I(z') = \frac{I_L}{2Z_0}\left[(Z_L + Z_0)e^{\gamma z'} - (Z_L - Z_0)e^{-\gamma z'}\right]$$

Finite Transmission Lines (3)

Recall:

$$V(z') = \frac{I_L}{2}\left[(Z_L + Z_0)e^{\gamma z'} + (Z_L - Z_0)e^{-\gamma z'}\right] \quad \leftarrow \frac{e^{\gamma z'} + e^{-\gamma z'}}{2} = \cosh \gamma z'$$

$$I(z') = \frac{I_L}{2Z_0}\left[(Z_L + Z_0)e^{\gamma z'} - (Z_L - Z_0)e^{-\gamma z'}\right] \quad \leftarrow \frac{e^{\gamma z'} - e^{-\gamma z'}}{2} = \sinh \gamma z',$$

$$\rightarrow V(z') = I_L (Z_L \cosh \gamma z' + Z_0 \sinh \gamma z')$$

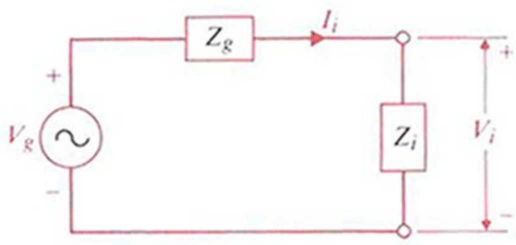
$$\rightarrow I(z') = \frac{I_L}{Z_0}(Z_L \sinh \gamma z' + Z_0 \cosh \gamma z')$$

Impedance:

$$Z(z') = \frac{V(z')}{I(z')} = Z_0 \frac{Z_L \cosh \gamma z' + Z_0 \sinh \gamma z'}{Z_L \sinh \gamma z' + Z_0 \cosh \gamma z'} = Z_0 \frac{Z_L + Z_0 \tanh \gamma z'}{Z_0 + Z_L \tanh \gamma z'}$$

Input impedance: $\leftarrow z = 0 \text{ or } z' = l$

$$Z_i = (Z)_{\substack{z=0 \\ z'=l}} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$$



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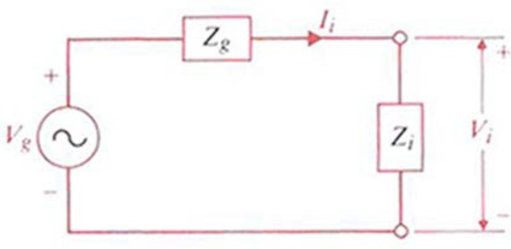
Finite Transmission Lines (4)



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$$\rightarrow V_i = \frac{Z_i}{Z_g + Z_i} V_g$$

$$\rightarrow I_i = \frac{V_g}{Z_g + Z_i}$$

Time-averaged power delivered to the input terminals:

$$\rightarrow (P_{av})_i = \frac{1}{2}\mathrm{Re}\left[V_i I_i^*\right]_{z=0,\, z'=l}$$

Time-averaged power delivered to the load:

$$\rightarrow (P_{av})_L = \frac{1}{2}\mathrm{Re}\left[V_L I_L^*\right]_{z=l,\, z'=0} = \frac{1}{2}\left|\frac{V_L}{Z_L}\right|^2 R_L = \frac{1}{2}\left|I_L\right|^2 R_L$$

For a lossless line: $(P_{av})_i = (P_{av})_L$

When a finite transmission line matched: $\leftarrow Z_L = Z_0$

$$\rightarrow Z(z') = Z_0 \frac{Z_L + Z_0 \tanh \gamma z'}{Z_0 + Z_L \tanh \gamma z'} = Z_0$$

Recall:

$$V(z) = \frac{I_L}{2}\left[(Z_L + Z_0)e^{\gamma(l-z)} + (Z_L - Z_0)e^{-\gamma(l-z)}\right] = \left(I_L Z_0 e^{\gamma l}\right)e^{-\gamma z} = V_i e^{-\gamma z}$$

$$I(z) = \frac{I_L}{2Z_0}\left[(Z_L + Z_0)e^{\gamma(l-z)} - (Z_L - Z_0)e^{-\gamma(l-z)}\right] = \left(I_L e^{\gamma l}\right)e^{-\gamma z} = I_i e^{-\gamma z}$$

Transmission Lines as Circuit Elements (1)

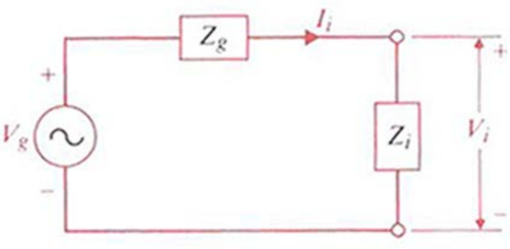
Ordinary lump-circuit elements possible for high frequencies, e.g. UHF (300 MHz to 3 GHz)?

$\rightarrow$ *Transmission-line segments*

Recall:

Input impedance: $z = 0$ *or* $z' = l$

$$Z_i = (Z)_{\substack{z=0 \\ z'=l}} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$$



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For lossless lines:

$$Z_i = R_0 \frac{Z_L + jR_0 \tan \beta l}{R_0 + jZ_L \tan \beta l}$$

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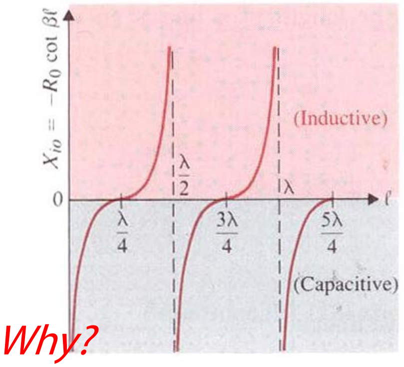

1. Open-circuit termination: $Z_L = \infty$

$$Z_{io} = -jR_0 \cot \beta l = jX_{io}$$

$\rightarrow$ *Purely reactive (capacitive or inductive)*

$\leftarrow$ *No ideal open-circuit termination: Why?*

For a short line: $\beta l << 1$

$$Z_{io} = jX_{io} \cong -j\frac{R_0}{\beta l} = -j\frac{\sqrt{L/C}}{\omega l \sqrt{LC}} = -j\frac{1}{\omega C l}$$

Transmission Lines as Circuit Elements (2)

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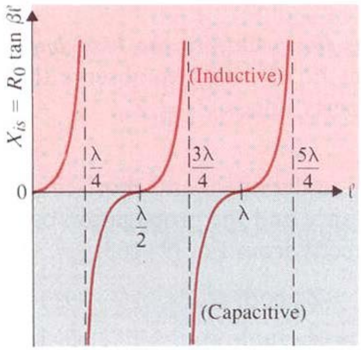

For lossless lines:

$$Z_i = R_0 \frac{Z_L + jR_0 \tan \beta l}{R_0 + jZ_L \tan \beta l}$$

2. Short-circuit termination: $Z_L \to 0$

$$Z_{is} = jR_0 \tan \beta l = jX_{is}$$

$\rightarrow$ Purely reactive (capacitive or inductive)

Shifted by an odd multiple of $\lambda/4$, compared to Z_{io}!

For a short line: $\beta l << 1$

$$Z_{is} = jX_{is} \cong jR_0 \beta l = j\sqrt{\frac{L}{C}}\omega\sqrt{LC}l \quad = j\omega L l$$

3. Quarter-wave section: $l = \lambda / 4, \ \beta l = \pi / 2$

$$\beta l = \frac{2\pi}{\lambda}(2n-1)\frac{\lambda}{4} \ = (2n-1)\frac{\pi}{2} \quad \rightarrow \tan \beta l = \pm\infty$$

$$\rightarrow Z_i = \frac{R_0^2}{Z_L}$$

4. Half-wave section: $l = \lambda / 2, \ \beta l = \pi$

$$\beta l = \frac{2\pi}{\lambda}\left(\frac{n\lambda}{2}\right) = n\pi \quad \rightarrow \tan \beta l = 0$$

$$\rightarrow Z_i = Z_L$$

Transmission Lines as Circuit Elements (3)

Input impedance: $Z_i = Z_0 \dfrac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}$

Open-circuited lines: $Z_L \to \infty \quad \to Z_{io} = Z_0 \coth \gamma l$

Short-circuited lines: $Z_L = 0 \quad \to Z_{is} = Z_0 \tanh \gamma l$

$$\to Z_0 = \sqrt{Z_{io} Z_{is}}$$

$$\to \gamma = \frac{1}{l} \tanh^{-1} \sqrt{\frac{Z_{is}}{Z_{io}}}$$

← *Technically very useful!*

For a lossy line with a short-circuit termination:

$$\to Z_{is} = Z_0 \tanh \gamma l \; = Z_0 \frac{\sinh(\alpha + j\beta)l}{\cosh(\alpha + j\beta)l} \; = Z_0 \frac{\sinh \alpha l \cos \beta l + j \cosh \alpha l \sin \beta l}{\cosh \alpha l \cos \beta l + j \sinh \alpha l \sin \beta l}$$

Series-resonant circuit condition: $l = n\lambda / 2 \;\to\; \beta l = n\pi \;\to\; \sin \beta l = 0$

$$\to Z_{is} = Z_0 \tanh \alpha l \; \cong Z_0 (\alpha l) \quad \text{if} \quad \alpha l << 1$$

Parallel-resonant circuit condition: $l = n\lambda / 4 \;\to\; \beta l = n\pi / 2 \;\to\; \cos \beta l = 0 \;\; (n : \text{odd int.})$

$$\to Z_{is} = Z_0 \coth \alpha l \; \cong Z_0 / \alpha l \quad \text{if} \quad \alpha l << 1$$

Transmission Lines as Circuit Elements (4)

$$l = n\lambda/4 \rightarrow \beta l = n\pi/2 \quad (n: odd\ int.)$$

Frequency selective characteristics (Parallel-resonant circuit):

$$\beta l = \frac{2\pi f}{u_p} l = \frac{2\pi(f_0 + \delta f)}{u_p} l$$

Resonant freq. (f_0)

$$= \frac{n\pi}{2} + \frac{n\pi}{2}\left(\frac{\delta f}{f_0}\right), \; n = odd\ int.$$

$$\rightarrow \cos\beta l = -\sin\left[\frac{n\pi}{2}\left(\frac{\delta f}{f_0}\right)\right] \cong -\frac{n\pi}{2}\left(\frac{\delta f}{f_0}\right)$$

$$\rightarrow \sin\beta l = \cos\left[\frac{n\pi}{2}\left(\frac{\delta f}{f_0}\right)\right] \cong 1$$

$$\rightarrow Z_{is} = Z_0 \frac{\sinh\alpha l\cos\beta l + j\cosh\alpha l\sin\beta l}{\cosh\alpha l\cos\beta l + j\sinh\alpha l\sin\beta l}$$

$$\alpha l << 1 \rightarrow$$

$$Z_{is} \cong \frac{Z_0}{\alpha l + j\frac{n\pi}{2}\left(\frac{\delta f}{f_0}\right)} \rightarrow |Z_{is}|^2 \cong \frac{|Z_0|^2}{(\alpha l)^2 + \left[\frac{n\pi}{2}\left(\frac{\delta f}{f_0}\right)\right]^2} \rightarrow \frac{|Z_{is}|^2}{|Z_{is}|^2_{\max}} \cong \frac{1}{1 + \left[\frac{n\pi}{2\alpha l}\left(\frac{\delta f}{f_0}\right)\right]^2}$$

Half-power frequencies: $\delta f = \pm\Delta f/2$

$$\rightarrow \frac{n\pi}{2\alpha l}\left(\frac{\delta f}{f_0}\right) = \frac{n\pi}{2\alpha l}\left(\frac{\Delta f}{2 f_0}\right) = \frac{\beta}{2\alpha}\left(\frac{\Delta f}{f_0}\right) = 1 \rightarrow Q = \frac{\omega W}{P_L} = \frac{\beta}{2\alpha} = \frac{f_0}{\Delta f}$$

For low-loss lines:

$$\rightarrow Q = \frac{\omega\sqrt{LC}}{R\sqrt{C/L} + G\sqrt{L/C}} = \frac{\omega L}{R + GL/C} = \frac{1}{R/\omega L + G/\omega C} \cong \frac{\omega L}{R}$$