Electromagnetics:
Transients on Transmission Lines
(9-5)

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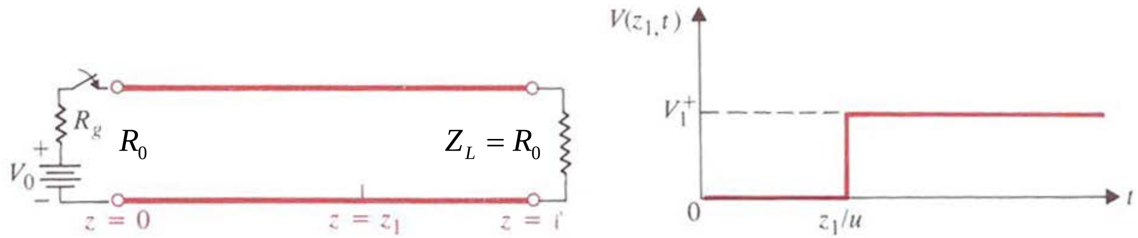
Transients on Transmission Lines (1)

Consider a dc source with R_g:



(a) Switch closed at $t = 0$. (b) Voltage at $z = z_1$.

D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

$$V_1^+ = \frac{R_0 V_0}{R_0 + R_g}, \quad I_1^+ = \frac{V_1^+}{R_0} = \frac{V_0}{R_0 + R_g}, \quad u = 1/\sqrt{LC}$$

No back-reflection! $\leftarrow Z_L = R_0$

Transients on Transmission Lines (2)

Consider: $R_L \neq R_0 \;\&\; R_g \neq R_0$

$$V_1^+ = \frac{R_0 V_0}{R_0 + R_g}, \quad V_1^- = \Gamma_L V_1^+ \;\leftarrow \Gamma_L = \frac{R_L - R_0}{R_L + R_0}$$

$$V_2^+ = \Gamma_g V_1^- = \Gamma_g \Gamma_L V_1^+ \qquad \leftarrow \Gamma_g = \frac{R_g - R_0}{R_g + R_0}$$

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$$\rightarrow V_L = V_1^+ + V_1^- + V_2^+ + V_2^- + V_3^+ + V_3^- + \cdots$$

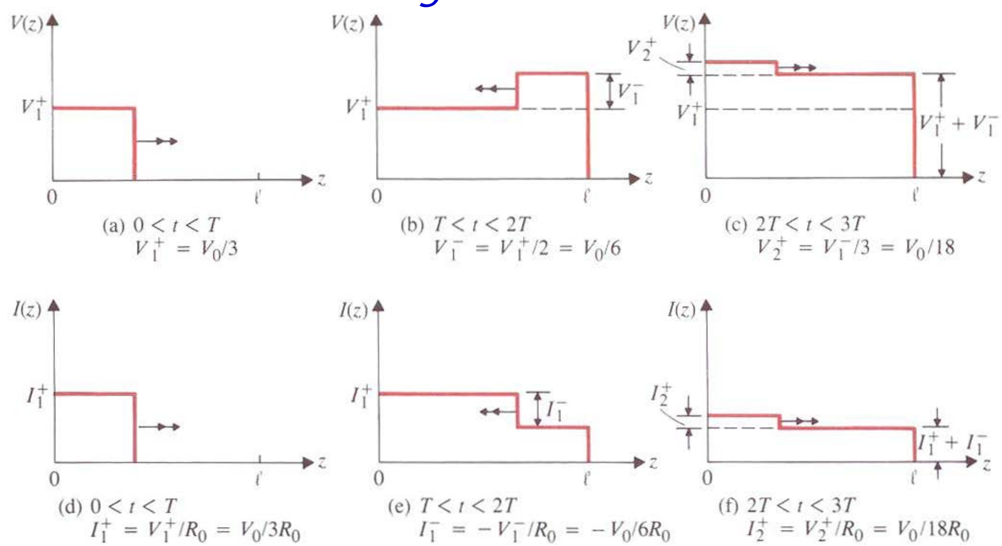
$$= V_1^+ (1 + \Gamma_L + \Gamma_g\Gamma_L + \Gamma_g\Gamma_L^2 + \Gamma_g^2\Gamma_L^2 + \Gamma_g^2\Gamma_L^3 + \cdots)$$

$$= V_1^+ \left[(1 + \Gamma_g\Gamma_L + \Gamma_g^2\Gamma_L^2 + \cdots) + \Gamma_L(1 + \Gamma_g\Gamma_L + \Gamma_g^2\Gamma_L^2 + \cdots)\right]$$

$$= V_1^+ \left(\frac{1 + \Gamma_L}{1 - \Gamma_g\Gamma_L}\right)$$

$$\rightarrow I_L = \frac{V_1^+}{R_0}\left(\frac{1 - \Gamma_L}{1 - \Gamma_g\Gamma_L}\right)$$

Transient voltage and current waves



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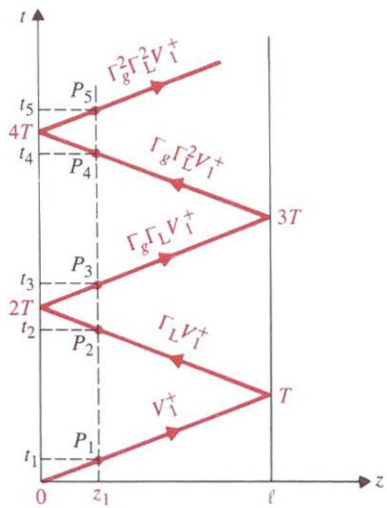
Reflection Diagrams

Voltage reflection diagram:



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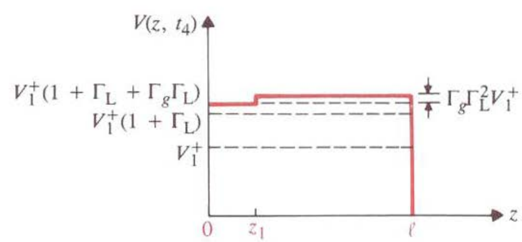
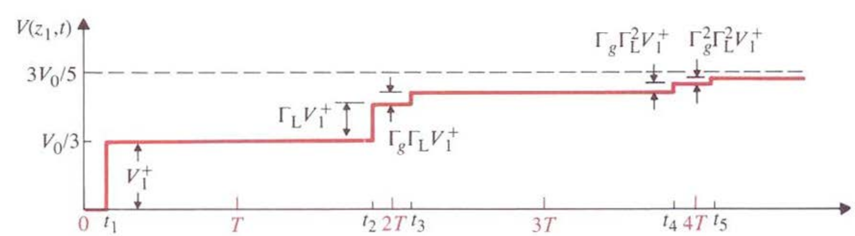
Time Range	Voltage	Voltage Discontinuity
$0 \le t < t_1 \ (t_1 = z_1/u)$	0	0
$t_1 \le t < t_2 \ (t_2 = 2T - t_1)$	V_1^+	V_1^+ at t_1
$t_2 \le t < t_3 \ (t_3 = 2T + t_1)$	$V_1^+(1 + \Gamma_L)$	$\Gamma_L V_1^+$ at t_2
$t_3 \le t < t_4 \ (t_4 = 4T - t_1)$	$V_1^+(1 + \Gamma_L + \Gamma_g\Gamma_L)$	$\Gamma_g\Gamma_L V_1^+$ at t_3
$t_4 \le t < t_5 \ (t_5 = 4T + t_1)$	$V_1^+(1 + \Gamma_L + \Gamma_g\Gamma_L + \Gamma_g\Gamma_L^2)$	$\Gamma_g\Gamma_L^2 V_1^+$ at t_4
⋮	⋮	⋮



(a) $V(z, t_4)$ versus z;
$\Gamma_L = \frac{1}{2}$, $\Gamma_g = \frac{1}{3}$, $V_1^+ = V_0/3$.



(b) $V(z_1,t)$ versus t; $V(z_1, \infty) = 3V_0/5$.

Pulse Excitation

"Heaviside" step function

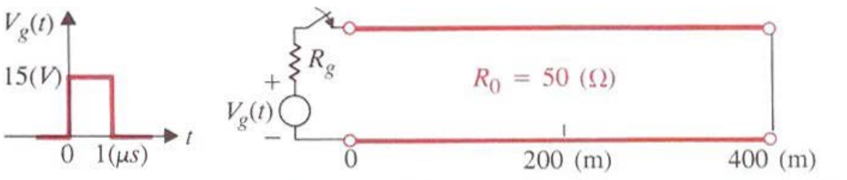
A step function:

$$v_g(t) = V_0 U(t) \qquad \leftarrow U(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

Pulse excitation:

$$v_g(t) = V_0 [U(t) - U(t - T_0)]$$

Superposition of two step functions!



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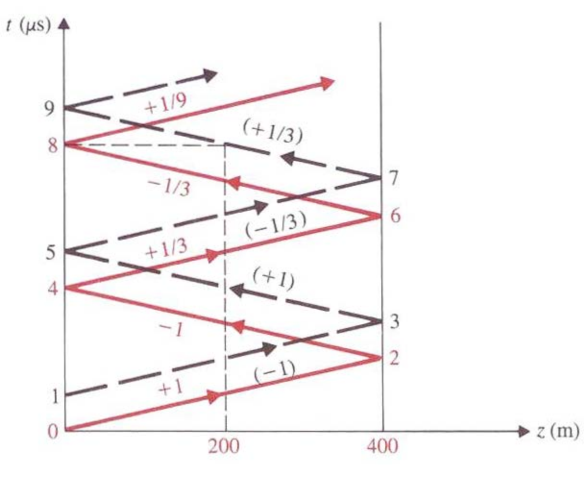

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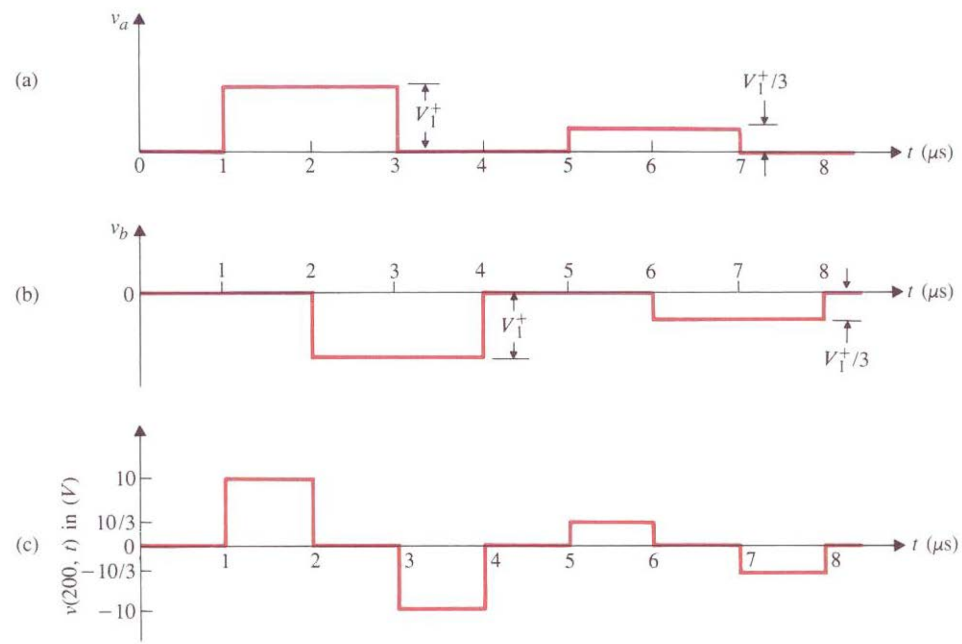

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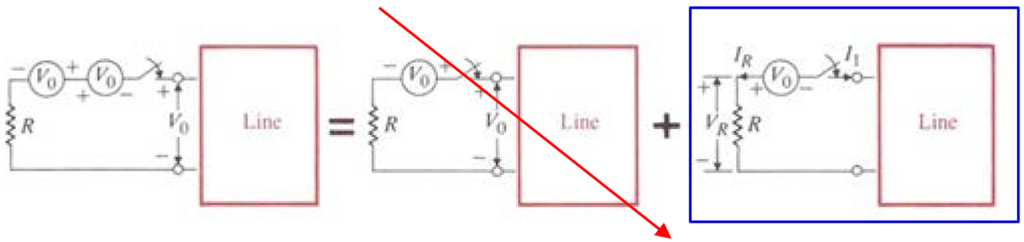
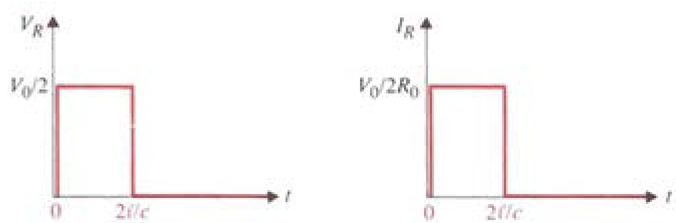
Initially Charged Line

Consider:

Superposition of two circuits:



→ *Of our interest*



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$$V_1^+ = -\frac{R_0}{R+R_0}V_0 = -\frac{V_0}{2} \quad \leftarrow R = R_0$$

$$V_1^- = V_1^+ = -\frac{V_0}{2} \quad \leftarrow \text{Open-circuited termination}$$

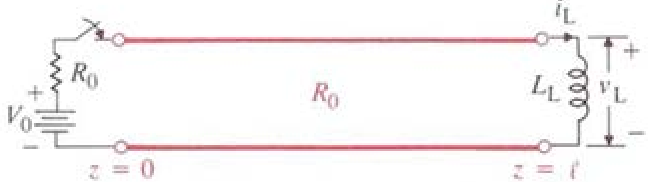
$$I_R = -I_1$$

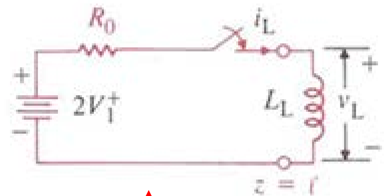
$$I_1 = I_1^+ = \frac{V_1^+}{R_0} = -\frac{V_0}{2R_0} \quad for\ 0 \le t < 2l/c$$

$$I_1^- = -I_1^+ = \frac{V_0}{2R_0}$$

Line with Reactive Load (1)

For a lossless line with an inductive termination:





← *Equivalent circuit*

$(t \geq T)$

D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

$$V_1^+ = \frac{V_0}{2}$$

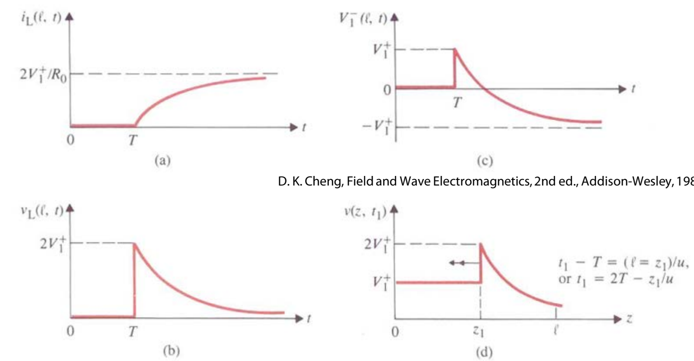
$$v_L(t) = V_1^+ + V_1^-(t) \quad at\ z = l\ \&\ t \geq T$$

$$i_L(t) = \frac{1}{R_0}\left[V_1^+ - V_1^-(t)\right]$$

$$v_L(t) = 2V_1^+ - R_0 i_L(t)$$

$$v_L(t) = L_L \frac{di_L(t)}{dt}$$

$$L_L \frac{di_L(t)}{dt} + R_0 i_L(t) = 2V_1^+, \quad t \geq T$$



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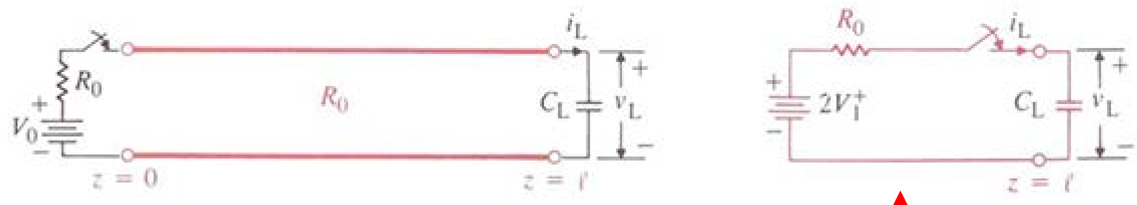
Solution:

$$i_L(t) = \frac{2V_1^+}{R_0}\left[1 - e^{-(t-T)R_0/L_L}\right], \quad t \geq T$$

$$v_L(t) = L_L \frac{di_L(t)}{dt} = 2V_1^+ e^{-(t-T)R_0/L_L}, \quad t \geq T$$

Line with Reactive Load (2)

For a lossless line with a capacitive termination:



← *Equivalent circuit*

$(t \geq T)$

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$$V_1^+ = \frac{V_0}{2}$$

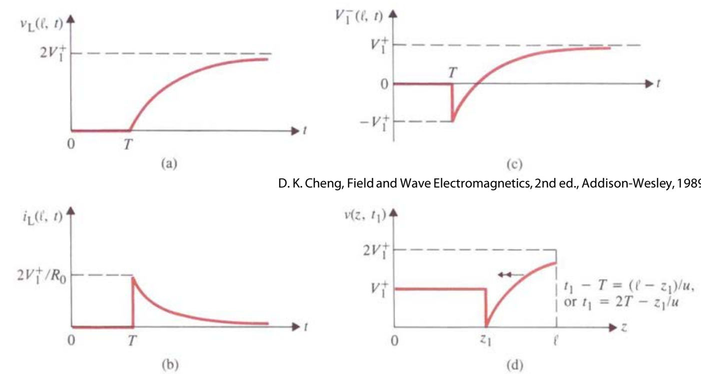
$$v_L(t) = V_1^+ + V_1^-(t) \quad at\ z = l\ \&\ t \geq T$$

$$i_L(t) = \frac{1}{R_0}\left[V_1^+ - V_1^-(t)\right]$$

$$v_L(t) = 2V_1^+ - R_0 i_L(t)$$

$$i_L(t) = C_L \frac{dv_L(t)}{dt}$$

$$C_L \frac{dv_L(t)}{dt} + \frac{1}{R_0} v_L(t) = \frac{2}{R_0} V_1^+, \quad t \geq T$$



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Solution:

$$v_L(t) = 2V_1^+\left[1 - e^{-(t-T)/R_0 C_L}\right], \quad t \geq T$$

$$i_L(t) = C_L \frac{dv_L(t)}{dt} = \frac{2V_1^+}{R_0} e^{-(t-T)/R_0 C_L}, \quad t \geq T$$