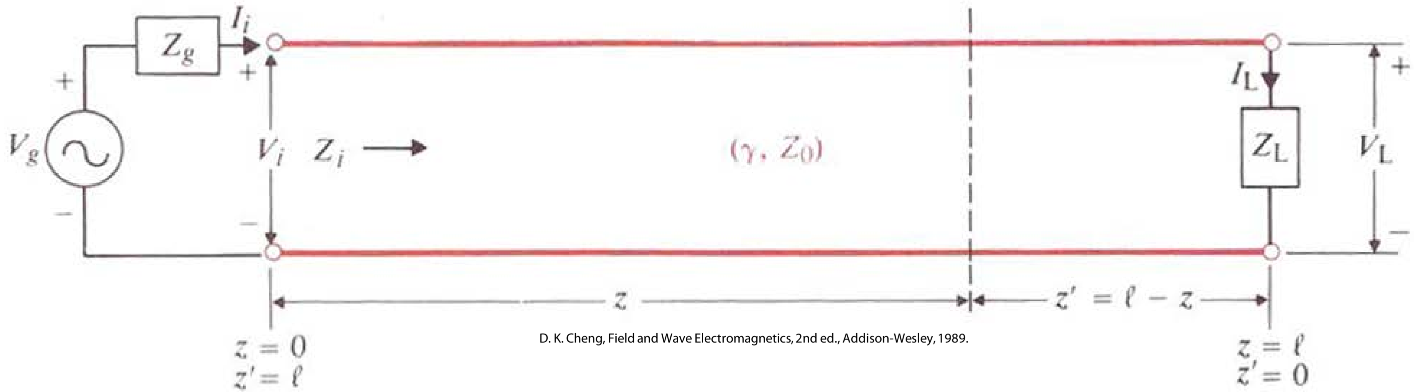
Electromagnetics:
Transmission-Line Impedance Matching
(9-7)

Yoonchan Jeong
School of Electrical Engineering, Seoul National University
Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953
Email: yoonchan@snu.ac.kr

Transmission-Line Impedance Matching

Transmission line:

→ *Used for the transmission of power and information*



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

What will happen if back-reflection occurs?

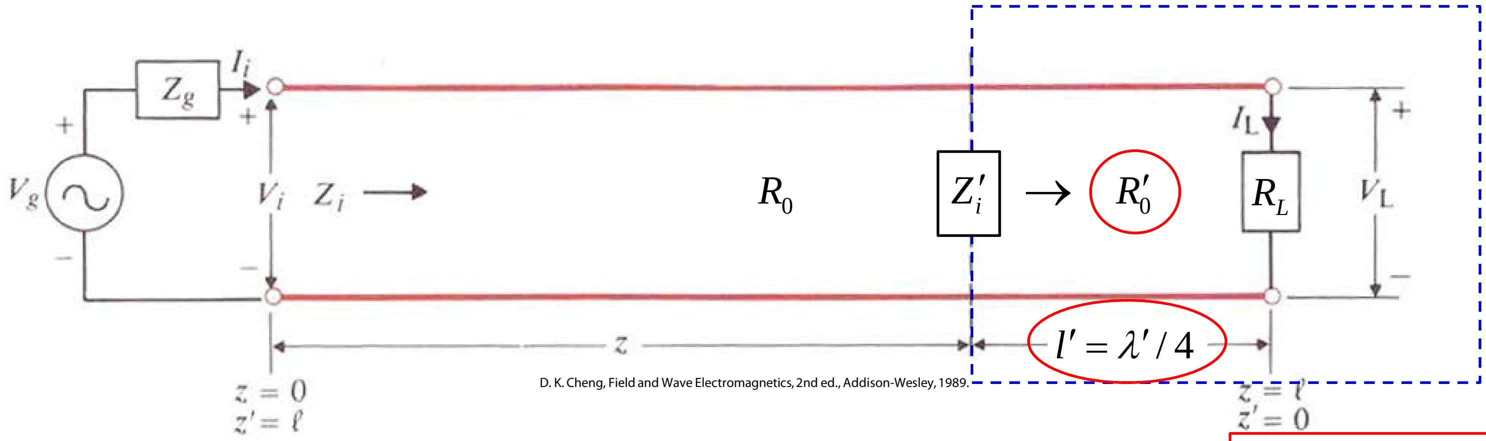
→ *Power loss & information distortion!!*

In order to remove back-reflection:

→ *The "effective" load impedance must be matched to Z_0!*

Impedance Matching by Quarter-Wave Transformer

Region of interest:



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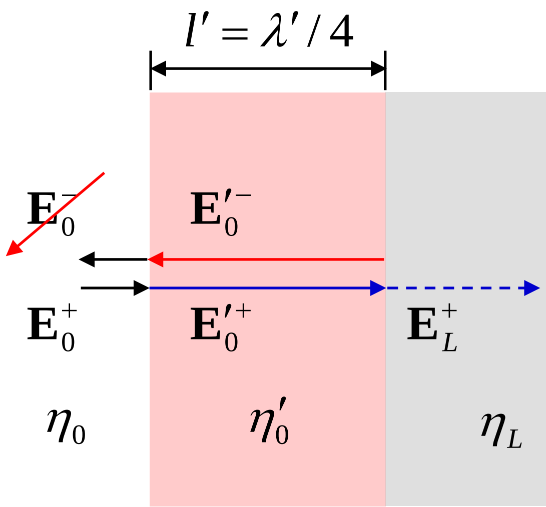
$\leftarrow R_L \neq R_0$

Recall a quarter-wave section: $Z_i = R_0 \dfrac{Z_L + jR_0 \tan \beta l}{R_0 + jZ_L \tan \beta l} \rightarrow Z_i' = \dfrac{R_0'^2}{R_L} = R_0$

Impedance matched: No back-reflection from Z_i'



$\lambda'/4 \rightarrow \Delta\phi = \pi/2 \rightarrow Round\ trip: 2\Delta\phi = \pi$

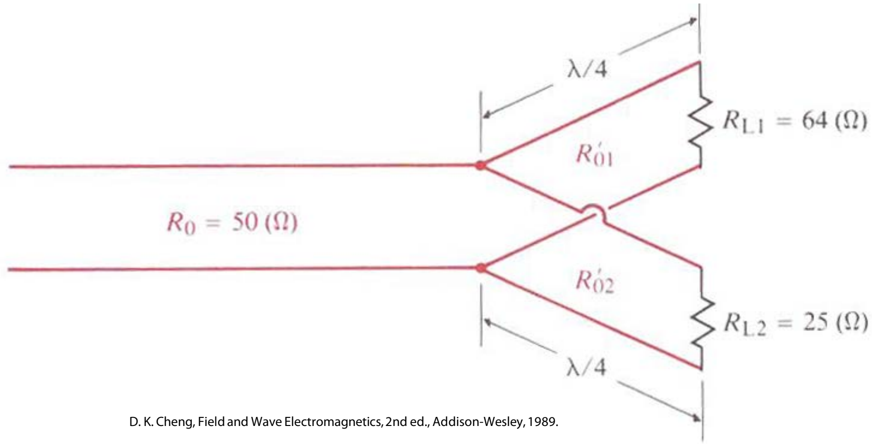
Destructive interference at $\eta_0 : \eta_0'$ *interface*

$\rightarrow$ *Back-reflection minimized: No backward-going wave*

$\eta_0 < \eta_0' < \eta_L$ or $\eta_0 > \eta_0' > \eta_L$

Phase-shifts by the front and rear back reflections cancelled out!

Example 9-17:



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Recall : $Z'_i = \frac{R_0'^2}{R_L} = R_0 \rightarrow \begin{cases} Z'_{01} = \frac{R_{01}'^2}{R_{L1}} = 2R_0 \\ Z'_{02} = \frac{R_{02}'^2}{R_{L2}} = 2R_0 \end{cases} \rightarrow Z'_i = \frac{1}{1/Z'_{01} + 1/Z'_{02}} = R_0$

Equal power transmission

$$\rightarrow \begin{cases} R'_{01} = \sqrt{2R_0R_{L1}} \\ R'_{02} = \sqrt{2R_0R_{L2}} \end{cases}$$

Note that if Z_L is a complex load, it becomes cumbersome to use quarter-wave transformers for impedance matching!

Impedance-to-Admittance Conversion

Load admittance: $Y_L = \dfrac{1}{Z_L}$

Normalized load admittance: $y_L = \dfrac{1}{z_L} = \dfrac{1}{Z_L / R_0}$ $\leftarrow y_L = Y_L / Y_0 = R_0 Y_L = g + jb$

Recall: $z_L = \dfrac{1+\Gamma}{1-\Gamma} \rightarrow y_L = \dfrac{1-\Gamma}{1+\Gamma} \rightarrow y_L = \dfrac{1+\Gamma e^{j\pi}}{1-\Gamma e^{j\pi}} = \dfrac{1+\Gamma''}{1-\Gamma''}$

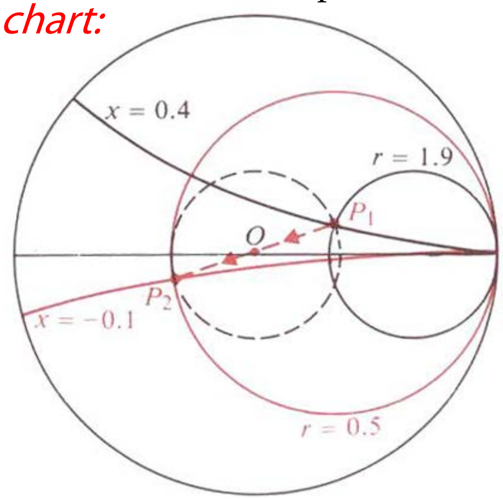
Impedance-to-admittance conversion: $\rightarrow |\Gamma''| = |\Gamma|$

$\rightarrow$ *Azimuthal shift by "π" or a "quarter-wave section" in the Smith chart!*

$(\because 2\beta\Delta z' = \pi)$

Smith chart:

Γ *or* Γ'' *– plane*



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Example 9-18: $Z_L \rightarrow Y_L$?

$\rightarrow z_L = Z_L / R_0 = 1.9 + j0.4$

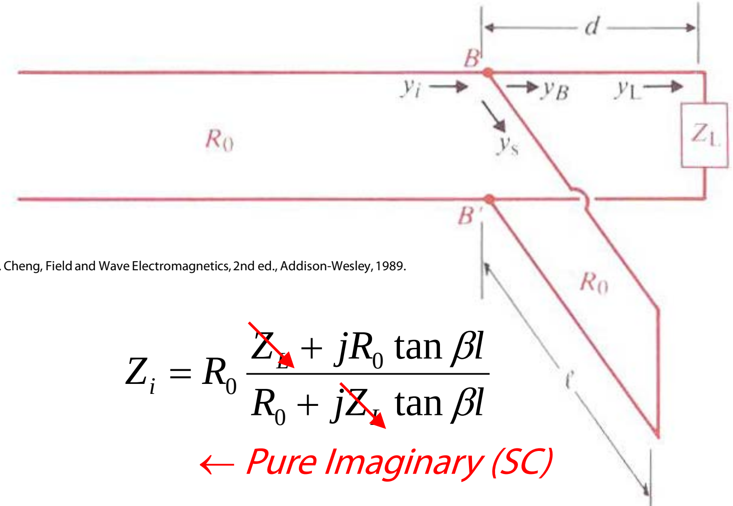
$\rightarrow P_1$ *for* z_L

$\rightarrow P_2$: *Diametrically opposite on the* $|\Gamma|$ *-circle*

$\rightarrow y_L = 0.5 - j0.1$

$\rightarrow Y_L = y_L / R_0$

Single-Stub Matching



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$$Z_i = R_0 \frac{Z_L + jR_0 \tan \beta l}{R_0 + jZ_L \tan \beta l}$$

← *Pure Imaginary (SC)*

Impedance-matching condition:

$$Y_i = Y_B + Y_s = Y_0 = \frac{1}{R_0}$$

In terms of normalized admittances:

$\rightarrow y_i = y_B + y_s = 1$

$\rightarrow y_B = 1 + jb_B$

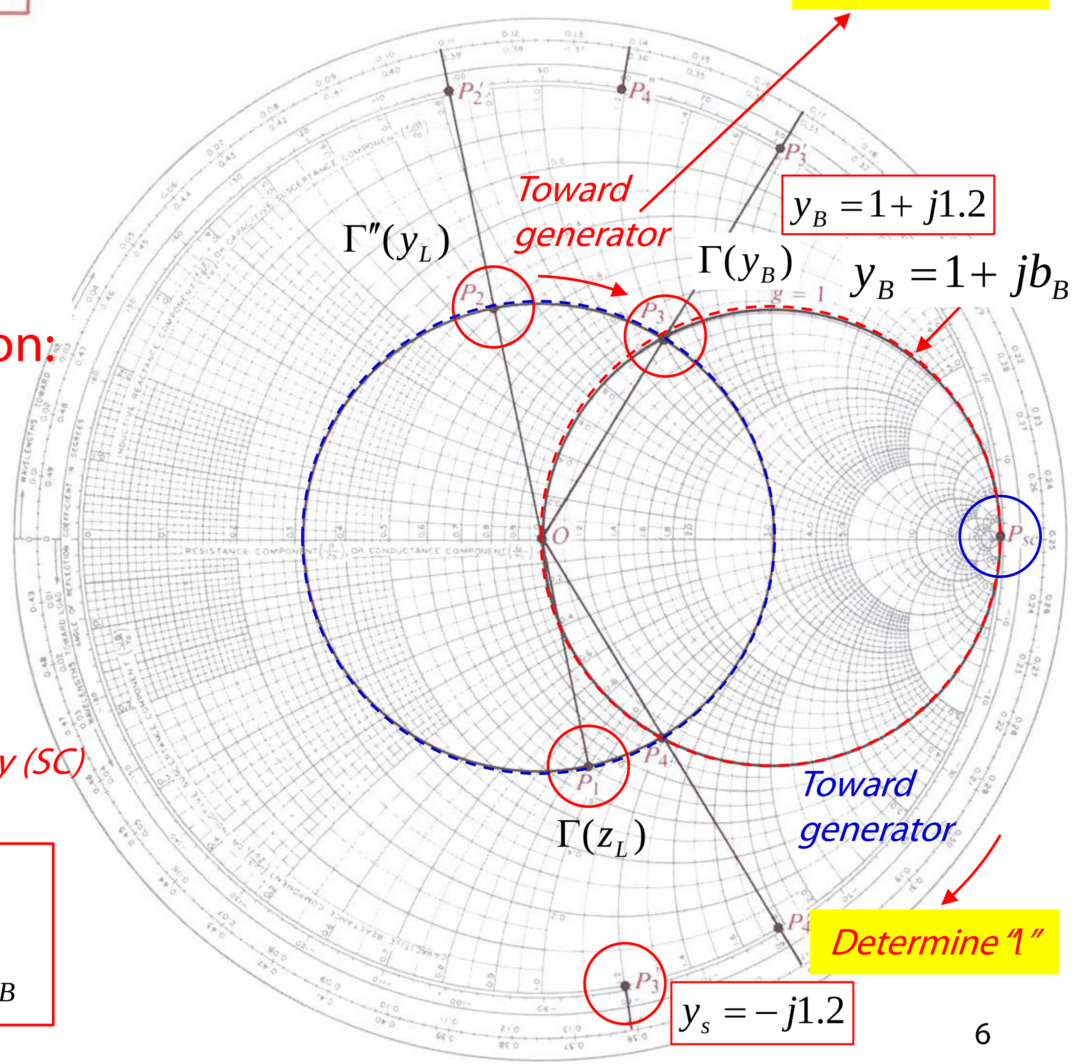
$\rightarrow y_s = -jb_B$ ← *Pure Imaginary (SC)*

1. Determine "d": $\mathrm{Re}(y_B) = 1$

2. Determine "l": To cancel out jb_B

Example 9-20: $z_L = 0.70 - j0.95$

Smith chart:



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Single-Stub Matching: Analytical Solutions

Recall: $Z_i = R_0 \dfrac{Z_L + jR_0 \tan\beta l}{R_0 + jZ_L \tan\beta l} \rightarrow z_B = \dfrac{(r_L + jx_L) + jt}{1 + j(r_L + jx_L)t} \leftarrow t = \tan\beta d$

Normalized input admittance:

$$y_B = \frac{1}{z_B} = g_B + jb_B \quad \rightarrow g_B = \frac{r_L(1 - x_L t) + r_L t(x_L + t)}{r_L^2 + (x_L + t)^2} \rightarrow b_B = \frac{r_L^2 t - (1 - x_L t)(x_L + t)}{r_L^2 + (x_L + t)^2}$$

Recall the impedance-matching condition:

$$1 = y_B + y_s \quad \rightarrow y_B = 1 + jb_B \quad \rightarrow y_s = -jb_B$$

$$\rightarrow g_B = \frac{r_L(1 - x_L t) + r_L t(x_L + t)}{r_L^2 + (x_L + t)^2} = 1$$

$$\rightarrow (r_L - 1)t^2 - 2tx_L + (r_L - r_L^2 - x_L^2) = 0$$

Solutions:

$$t = \begin{cases} \dfrac{1}{r_L - 1}\left\{x_L \pm \sqrt{r_L[(1 - r_L)^2 + x_L^2]}\right\}, & r_L \neq 1 \\ -\dfrac{x_L}{2}, & r_L = 1 \end{cases}$$

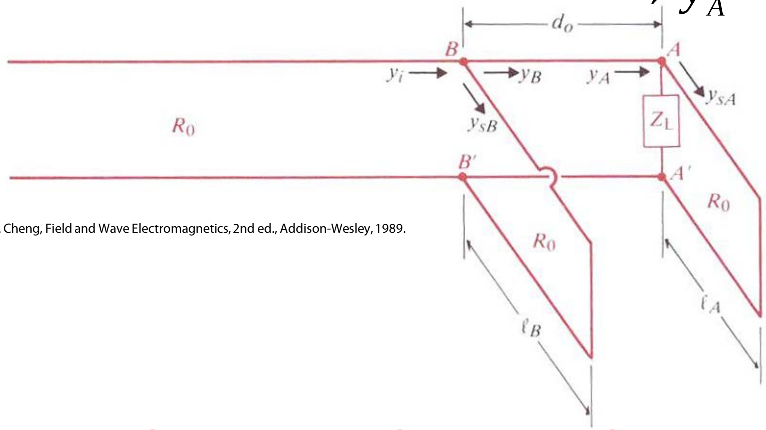
Recall: $z_{is} = j\tan\beta l \rightarrow y_{is} = 1/(j\tan\beta l)$

$$\rightarrow \frac{d}{\lambda} = \begin{cases} \dfrac{1}{2\pi}\tan^{-1} t, & t \geq 0 \\ \dfrac{1}{2\pi}(\pi + \tan^{-1} t), & t < 0 \end{cases}$$

$$\rightarrow \frac{l}{\lambda} = \begin{cases} \dfrac{1}{2\pi}\tan^{-1}\left(\dfrac{1}{b_B}\right), & b_B \geq 0 \\ \dfrac{1}{2\pi}\left[\pi + \tan^{-1}\left(\dfrac{1}{b_B}\right)\right], & b_B < 0 \end{cases}$$

Double-Stub Matching

$\rightarrow y_A = y_L + y_{sA}$



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Impedance-matching condition:

$$Y_i = Y_B + Y_{sB} = Y_0 = \frac{1}{R_0}$$

$$\rightarrow 1 = y_B + y_{sB}$$

$$\rightarrow y_B = 1 + jb_B$$

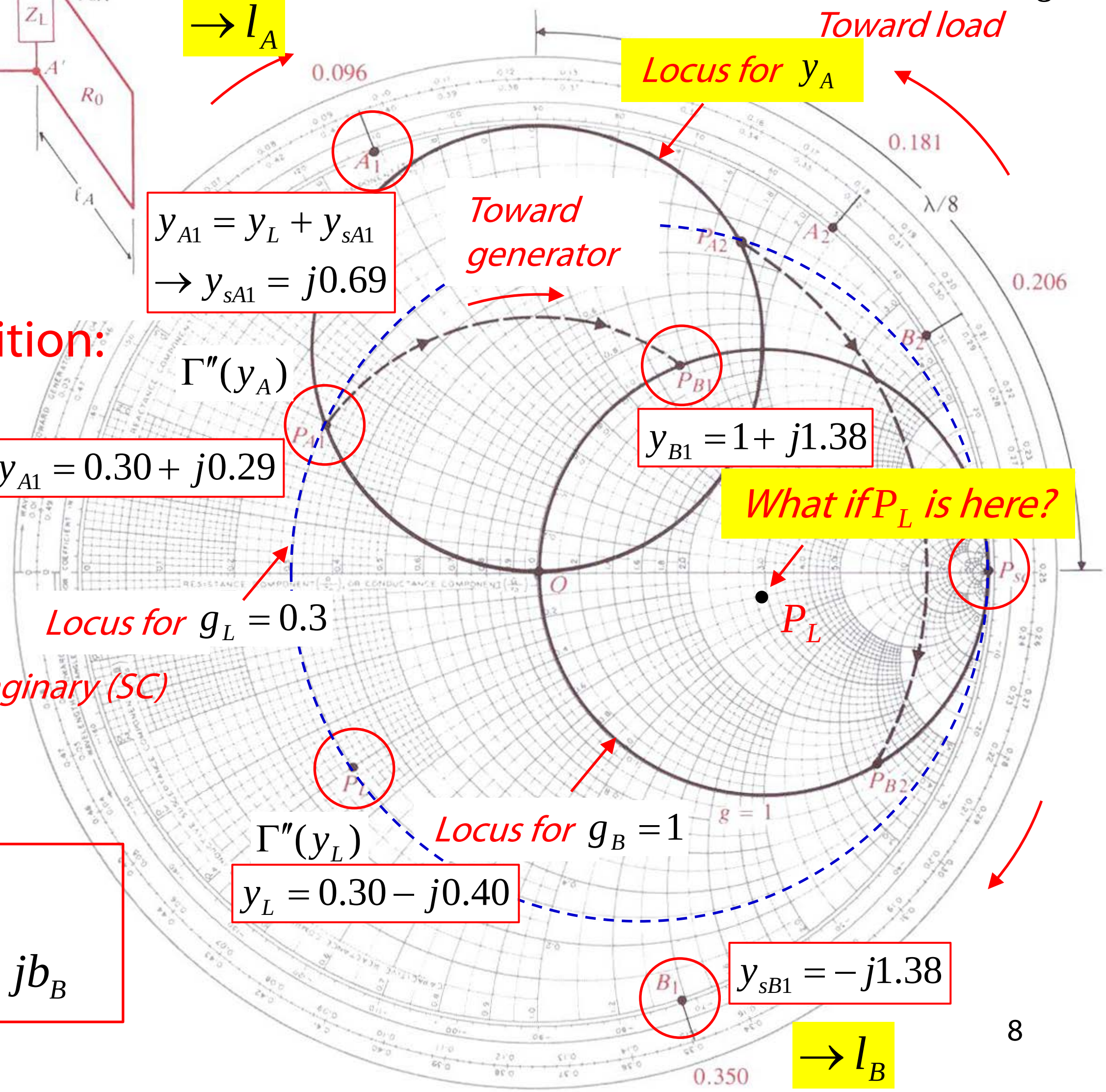
$$\rightarrow y_{sB} = -jb_B$$ ← *Pure Imaginary (SC)*

Note: d_0 is a fixed value.

$y_L = 0.30 - j0.40$

Smith chart:

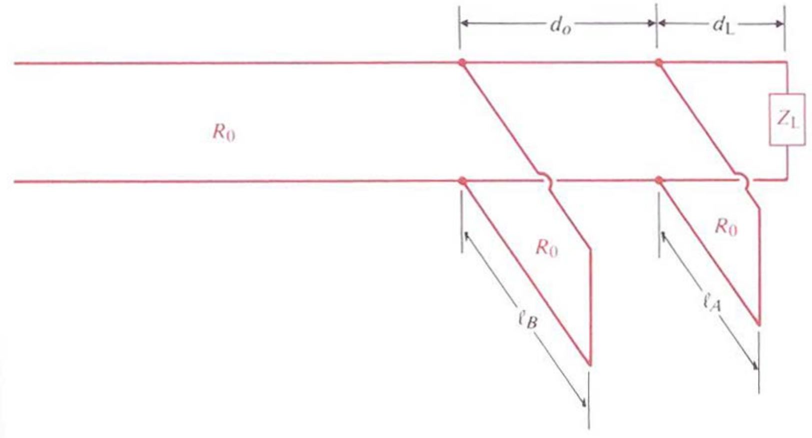
$\rightarrow d_o = \frac{\lambda}{8}$



1. Determine "l_A": $\mathrm{Re}(y_B) = 1$

2. Determine "l_B" To cancel out jb_B

Double-Stub Matching with an Added Load-Line Section



How does it work?

It must be worthwhile for you to figure it out for yourself (HW)!