

# Electromagnetics:

Introduction to Antennas and Radiating Systems

Radiation Fields of Elemental Dipoles

(11-1, 11-2)

Yoonchan Jeong

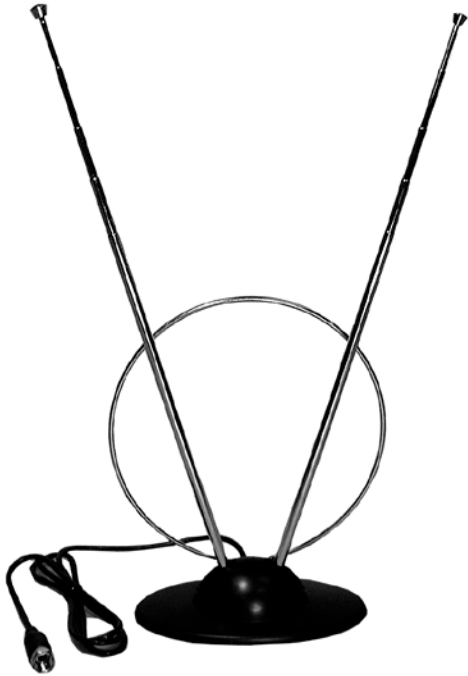
School of Electrical Engineering, Seoul National University

Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953

Email: [yunchan@snu.ac.kr](mailto:yunchan@snu.ac.kr)

# Antennas

Various types of antennas:



<http://www.farnsworthelectronics.com/part%20images/antenn16.gif>



<http://deepspace.jpl.nasa.gov/dsn/antennas/70m.html>

**Antennas:** Structures designed for radiating or collecting electromagnetic energy!

# Some Equations To Recall (Not Too Difficult)

Vector magnetic potential:

$$\begin{aligned}\nabla \cdot \mathbf{B} = 0 &\rightarrow \mathbf{B} = \nabla \times \mathbf{A} \rightarrow \mathbf{H} = \frac{1}{\mu} \nabla \times \mathbf{A} \\ \rightarrow \nabla \times \mathbf{E} = -\frac{\partial}{\partial t}(\nabla \times \mathbf{A}) &\rightarrow \nabla \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t}\right) = 0 \\ \rightarrow \mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla V &\rightarrow \mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \rightarrow \mathbf{E} = -\nabla V - j\omega \mathbf{A}\end{aligned}$$

Retarded potentials:

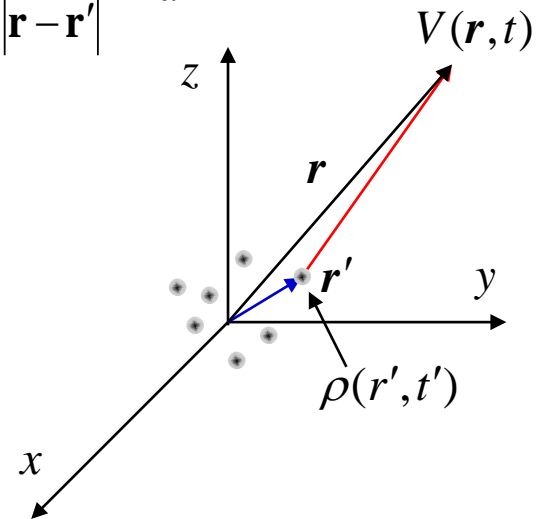
$$\rightarrow \mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}') e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} d^3\mathbf{r}' \quad \rightarrow V(\mathbf{r}) = \frac{1}{4\pi\epsilon} \int \frac{\rho(\mathbf{r}') e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} d^3\mathbf{r}'$$

Principle of conservation of charge:

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t} \quad \rightarrow \nabla \cdot \mathbf{J} = -j\omega \rho$$

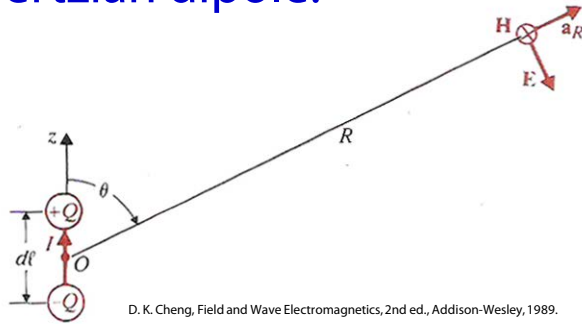
For source-free regions:

$$\mathbf{E} = \frac{1}{j\omega\epsilon} \nabla \times \mathbf{H}$$



# Elemental Electric Dipole (1)

Hertzian dipole:



$$i(t) = I \cos \omega t = \text{Re}[Ie^{j\omega t}]$$

$$\rightarrow i(t) = \pm \frac{dq(t)}{dt} \quad \leftarrow q(t) = \text{Re}[Qe^{j\omega t}]$$

$$\rightarrow I = \pm j\omega Q$$

$$\rightarrow Q = \pm \frac{I}{j\omega}$$

Electric dipole moment:  $\rightarrow \mathbf{p} = \mathbf{a}_z Q dl$

Magnetic vector potential:

$$\rightarrow \mathbf{A} = \mathbf{a}_z \frac{\mu_0 I dl}{4\pi} \left( \frac{e^{-j\beta R}}{R} \right) \quad \leftarrow \beta = k_0 = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

With spherical coordinates:

$$\rightarrow A_R = A_z \cos \theta = \frac{\mu_0 I dl}{4\pi} \left( \frac{e^{-j\beta R}}{R} \right) \cos \theta$$

$$\rightarrow A_\theta = -A_z \sin \theta = -\frac{\mu_0 I dl}{4\pi} \left( \frac{e^{-j\beta R}}{R} \right) \sin \theta$$

$$\rightarrow A_\phi = 0$$

# Elemental Electric Dipole (2)

$$A_R = A_z \cos \theta = \frac{\mu_0 Idl}{4\pi} \left( \frac{e^{-j\beta R}}{R} \right) \cos \theta$$

$$A_\theta = -A_z \sin \theta = -\frac{\mu_0 Idl}{4\pi} \left( \frac{e^{-j\beta R}}{R} \right) \sin \theta$$

$$A_\phi = 0$$

$$\rightarrow \mathbf{H} = \frac{1}{\mu_0} \nabla \times \mathbf{A}$$

$$= \mathbf{a}_\phi \frac{1}{\mu_0 R} \left[ \frac{\partial}{\partial R} (R A_\theta) - \frac{\partial A_R}{\partial \theta} \right]$$

$$= -\mathbf{a}_\phi \frac{Idl}{4\pi} \beta^2 \sin \theta \left[ \frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} \right] e^{-j\beta R}$$

$$\rightarrow \mathbf{E} = \frac{1}{j\omega\epsilon_0} \nabla \times \mathbf{H}$$

$$= \frac{1}{j\omega\epsilon_0} \left[ \mathbf{a}_R \frac{1}{R \sin \theta} \frac{\partial}{\partial \theta} (H_\phi \sin \theta) - \mathbf{a}_\theta \frac{1}{R} \frac{\partial}{\partial R} (R H_\phi) \right]$$

$$\rightarrow E_R = -\frac{Idl}{4\pi} \eta_0 \beta^2 2 \cos \theta \left[ \frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3} \right] e^{-j\beta R}$$

$$\rightarrow E_\theta = -\frac{Idl}{4\pi} \eta_0 \beta^2 \sin \theta \left[ \frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3} \right] e^{-j\beta R}$$

$$\rightarrow E_\phi = 0$$

# Elemental Electric Dipole (3)

$$H_\phi = -\frac{Idl}{4\pi} \beta^2 \sin \theta \left[ \frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} \right] e^{-j\beta R}$$

$$E_R = -\frac{Idl}{4\pi} \eta_0 \beta^2 2 \cos \theta \left[ \frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3} \right] e^{-j\beta R}$$

$$E_\theta = -\frac{Idl}{4\pi} \eta_0 \beta^2 \sin \theta \left[ \frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3} \right] e^{-j\beta R}$$

*Wavelength dependent!*



**Near fields:**  $\rightarrow \beta R = 2\pi R / \lambda \ll 1$

**Far fields:**  $\rightarrow \beta R = 2\pi R / \lambda \gg 1$

$$H_\phi = \frac{Idl}{4\pi R^2} \sin \theta$$

$$E_R = \frac{p}{4\pi \epsilon_0 R^3} 2 \cos \theta$$

$$E_\theta = \frac{p}{4\pi \epsilon_0 R^3} \sin \theta$$

$$\leftarrow p = Qdl = \frac{Idl}{j\omega}$$

$\rightarrow$  *Quasi-static fields*

$$H_\phi = j \frac{Idl}{4\pi} \left( \frac{e^{-j\beta R}}{R} \right) \beta \sin \theta$$

$$E_R = 0$$

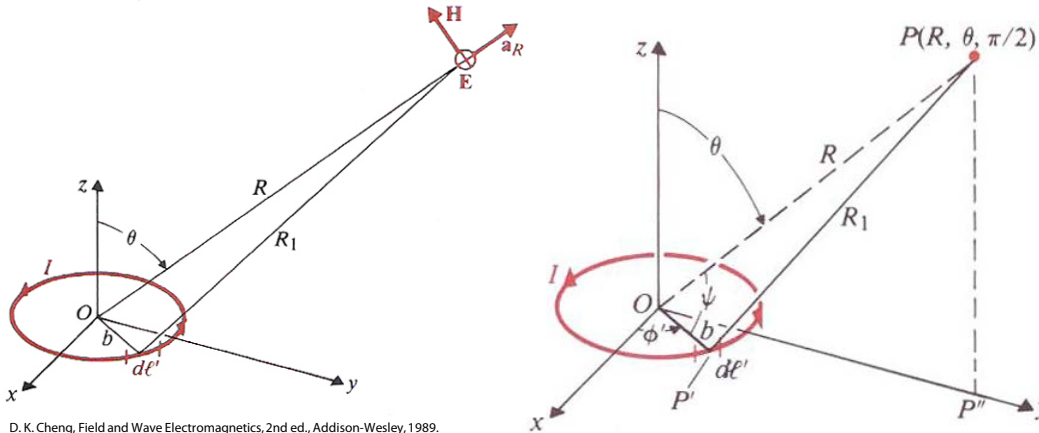
$$E_\theta = j \frac{Idl}{4\pi} \left( \frac{e^{-j\beta R}}{R} \right) \eta_0 \beta \sin \theta$$

$$\rightarrow E_\theta / H_\phi = \eta_0$$

$$\rightarrow \mathbf{E} \perp \mathbf{H}$$

$\rightarrow$  *In space quadrature*  $\rightarrow$  *In time phase* <sup>6</sup>

# Elemental Magnetic Dipole (1)



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

(See pp. 239-240)

Magnetic vector potential:

$$\mathbf{A} = \frac{\mu_0 I}{4\pi} \oint \frac{e^{-j\beta R_1}}{R_1} d\mathbf{l}'$$

$$\leftarrow e^{-j\beta R_1} = e^{-j\beta R} e^{-j\beta(R_1 - R)}$$

$$\cong e^{-j\beta R} [1 - j\beta(R_1 - R)]$$

Magnetic dipole moment:

$$\rightarrow \mathbf{m} = \mathbf{a}_z I \pi b^2$$

$$\rightarrow \mathbf{A} = \frac{\mu_0 I}{4\pi} e^{-j\beta R} \left[ (1 + j\beta R) \oint \frac{d\mathbf{l}'}{R_1} - j\beta \oint d\mathbf{l}' \right]$$

$$\leftarrow d\mathbf{l}' = (-\mathbf{a}_x \sin \phi' + \mathbf{a}_y \cos \phi') b d\phi'$$

$$\rightarrow \mathbf{a}_\phi$$

$$\rightarrow \mathbf{A} = \mathbf{a}_\phi \frac{\mu_0 I}{4\pi} e^{-j\beta R} (1 + j\beta R) \int_0^{2\pi} \frac{b \sin \phi'}{R_1} d\phi'$$

$$= \mathbf{a}_\phi \frac{\mu_0 m}{4\pi R^2} (1 + j\beta R) e^{-j\beta R} \sin \theta$$

$$\frac{1}{R_1} = (R^2 + b^2 - 2bR \cos \psi)^{-1/2}$$

$$\cong \frac{1}{R} \left( 1 + \frac{b}{R} \cos \psi \right) \cong \frac{1}{R} \left( 1 + \frac{b}{R} \sin \phi' \sin \theta \right)$$

# Elemental Magnetic Dipole (2)

$$\rightarrow \mathbf{H} = \frac{1}{\mu_0} \nabla \times \mathbf{A}$$

$$\rightarrow \mathbf{E} = \frac{1}{j\omega\epsilon_0} \nabla \times \mathbf{H}$$

$$E_\phi = \frac{j\omega\mu_0 m}{4\pi} \beta^2 \sin\theta \left[ \frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} \right] e^{-j\beta R}$$

$$H_R = -\frac{j\omega\mu_0 m}{4\pi\eta_0} \beta^2 2\cos\theta \left[ \frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3} \right] e^{-j\beta R}$$

$$H_\theta = -\frac{j\omega\mu_0 m}{4\pi\eta_0} \beta^2 \sin\theta \left[ \frac{1}{j\beta R} + \frac{1}{(j\beta R)^2} + \frac{1}{(j\beta R)^3} \right] e^{-j\beta R}$$

Far fields:

$$E_\phi = \frac{\omega\mu_0 m}{4\pi} \left( \frac{e^{-j\beta R}}{R} \right) \beta \sin\theta$$

$$H_\theta = -\frac{\omega\mu_0 m}{4\pi\eta_0} \left( \frac{e^{-j\beta R}}{R} \right) \beta \sin\theta$$

*Recall: Principle of duality*

$$\mathbf{E}_e = \eta_0 \mathbf{H}_m$$

$$\mathbf{H}_e = -\frac{\mathbf{E}_m}{\eta_0}$$

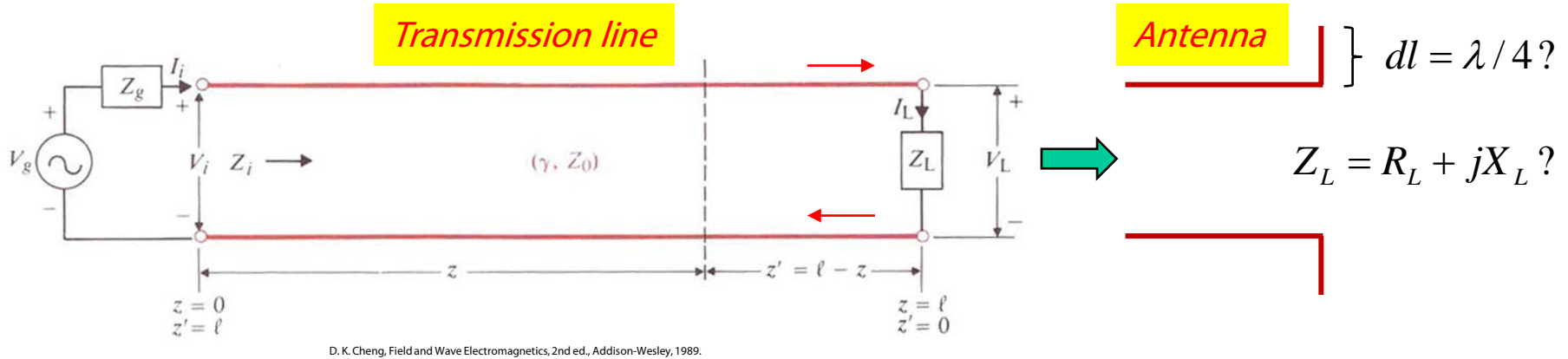
*Recall:* Hertzian dipole

$$E_\theta = j \frac{Idl}{4\pi} \left( \frac{e^{-j\beta R}}{R} \right) \eta_0 \beta \sin\theta$$

*Note:  $E_{\theta,p}$  &  $E_{\phi,m}$  are in both space and time quadrature!*



# Antenna vs Transmission Line



Input impedance (lossless):  $z = 0$  or  $z' = l$

$$Z_i = R_0 \frac{Z_L + jR_0 \tan \beta l}{R_0 + jZ_L \tan \beta l}$$

Open-circuit termination:  $Z_L = \infty$

$$Z_{io} = -jR_0 \cot \beta l = jX_{io}$$

→ Purely reactive  
(capacitive or inductive)

Recall:

$$\rightarrow C = \epsilon \frac{w}{d} \quad \rightarrow L = \mu \frac{d}{w}$$

