

**Electromagnetics:**  
**Antenna Patterns and Antenna Parameters**  
**Thin Linear Antennas**  
**Antenna Arrays**  
**(11-3, 11-4, 11-5)**

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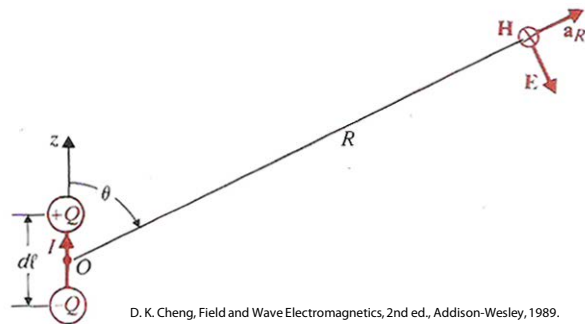
# Radiation Pattern or Antenna Pattern

Relative far-zone field strength at a **fixed distance** from an antenna:

→ *E-plane pattern*: Normalized field strength vs  $\theta$  for a constant  $\phi$

→ *H-plane pattern*: Normalized field strength vs  $\phi$  for  $\theta = \pi/2$

Herzian dipole:



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

Far fields:  $\beta R = 2\pi R / \lambda \gg 1$

$$E_{\theta} = j \frac{Idl}{4\pi} \left( \frac{e^{-j\beta R}}{R} \right) \eta_0 \beta \sin \theta$$

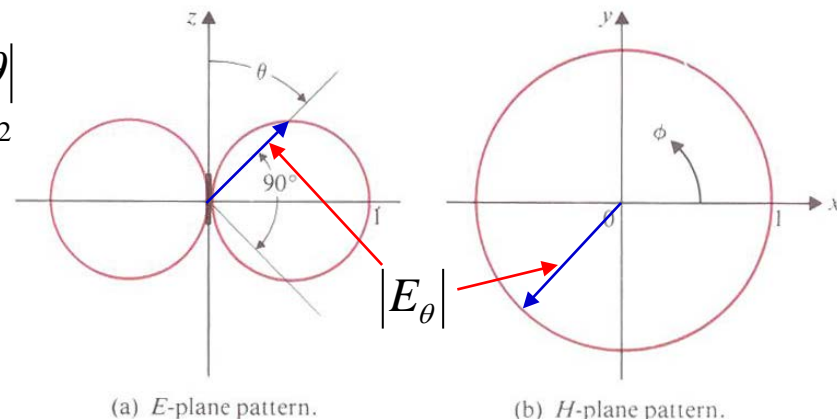
$$H_{\phi} = j \frac{Idl}{4\pi} \left( \frac{e^{-j\beta R}}{R} \right) \beta \sin \theta$$

**E-plane pattern:** → Normalized  $|E_{\theta}| = |\sin \theta|$

$$\rightarrow |E_{\theta}| = \left| \frac{x}{\sqrt{x^2 + z^2}} \right| \rightarrow \left( x \pm \frac{1}{2} \right)^2 + z^2 = \left( \frac{1}{2} \right)^2$$

**H-plane pattern:**

$$\rightarrow \text{Normalized } |E_{\theta}| = |\sin \theta|_{\theta=\pi/2} = 1$$

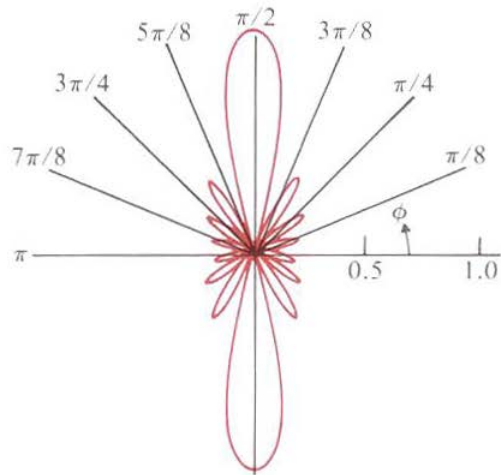


(a) E-plane pattern.

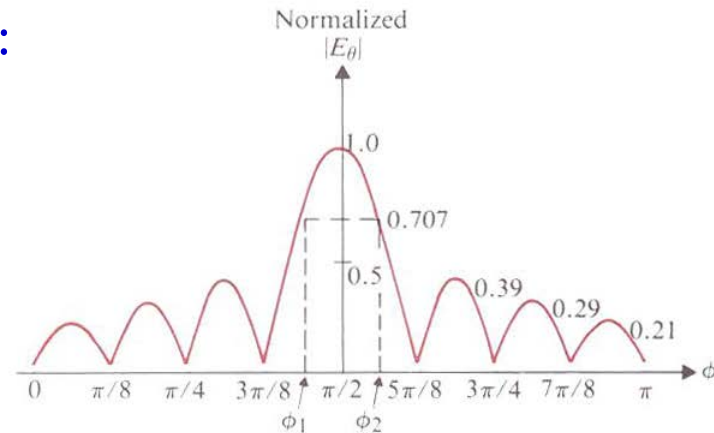
(b) H-plane pattern.

# Antenna Patterns

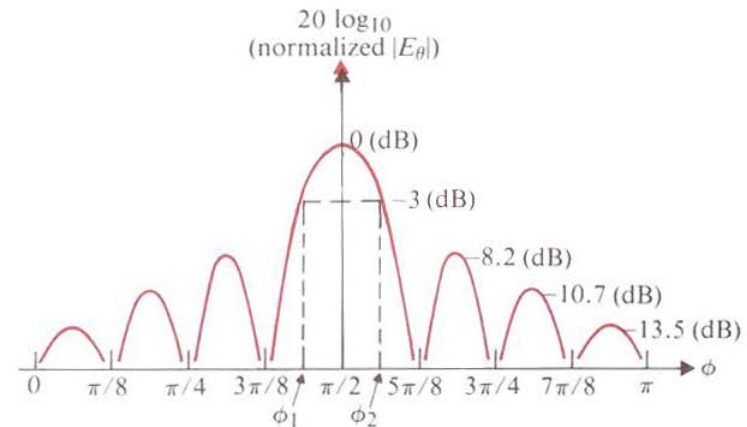
Typical H-plane radiation patterns:



(a) A typical radiation pattern in polar coordinates.



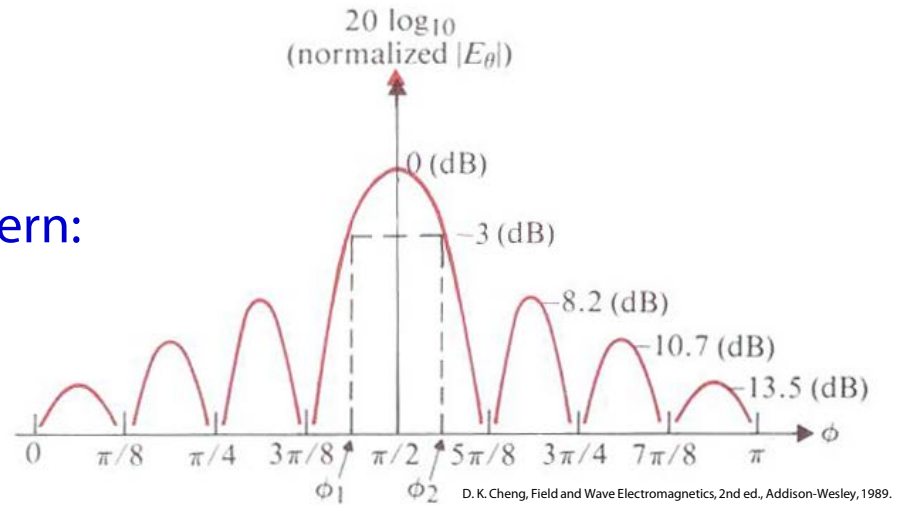
(b) Radiation pattern in rectangular coordinates.



(c) Radiation pattern in rectangular coordinates plotted in dB scale.

# Antenna Parameters (1)

A typical H-plane radiation pattern:



(c) Radiation pattern in rectangular coordinates plotted in dB scale.

1. Bandwidth (Width of the main beam):

→ Angular width between the half-power or  $-3$ -dB points (occasionally,  $-10$ -dB points)

2. Sidelobe levels: → Typically,  $-40$ -dB required for radar applications

3. Directivity: → Radiation intensity (time-averaged power per unit solid angle):  $U = R^2 P_{av}$

→ Total time-average power radiated:  $P_r = \oint \mathbf{P}_{av} \cdot d\mathbf{s} = \oint U d\Omega$  Time-average Poynting vector

→ Directive gain:  $G_D(\theta, \phi) = \frac{U(\theta, \phi)}{U_{av}} = \frac{U(\theta, \phi)}{\oint U d\Omega / 4\pi} = \frac{U(\theta, \phi)}{P_r / 4\pi}$

→ Directivity:  $D = \frac{U_{max}}{U_{av}} = \frac{U_{max}}{P_r / 4\pi} = \frac{4\pi |E_{max}|^2}{\int_0^{2\pi} \int_0^\pi |E(\theta, \phi)|^2 \sin \theta d\theta d\phi}$

## Antenna Parameters (2)

4. Power gain:

$$G_p = \frac{U_{\max}}{P_i / 4\pi}$$

*Radiated power*

$\leftarrow P_i = P_r + P_l$  *Power loss (Ohmic, structural, etc.)*

*Total input power*

5. Radiation efficiency:

$$\eta_r = \frac{P_r}{P_i} = \frac{G_p}{D}$$

$$D = \frac{U_{\max}}{U_{av}} = \frac{U_{\max}}{P_r / 4\pi}$$

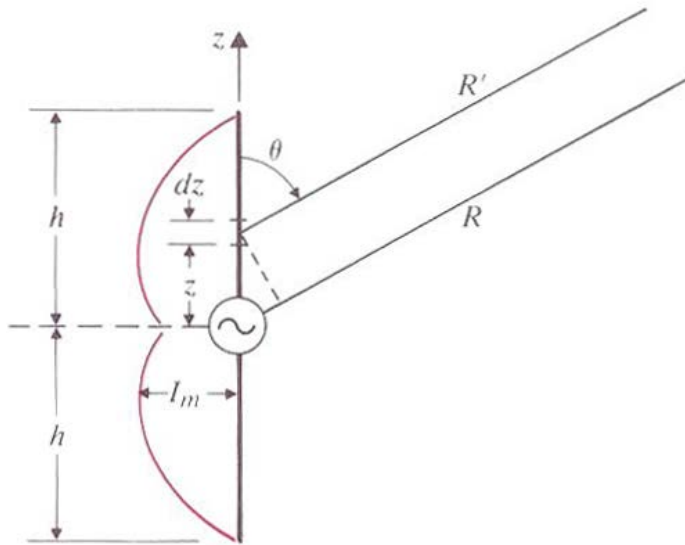
6. Radiation resistance:

$$P_r = \frac{1}{2} I^2 R_r$$

7. Loss resistance:

$$P_l = \frac{1}{2} I^2 R_l$$

# Thin Linear Antennas (1)



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

Current distribution:

$$\rightarrow I(z) = I_m \sin \beta(h - |z|)$$

Why?

Recall: **Hertzian dipole** ( $\beta R = 2\pi R / \lambda \gg 1$ )

$$E_\theta = j \frac{Idl}{4\pi} \left( \frac{e^{-j\beta R}}{R} \right) \eta_0 \beta \sin \theta$$

$$H_\phi = j \frac{Idl}{4\pi} \left( \frac{e^{-j\beta R}}{R} \right) \beta \sin \theta$$

Elemental contribution

Far fields:

$$\rightarrow dE_\theta = \eta_0 dH_\phi = j \frac{Idz}{4\pi} \left( \frac{e^{-j\beta R'}}{R'} \right) \eta_0 \beta \sin \theta \quad \leftarrow R' = (R^2 + z^2 - 2Rz \cos \theta)^{1/2} \cong R - z \cos \theta$$

$$\rightarrow E_\theta = \eta_0 H_\phi \cong j \frac{I_m \eta_0 \beta \sin \theta}{4\pi R} e^{-j\beta R} \int_{-h}^h \sin \beta(h - |z|) e^{j\beta z \cos \theta} dz$$

Why?

$$= j \frac{I_m \eta_0 \beta \sin \theta}{4\pi R} e^{-j\beta R} \int_{-h}^h \sin \beta(h - |z|) [\cos(\beta z \cos \theta) + j \sin(\beta z \cos \theta)] dz$$

$$= j \frac{I_m \eta_0 \beta \sin \theta}{2\pi R} e^{-j\beta R} \int_0^h \sin \beta(h - z) \cos(\beta z \cos \theta) dz$$

# Thin Linear Antennas (2)

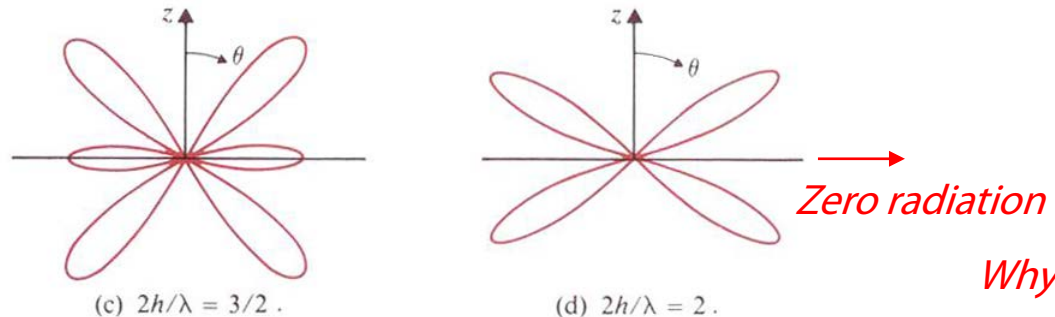
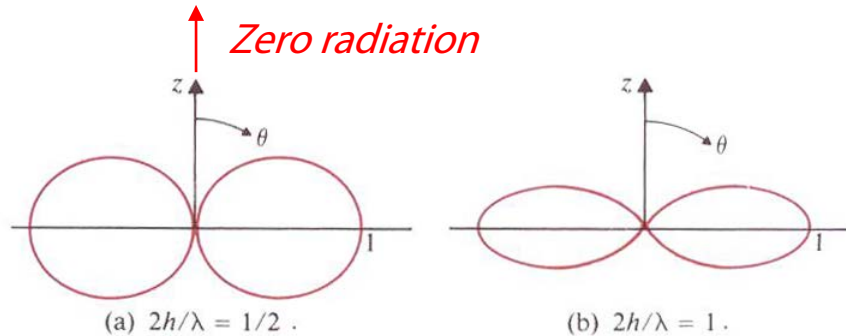
Far fields:

$$\begin{aligned} \rightarrow E_\theta &= \eta_0 H_\phi \cong j \frac{I_m \eta_0 \beta \sin \theta}{2\pi R} e^{-j\beta R} \int_0^h \sin \beta(h-z) \cos(\beta z \cos \theta) dz \\ &= j \frac{60 I_m}{R} e^{-j\beta R} F(\theta) \quad \leftarrow F(\theta) = \frac{\cos(\beta h \cos \theta) - \cos \beta h}{\sin \theta} \end{aligned}$$

*← Pattern function*

Radiation pattern:

$$\rightarrow \beta h = \frac{2\pi}{\lambda} h$$



*Why?*

# Half-Wave Dipole (1)

E-plane Pattern function:

$$F(\theta) = \frac{\cos(\beta h \cos \theta) - \cos \beta h}{\sin \theta}$$

For a half-wave dipole:  $2h = \lambda / 2 \rightarrow \beta h = 2\pi h / \lambda = \pi / 2$

Far-zone fields:  $\rightarrow E_{\theta} = \frac{j60I_m}{R} e^{-j\beta R} \left\{ \frac{\cos[(\pi / 2) \cos \theta]}{\sin \theta} \right\}$

$$\rightarrow H_{\phi} = \frac{jI_m}{2\pi R} e^{-j\beta R} \left\{ \frac{\cos[(\pi / 2) \cos \theta]}{\sin \theta} \right\}$$

Time-average Poynting vector:

$$P_{av} = \frac{1}{2} E_{\theta} H_{\phi}^* = \frac{15I_m^2}{\pi R^2} \left\{ \frac{\cos[(\pi / 2) \cos \theta]}{\sin \theta} \right\}^2$$

Total power radiated:

$$\begin{aligned} P_r &= \int_0^{2\pi} \int_0^{\pi} P_{av} R^2 \sin \theta d\theta d\phi \\ &= 30I_m^2 \int_0^{\pi} \frac{\cos^2[(\pi / 2) \cos \theta]}{\sin \theta} d\theta \cong 36.54I_m^2 \end{aligned}$$



# Half-Wave Dipole (2)

Radiation resistance:

$$R_r = \frac{2P_r}{I_m^2} = 73.1 \text{ } (\Omega) \leftarrow P_r = \frac{1}{2} I_m^2 R_r$$

Directivity:

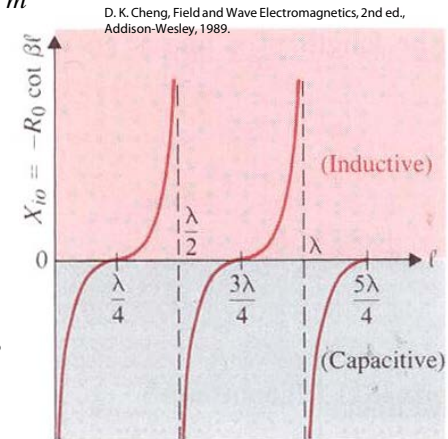
$$U_{\max} = R^2 P_{av} (\theta = \pi/2) = \frac{15}{\pi} I_m^2 \leftarrow P_{av} = \frac{15 I_m^2}{\pi R^2} \left\{ \frac{\cos[(\pi/2) \cos \theta]}{\sin \theta} \right\}^2$$

$$\rightarrow D = \frac{U_{\max}}{P_r / 4\pi} = \frac{60}{36.54} = 1.64 \leftarrow P_r \cong 36.54 I_m^2$$

Half-power bandwidth:

$$E_\theta = \frac{j60 I_m}{R} e^{-j\beta R} \left\{ \frac{\cos[(\pi/2) \cos \theta]}{\sin \theta} \right\}$$

$$\rightarrow \frac{\cos[(\pi/2) \cos \theta]}{\sin \theta} = \frac{1}{\sqrt{2}} \rightarrow \theta_{FWHM} \approx 78^\circ$$



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

*What about the total impedance?*

$$Z_r = R_r + jX_r$$

*For the impedance matching, the dipole length should be slightly shorter than  $\lambda/4$ !*

# Quarter-Wave Monopole

Recall: Image method ← Radiation limited into the upper half-space!

Total power radiated:

$$\rightarrow P_r = \frac{1}{2} \times (36.54 I_m^2)$$

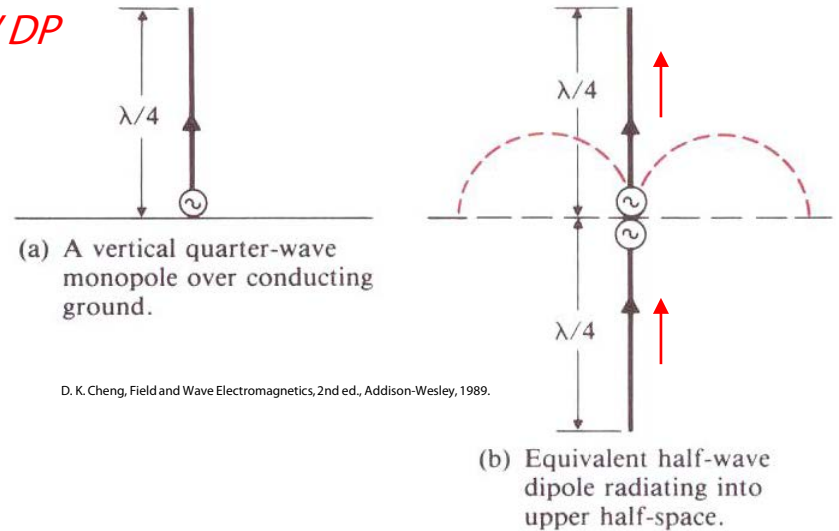
*For HW DP*

Radiation resistance:

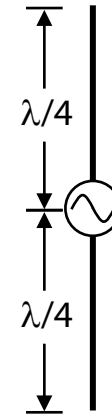
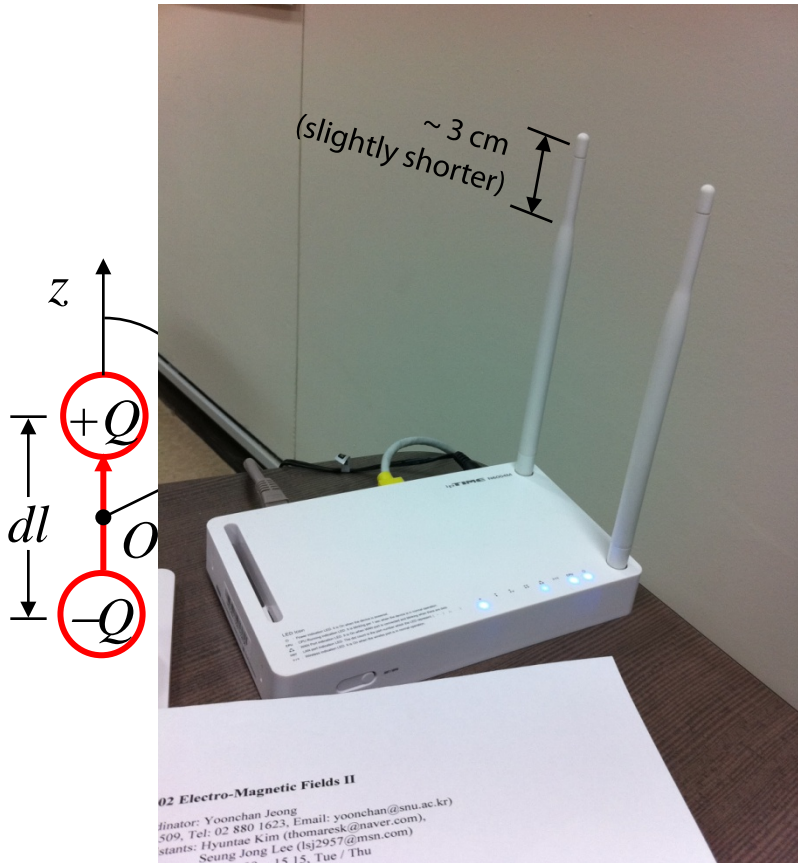
$$\rightarrow R_r = \frac{2P_r}{I_m^2} = 36.54 \text{ } (\Omega)$$

Directivity:

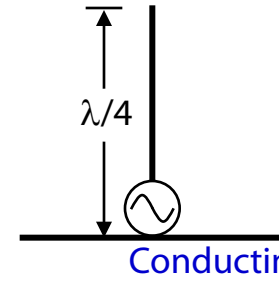
$$\rightarrow D = \frac{U_{max}}{U_{av}} = \frac{U_{max}}{P_r / 2\pi} = 1.64$$



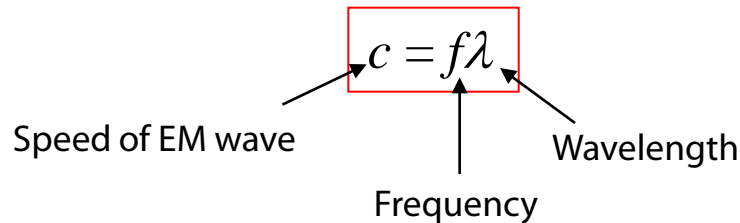
# Recall: Electromagnetic Waves (RF/MW)



Half-wave dipole antenna



Quarter-wave Monopole antenna



WiFi router ( $f = 2.4 \text{ GHz}$ )

$$\lambda/4 = 3 \times 10^8 / 2.4 \times 10^9 / 4 \approx 0.031 \text{ m}$$

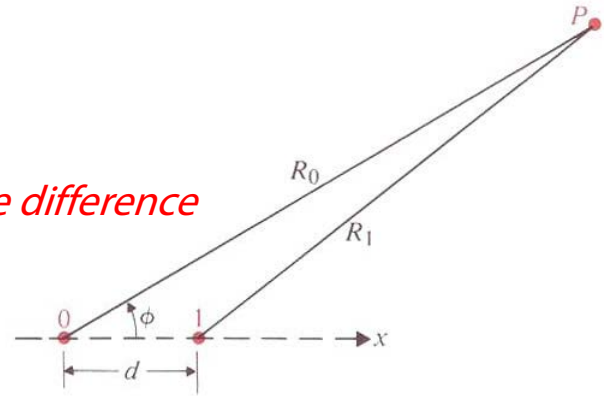
# Antenna Arrays

Two-element arrays:

$$E_0 = E_m F(\theta, \phi) \frac{e^{-j\beta R_0}}{R_0}$$

$$E_1 = E_m F(\theta, \phi) \frac{e^{j\xi} e^{-j\beta R_1}}{R_1}$$

*Initial phase difference*



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

Total electric field:

$$E = E_0 + E_1 = E_m F(\theta, \phi) \left( \frac{e^{-j\beta R_0}}{R_0} + \frac{e^{j\xi} e^{-j\beta R_1}}{R_1} \right) \quad \leftarrow R_1 \cong R_0 - d \sin \theta \cos \phi$$

*Why?*

$$\begin{aligned} \rightarrow E &= E_m \frac{F(\theta, \phi)}{R_0} e^{-j\beta R_0} (1 + e^{j\beta d \sin \theta \cos \phi} e^{j\xi}) = E_m \frac{F(\theta, \phi)}{R_0} e^{-j\beta R_0} (1 + e^{j\psi}) \\ &= E_m \frac{F(\theta, \phi)}{R_0} e^{-j\beta R_0} e^{j\psi/2} \left( 2 \cos \frac{\psi}{2} \right) \quad \leftarrow \psi = \beta d \sin \theta \cos \phi + \xi \end{aligned}$$

$$\rightarrow |E| = \frac{2E_m}{R_0} |F(\theta, \phi)| \left| \cos \frac{\psi}{2} \right|$$

*Array factor*

*Element factor*

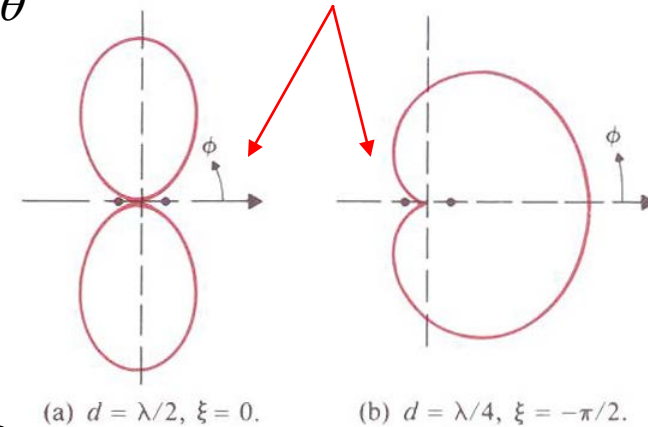
# Multi-Element Parallel Dipole Arrays

For two parallel half-wave dipoles:

$$|E| = \frac{2E_m}{R_0} |F(\theta, \phi)| \left| \cos \frac{\psi}{2} \right| \leftarrow F(\theta) = \frac{\cos(\beta h \cos \theta) - \cos \beta h}{\sin \theta}$$

$$= \frac{2E_m}{R_0} \left| \frac{\cos[(\pi/2) \cos \theta]}{\sin \theta} \right| \left| \cos \frac{\psi}{2} \right|$$

Why zero?



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

→ Normalized array factor:

$$|A(\psi)| = \left| \cos \frac{\psi}{2} \right| = \left| \cos \frac{1}{2} (\beta d \cos \phi + \xi) \right|$$

$$\leftarrow \psi = \beta d \sin \theta \cos \phi + \xi \quad \leftarrow \theta = \pi/2$$

Arrays of equal magnitude and a uniform progressive phase shift:

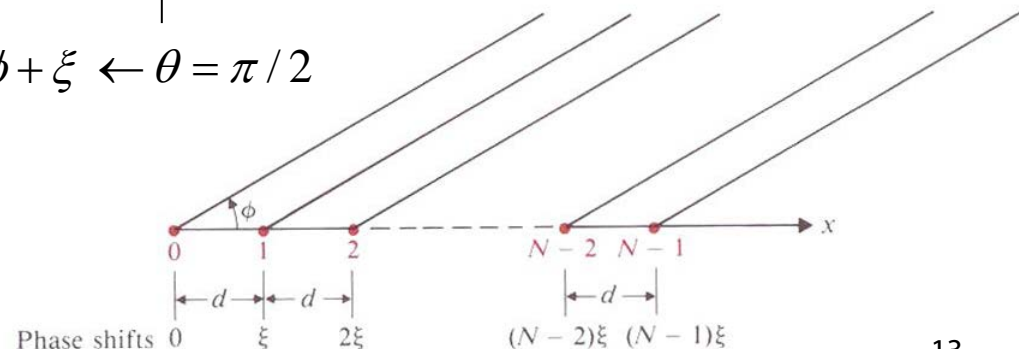
$$|A(\psi)| = \frac{1}{N} \left| 1 + e^{j\psi} + e^{j2\psi} + \dots + e^{j(N-1)\psi} \right|$$

→ Broadside or endfire?

$$\leftarrow \psi = \beta d \cos \phi + \xi \quad \leftarrow \theta = \pi/2$$

$$\rightarrow |A(\psi)| = \frac{1}{N} \left| \frac{1 - e^{jN\psi}}{1 - e^{j\psi}} \right|$$

$$= \frac{1}{N} \left| \frac{\sin(N\psi/2)}{\sin(\psi/2)} \right|$$



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.