Electro-Optics:

Polarization of Light Waves (1)

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Polarization of Monochromatic Plane Waves (1)

Electric field vector:

$$\mathbf{E}(z,t) = \mathrm{Re}[\mathbf{A}e^{i(\omega t - kz)}] \quad \leftarrow \quad \mathbf{A} = \hat{\mathbf{x}}A_x e^{i\delta_x} + \hat{\mathbf{y}}A_y e^{i\delta_y}$$

$$\rightarrow \; E_x = A_x \cos(\omega t - kz + \delta_x)$$

$$\rightarrow \; E_y = A_y \cos(\omega t - kz + \delta_y)$$

Derivation of the time-evolution locus:

$$\left\{ \begin{array}{l} \rightarrow \; E_x = A_x[\cos(\omega t - kz)\cos\delta_x - \sin(\omega t - kz)\sin\delta_x] \\ \rightarrow \; E_y = A_y[\cos(\omega t - kz)\cos\delta_y - \sin(\omega t - kz)\sin\delta_y] \end{array} \right.$$

$$\left\{ \begin{array}{l} \rightarrow \; \dfrac{E_x}{A_x}\cos\delta_y = \cos(\omega t - kz)\cos\delta_x\cos\delta_y - \sin(\omega t - kz)\sin\delta_x\cos\delta_y \\ \rightarrow \; \dfrac{E_y}{A_y}\cos\delta_x = \cos(\omega t - kz)\cos\delta_x\cos\delta_y - \sin(\omega t - kz)\cos\delta_x\sin\delta_y \end{array} \right.$$

$$\left\{ \begin{array}{l} \rightarrow \; \dfrac{E_x}{A_x}\sin\delta_y = \cos(\omega t - kz)\cos\delta_x\sin\delta_y - \sin(\omega t - kz)\sin\delta_x\sin\delta_y \\ \rightarrow \; \dfrac{E_y}{A_y}\sin\delta_x = \cos(\omega t - kz)\sin\delta_x\cos\delta_y - \sin(\omega t - kz)\sin\delta_x\sin\delta_y \end{array} \right.$$

Polarization of Monochromatic Plane Waves (2)

Continued:

$$\left\{\begin{array}{l} \rightarrow \dfrac{E_x}{A_x}\cos\delta_y - \dfrac{E_y}{A_y}\cos\delta_x = \sin(\omega t - kz)[\cos\delta_x \sin\delta_y - \sin\delta_x \cos\delta_y] \\ \rightarrow \dfrac{E_x}{A_x}\sin\delta_y - \dfrac{E_y}{A_y}\sin\delta_x = \cos(\omega t - kz)\underbrace{[\cos\delta_x \sin\delta_y - \sin\delta_x \cos\delta_y]} \end{array}\right.$$

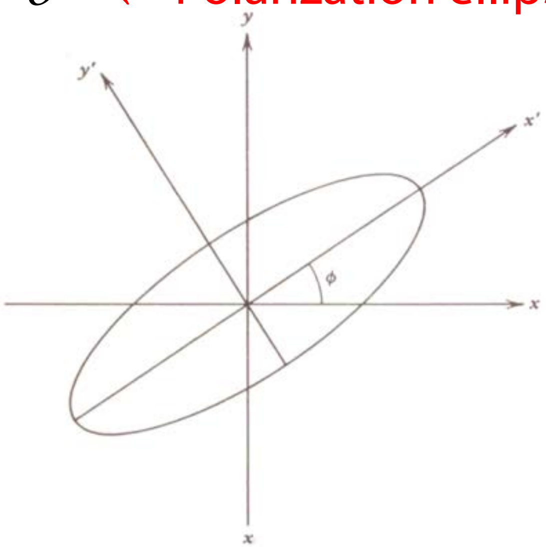
$$\sin\delta \quad \leftarrow \quad \delta = \delta_y - \delta_x$$

Time-evolution locus:

$$\rightarrow \left(\frac{E_x}{A_x}\right)^2 + \left(\frac{E_y}{A_y}\right)^2 - 2\frac{\cos\delta}{A_x A_y}E_x E_y = \sin^2\delta$$ ← Polarization ellipse

In the principal coordinate system:

$$\rightarrow \begin{pmatrix} E_{x'} \\ E_{y'} \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

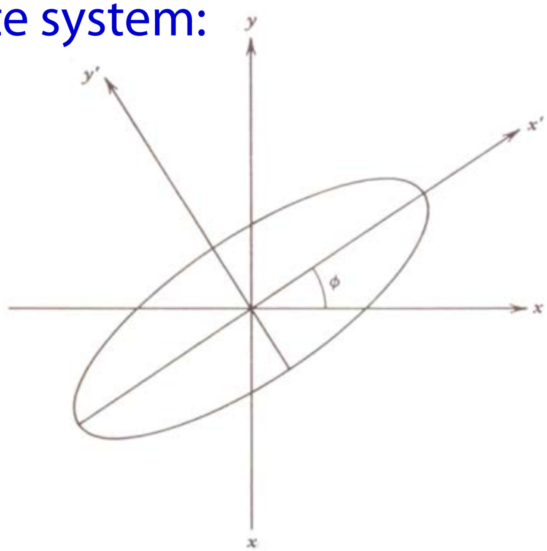


A. Yariv and P. Yeh, *Optical Waves in Crystals*, 1984

Polarization of Monochromatic Plane Waves (3)

Time-evolution locus in the principal coordinate system:

$$\rightarrow \left(\frac{E_{x'}}{a}\right)^2 + \left(\frac{E_{y'}}{b}\right)^2 = 1 \quad \leftarrow \text{H.W.}$$



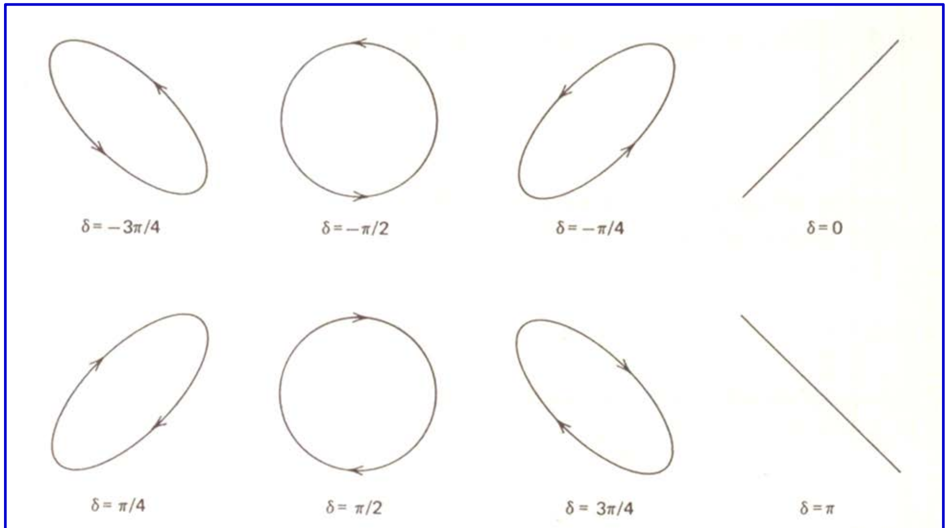
A. Yariv and P. Yeh, *Optical Waves in Crystals*, 1984

$$\leftarrow \tan 2\phi = \frac{2A_x A_y}{A_x^2 - A_y^2}\cos\delta$$

$$\leftarrow \; a^2 = A_x^2\cos^2\phi + A_y^2\sin^2\phi + 2A_x A_y\cos\delta\cos\phi\sin\phi$$

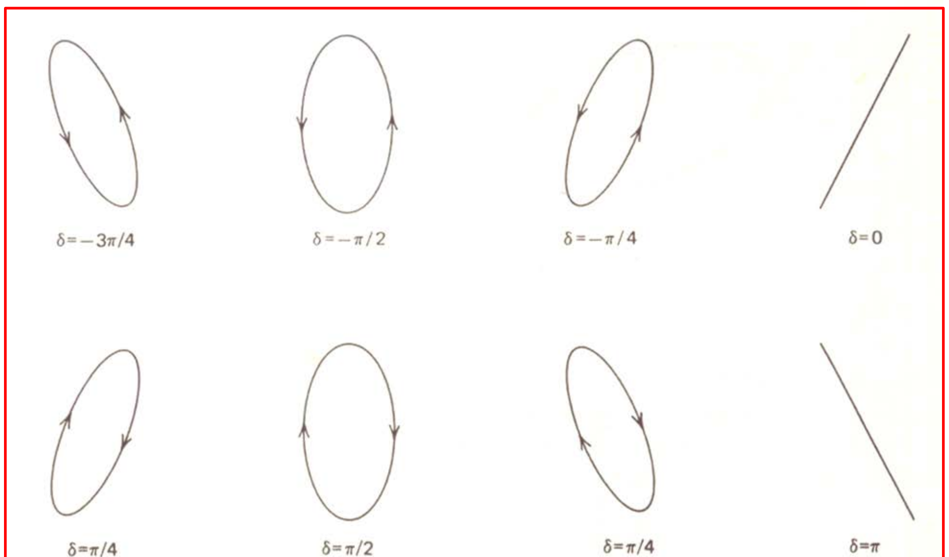
$$\leftarrow \; b^2 = A_x^2\sin^2\phi + A_y^2\cos^2\phi - 2A_x A_y\cos\delta\cos\phi\sin\phi$$

Polarization Ellipses



A. Yariv and P. Yeh, *Optical Waves in Crystals*, 1984

$$\leftarrow \quad E_x = \cos(\omega t - kz)$$
$$E_y = \cos(\omega t - kz + \delta)$$



$$\leftarrow \quad E_x = \frac{1}{2}\cos(\omega t - kz)$$
$$E_y = \cos(\omega t - kz + \delta)$$

Linear and Circular Polarizations

Recall:

$$\rightarrow E_x = A_x \cos(\omega t - kz + \delta_x)$$

$$\rightarrow E_y = A_y \cos(\omega t - kz + \delta_y)$$

Linear polarization:

$$\delta = \delta_y - \delta_x = m\pi \quad (m = 0, 1)$$

$$\rightarrow \frac{E_y}{E_x} = (-1)^m \frac{A_y}{A_x}$$

Circular polarization:

$$\delta = \delta_y - \delta_x = \pm\frac{1}{2}\pi \quad \& \; A_y = A_x$$

$$\rightarrow \delta = -\frac{1}{2}\pi$$ ← Right-hand circularly polarized

$$\rightarrow \delta = +\frac{1}{2}\pi$$ ← Left-hand circularly polarized

Elliptical polarization:

$$e = \pm\frac{b}{a}$$ ← Ellipticity of a polarization ellipse

(±: Right- & left-handed rotation)