Electro-Optics:

Electromagnetic Propagation in Anisotropic Media (3)

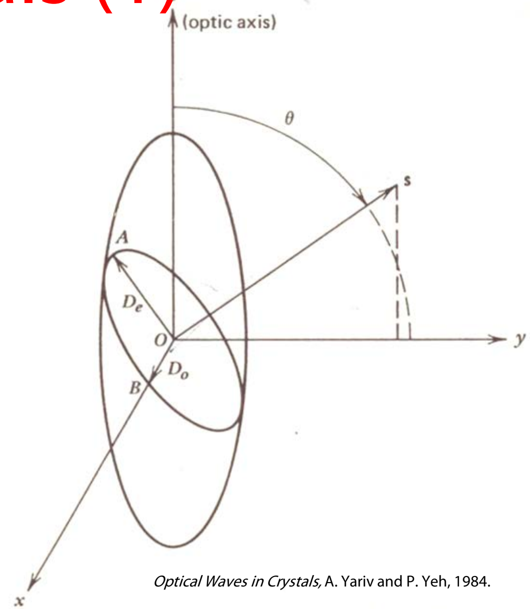
Yoonchan Jeong

School of Electrical Engineering, Seoul National University

Tel: +82 (0)2 880 1623, Fax: +82 (0)2 873 9953

Email: yoonchan@snu.ac.kr

Light Propagation in Uniaxial Crystals (1)



Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

Index ellipsoid:

$$\frac{x^2}{n_o^2}+\frac{y^2}{n_o^2}+\frac{z^2}{n_e^2}=1$$

Polarizations for the displacement vector:

$$\mathbf{d}_o=\frac{\mathbf{k}\times\mathbf{c}}{|\mathbf{k}\times\mathbf{c}|} \leftarrow \text{Ordinary polarization}$$

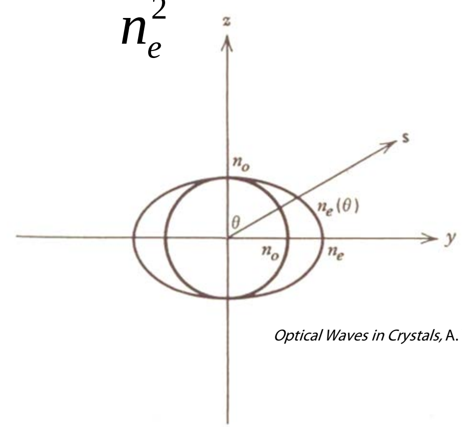
$$\mathbf{d}_e=\frac{\mathbf{d}_o\times\mathbf{k}}{|\mathbf{d}_o\times\mathbf{k}|} \leftarrow \text{Extraordinary polarization}$$

For $k_x = 0$:

$$\frac{x^2}{n_o^2}+\frac{y^2}{n_o^2}+\frac{z^2}{n_e^2}=1 \;\rightarrow\; \frac{1}{n_e^2(\theta)}=\frac{\cos^2\theta}{n_o^2}+\frac{\sin^2\theta}{n_e^2}$$

Extraordinary electric field:

$$\rightarrow \begin{pmatrix} 0 \\ \dfrac{\sin\theta}{n_e^2(\theta)-n_o^2} \\ \dfrac{\cos\theta}{n_e^2(\theta)-n_e^2} \end{pmatrix}$$



Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

Light Propagation in Uniaxial Crystals (2)

Surface of constant energy density:

$$U_e = \frac{1}{2}\mathbf{E}\cdot\mathbf{D} \;\rightarrow\; \frac{D_x^2}{\varepsilon_x} + \frac{D_y^2}{\varepsilon_y} + \frac{D_z^2}{\varepsilon_z} = 2U_e \;\leftarrow\; n_i^2 = \varepsilon_i / \varepsilon_0$$

$$\leftarrow\; x_i \equiv \frac{D_i}{\sqrt{2U_e\varepsilon_0}}$$

Index ellipsoid:

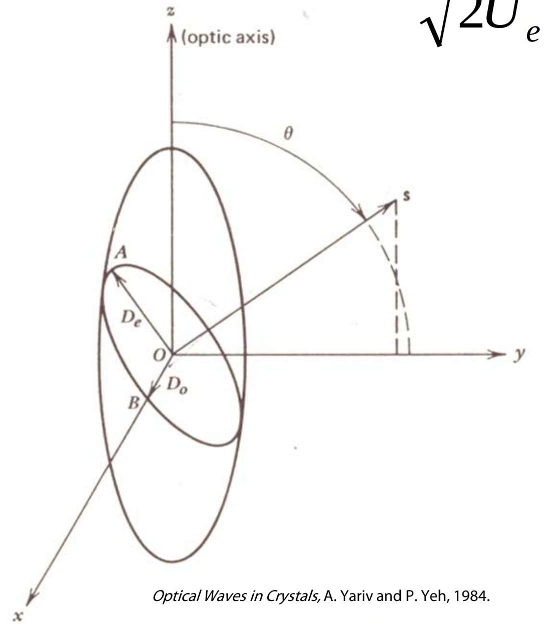
$$\rightarrow\; \frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1$$

Impermeability tensor:

$$\eta_{ij} = \varepsilon_0(\varepsilon^{-1})_{ij} \;\rightarrow\; \mathbf{E} = \frac{1}{\varepsilon_0}\eta\mathbf{D}$$

For uniaxial crystals:

$$\rightarrow\; \frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1$$



Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

Wave equation:

$$\mathbf{k}\times(\mathbf{k}\times\mathbf{E}) + \omega^2\mu\varepsilon\mathbf{E} = 0 \;\rightarrow\; \mathbf{s}\times(\mathbf{s}\times\eta\mathbf{D}) + \frac{1}{n^2}\mathbf{D} = 0$$

$$\rightarrow\; \mathbf{s}(\mathbf{s}\cdot\eta\mathbf{D}) - \eta\mathbf{D} + \frac{1}{n^2}\mathbf{D} = 0$$

"back-cab" rule

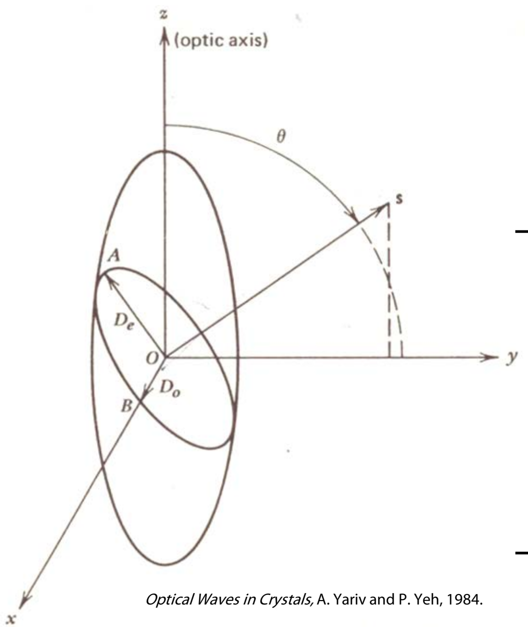
$$\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$$

Light Propagation in Uniaxial Crystals (3)

Wave equation:

$$\rightarrow \mathbf{s}(\mathbf{s}\cdot\eta\mathbf{D}) - \eta\mathbf{D} + \frac{1}{n^2}\mathbf{D} = 0$$

$$\leftarrow \mathbf{s} = (0 \quad \sin\theta \quad \cos\theta)^T \qquad \leftarrow \eta = \begin{pmatrix} \frac{1}{n_o^2} & 0 & 0 \\ 0 & \frac{1}{n_o^2} & 0 \\ 0 & 0 & \frac{1}{n_e^2} \end{pmatrix}$$



Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

$$\rightarrow \mathbf{s}\cdot\eta\mathbf{D} = \frac{D_y}{n_o^2}\sin\theta + \frac{D_z}{n_e^2}\cos\theta$$

$$\rightarrow \eta\mathbf{D} - \mathbf{s}(\mathbf{s}\cdot\eta\mathbf{D}) = \begin{pmatrix} \frac{1}{n_o^2} & 0 & 0 \\ 0 & \frac{\cos^2\theta}{n_o^2} & -\frac{\cos\theta\sin\theta}{n_e^2} \\ 0 & -\frac{\cos\theta\sin\theta}{n_o^2} & \frac{\sin^2\theta}{n_e^2} \end{pmatrix}\mathbf{D}$$

Eigenvalues and eigenvectors:

$$\rightarrow n_1 = n_o \quad \rightarrow \mathbf{D}_o = (n_o \quad 0 \quad 0)^T$$

← Exactly equivalent to the IE method

$$\rightarrow \frac{1}{n_2^2} = \frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2} \quad \rightarrow n_2 \equiv n_e(\theta)$$

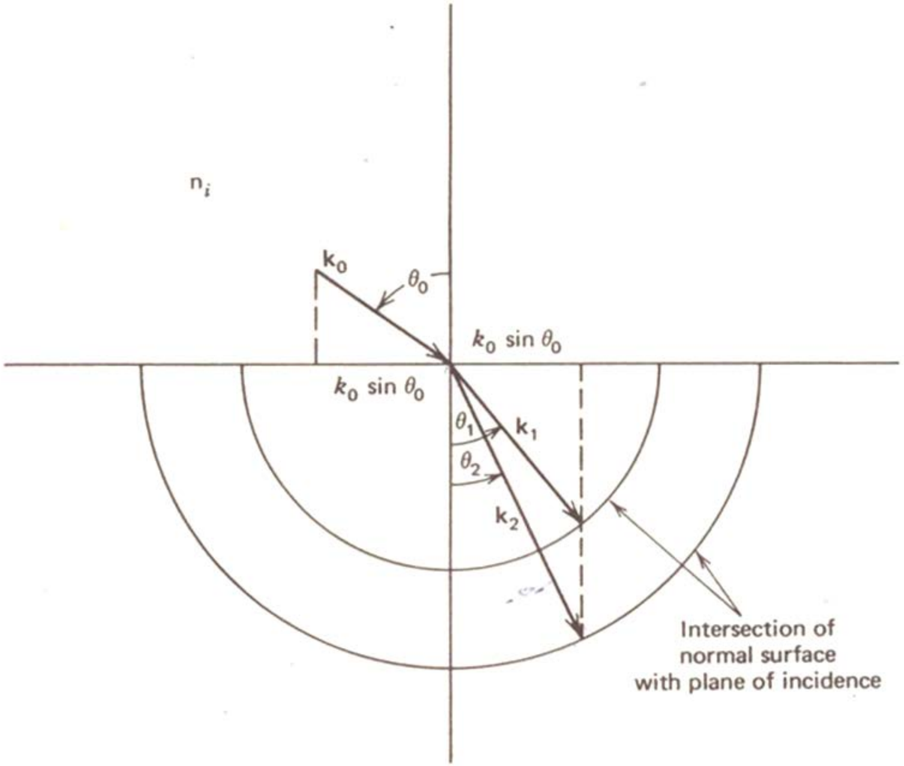
$$\rightarrow \mathbf{D}_e = [0 \quad n_e(\theta)\cos\theta \quad n_e(\theta)\sin\theta]^T$$

Double Refraction at a Boundary

Boundary condition:

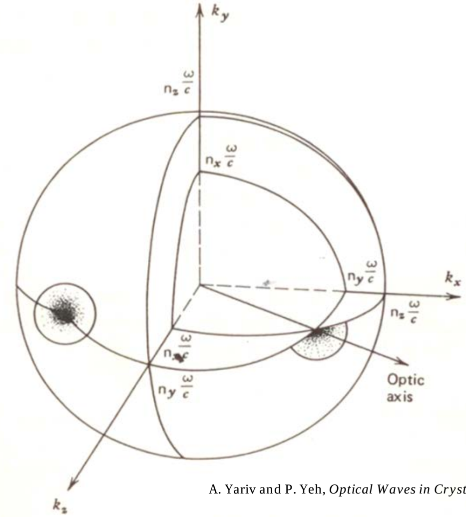
$$k_0 \sin\theta_0 = k_1 \sin\theta_1 = k_2 \sin\theta_2$$

Snell's law: $\rightarrow\ n_i \sin\theta_0 = n_o \sin\theta_1 = n_e(\theta_2)\sin\theta_2$



Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

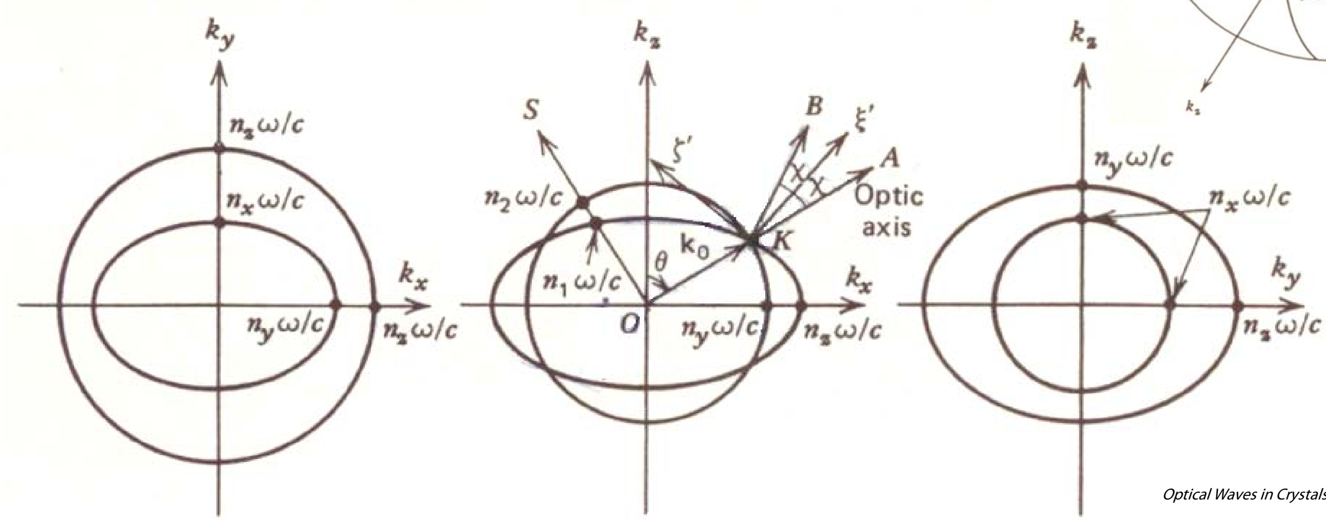
Light Propagation in Biaxial Crystals



A. Yariv and P. Yeh, *Optical Waves in Crystals*, 1984.

Biaxial media: $\rightarrow \; n_x < n_y < n_z$

Normal surface:



Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

k_x-k_y plane:

$$\rightarrow \frac{k_x^2 + k_y^2}{n_z^2} = \left(\frac{\omega}{c}\right)^2$$

$$\rightarrow \frac{k_x^2}{n_y^2} + \frac{k_y^2}{n_x^2} = \left(\frac{\omega}{c}\right)^2$$

k_z-k_x plane:

$$\rightarrow \frac{k_z^2 + k_x^2}{n_y^2} = \left(\frac{\omega}{c}\right)^2$$

$$\rightarrow \frac{k_z^2}{n_x^2} + \frac{k_x^2}{n_z^2} = \left(\frac{\omega}{c}\right)^2$$

k_y-k_z plane:

$$\rightarrow \frac{k_y^2 + k_z^2}{n_x^2} = \left(\frac{\omega}{c}\right)^2$$

$$\rightarrow \frac{k_y^2}{n_z^2} + \frac{k_z^2}{n_y^2} = \left(\frac{\omega}{c}\right)^2$$

Conical Refraction (1)

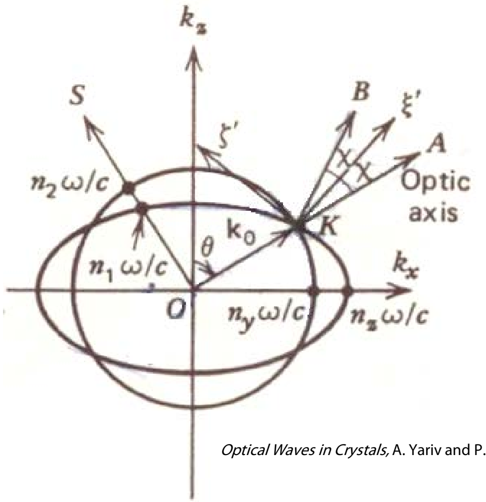
For light propagation in the direction of the optic axis:

$$\mathbf{k}_0 = \hat{\mathbf{x}}k_{x0} + \hat{\mathbf{y}}k_{y0} + \hat{\mathbf{z}}k_{z0}$$

$$\rightarrow k_{x0} = n_y \frac{\omega}{c}\sin\theta$$

$$\rightarrow k_{y0} = 0 \qquad \rightarrow \tan\theta = \frac{n_z}{n_x}\left(\frac{n_y^2 - n_x^2}{n_z^2 - n_y^2}\right)^{1/2}$$

$$\rightarrow k_{z0} = n_y \frac{\omega}{c}\cos\theta$$



Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

Taylor series expansion:

$$\rightarrow k_x = k_{x0} + \xi$$

$$\rightarrow k_y = k_{y0} + \eta \qquad \rightarrow k_x^2 YZ + k_y^2 ZX + k_z^2 XY - XYZ = 0$$

$$\rightarrow k_z = k_{z0} + \varsigma$$

$$\rightarrow 4(k_{x0}\xi + k_{z0}\zeta)(n_x^2 k_{x0}\xi + n_z^2 k_{z0}\varsigma) + \eta^2 (n_y^2 - n_x^2)(n_y^2 - n_z^2)\left(\frac{\omega}{c}\right)^2 = 0$$

Rotation of the coordinates:

$$\rightarrow \alpha\xi'^2 + \beta\varsigma'^2 + \eta^2 (n_y^2 - n_x^2)(n_y^2 - n_z^2) = 0$$

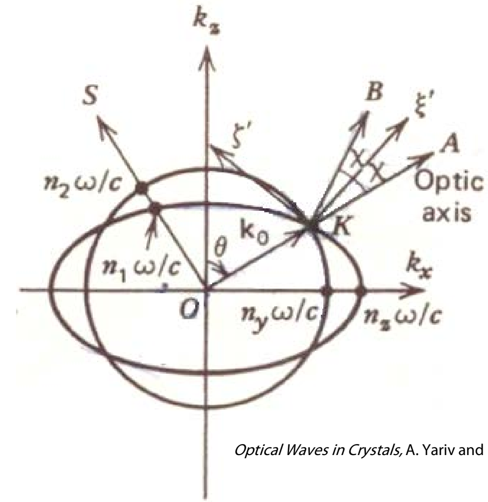
Conical Refraction (2)

Rotation of the coordinates:

$$\rightarrow \alpha \xi'^2 + \beta \varsigma'^2 + \eta^2 (n_y^2 - n_x^2)(n_y^2 - n_z^2) = 0$$

$$\rightarrow \begin{pmatrix} \xi' \\ \varsigma' \end{pmatrix} = \begin{pmatrix} \cos(\frac{\pi}{2} + \chi - \theta) & \sin(\frac{\pi}{2} + \chi - \theta) \\ -\sin(\frac{\pi}{2} + \chi - \theta) & \cos(\frac{\pi}{2} + \chi - \theta) \end{pmatrix} \begin{pmatrix} \xi \\ \varsigma \end{pmatrix}$$

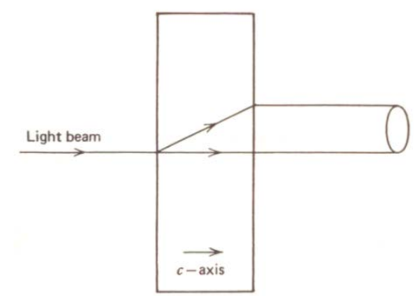
$$= \begin{pmatrix} -\sin(\chi - \theta) & \cos(\chi - \theta) \\ -\cos(\chi - \theta) & -\sin(\chi - \theta) \end{pmatrix} \begin{pmatrix} \xi \\ \varsigma \end{pmatrix}$$



Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

$$\rightarrow \varsigma'^2 + \frac{1}{1 - \tan^2 \chi} \eta^2 = \xi'^2 \cot^2 \chi$$ ← Eq. of an elliptical cone

$$\rightarrow \tan^2 2\chi = \frac{(n_z^2 - n_y^2)(n_y^2 - n_x^2)}{n_x^2 n_z^2}$$



Optical Waves in Crystals, A. Yariv and P. Yeh, 1984.

Energy flow: $\rightarrow \varsigma'^2 + (1 - \tan^2 \chi)\eta^2 = \xi'^2 \tan^2 \chi$

$$\leftarrow \begin{pmatrix} \varsigma' & \frac{1}{1 - \tan^2 \chi} \eta & -\xi' \cot^2 \chi \end{pmatrix} \cdot \begin{pmatrix} \varsigma' & \eta & \xi' \end{pmatrix} = 0$$