

Electro-Optics

Noise in Optical Detection and Generation (1)

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Limitations Due to Noise Power (1)

Noise in optical detection:

The noise causes fluctuations in the measurement, thus placing a lower limit on the smallest amount of power that can be measured.

Thermal noise:

Generated by thermally agitated charge carriers

→ Thermodynamic treatment & statistical analysis

Shot noise:

Generation-recombination noise in photoconductive detectors

→ Uncertainty principle in quantum mechanics

Measurement of optical power:

Optical signal field: $v_s(t) = V_s \cos \omega t$

Noise field: $v_N(t) = V_{NC}(t) \cos \omega t + V_{NS}(t) \sin \omega t$

Total field at the detector: $v(t) = v_s(t) + v_N(t)$

$$= \operatorname{Re}\{[V_s + V_{NC}(t) - iV_{NS}(t)]e^{i\omega t}\}$$

$$\equiv \operatorname{Re}[V(t)e^{i\omega t}]$$

Limitations Due to Noise Power (2)

Probability function (Gaussian) :

$$p(V_{NC})dV_{NC} = \frac{1}{(2\pi)^{1/2}\sigma} e^{-V_{NC}^2/2\sigma^2} dV_{NC} \rightarrow \int_{-\infty}^{+\infty} p(V_{NC})dV_{NC} = 1$$

Ensemble average:

$$\begin{aligned} \rightarrow \overline{V_{NC}^2} &= \int_{-\infty}^{+\infty} V_{NC}^2 p(V_{NC})dV_{NC} = \sigma^2 & \rightarrow \overline{V_{NS}^2} &= \sigma^2 \\ \rightarrow \overline{V_{NC}} &= 0 & \rightarrow \overline{V_{NS}} &= 0 \end{aligned}$$

Field "power":

$$\begin{aligned} P(t) &\equiv [V(t)e^{i\omega t}][V^*(t)e^{-i\omega t}] = V_S^2 + 2V_S V_{NC} + V_{NC}^2 + V_{NS}^2 \\ \rightarrow \overline{P} &\equiv \overline{P(t)} = \overline{V_S^2} + \overline{V_{NC}^2} + \overline{V_{NS}^2} = \overline{V_S^2} + 2\sigma^2 \end{aligned}$$

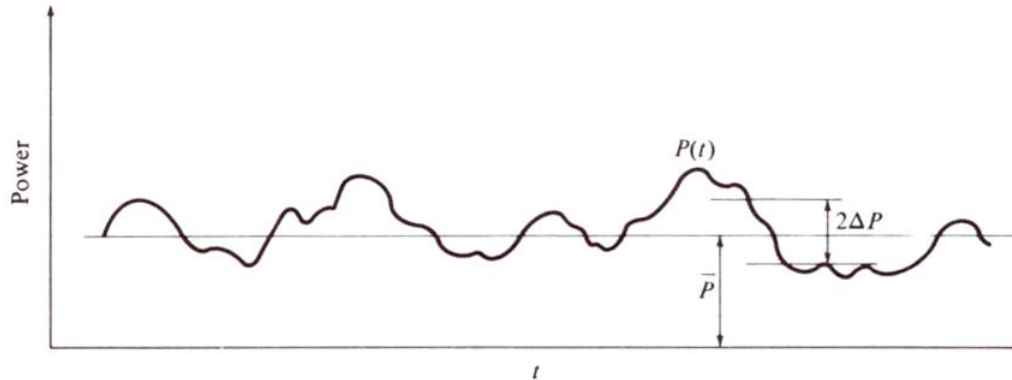
Uncertainty in power measurement: \rightarrow Root mean square power deviation

$$\begin{aligned} \Delta P &\equiv \{\overline{[P(t) - \overline{P}]^2}\}^{1/2} = (4\overline{V_S^2 V_{NC}^2} + 2\overline{V_{NC}^4} - 2\overline{V_{NS}^2 V_{NC}^2})^{1/2} \\ &= 2\sigma(\overline{V_S^2} + \sigma^2)^{1/2} & \leftarrow \overline{V_{NC}^4} &= \int_{-\infty}^{+\infty} V_{NC}^4 p(V_{NC})dV_{NC} = 3\sigma^4 \\ &= 2\sigma(P_s + \sigma^2)^{1/2} \end{aligned}$$

Limitations Due to Noise Power (3)

Minimum signal power measurable in the presence of noise:

$$\Delta P = 2\sigma(P_s + \sigma^2)^{1/2}$$



A. Yariv, *Optical Electronics*, 4th ed. Saunders, 1991.

$$\begin{aligned}\rightarrow P_{\text{limit}} &= 2\sigma(P_{\text{limit}} + \sigma^2)^{1/2} \\ &= 2\sigma^2(1 + \sqrt{2}) = P_N(1 + \sqrt{2})\end{aligned}$$

Noise power

Noise – Basic Definitions and Theorems

A real function and its Fourier transform:

$$V(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} v(t) e^{-i\omega t} dt \quad \rightarrow \quad v(t) = \int_{-\infty}^{+\infty} V(\omega) e^{i\omega t} d\omega$$

In case: $v(t) = 0$ for $t \leq -T/2$ & $t \geq T/2$

$$V_T(\omega) = \frac{1}{2\pi} \int_{-T/2}^{+T/2} v(t) e^{-i\omega t} dt \quad \rightarrow \quad V_T(\omega) = V_T^*(-\omega)$$

Average power:

$$\begin{aligned} P &= \frac{1}{T} \int_{-T/2}^{+T/2} v^2(t) dt = \frac{1}{T} \int_{-T/2}^{+T/2} v(t) \left[\int_{-\infty}^{+\infty} V_T(\omega) e^{i\omega t} d\omega \right] dt \\ &= \frac{1}{T} \int_{-T/2}^{+T/2} \left[\int_{-\infty}^{+\infty} V_T(\omega') e^{i\omega' t} d\omega' \right] \left[\int_{-\infty}^{+\infty} V_T(\omega) e^{i\omega t} d\omega \right] dt \\ &\quad \leftarrow \lim_{T \rightarrow \infty} \frac{1}{2\pi} \int_{-T/2}^{+T/2} \exp[i(\omega + \omega')t] dt = \delta(\omega + \omega') \\ &= \frac{2\pi}{T} \int_{-\infty}^{+\infty} |V_T(\omega)|^2 d\omega = \frac{4\pi}{T} \int_0^{+\infty} |V_T(\omega)|^2 d\omega \end{aligned}$$

Spectral density function:

$$S_v(\omega) = \lim_{T \rightarrow \infty} \frac{4\pi |V_T(\omega)|^2}{T}$$

Wiener-Khintchine Theorem

Autocorrelation function:

$$\begin{aligned}C_v(\tau) &= \overline{v(t)v(t+\tau)} = \frac{1}{T} \int_{-T/2}^{+T/2} v(t)v(t+\tau) dt \\&= \frac{1}{T} \int_{-T/2}^{+T/2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} V_T(\omega)V_T(\omega') e^{i(\omega+\omega')t} e^{i\omega\tau} dt d\omega d\omega' \\&= \frac{1}{2} \int_{-\infty}^{+\infty} \frac{4\pi |V_T(\omega)|^2}{T} e^{i\omega\tau} d\omega\end{aligned}$$

Spectral density function

Wiener-Khintchine theorem:

$$\begin{aligned}\rightarrow C_v(\tau) &= \frac{1}{2} \int_{-\infty}^{+\infty} S_v(\omega) e^{i\omega\tau} d\omega \\ \rightarrow S_v(\omega) &= \frac{1}{\pi} \int_{-\infty}^{+\infty} C_v(\tau) e^{-i\omega\tau} d\tau\end{aligned}$$

Fourier transform pair

Spectral Density Function of Random Events (1)

Time-dependent random variable:

$$i_T(t) = \sum_{i=1}^{N_T} f(t-t_i) \quad 0 \leq t \leq T$$

$$\rightarrow I_T(\omega) = \sum_{i=1}^{N_T} F_i(\omega) \quad \leftarrow F_i(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t-t_i) e^{-i\omega t} dt$$

Ensemble average:

$$\rightarrow \overline{|I_T(\omega)|^2} = \overline{|F(\omega)|^2 \sum_{i=1}^{N_T} \sum_{j=1}^{N_T} e^{-i\omega(t_i-t_j)}}$$

$$= \frac{e^{-i\omega t_i}}{2\pi} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$

$$= e^{-i\omega t_i} F(\omega)$$

$$= |F(\omega)|^2 \left(N_T + \sum_{i \neq 1}^{N_T} \sum_{j=1}^{N_T} e^{-i\omega(t_i-t_j)} \right) = \overline{N_T} |F(\omega)|^2 = \overline{NT} |F(\omega)|^2$$

Average rate

Recall: Spectral density function

$$S_v(\omega) = \frac{4\pi |V_T(\omega)|^2}{T} \quad \rightarrow S(\omega) = 4\pi \overline{N} |F(\omega)|^2$$

Spectral Density Function of Random Events (2)

Carson's theorem:

$$S(\omega) = 4\pi\bar{N}|F(\omega)|^2$$
$$\rightarrow S(\nu) = 8\pi^2\bar{N}|F(2\pi\nu)|^2 \quad \leftarrow S(\nu)d\nu = S(\omega)d\omega$$

For subclass events: $f_\alpha(t - t_i)$

$$\rightarrow S_\alpha(\nu)d\alpha = 8\pi^2\bar{N}(\alpha)|F_\alpha(2\pi\nu)|^2 d\alpha \quad \leftarrow \int_{-\infty}^{+\infty}\bar{N}(\alpha)d\alpha = \bar{N}$$

Probability distribution function:

$$\rightarrow \int_{-\infty}^{+\infty} p(\alpha)d\alpha = \frac{1}{\bar{N}} \int_{-\infty}^{+\infty}\bar{N}(\alpha)d\alpha = 1$$

Overall "spectral density function":

$$\begin{aligned} \rightarrow S(\nu) &= \int_{-\infty}^{+\infty} S_\alpha(\nu)d\alpha = \int_{-\infty}^{+\infty} 8\pi^2\bar{N}(\alpha)|F_\alpha(2\pi\nu)|^2 d\alpha \\ &= 8\pi^2\bar{N} \int_{-\infty}^{+\infty} |F_\alpha(2\pi\nu)|^2 p(\alpha)d\alpha \\ &= 8\pi^2\bar{N}|F_\alpha(2\pi\nu)|^2 \end{aligned}$$

Shot Noise

Random electron flow between two electrodes:

$$\bar{N} = \bar{I} / q \leftarrow \text{Average rate of electron emission}$$

Current pulse due to a "single" electron:

$$i_e(t) = \frac{qv(t)}{d}$$

Instantaneous velocity

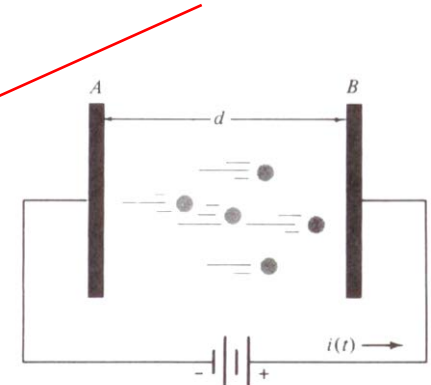
Fourier transform of a single current pulse:

$$F(\omega) = \frac{q}{2\pi d} \int_0^{t_a} v(t) e^{-i\omega t} dt \leftarrow \omega t_a \ll 1$$

Arrival time

(Small transit time)

$$\cong \frac{q}{2\pi d} \int_0^{t_a} \frac{dx}{dt} dt = \frac{q}{2\pi}$$



A. Yariv, *Optical Electronics*, 4th ed. Saunders, 1991.

Spectral density function:

$$S(\nu) = 8\pi^2 \bar{N} |F(2\pi\nu)|^2 = 8\pi^2 \bar{N} \left(\frac{q}{2\pi} \right)^2 = 2q\bar{I}$$

"Shot noise" power by an equivalent noise generator:

$$\overline{i_N^2}(\nu) = S(\nu) \Delta\nu \rightarrow \text{Proportional to the "magnitude of an individual charge" even for the same } \bar{I}$$

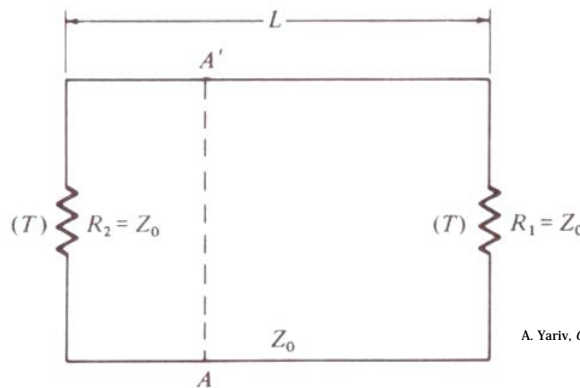
→ Caused by quantum fluctuations in the current

Thermal Noise (1)

Johnson-Nyquist noise:

Fluctuations in the voltage across a dissipative circuit element

Transmission line connected by two similar resistances at the same T :



→ Matched impedance
having no reflection

A. Yariv, Optical Electronics, 4th ed. Saunders, 1991.

Traveling voltage wave: $v(t) = A \cos(\omega t \pm kz)$

$$\rightarrow v(t) = A \cos[\omega t \pm k(z + L)] = A \cos(\omega t \pm kz)$$

$$\rightarrow kL = 2m\pi, \quad m = 1, 2, 3, \dots$$

$$\rightarrow \Delta k = \frac{2\pi}{L} \rightarrow \text{Mode spacing}$$

Number of modes between 0 to k :

$$\rightarrow N_k = \frac{k}{\Delta k} = \frac{kL}{2\pi} \rightarrow N(\nu) = \frac{\nu L}{c}$$

Thermal Noise (2)

Number of mode per unit frequency:

$$p(\nu) = \frac{dN(\nu)}{d\nu} = \frac{L}{c}$$

Power transmission:

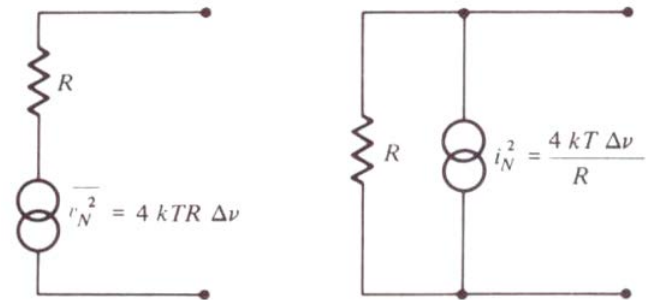
$$\text{power} = \frac{\text{energy}}{\text{distance}} (\text{velocity of energy})$$

$$\begin{aligned} \rightarrow P &= \left(\frac{c}{L}\right) (\text{number of modes between } \nu \text{ and } \nu + \Delta\nu) (\text{energy per mode}) \\ &= \left(\frac{c}{L}\right) \left(\frac{L}{c} \Delta\nu\right) \left(\frac{h\nu}{e^{h\nu/kT} - 1}\right) = \frac{h\nu\Delta\nu}{e^{h\nu/kT} - 1} \cong kT\Delta\nu \quad (kT \gg h\nu) \end{aligned}$$

Equivalent noise generator:

$$\rightarrow \overline{v_N^2}(\nu) = \frac{4h\nu R \Delta\nu}{e^{h\nu/kT} - 1} \cong_{kT \gg h\nu} 4kTR\Delta\nu$$

$$\rightarrow \overline{i_N^2}(\nu) = \frac{4h\nu\Delta\nu}{R(e^{h\nu/kT} - 1)} \cong_{kT \gg h\nu} \frac{4kT\Delta\nu}{R}$$



A. Yariv, *Optical Electronics*, 4th ed. Saunders, 1991.