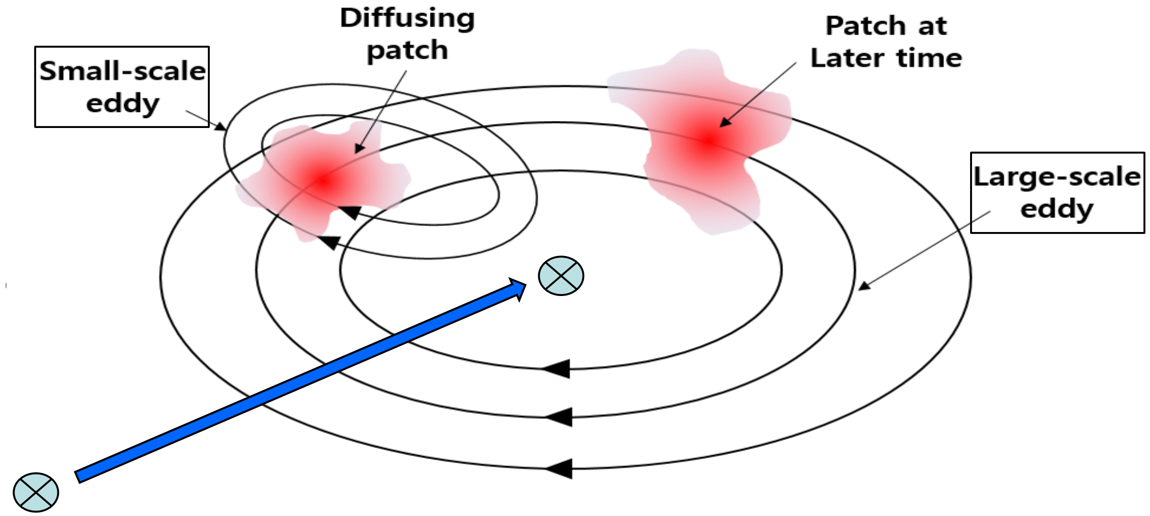
Chapter 3

Fluid Transport



Chapter 3 Fluid Transport

Contents

Objectives

- Introduce the concept of fluid transport
- Study analogy between mass, heat, and momentum transport
- Derive a general equation of fluid transport

3.1 Introduction

Fluid transport phenomena

- Transport

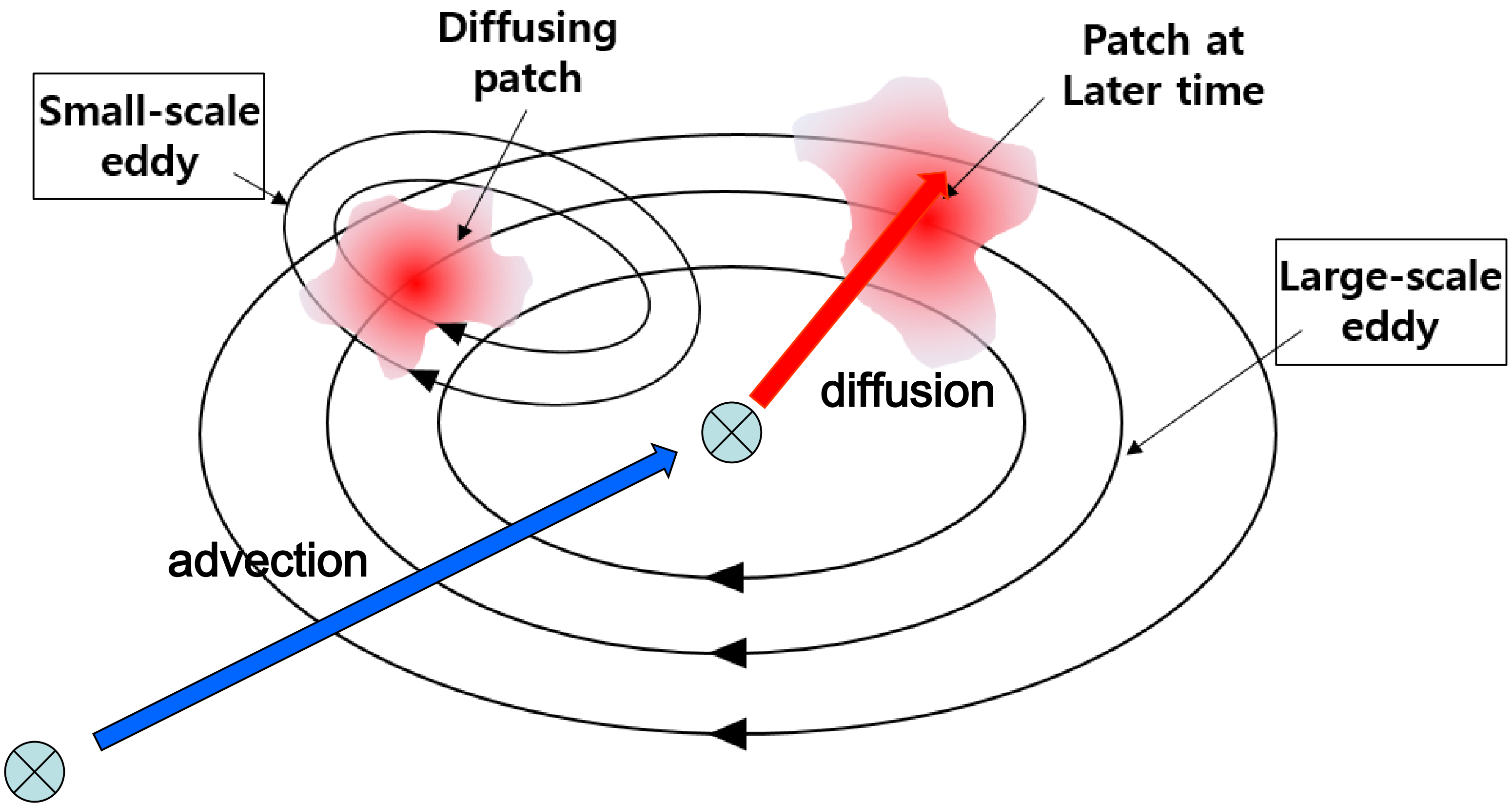
= ability of fluids in motion to convey materials and properties from place to place

= mechanism by which materials and properties are diffused and transmitted through the fluid medium

= advection + diffusion

- Advection = transport by imposed current (velocity)
- Diffusion = movement of mass or heat or momentum in the direction of decreasing concentration of mass, temperature, or momentum

Introduction



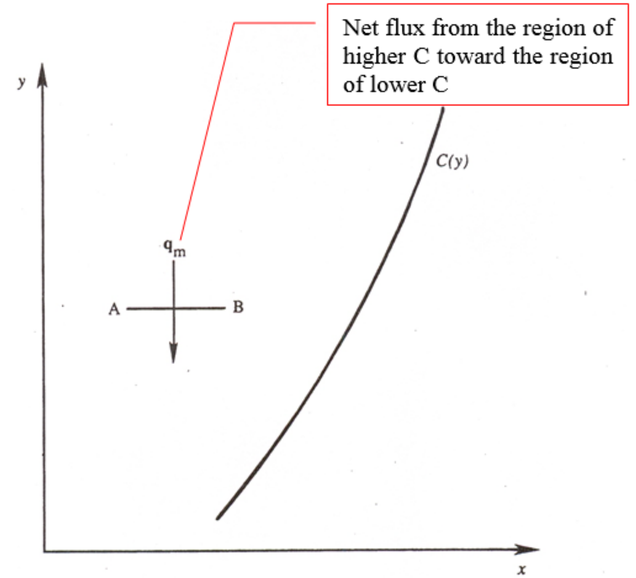
Introduction

- Diffusion

Transport of materials and properties in the direction of decreasing mass, temperature, momentum

$$q = \left\{ \frac{dM / dt}{area} \right\} \propto \left\{ \frac{d\left(M / vol\right)}{ds} \right\}$$

Flux = quantity per unit time per area

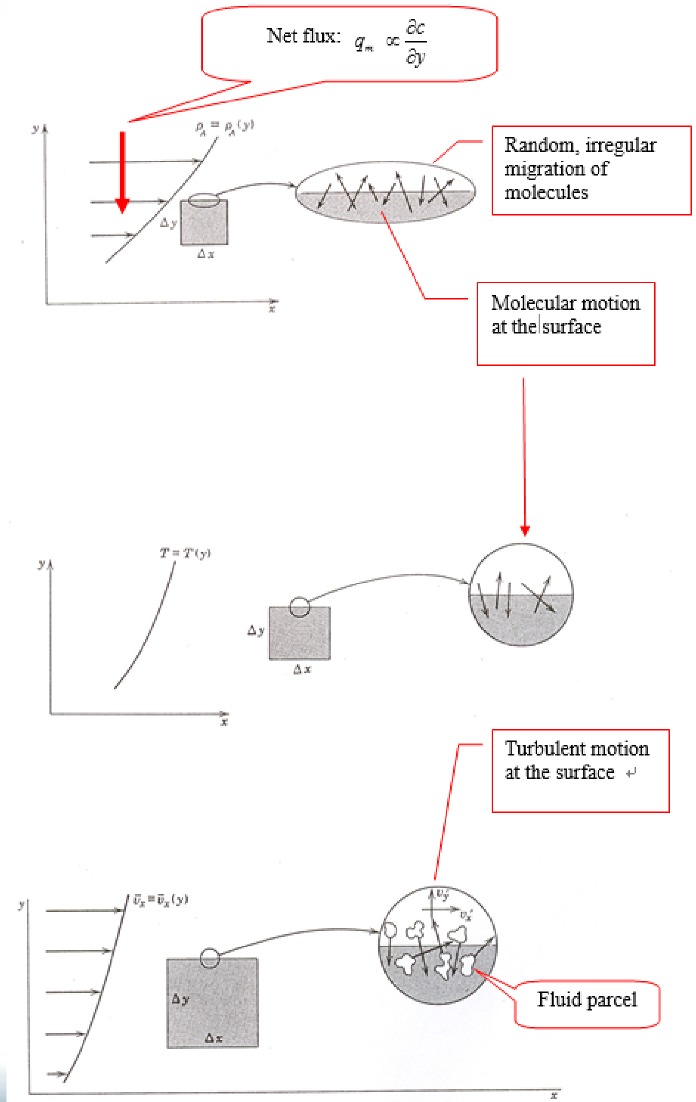


Introduction

Mass diffusion

Heat diffusion

Momentum diffusion



3.2 Transport Analogies

- **Diffusion:** driving force = gradient

$$flux = \left\{\frac{dM/dt}{area}\right\} \propto \left\{\frac{d\left(M/vol\right)}{ds}\right\}$$

mass, heat, momentum

Flux = Time rate of transport of M per unit area normal to transport direction

Gradient of *M* per unit volume of fluid in the transport direction

$$\frac{dM/dt}{A} = K\frac{d\left(M/vol\right)}{ds} \qquad (3.3)$$

Transport Analogies

where K = diffusivity constant $\left(m^2 / S\right)\left[L^2 / t\right]$

$K = f$ (modes of fluid motion, i.e., laminar and turbulent flow)

- molecular diffusivity for laminar flow
- turbulent diffusivity for turbulent flow

Transport Analogies

1) Momentum transport

Set $M = \text{momentum} = \Delta mu$

$$\therefore \quad \frac{d(\Delta mu)}{dt}\frac{1}{\Delta x \Delta z} = K\frac{d}{dy}\left(\frac{\Delta mu}{\Delta vol}\right)$$

Now, apply Newton's 2nd law to LHS

$$\frac{d}{dt}(mu) = m\frac{du}{dt} = ma = F$$

$$\therefore \quad \frac{d(\Delta mu)}{dt} = \Delta F_x$$

$$\therefore \quad LHS = \frac{\dfrac{d(\Delta mu)}{dt}}{\Delta x \Delta z} = \frac{\Delta F_x}{\Delta x \Delta z} = \tau_{yx} \qquad \text{(i)}$$

Transport Analogies

$\tau_{yx} =$ shear stress parallel to the x-direction acting on a plane whose normal is parallel to *y*-direction

RHS:

$$\frac{\Delta m}{\Delta vol} = \rho$$

$$\therefore\ RHS = K\frac{d}{dy}\left(\frac{\Delta mu}{\Delta vol}\right) = K\frac{d(\rho u)}{dy} \qquad \text{(ii)}$$

Combine (i) and (ii)

$$\tau_{yx} = K\frac{d(\rho u)}{dy} \qquad (3.4)$$

Transport Analogies

If ρ = constant

$$\tau_{yx} = \rho K \frac{du}{dy} \qquad (3.5)$$

K = molecular diffusivity constant $\left(\mathrm{m}^2/\mathrm{s}\right)$

If $K \equiv v = \dfrac{\mu}{\rho} = \text{kinematic viscosity}$

Then,

$$\boxed{\tau_{yx} = \rho v \frac{du}{dy} = \mu \frac{du}{dy}} \qquad (3.6)$$

Transport Analogies

2) Heat transport

upper plate ~ high temperature

lower plate ~ low temperature

conduction of heat within fluid

Set

$$M = \text{heat} = Q = \Delta m C_p T \qquad (3.7)$$

where C_p = specific heat at constant pressure

Then, Eq. (3.3) becomes

$$\frac{dQ}{dt}\frac{1}{\Delta x \Delta z} = q_{H_y} = -K\frac{d}{dy}\left[\frac{\Delta m C_p T}{\Delta vol}\right] \qquad (3.8)$$

Transport Analogies

$q_H =$ time rate of heat transfer per unit area normal to the direction of transport $(j/\sec - \mathrm{m}^2)$

$K = \alpha =$ thermal diffusivity $(\mathrm{m}^2/\sec)$

If $\rho (= \frac{\Delta m}{\Delta vol})$ and C_p = const.

$$\therefore \quad q_{Hy} = -\rho C_p K \frac{dT}{dy} = -k \frac{dT}{dy} \tag{3.9}$$

where $k = \rho C_p K =$ <u>thermal conductivity</u> $(j/\sec - \mathrm{m} - K)$

Transport Analogies

3) Mass transport

Set $M = \text{dissolved mass of substancs} = \Delta m_f C_M$ (3.10)

where $\Delta m_f =$ mass of fluid

$C_M =$ concentration

$\equiv$ mass of dissolved substance /unit mass of fluid

[Cf] $C_s = \dfrac{\Delta m_s}{\Delta vol_f} \quad (\text{mg}/l, ppm)$

Transport Analogies

Then, Eq. (3.3) becomes

$$\frac{d\left(\Delta m_f C_M\right)}{dt}\frac{1}{\Delta x \Delta z} = j_{M_y} = -K\frac{d}{dy}\left[\frac{\Delta m_f C_M}{\Delta vol}\right] \tag{3.11}$$

$j_M =$ time rate or mass transfer per unit area normal to the direction of transport $\mathrm{kg/m^2 \cdot s}$

If $\rho = \dfrac{\Delta m}{\Delta vol} = \mathrm{const.} = \dfrac{\Delta m_f}{\Delta vol_f}$

$$j_{M_y} = -\rho K \frac{dC_M}{dy} \tag{3.12}$$

Transport Analogies

$$= -K\frac{dC_M \cdot \rho}{dy} = -K\frac{d\left(\dfrac{\Delta m_s}{\Delta m_f}\cdot\dfrac{\Delta m_f}{\Delta vol_f}\right)}{dy} = -K\frac{d\left(\dfrac{\Delta m_s}{\Delta vol_f}\right)}{dy} = -K\frac{dC_s}{dy}$$

Set $K = D =$ molecular diffusion coefficient $(\mathrm{m}^2/\mathrm{sec})$

$$j_{M_y} = -\rho K\frac{dC_M}{dy} = -D\frac{dC_s}{dy}$$

Transport Analogies

Flux	Driving force	Law	Relation
Mass flux q_m	Concentration gradient $\frac{\partial c}{\partial x_j}$	Fick's law	$q_m = -D\frac{\partial c}{\partial x_j} = -D\nabla c$
Heat flux q_H	Temperature gradient $\frac{\partial T}{\partial x_j}$	Fourier's law	$q_H = -k\frac{\partial T}{\partial x_j} = -k\nabla T$
Momentum Flux, q_{mo}	Velocity gradient $\frac{\partial u_i}{\partial x_j}$	Newton's law	$\tau = \mu\frac{\partial u_i}{\partial x_j}$

Transport Analogies

Mass flux

q_m

Heat flux

q_H

$$flux \propto \frac{d\left(M/vol\right)}{ds}$$

$$\tau = \mu \frac{\partial u_i}{\partial x_j}$$

mass/heat transport:

c, T – scalar

q_m, q_H - vector

momentum transport:

u_i – vector

$q_{mo}\,(=\tau)$ - tensor

Transport Analogies

Transport process	Kinematic fluid property (m^2 / s)
Momentum	ν (kinematic viscosity)
Heat	α (thermal diffusivity)
Mass	D (diffusion coefficient)

3.3 Mass Transport

All fluid motions must satisfy the principle of conservation of matter.

- homogeneous fluid — single phase
- non-homogeneous fluid
  - multi-phase: air-liquid, liquid-solid
  - multi-species: fresh water - salt water

Continuity equation: relation for temporal and spatial variation of velocity and density

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{q}) = 0$$

Mass Transport

Homogeneous fluid	Non-homogenous fluid
single phase	multi phase
single species	single phase & multi species
Continuity Equation [Ch. 4]	mass transport due to local velocity + mass transport due to diffusion → Advection-Diffusion Equation [Advanced Environmental Hydraulics I] [Ch. 16]

Mass Transport

Advection-Diffusion Equation

= mass conservation + Fick's law

$$u = \overline{u} + u'$$

mean motion

fluctuation

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x}\left(uc - D\frac{\partial c}{\partial x} \right) = 0$$

advection

diffusion

$$q_d = \overline{u'c'} = D\frac{\partial c}{\partial x}$$

3.4 Heat Transport

thermodynamics ~ non-flow processes

equilibrium states of matter

fluid dynamics ~ transport of heat (scalar) by fluid motion

- Apply conservation of energy to flow process (= 1^{st} law of thermodynamics)

~ relation between pressure, density, temperature, velocity, elevation, mechanical work, and heat input (or output).

~ since heat capacity of fluid is large compared to its kinetic energy, temperature and density remain constant even though large amounts of kinetic energy are dissipated by friction.

→ simplified energy equation

Heat Transport

- Heat transfer in flow process

  1) convection: due to velocity of the flow → advection

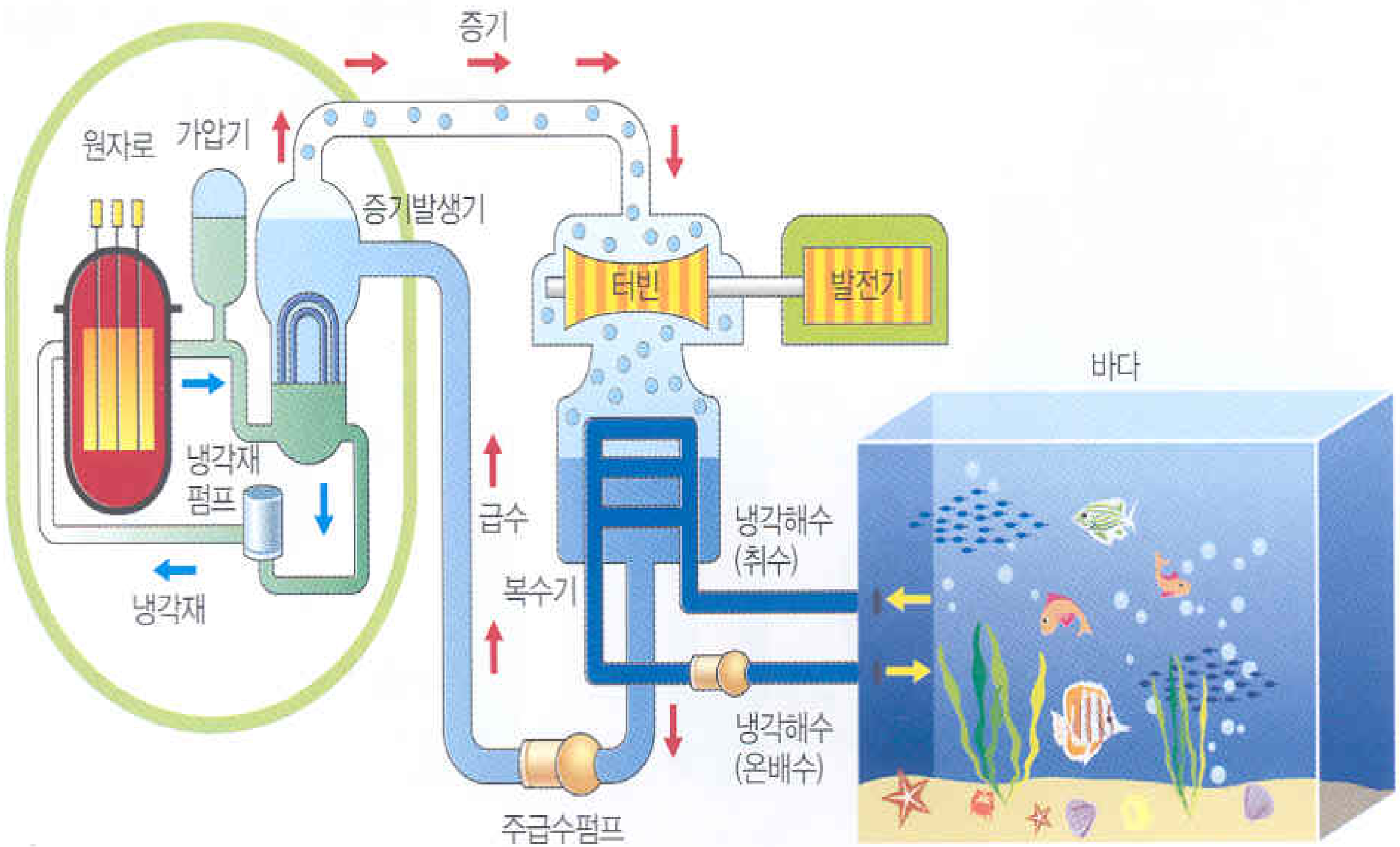
  2) conduction: analogous to diffusion, tendency for heat to move in the direction of decreasing temperature

- Application

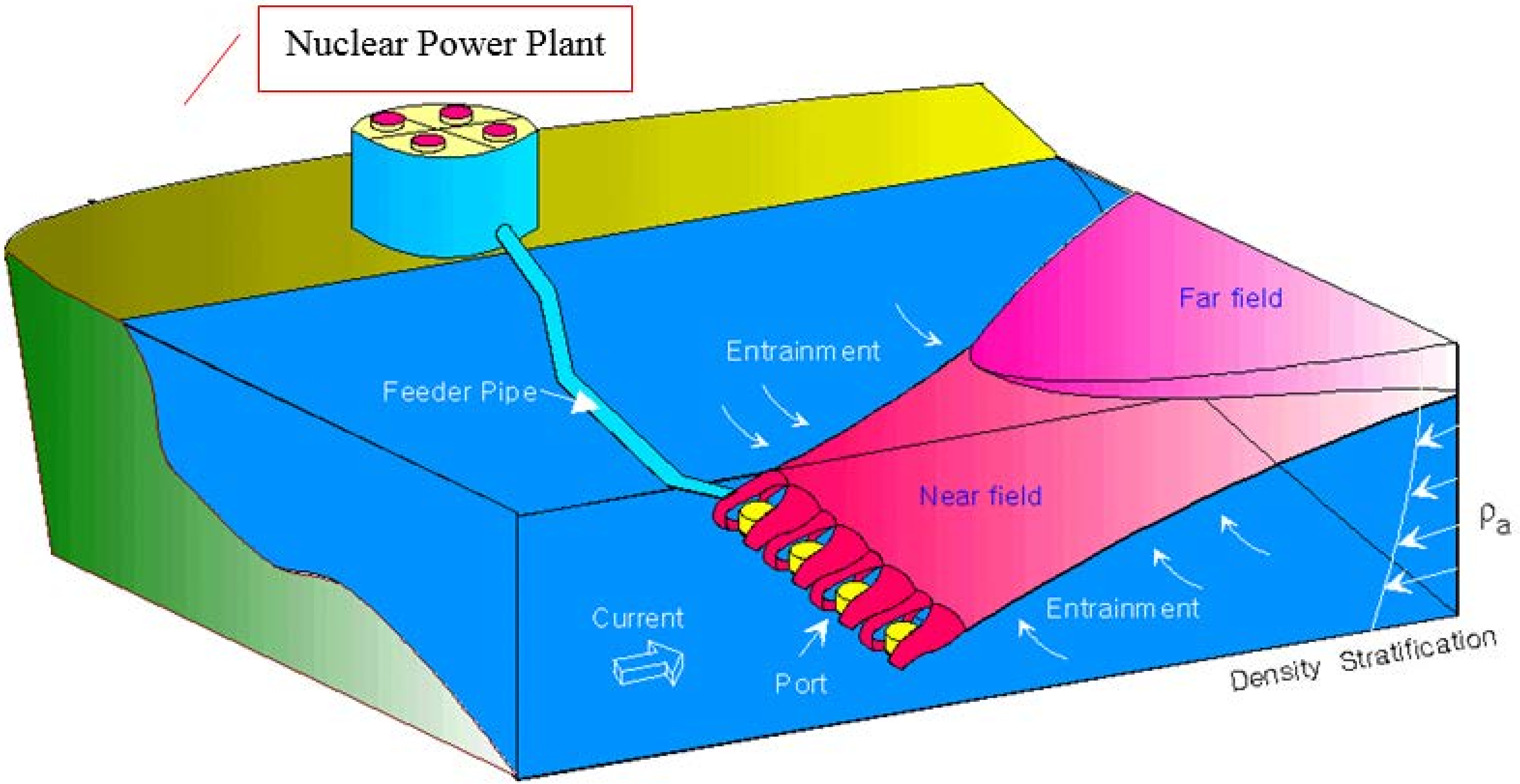
  1) Fluid machine (compressors, pumps, turbines): energy transfer in flow processes

  2) Heat pollution: discharge of heated water for nuclear power plant
     discharge of cooled water for LNG plant

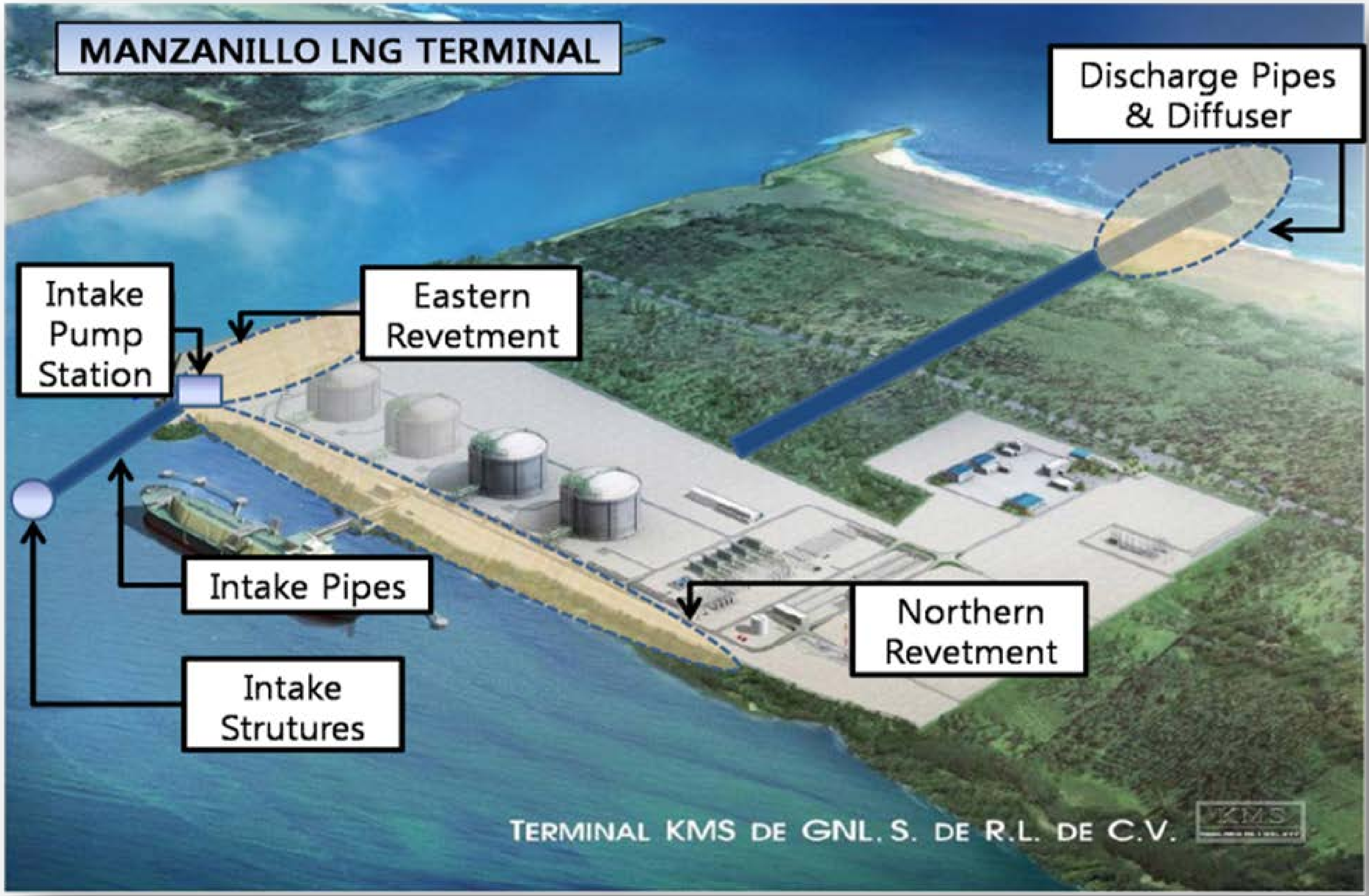
Heat Transport



Heat Transport



Heat Transport



3.5 Momentum Transport

Momentum transport phenomena

~ encompass the mechanisms of fluid resistance, boundary and internal shear stresses, and propulsion and forces on immersed bodies.

Momentum $= \text{mass} \cdot \text{velocity vector} = m\vec{u}$

Adopt Newton's 2nd law

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{u}}{dt} = \frac{d}{dt}(m\vec{u}) \qquad (3.2)$$

→ Equation of motion

Momentum Transport

- Effect of velocity gradient $\frac{\partial u}{\partial y}$

- macroscopic fluid velocity tends to become uniform due to the random motion of molecules because of intermolecular collisions and the consequent exchange of molecular momentum

→ the velocity distribution tends toward the dashed line

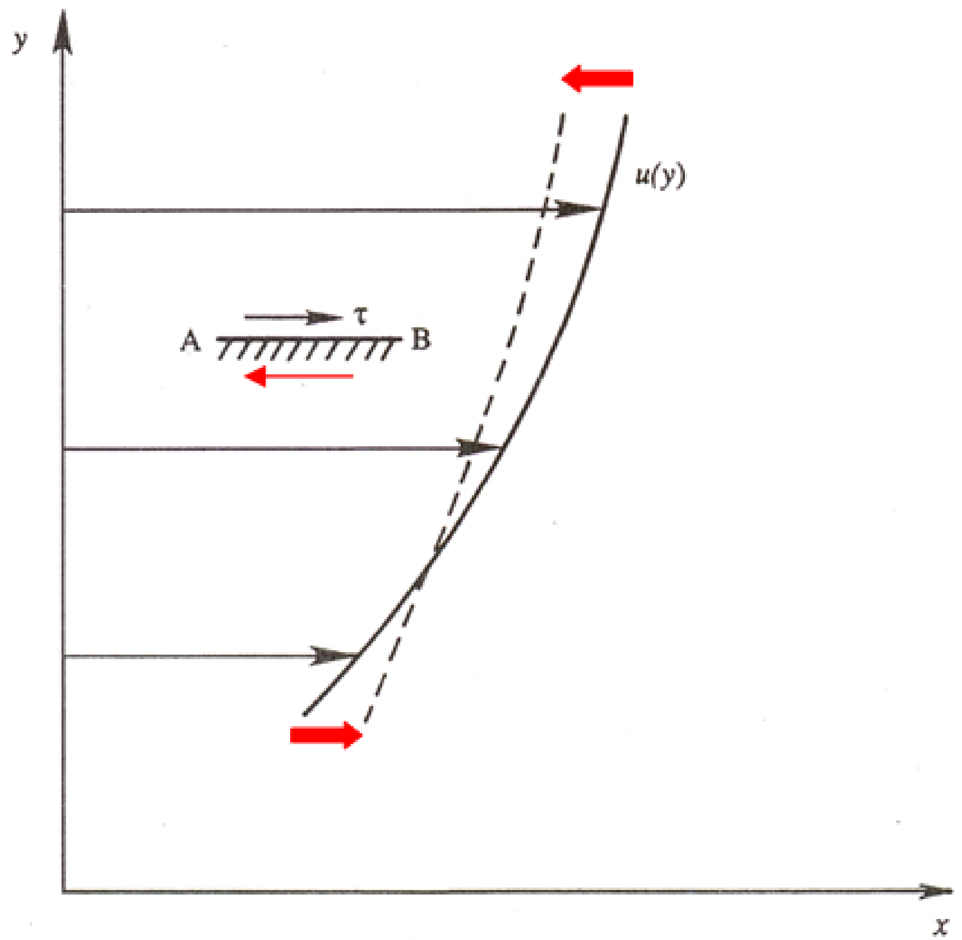
→ momentum flux is equivalent to the existence of the shear stress

$$\tau \propto \frac{\partial u}{\partial y}$$

$$\tau = \mu \frac{\partial u}{\partial y}$$

→ Newton's law of friction

Momentum Transport



Momentum Transport

Momentum transport for Couette flow

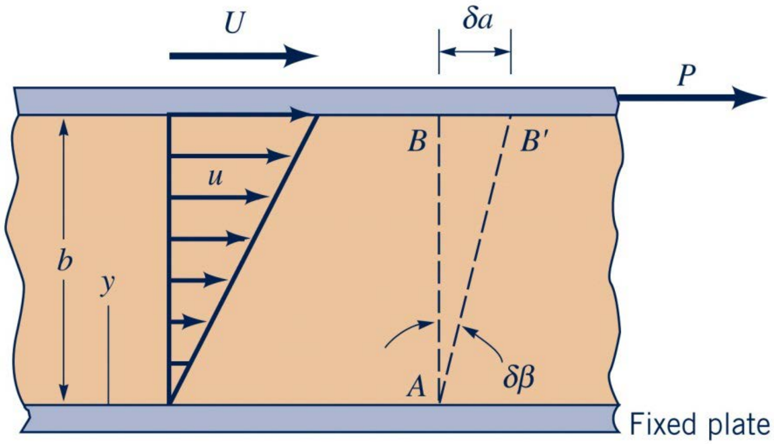
Couette flow – laminar flow between two plates

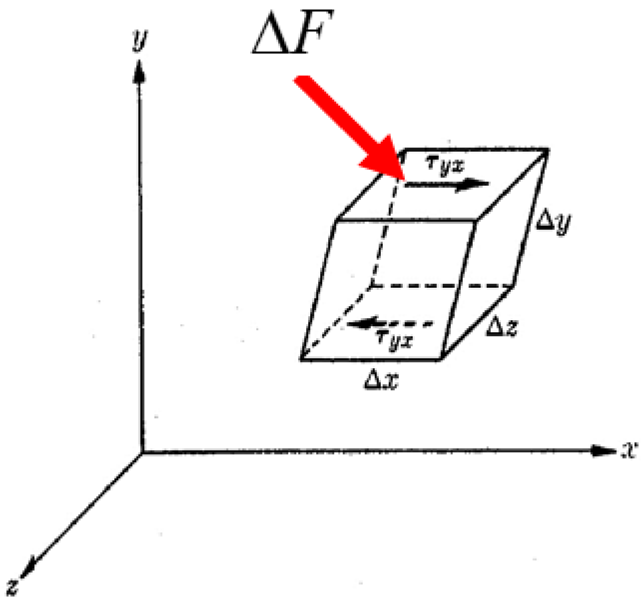
transverse transport of longitudinal momentum (mv)

$\propto$ transverse gradient of longitudinal velocity $\left(\dfrac{dv}{dy}\right)$

in the direction of decreasing velocity (longitudinal momentum)

Momentum Transport



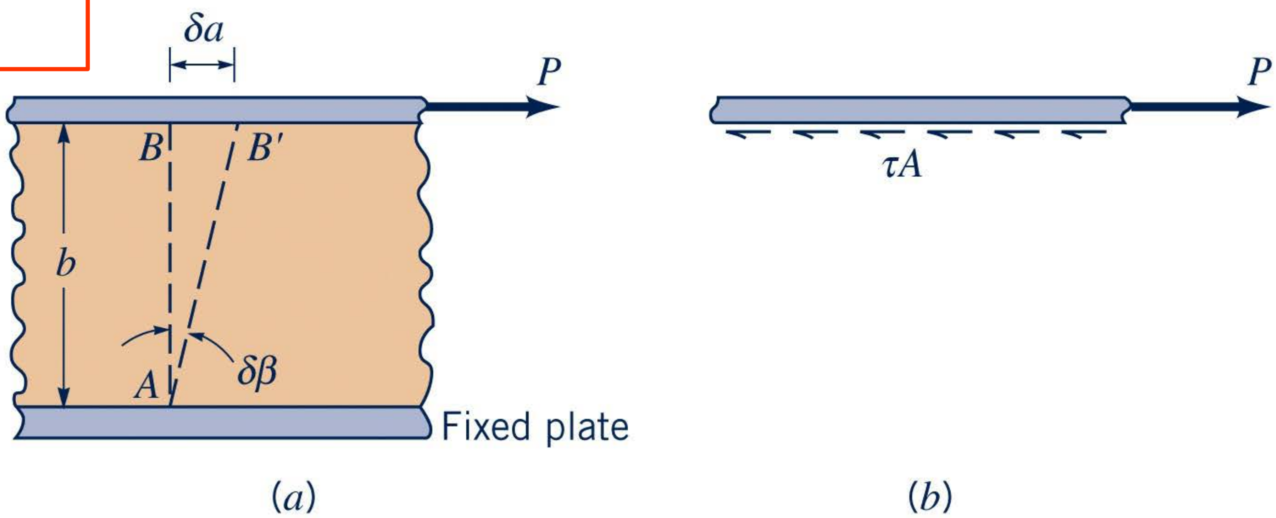


Momentum Transport

velocity gradient of Couette flow - linear

$$\frac{dv}{dy} = \frac{U}{b}$$

$$\tau = \mu \frac{U}{b}$$



Homework Assignment No. 3

1. Derive an one-dimensional advection-diffusion equation given below by combining the conservation of mass and Fick's law for molecular diffusion.

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x}\left(uc - D\frac{\partial c}{\partial x} \right) = 0$$