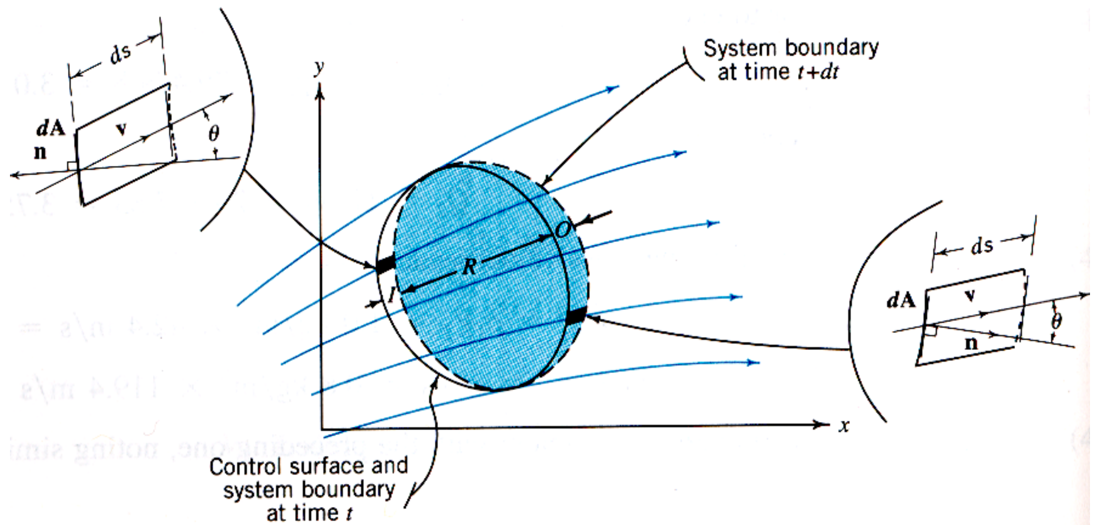
Chapter 4

Continuity, Energy, and Momentum Equations



Chapter 4 Continuity, Energy, and Momentum Equations

Contents

Objectives

- Apply finite control volume to get integral form of continuity, energy, and momentum equations
- Compare integral and point form equations
- Derive the simplified equations for continuity, energy, and momentum equations

Particle and Control-Volume Concepts

Infinitesimal elements and control volumes

- Each of the observational laws of mass, heat, and momentum transport may be formulated in the Eulerian sense of focusing attention on a fixed point in space.
- There are two basic method of arriving the Eulerian equation.
  - Material method (Particle approach)
  - Control volume method:
    - ✓ Finite control volume
    - ✓ Differential control volume
  - If fluid is considered as <u>a continuum</u>, end result of either method is <u>identical</u>.

Particle and Control-Volume Concepts

- Material method (Particle approach)
  - Describe flow characteristics at a fixed point (x, y, z) by observing the motion of a material particle of a infinitesimal mass
  - Laws of conservation of mass, momentum, and energy can be stated in the differential form, applicable at a point.
  - Newton's 2nd law

$$d\vec{F} = dm\vec{a}$$

  - Material-particle approach is used to develop a stress-strain relationship in Ch. 5.

Particle and Control-Volume Concepts

- **Control volume method**

① Finite control volume – arbitrary control volume

② Differential (infinitesimal) control volume – parallelepiped control volume

[Re] Control volume

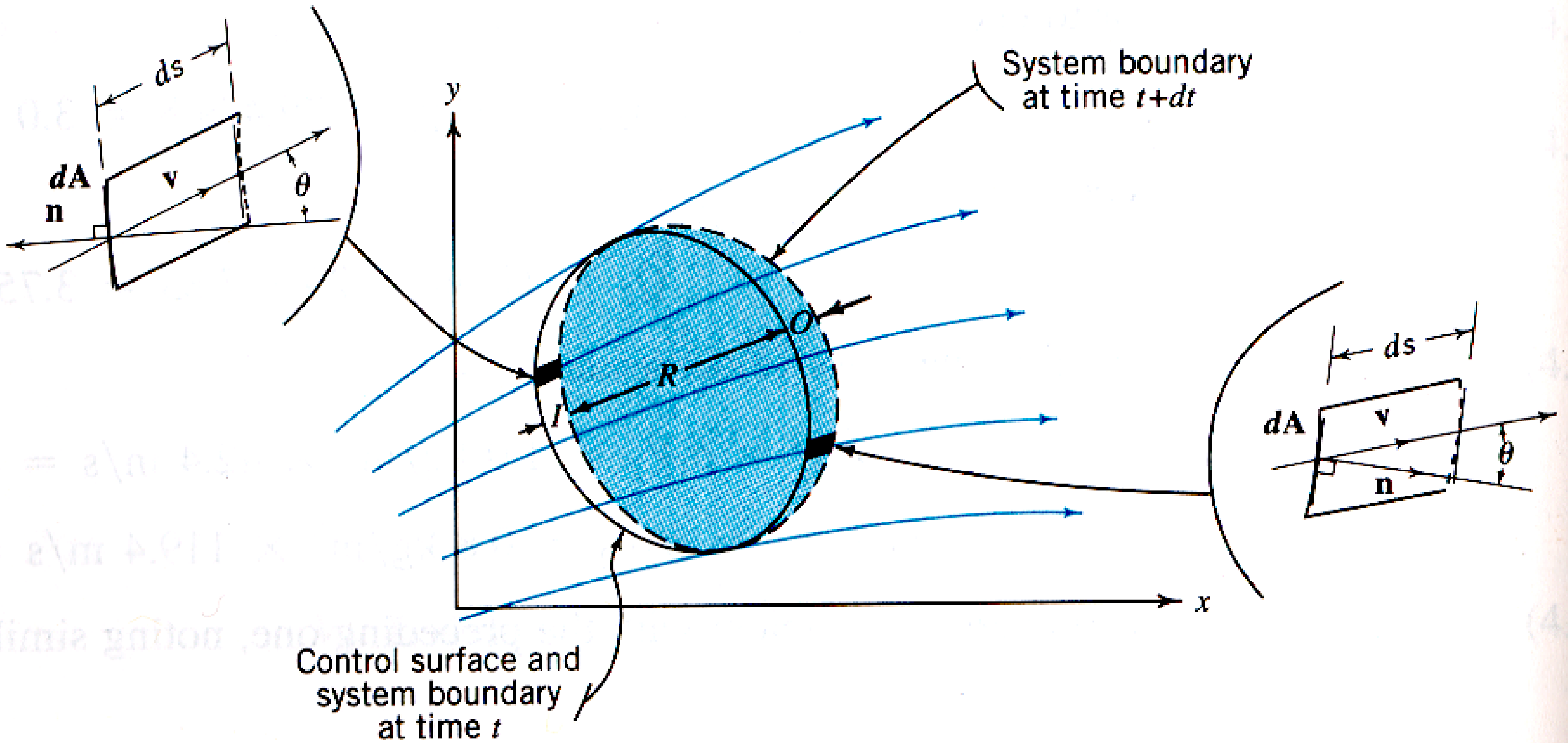
- fixed volume which consists of the same fluid particles and whose bounding surface moves with the fluid

Particle and Control-Volume Concepts

- Finite control volume method
  - Frequently used for 1D analysis (Ch. 4)
  - Gross descriptions of flow
  - Analytical formulation is easier than differential control volume method
  - Integral form of equations for conservation of mass, momentum, and energy
  - Continuity equation: conservation of mass

$$\int_{CV} \frac{\partial \rho}{\partial t} dV + \oint_{CS} \rho \vec{q} \cdot d\vec{A} = 0$$

Particle and Control-Volume Concepts



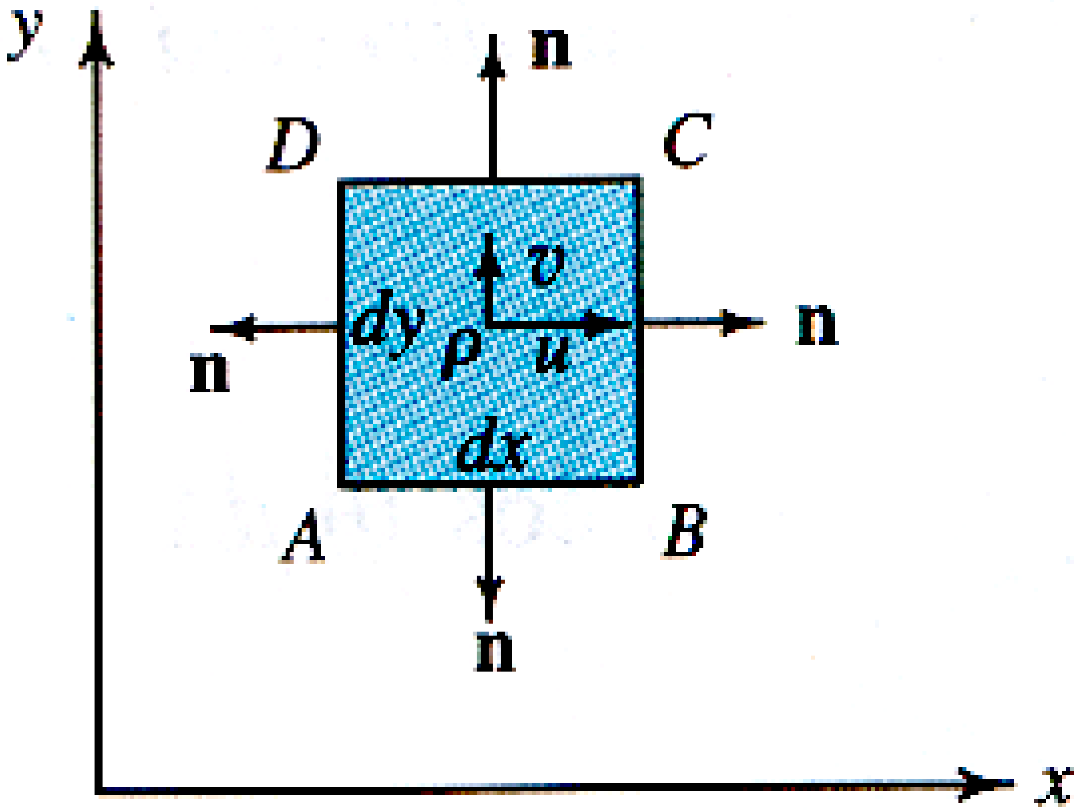
Particle and Control-Volume Concepts

- Differential control volume method
  - Concerned with a fixed differential control volume (=ΔxΔyΔz) of fluid
  - 2D or 3D analysis (Ch. 6)

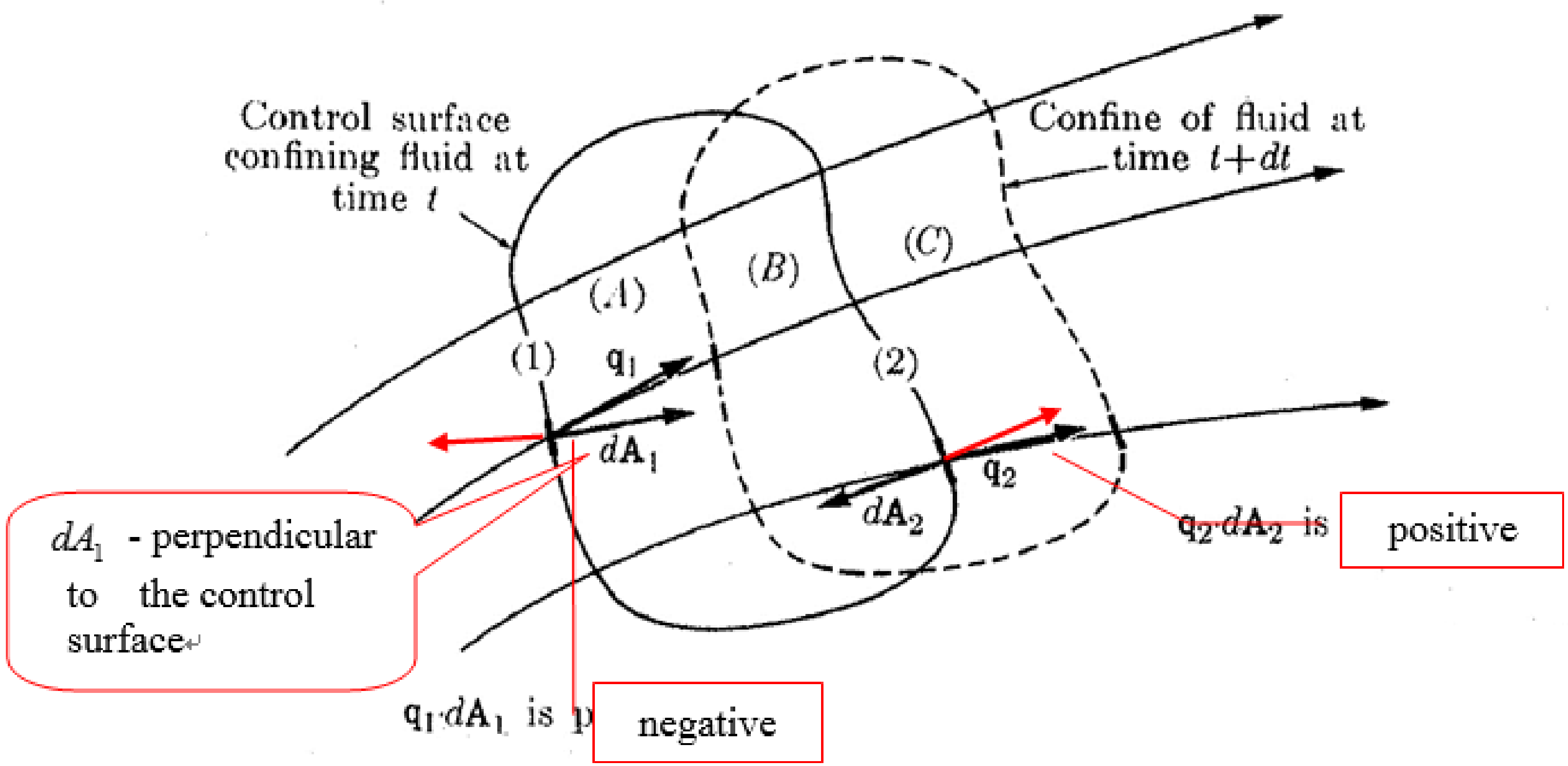
$$\Delta \vec{F} = \frac{d}{dt}\left(\Delta m \vec{q}\right) = \frac{d}{dt}\left(\rho \Delta x \Delta y \Delta z \vec{q}\right)$$

  - Δx, Δy, Δz become vanishingly small
  - Point form of equations for conservation of mass, momentum, and energy

Particle and Control-Volume Concepts



4.1 Conservation of Matter in Homogeneous Fluids



4.1 Conservation of Matter in Homogeneous Fluids

4.1.1 Finite control volume method-arbitrary control volume

- Consider an arbitrary control volume
- Although control volume remains fixed, mass of fluid originally enclosed (regions A+B) occupies the volume within the dashed line (regions B+C).
- Since mass m is conserved:

$$\left(m_A\right)_t + \left(m_B\right)_t = \left(m_B\right)_{t+dt} + \left(m_C\right)_{t+dt} \qquad (4.1)$$

$$\frac{\left(m_B\right)_{t+dt} - \left(m_B\right)_t}{dt} = \frac{\left(m_A\right)_t - \left(m_C\right)_{t+dt}}{dt} \qquad (4.2)$$

4.1 Conservation of Matter in Homogeneous Fluids

- LHS of Eq. (4.2) = time rate of change of mass in the original control volume in the limit

$$\frac{(m_B)_{t+dt} - (m_B)_t}{dt} \approx \frac{\partial (m_B)}{\partial t} = \frac{\partial}{\partial t}\int_{CV}(\rho\, dV) \qquad (4.3)$$

where dV = volume element

- RHS of Eq. (4.2)

= net flux of matter through the control surface

= flux in – flux out

$$= \int \rho q_n \, dA_1 - \int \rho q_n \, dA_2$$

4.1 Conservation of Matter in Homogeneous Fluids

where q_n = component of velocity vector normal to the surface

of $CV = |\vec{q}|\cos\phi$

$$\therefore \quad \frac{\partial}{\partial t}\int_{CV}(\rho\, dV) = \int_{CS}\rho q_n dA_1 - \int_{CS}\rho q_n dA_2 \qquad (4.4)$$

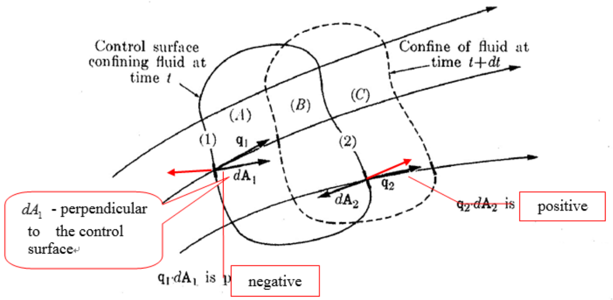
※ Flux (= mass/time) is due to velocity of the flow.

• Vector form is

$$\frac{\partial}{\partial t}\int_{CV}(\rho\, dV) = -\oint_{CS}\rho\vec{q}\cdot d\vec{A} \qquad (4.5)$$

4.1 Conservation of Matter in Homogeneous Fluids

where $d\vec{A}$ = vector differential area pointing in the outward direction over an enclosed control surface



$$\therefore \quad \vec{q} \cdot d\vec{A} = |\vec{q}|\ |d\vec{A}|\ \cos\phi$$

$$= \begin{cases} \text{positive for an outflow from cv,} \quad \phi \le 90^\circ \\ \text{negative for inflow into cv,} \quad 90^\circ \le \phi \le 180^\circ \end{cases}$$

If fluid continues to occupy the entire control volume at subsequent times

→ time independent

LHS: $$\frac{\partial}{\partial t}\int_{CV}(\rho\, dV) \Rightarrow \int_{CV}\frac{\partial \rho}{\partial t} dV \qquad (4.5a)$$

4.1 Conservation of Matter in Homogeneous Fluids

Eq. (4.4) becomes

$$\boxed{\int_{CV} \frac{\partial \rho}{\partial t} dV + \oint_{CS} \rho \vec{q} \cdot d\vec{A} = 0} \qquad (4.6)$$

→ General form of continuity equation→ Integral form

[Re] Differential form

Use Gauss divergence theorem

$$\int_V \frac{\partial F}{\partial x_i} dV = \int_A F dA_i$$

4.1 Conservation of Matter in Homogeneous Fluids

Transform surface integral of Eq. (4.6) into volume integral

$$\oint_{CS} \rho \vec{q} \cdot d\vec{A} = \int_{CV} \nabla \cdot (\rho \vec{q}\,) dV$$

Then, Eq. (4.6) becomes

$$\int_{CV} \left[\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{q}\,) \right] dV \;=\; 0 \qquad (4.6a)$$

Eq. (4.6a) holds for any volume only if the integrand vanishes at every point.

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{q}\,) = 0} \qquad (4.6b)$$

→ Differential (point) form

4.1 Conservation of Matter in Homogeneous Fluids

Simplified form of continuity equation

- Steady flow of a compressible fluid

$$\int_{CV} \frac{\partial \rho}{\partial t} dv = 0$$

Therefore, Eq. (4.6) becomes

$$\boxed{\oint_{CS} \rho \vec{q} \cdot d\vec{A} = 0} \qquad (4.7)$$

- Incompressible fluid (for both steady and unsteady conditions)

$$\rho = \text{const.} \;\rightarrow\; \frac{\partial \rho}{\partial t} = 0, \; \frac{d\rho}{dt} = 0$$

4.1 Conservation of Matter in Homogeneous Fluids

Therefore, Eq. (4.6) becomes

$$\boxed{\oint_{CS} \vec{q} \cdot d\vec{A} = 0} \qquad (4.8)$$

4.1 Conservation of Matter in Homogeneous Fluids

[Cf] Non-homogeneous fluid mixture

- Conservation of mass equations for the individual species

→ Advection-diffusion equation

= conservation of mass equation + <u>mass flux equation</u> due to advection and diffusion

$$\frac{\partial c}{\partial t} + \frac{\partial q}{\partial x} = 0 \qquad\qquad q = uc - D\frac{\partial c}{\partial x}$$

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x}\left(uc - D\frac{\partial c}{\partial x}\right) = 0 \quad \rightarrow \quad \frac{\partial c}{\partial t} + \frac{\partial uc}{\partial x} = \frac{\partial}{\partial x}\left(D\frac{\partial c}{\partial x}\right)$$

4.1 Conservation of Matter in Homogeneous Fluids

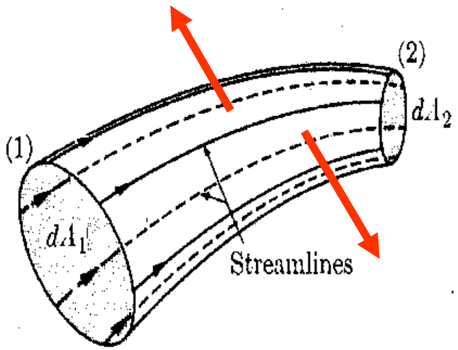
4.1.2 Stream - tube control volume analysis for steady flow

- Steady flow: There is no flow across the longitudinal boundary of the stream tube.
- Eq. (4.7) becomes

$$\oint \rho \vec{q} \cdot d\vec{A} = -\rho_1 q_1 dA_1 + \rho_2 q_2 dA_2 = 0$$

$$\rho q dA = \text{const.} \qquad (4.9)$$

Flux = 0



4.1 Conservation of Matter in Homogeneous Fluids

If density = const.

$$q_1 dA_1 = q_2 dA_2 = dQ \tag{4.10}$$

where dQ = volume rate of flow

- For flow in conduit with variable density

$$V = \frac{\int q dA}{A} \rightarrow \text{average velocity}$$

$$\bar{\rho} = \frac{\int \rho dQ}{Q} \rightarrow \text{average density}$$

$$\bar{\rho}_1 V_1 A_1 = \bar{\rho}_2 V_2 A_2 \tag{4.11}$$

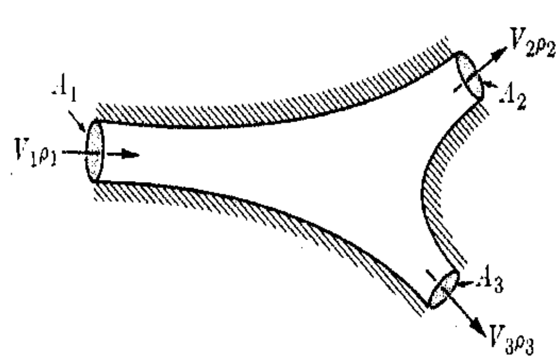
4.1 Conservation of Matter in Homogeneous Fluids

- For a branching conduit

$$\oint \rho \vec{q} \cdot d\vec{A} = 0$$

$$-\int_{A_1} \rho_1 q_1 dA_1 + \int_{A_2} \rho_2 q_2 dA_2 + \int_{A_3} \rho_3 q_3 dA_3 = 0$$

$$\bar{\rho}_1 V_1 A_1 = \bar{\rho}_2 V_2 A_2 + \bar{\rho}_3 V_3 A_3 \qquad (4.12)$$



4.1 Conservation of Matter in Homogeneous Fluids

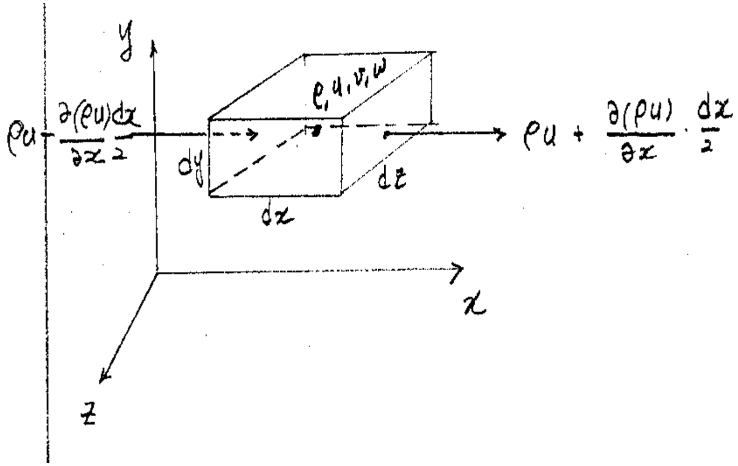
◆ Equation of Continuity

Use Infinitesimal (differential) control volume method

- At the centroid of the control volume,

  $\rho,\ u,\ v,\ w$

- rate of mass flux across the surface perpendicular to x is



4.1 Conservation of Matter in Homogeneous Fluids

$$\text{flux in} = \left\{ \rho u - \frac{\partial(\rho u)}{\partial x}\frac{dx}{2} \right\} dydz$$

$$\text{flux out} = \left\{ \rho u + \frac{\partial(\rho u)}{\partial x}\frac{dx}{2} \right\} dydz$$

$$\text{net flux} = \text{flux in} - \text{flux out} = -\frac{\partial(\rho u)}{\partial x} dxdydz$$

net mass flux across the surface perpendicular to $y = -\frac{\partial(\rho v)}{\partial y} dydxdz$

net mass flux across the surface perpendicular to $z = -\frac{\partial(\rho w)}{\partial z} dzdxdy$

Time rate of change of mass inside the c.v. $= \frac{\partial(\rho dxdydz)}{\partial t}$

4.1 Conservation of Matter in Homogeneous Fluids

Time rate of change of mass inside = sum of three net rates

$$\frac{\partial\left(\rho dxdydz\right)}{\partial t} = -\left[\frac{\partial\left(\rho u\right)}{\partial x} + \frac{\partial\left(\rho v\right)}{\partial y} + \frac{\partial\left(\rho w\right)}{\partial z}\right] dxdydz$$

By taking limit *dV = dxdydz*

$$-\frac{\partial \rho}{\partial t} = \frac{\partial\left(\rho u\right)}{\partial x} + \frac{\partial\left(\rho v\right)}{\partial y} + \frac{\partial\left(\rho w\right)}{\partial z} = \nabla \cdot \rho\vec{q} = div\left(\rho\vec{q}\right)$$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot \rho\vec{q} = 0} \qquad \text{(A1)}$$

→ point (differential) form of Continuity Equation (the same as Eq. 4.6b)

4.1 Conservation of Matter in Homogeneous Fluids

[Re] $\dfrac{\partial(\rho u)}{\partial x} + \dfrac{\partial(\rho v)}{\partial y} + \dfrac{\partial(\rho w)}{\partial z} = \nabla \cdot \rho\vec{q} = div\,(\rho\vec{q})$

By the way,

$$\nabla \cdot \rho\vec{q} = \vec{q} \cdot \nabla\rho + \rho\nabla \cdot \vec{q}$$

Thus, (A1) becomes

$$\frac{\partial\rho}{\partial t} + \vec{q} \cdot \nabla\rho + \rho\nabla \cdot \vec{q} = 0 \qquad \text{(A2)}$$

4.1 Conservation of Matter in Homogeneous Fluids

1) For incompressible fluid

$$\frac{d\rho}{dt} = 0 \ (\rho = \text{const.})$$

$$\rightarrow \quad \frac{\partial \rho}{\partial t} + \vec{q} \cdot \nabla \rho = \frac{d\rho}{dt} = 0$$

Therefore Eq. (A2) becomes

$$\rho \nabla \cdot \vec{q} = 0 \quad \rightarrow \quad \boxed{\nabla \cdot \vec{q} = 0} \qquad \text{(A3)}$$

In scalar form,

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0} \qquad \text{(A4)}$$

→ Continuity Eq. for 3D incompressible fluid

4.1 Conservation of Matter in Homogeneous Fluids

For 2D incompressible fluid,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

2) For steady flow,

$$\frac{\partial \rho}{\partial t} = 0$$

Thus, (A1) becomes

$$\nabla \cdot \rho \vec{q} = \vec{q} \cdot \nabla \rho + \rho \nabla \cdot \vec{q} = 0 \qquad (4.13)$$

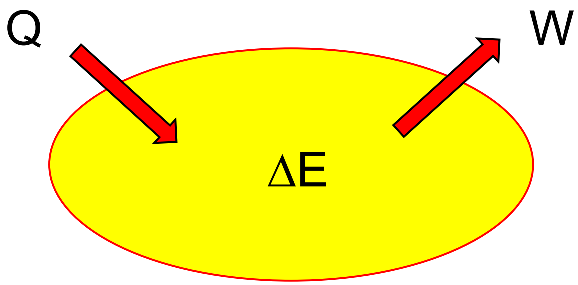
4.2 The General Energy Equation

4.2.1 The 1st law of thermodynamics

- The 1st law of thermodynamics:

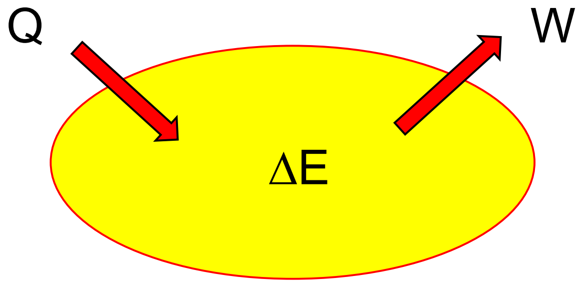
The difference between the heat added to a system of masses and the work done by the system depends only on the initial and final states of the system (→ change in energy).

→ Conservation of energy



4.2 The General Energy Equation

$$\delta Q - \delta W = dE \qquad (4.14)$$



where δQ = heat added to the system from surroundings

δW = work done by the system on its surroundings

δE = increase in energy of the system

4.2 The General Energy Equation

[Re]

- property of a system: position, velocity, pressure, temperature, mass, volume
- state of a system: condition as identified through properties of the system

Consider time rate of change

$$\frac{\delta Q}{dt} - \frac{\delta W}{dt} = \frac{dE}{dt} \tag{4.15}$$

4.2 The General Energy Equation

- **Work**

$W_{pressure}$ = work of normal stresses acting on the system boundary

W_{shear} = work of tangential stresses done at the system boundary on adjacent external fluid in motion

W_{shaft} = shaft work done on a rotating element in the system

- **Energy**

Consider e = energy per unit mass = *E/mass*

e_u = internal energy associated with fluid temperature = u

e_p = potential energy per unit mass = gh

where h = local elevation of the fluid

e_q = kinetic energy per unit mass = $\dfrac{q^2}{2}$

4.2 The General Energy Equation

$$u + \frac{p}{\rho} = \text{enthalpy}$$

$$\boxed{e = e_u + e_p + e_q = u + gh + \frac{q^2}{2}} \qquad (4.16)$$

- Internal energy

  = activity of the molecules comprising the substance

  = force existing between the molecules

  ~ depend on <u>temperature and change in phase</u>

4.2 The General Energy Equation

4.2.2 General energy equation

$$\frac{\delta Q}{dt} - \frac{\delta W}{dt} = \frac{dE}{dt} \qquad (4.15)$$

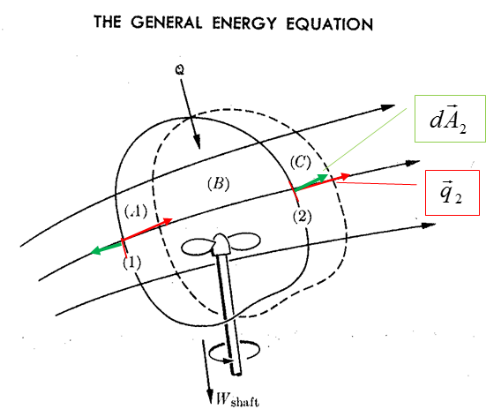
Consider work done

$$\frac{\delta W}{dt} = \frac{\delta W_{pressure}}{dt} + \frac{\delta W_{shaft}}{dt} + \frac{\delta W_{shear}}{dt} \qquad (4.15a)$$

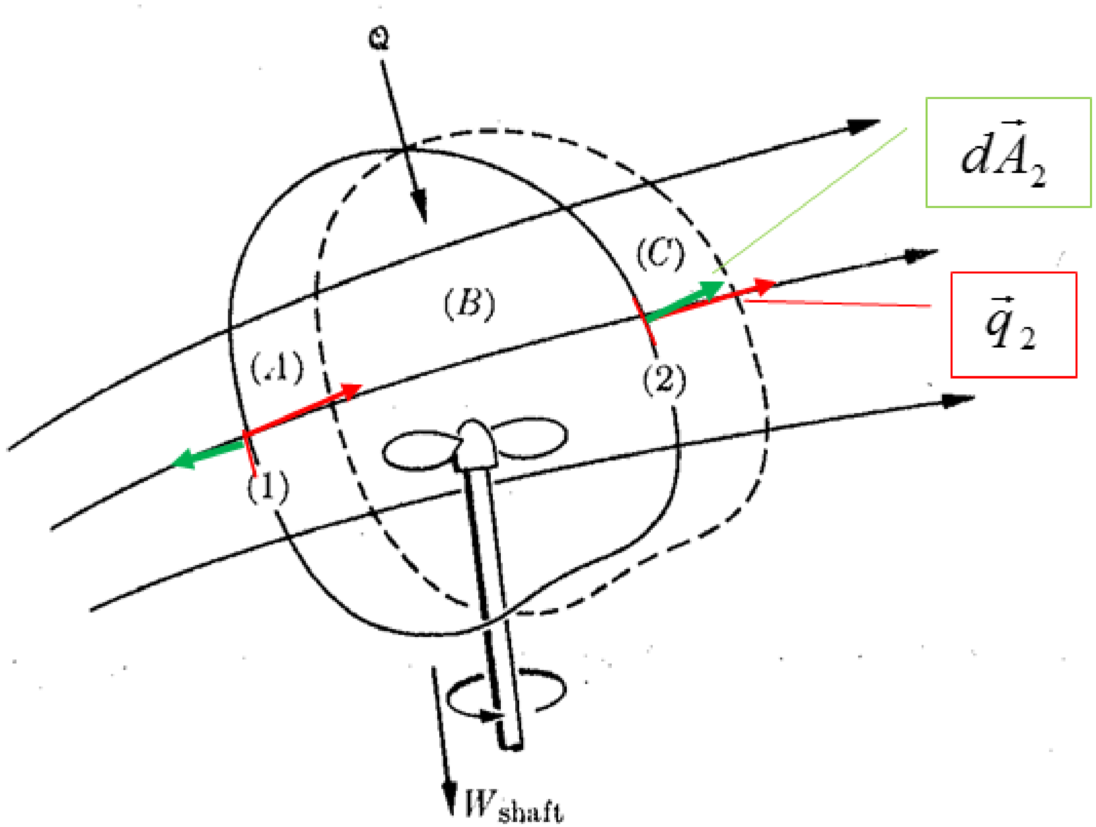
$\dfrac{\delta W_{pressure}}{dt}$ = net rate at which work of pressure is done by the fluid on the surroundings

$\boxed{= \text{work flux}_{out} - \text{work flux}_{in}}$

$$= \oint_{CS} p\left(\vec{q} \cdot d\vec{A}\right)$$



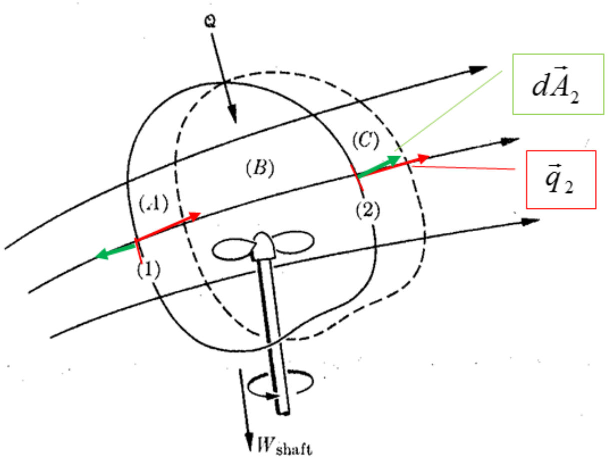
4.2 The General Energy Equation



4.2 The General Energy Equation

p = pressure acting on the surroundings = $F/A = F/L^2$

$$\vec{q}\cdot d\vec{A} = \begin{cases} \text{positive for outflow into CV} \\ \\ \text{negative for inflow} \end{cases}$$



$$\vec{q}\cdot d\vec{A} = Q = L^3/t$$

$$p\left(\vec{q}\cdot d\vec{A}\right) = \frac{F}{L^2}\frac{L^3}{t} = FL/t = E/t$$

Thus, (4.15a) becomes

$$\frac{\delta W}{dt} = \oint_{CS} p\left(\vec{q}\cdot d\vec{A}\right) + \frac{\delta W_{shaft}}{dt} + \frac{\delta W_{shear}}{dt} \qquad (4.15b)$$

4.2 The General Energy Equation

Now, consider energy change term

$\frac{dE}{dt}$ = total rate change of stored energy

= net rate of energy flux through C.V.

+ time rate of change inside C.V.

$$\boxed{= \oint_{CS} e\rho\left(\vec{q}\cdot d\vec{A}\right) + \frac{\partial}{\partial t}\int_{CV}\left(e\rho\, dV\right)} \quad (4.15c)$$

$$e = E\,/\,mass;\ \rho\left(\vec{q}\cdot d\vec{A}\right) = mass\,/\,time$$

$$e\rho\left(\vec{q}\cdot d\vec{A}\right) = E\,/\,t$$

4.2 The General Energy Equation

Substituting (4.15b) and (4.15c) into Eq. (4.15) yields

$$\frac{\delta Q}{dt} - \frac{\delta W_{shaft}}{dt} - \frac{\delta W_{shear}}{dt} - \oint_{CS} p\left(\vec{q}\cdot d\vec{A}\right)$$

$$= \oint_{CS} e\rho\left(\vec{q}\cdot d\vec{A}\right) + \frac{\partial}{\partial t}\int_{CV}\left(e\rho\, dV\right)$$

$$\frac{\delta Q}{dt} - \frac{\delta W_{shaft}}{dt} - \frac{\delta W_{shear}}{dt} \tag{4.16}$$

$$= \oint_{CS}\left(\frac{p}{\rho} + e\right)\rho\left(\vec{q}\cdot d\vec{A}\right) + \frac{\partial}{\partial t}\int_{CV}\left(e\rho\, dV\right) \tag{4.17}$$

4.2 The General Energy Equation

Assume potential energy $e_p = gh$ (due to gravitational field of the earth)

Then $e = u + gh + \dfrac{q^2}{2}$

Then, Eq. (4.17) becomes

$$\frac{\delta Q}{dt} - \frac{\delta W_{shaft}}{dt} - \frac{\delta W_{shear}}{dt} = \oint_{CS}\left(\frac{p}{\rho} + u + gh + \frac{q^2}{2}\right)\rho\left(\vec{q}\cdot d\vec{A}\right) + \frac{\partial}{\partial t}\int_{CV} e\rho\, dV \qquad (4.17)$$

4.2 The General Energy Equation

- Application: generalized apparatus

At boundaries normal to flow lines → no shear

$$\rightarrow W_{shear} = 0 \tag{4.18}$$

$$\frac{\delta Q}{dt} - \frac{\delta W_{shaft}}{dt} = \oint_{CS} \left(\frac{p}{\rho} + u + gh + \frac{q^2}{2} \right) \rho \left(\vec{q} \cdot d\vec{A} \right) + \frac{\partial}{\partial t} \int_{CV} e\rho \, dV \tag{4.19}$$

For steady motion,

$$\boxed{\frac{\delta Q}{dt} - \frac{\delta W_{shaft}}{dt} = \oint_{CS} \left(\frac{p}{\rho} + u + gh + \frac{q^2}{2} \right) \rho \left(\vec{q} \cdot d\vec{A} \right)} \tag{4.20}$$

4.2 The General Energy Equation

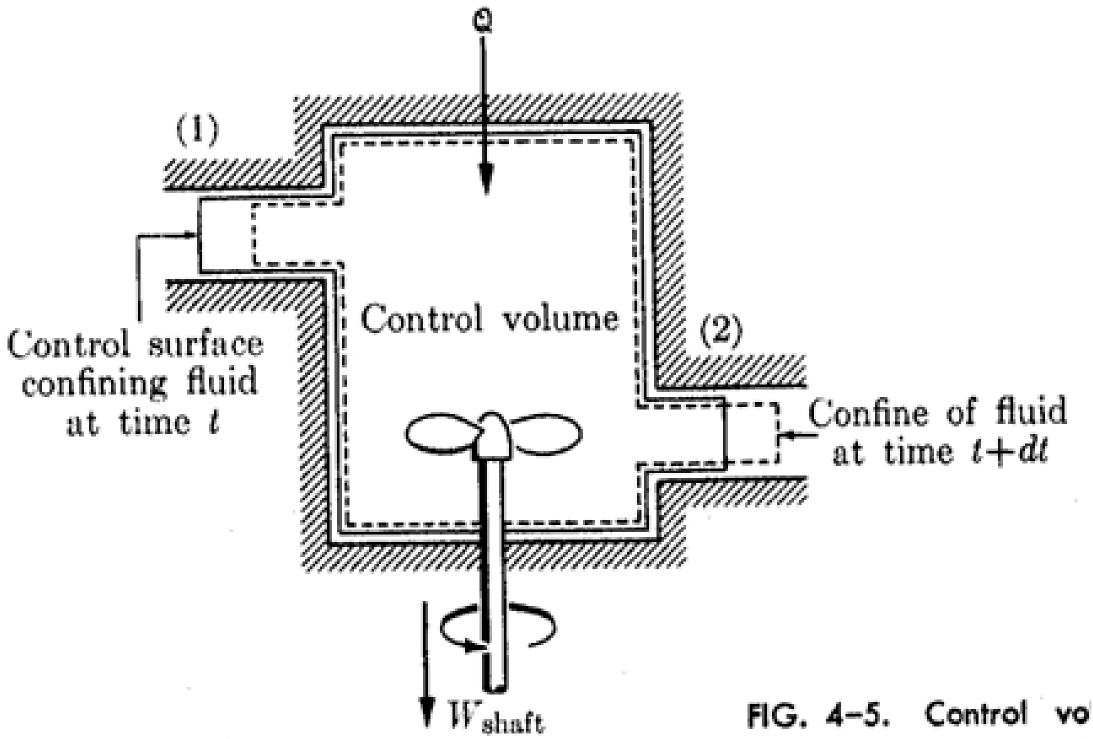


FIG. 4-5. Control vo

4.2 The General Energy Equation

❖ Effect of friction

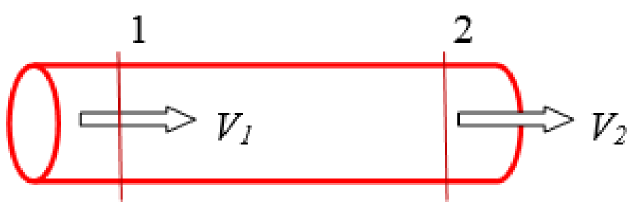
- This effect is accounted for implicitly.
- This results in a degradation of mechanical energy into heat which may be transferred away (Q, heat transfer), or may cause a temperature change → modification of internal energy.
- Thus, Eq. (4.20) can be applied to both viscous fluids and non-viscous fluids (ideal frictionless processes).

4.2 The General Energy Equation

4.2.3 1 D Steady flow equations

For flow through conduits, properties are uniform normal to the flow direction.

→ one-dimensional steady flow



$$\frac{\delta Q}{dt} - \frac{\delta W_{shaft}}{dt} = \oint_{CS} \left(u + \frac{p}{\rho} + gh + \frac{q^2}{2} \right) \rho \left(\vec{q} \cdot d\vec{A} \right)$$

Integrated form of Eq. (4.20) = ② − ①

$$\frac{\delta Q}{dt} - \frac{\delta W_{shaft}}{dt} = \left[u + \frac{p}{\rho} + gh + \frac{V^2}{2} \right]_{②} \rho Q - \left[u + \frac{p}{\rho} + gh + \frac{V^2}{2} \right]_{①} \rho Q$$

4.2 The General Energy Equation

where $\dfrac{V^2}{2}$ = average kinetic energy per unit mass

$$Section\,1:\ \int_1 \rho\left(\vec{q}\cdot d\vec{A}\right) = -\rho Q = \text{mass flow rate into CV}$$

$$Section\ 2:\ \int_2 \rho\left(\vec{q}\cdot d\vec{A}\right) = \rho Q = \text{mass flow from CV}$$

$$\boxed{M = \rho Q\ dt}$$

Divide by ρQ (mass/time)

$$\frac{heat\ transfer}{mass} - \frac{W_{shaft}}{mass} = \left[u + \frac{p}{\rho} + gh + \frac{V^2}{2}\right]_{②} - \left[u + \frac{p}{\rho} + gh + \frac{V^2}{2}\right]_{①}$$

4.2 The General Energy Equation

Divide by g

$$\frac{heat\ transfer}{weight} - \frac{W_{shaft}}{weight} = \left[\frac{u}{g} + \frac{p}{\gamma} + h + \frac{V^2}{2g}\right]_{②} - \left[\frac{u}{g} + \frac{p}{\gamma} + h + \frac{V^2}{2g}\right]_{①}$$

(4.21)

- Energy Equation for 1-D steady flow: Eq. (4.21)
  - use average values for p, γ, h, u, and V at each flow section
  - use K_e (energy correction coeff.) to account for non-uniform velocity distribution over flow cross section

4.2 The General Energy Equation

$$K_e \frac{\rho}{2} V^2 Q = \int \frac{\rho}{2} q^2 dQ$$ ---- kinetic energy/time $= \frac{1}{2}\frac{mV^2}{t}$

$$K_e = \frac{\int \frac{\rho}{2} q^2 dQ}{\frac{\rho}{2} V^2 Q} \geq 1 \tag{4.22}$$

$$\frac{heat\ transfer}{weight} - \frac{W_{shaft}}{weight} = \left[\frac{p}{\gamma} + h + K_e \frac{V^2}{2g}\right]_{②} - \left[\frac{p}{\gamma} + h + K_e \frac{V^2}{2g}\right]_{①} + \frac{u_2 - u_1}{g} \tag{4.23}$$

$K_e = \begin{cases} \text{2, for laminar flow (parabolic velocity distribution)} \\ \text{1.06, for turbulent flow (smooth pipe)} \end{cases}$

4.2 The General Energy Equation

For a fluid of uniform density γ

$$\frac{p_1}{\gamma} + h_1 + K_{e_1}\frac{V_1^2}{2g} = \frac{p_2}{\gamma} + h_2 + K_{e_2}\frac{V_2^2}{2g} + \frac{W_{shaft}}{weight} \boxed{- \frac{heat\ transfer}{weight} + \frac{u_2 - u_1}{g}} \tag{4.24}$$

→ unit: m (energy per unit weight)

For viscous fluid;

$$-\frac{heat\ transfer}{weight} + \frac{u_2 - u_1}{g} = H_{L_{1-2}}$$

→ loss of mechanical energy

~ irreversible in liquid

4.2 The General Energy Equation

Then, Eq. (4.24) becomes

$$\boxed{\frac{p_1}{\gamma} + h_1 + K_{e_1}\frac{V_1^2}{2g} = \frac{p_2}{\gamma} + h_2 + K_{e_2}\frac{V_2^2}{2g} + \Delta H_M + \Delta H_{L_{1-2}}} \tag{4.24a}$$

where ΔH_M = shaft work transmitted from the system to the outside

$$H_1 = H_2 + \Delta H_M + \Delta H_{L_{1-2}} \tag{4.24b}$$

where H_1 , H_2 = weight flow rate average values of total head

4.2 The General Energy Equation

❖ Bernoulli Equation

Assume

① ideal fluid → friction losses are negligible

② no shaft work → $\Delta H_M = 0$

③ no heat transfer and internal energy is constant → $\Delta H_{L_{1-2}} = 0$

$$\frac{p_1}{\gamma} + h_1 + K_{e_1}\frac{V_1^{\,2}}{2g} = \frac{p_2}{\gamma} + h_2 + K_{e_2}\frac{V_2^{\,2}}{2g} \qquad (4.25)$$

$$H_1 = H_2$$

4.2 The General Energy Equation

If $K_{e1} = K_{e2} = 1$, then Eq. (4.25) reduces to

work

Pressure head

Potential head

Velocity head

$$\boxed{H = \frac{p_1}{\gamma} + h_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + h_2 + \frac{V_2^2}{2g}} \qquad (4.26)$$

~ total head along a conduct is constant

4.2 The General Energy Equation

- Grade lines

1) Energy (total head) line (E.L) ~ H above datum

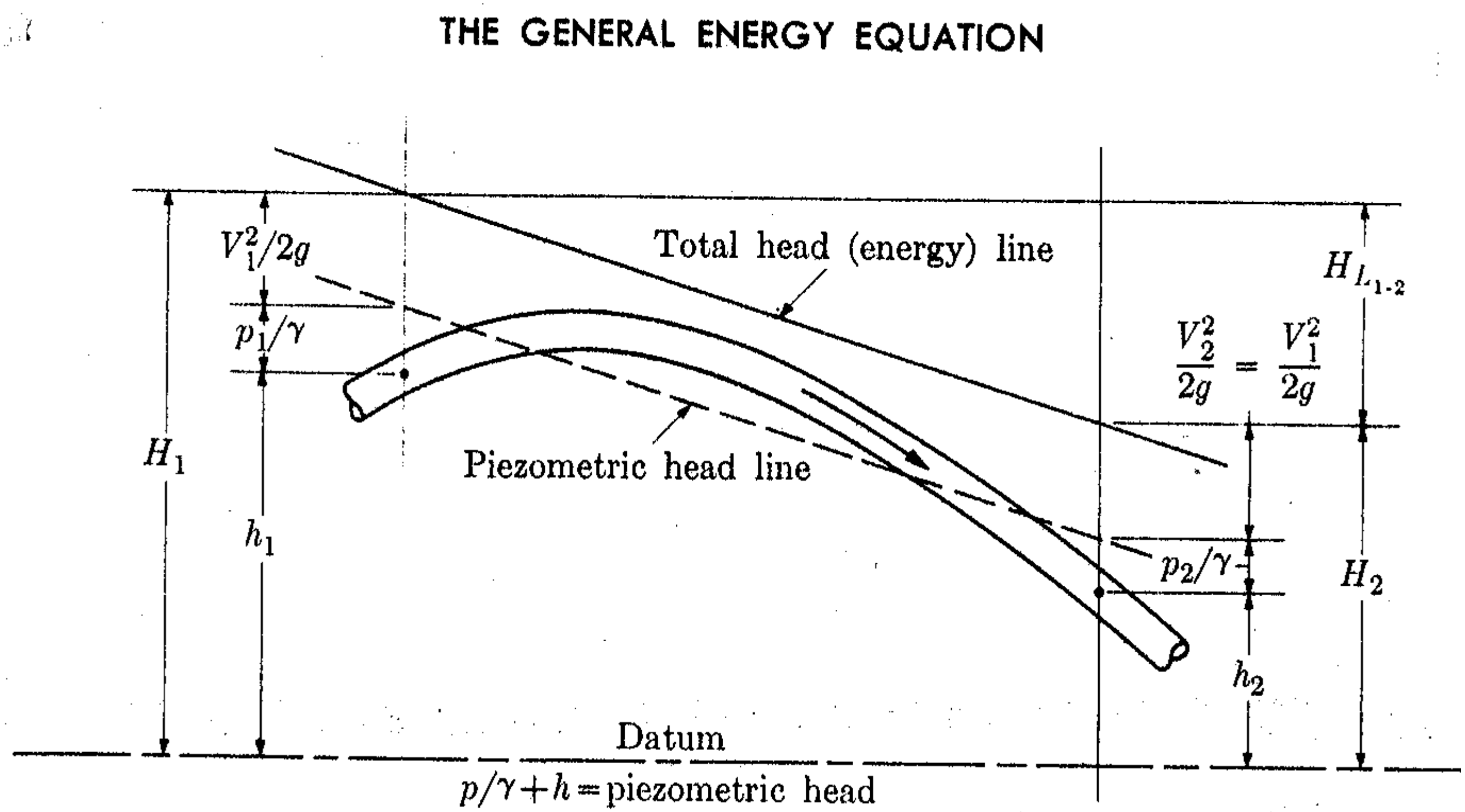
2) Hydraulic (piezometric head) grade line (H.G.L.)

$$= \left(\frac{p}{\gamma} + h \right) \text{above datum}$$

For flow through a pipe with a constant diameter

$$V_1 = V_2 \quad \rightarrow \quad \frac{V_1^{\,2}}{2g} = \frac{V_2^{\,2}}{2g}$$

4.2 The General Energy Equation



4.2 The General Energy Equation

1) If the fluid is real (viscous fluid) and if no energy is being added, then the energy line may never be horizontal or slope upward in the direction of flow.

2) Vertical drop in energy line represents the head loss or energy dissipation.

4.3 Linear Momentum Equation for Finite Control Volumes

4.3.1 Momentum Principle

- The momentum equation can be derived from Newton's 2nd law of motion

$$\vec{F} = m\vec{a} = m\frac{d\vec{q}}{dt} = \frac{d\left(m\vec{q}\right)}{dt} = \frac{d\vec{M}}{dt} \tag{4.27}$$

$\vec{M}$ = linear momentum vector $= m\vec{q}$

$\vec{F}$ = external force

= boundary (surface) forces: normal to boundary - pressure, $\vec{F}_p$; tangential to boundary - shear, $\vec{F}_s$

body forces - force due to gravitational or magnetic fields, $\vec{F}_b$

4.3 Linear Momentum Equation for Finite Control Volumes

$$\vec{F}_p + \vec{F}_s + \vec{F}_b = \frac{d\vec{M}}{dt} \tag{4.28}$$

$\vec{F}_b = \int_{CV} f_b \left(\rho\, dv\right),$ where f_b = body force per unit mass

4.3 Linear Momentum Equation for Finite Control Volumes

4.3.2 The general linear momentum equation

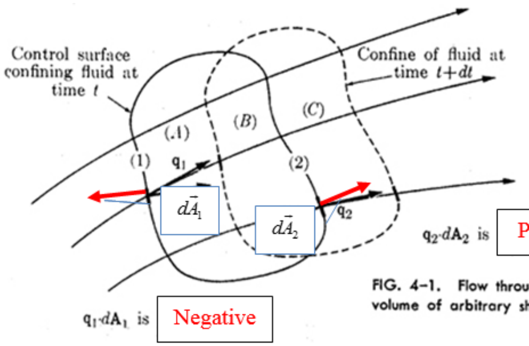


FIG. 4-1. Flow through a finite control volume of arbitrary shape.

Consider change of momentum

$$\frac{d\vec{M}}{dt}$$ = total rate of change of momentum

= net momentum flux across the CV boundaries

+ time rate of increase of momentum within CV

$$= \oint_{CS} \vec{q}\rho\left(\vec{q}\cdot d\vec{A}\right) + \frac{\partial}{\partial t}\int_{CV} \vec{q}\rho\, dV \qquad (4.29)$$

where $\vec{q}\rho\left(\vec{q}\cdot d\vec{A}\right)$ = momentum flux = $\text{velocity} \times \text{mass per time}$

$d\vec{A}$ = vector unit area pointing outward over the control surface

4.3 Linear Momentum Equation for Finite Control Volumes

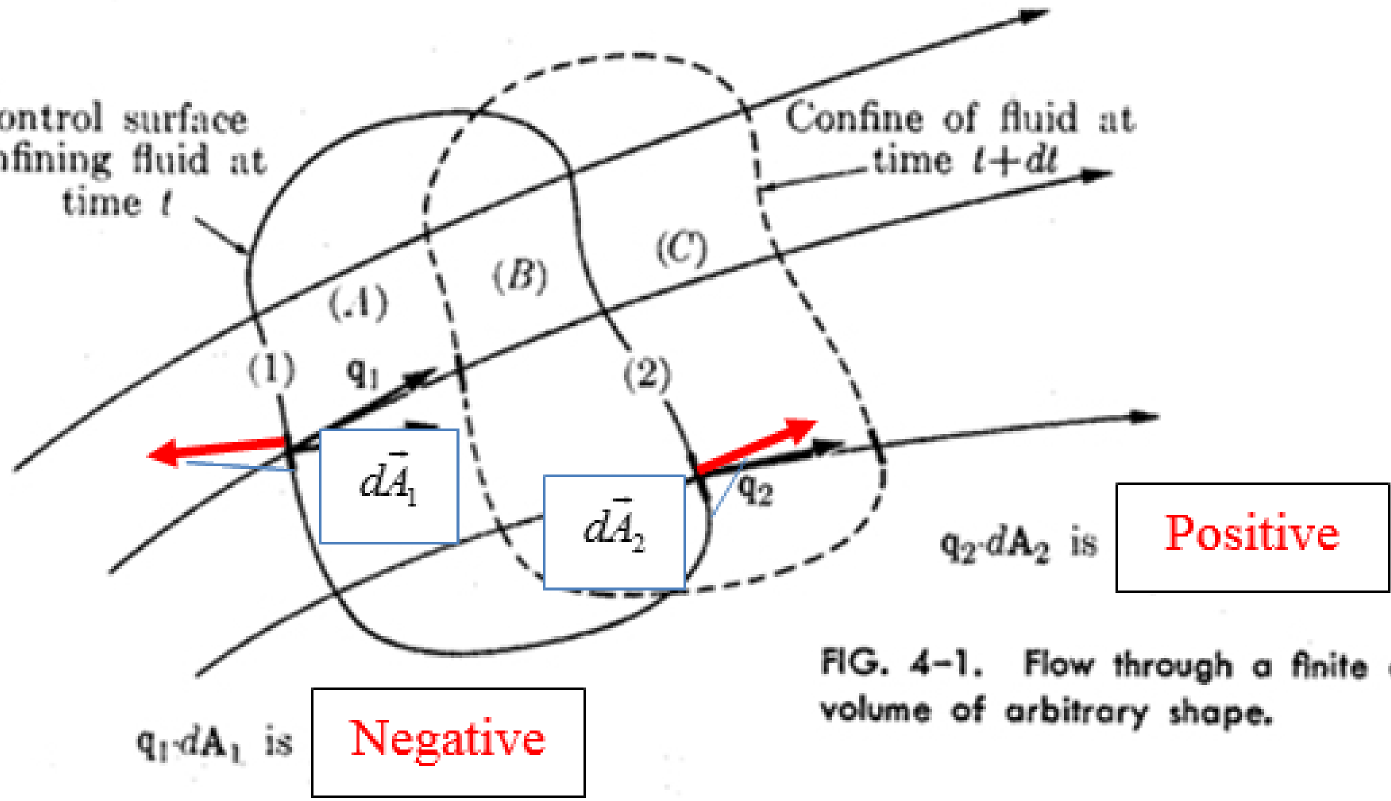


FIG. 4-1. Flow through a finite control volume of arbitrary shape.

4.3 Linear Momentum Equation for Finite Control Volumes

Substitute (4.29) into (4.28)

$$\boxed{\vec{F}_p + \vec{F}_s + \vec{F}_b = \oint_{CS} \vec{q}\rho\left(\vec{q}\cdot d\vec{A}\right) + \frac{\partial}{\partial t}\int_{CV} \vec{q}\rho\, dV} \qquad (4.30)$$

For steady flow and negligible body forces

$$\boxed{\vec{F}_p + \vec{F}_s = \oint_{CS} \vec{q}\rho\left(\vec{q}\cdot d\vec{A}\right)} \qquad (4.31)$$

- Eq. (4.30)
  - It is applicable to both ideal fluid systems and viscous fluid systems involving friction and energy dissipation.
  - It is applicable to both compressible fluid and incompressible fluid.

4.3 Linear Momentum Equation for Finite Control Volumes

- Combined effects of friction, energy loss, and heat transfer appear implicitly in the magnitude of the external forces, with corresponding effects on the local flow velocities.
- Knowledge of the internal conditions is not necessary.
- We can consider only external conditions.

4.3 Linear Momentum Equation for Finite Control Volumes

4.3.3 Inertial control volume for a generalized apparatus

- Three components of the forces

$$x-dir.:\ \vec{F}_{p_x}+\vec{F}_{s_x}+\vec{F}_{b_x}=\oint_{CS}u\rho\left(\vec{q}\cdot d\vec{A}\right)+\frac{\partial}{\partial t}\int_{CV}u\rho\,dV$$

$$y-dir.:\ \vec{F}_{p_y}+\vec{F}_{s_y}+\vec{F}_{b_y}=\oint_{CS}v\rho\left(\vec{q}\cdot d\vec{A}\right)+\frac{\partial}{\partial t}\int_{CV}v\rho\,dV$$

$$z-dir.:\ \vec{F}_{p_z}+\vec{F}_{s_z}+\vec{F}_{b_z}=\oint_{CS}w\rho\left(\vec{q}\cdot d\vec{A}\right)+\frac{\partial}{\partial t}\int_{CV}w\rho\,dV \qquad (4.32)$$

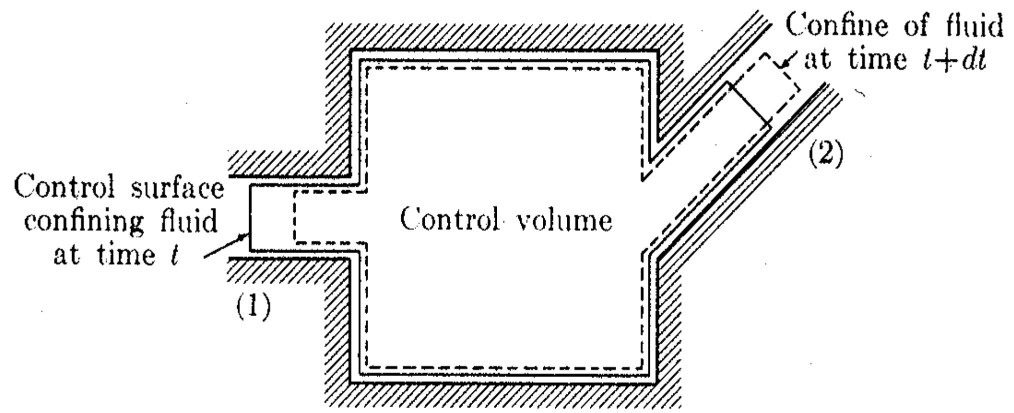
4.3 Linear Momentum Equation for Finite Control Volumes

- For flow through generalized apparatus

$$x-dir.:\ \vec{F}_{p_x}+\vec{F}_{s_x}+\vec{F}_{b_x}=\int_2 u\rho\, dQ-\int_1 u\rho\, dQ+\frac{\partial}{\partial t}\int_{CV} u\rho\, dV$$

- For 1D steady flow,

$$\frac{\partial}{\partial t}\int_{CV} q\rho\, dV=0$$



4.3 Linear Momentum Equation for Finite Control Volumes

- Velocity and density are constant normal to the flow direction.

$$x-dir.:\ \vec{F}_{p_x}+\vec{F}_{s_x}+\vec{F}_{b_x}=\sum F_x=(V_x\rho Q)_2-(V_x\rho Q)_1$$

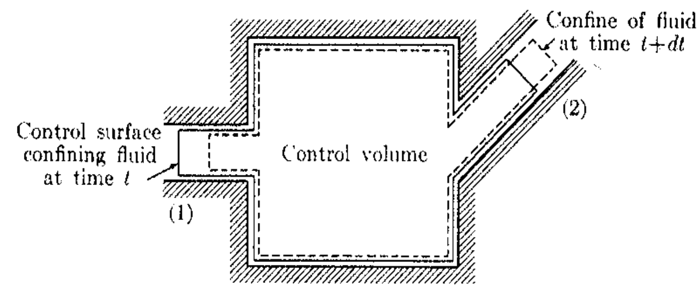
$$=V_{x_2}\rho_2 Q_2-V_{x_1}\rho_1 Q_1=Q\rho\left(V_{x_2}-V_{x_1}\right)=Q\rho\left(V_{x_{out}}-V_{x_{in}}\right)$$

$$\boxed{\rho_1 Q_1=\rho_2 Q_2=Q\rho \quad (4.12)}$$

$$y-dir.:\ \sum F_y=(V_y\rho Q)_2-(V_y\rho Q)_1$$

$$z-dir.:\ \sum F_z=(V_z\rho Q)_2-(V_z\rho Q)_1$$

where V = average velocity in flow direction



4.3 Linear Momentum Equation for Finite Control Volumes

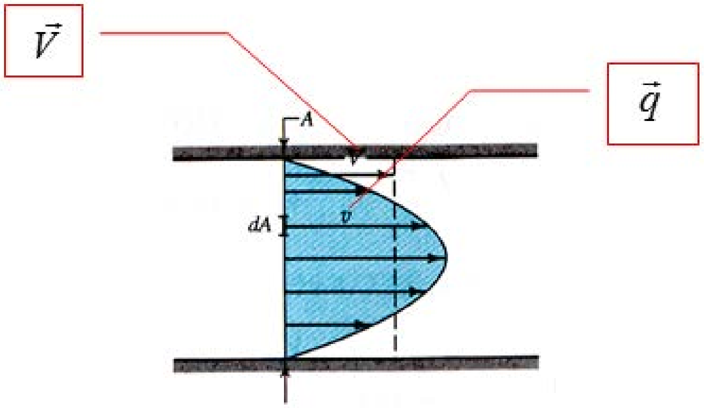
- Non-uniform velocity profile

If velocity varies over the cross section, then introduce momentum flux coefficient

$$\int \vec{q}\rho\left(\vec{q}\cdot d\vec{A}\right) = K_m \vec{V}\left(\rho V A\right)$$

$$\int \vec{q}\rho\, dQ = K_m \vec{V}\rho Q$$

$$K_m = \frac{\int \vec{q}\rho dQ}{\vec{V}\rho Q}$$



4.3 Linear Momentum Equation for Finite Control Volumes

where

V = magnitude of average velocity over cross section = Q/A

$\vec{V}$ = average velocity vector

K_m = momentum flux coefficient ≥ 1

= 1.33 for laminar flow (pipe flow)

1.03-1.04 for turbulent flow (smooth pipe)

$$\boxed{\begin{aligned} \sum F_x &= \left(K_m V_x \rho Q\right)_2 - \left(K_m V_x \rho Q\right)_1 \\ \sum F_y &= \left(K_m V_y \rho Q\right)_2 - \left(K_m V_y \rho Q\right)_1 \\ \sum F_z &= \left(K_m V_z \rho Q\right)_2 - \left(K_m V_z \rho Q\right)_1 \end{aligned}}$$

4.3 Linear Momentum Equation for Finite Control Volumes

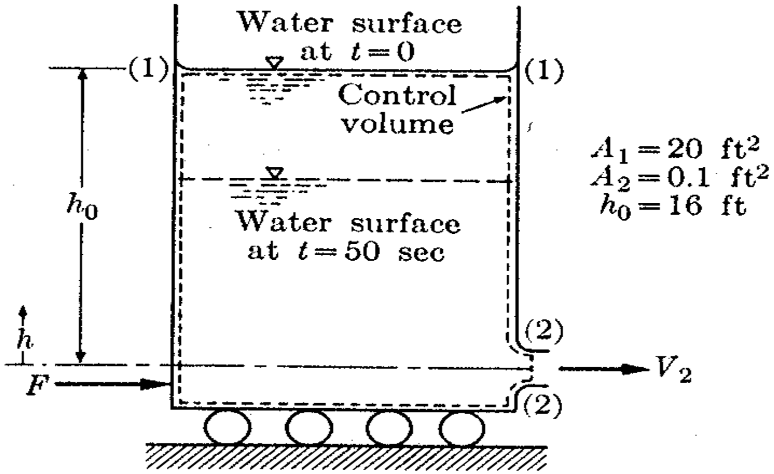
[Cf] Energy correction coefficient

$$K_e = \frac{\int \frac{\rho}{2} q^2 dQ}{\frac{\rho}{2} \vec{V} Q}$$

4.3 Linear Momentum Equation for Finite Control Volumes

[Example 4-4] Continuity, energy, and linear momentum with unsteady flow

A large tank mounted on rollers is filled with water to a depth of 16 ft above a discharge port. At time $t=0$, the fast-acting valve on the discharge nozzle is opened. Determine depth h, discharge rate Q, and force F necessary to keep the tank stationary at $t = 50$ *sec* .



4.3 Linear Momentum Equation for Finite Control Volumes

Continuity, energy, and linear momentum equations

(4.6)
$$\int_{CV} \frac{\partial \rho}{\partial t} dV + \oint_{CS} \rho \vec{q} \cdot d\vec{A} = 0$$

(4.17)
$$\frac{\delta Q}{dt} - \frac{\delta W_{shaft}}{dt} - \frac{\delta W_{shear}}{dt} = \oint_{CS} \left(\frac{p}{\rho} + u + gh + \frac{q^2}{2} \right) \rho \left(\vec{q} \cdot d\vec{A} \right) + \frac{\partial}{\partial t} \int_{CV} e \rho \, dV$$

(4.30)
$$\vec{F}_p + \vec{F}_s + \vec{F}_b = \oint_{CS} \vec{q} \rho \left(\vec{q} \cdot d\vec{A} \right) + \frac{\partial}{\partial t} \int_{CV} \vec{q} \rho \, dV$$

4.3 Linear Momentum Equation for Finite Control Volumes

i) Use integral form of continuity equation, Eq. (4.6)

$$\frac{\partial}{\partial t}\int_{CV} \rho \, dV = \int \rho q_n dA_1 - \int \rho q_n dA_2$$

$dV = A_1 dh$, $\rho q_n dA_1 = 0$ (because no inflow across the Section 1)

$$\therefore \quad \rho A_1 \frac{\partial}{\partial t}\int_0^h dh = -\rho V_2 A_2$$

$$A_1 \frac{dh}{dt} = -V_2 A_2 \qquad \text{(A)}$$

ii) Energy equation, Eq. (4.17)

~ no shaft work

~ heat transfer and temperature changes <u>due to friction are negligible</u>

4.3 Linear Momentum Equation for Finite Control Volumes

$$\cancel{\frac{\delta Q}{dt}} - \cancel{\frac{\delta W_{shaft}}{dt}} - \cancel{\frac{\delta W_{shear}}{dt}}$$

$$= \underbrace{\oint_{CS}\left(\frac{p}{\rho} + u + gh + \frac{q^2}{2}\right)\rho\left(\vec{q}\cdot d\vec{A}\right)}_{\mathrm{I}} + \underbrace{\frac{\partial}{\partial t}\int_{CV} e\rho\, dV}_{\mathrm{II}}$$

e = energy per unit mass = $u + gh + \dfrac{q^2}{2}$

$$\mathrm{I} = \oint_{CS}\left(u + \frac{p}{\rho} + gh + \frac{q^2}{2}\right)\rho\left(\vec{q}\cdot d\vec{A}\right)$$

$$= \left(u + \frac{p}{\rho} + gh + \frac{q^2}{2}\right)_2 \rho V_2 A_2 - \left(u + \frac{p}{\rho} + gh + \frac{q^2}{2}\right)_1 \rho V_1 A_1$$

4.3 Linear Momentum Equation for Finite Control Volumes

$$= \left(u + \frac{p}{\rho} + gh + \frac{q^2}{2} \right)_2 \rho V_2 A_2 \qquad (V_1 \approx 0)$$

$$\mathrm{II} = \frac{\partial}{\partial t}\int_{CV} e\rho\, dV = \frac{\partial}{\partial t}\int_{CV} \left(u + gh + \cancel{\frac{q^2}{2}} \right) \rho\, dV$$

$A_1\, dh$

∵ nearly constant in the tank except near the nozzle

$$= A_1 \rho \frac{\partial}{\partial t}\int_0^h (u + gh)\, dh$$

$$\therefore\ 0 = \left(u + \frac{p}{\rho} + gh + \frac{q^2}{2} \right)_2 \rho V_2 A_2 + A_1 \rho \frac{\partial}{\partial t}\int_0^h (u + gh)\, dh$$

4.3 Linear Momentum Equation for Finite Control Volumes

Assume $\rho = \text{const.}\ ,\quad p_2 = p_{atm} = 0\ ,\quad h_2 = 0\ (\text{datum})$

$$0 = uV_2A_2 + \frac{V_2^2}{2}V_2A_2 + uA_1\frac{dh}{dt} + A_1gh\frac{dh}{dt} \qquad \text{(B)}$$

$$\boxed{A_1\frac{dh}{dt} = -V_2A_2}$$

Substitute (A) into (B)

$$0 = \cancel{uV_2A_2} + \frac{V_2^2}{2}V_2A_2 + \cancel{u\left(-V_2A_2\right)} + gh\left(-V_2A_2\right)$$

$$\therefore \quad \frac{V_2^2}{2}V_2A_2 = ghV_2A_2$$

$$V_2 = \sqrt{2gh} \qquad \text{(C)}$$

4.3 Linear Momentum Equation for Finite Control Volumes

Substitute (C) into (A)

$$A_2\sqrt{2gh} = -A_1\frac{dh}{dt}$$

$$\frac{dh}{\sqrt{h}} = -\frac{A_2}{A_1}\sqrt{2g}\,dt$$

Integrate

$$\int_{h_0}^{h}\frac{dh}{\sqrt{h}} = \int_0^t -\frac{A_2}{A_1}\sqrt{2g}\,dt \qquad \left\{\int_{h_0}^{h} h^{-\frac{1}{2}}dh = \left[2h^{\frac{1}{2}}\right]_{h_0}^{h}\right\}$$

$$h = \left(h_0^{\frac{1}{2}} - \frac{A_2}{A_1}\frac{\sqrt{2g}}{2}t\right)^2$$

4.3 Linear Momentum Equation for Finite Control Volumes

$$h = \left(\sqrt{16} - \frac{0.1}{20} \frac{\sqrt{2(32.2)}}{2} t \right)^2$$

$$= (4 - 0.0201t)^2$$

At $t = 50\sec$, $h = (4 - 0.0201 \times 50)^2 = 8.98\,ft$

$$V_2 = \sqrt{2gh} = \sqrt{2(32.2)(8.98)} = 24.05 \text{ fps}$$

$$Q_2 = (VA)_2 = 24.05(0.1) = 2.405 \text{ cfs}$$

4.3 Linear Momentum Equation for Finite Control Volumes

iii) Momentum equation, Eq. (4.30)

$$\vec{F}_p + \cancel{\vec{F}_s} + \cancel{\vec{F}_b} = \underbrace{\oint_{CS} \vec{q}\rho\left(\vec{q}\cdot d\vec{A}\right)}_{\boxed{\mathrm{I}}} + \underbrace{\frac{\partial}{\partial t}\int_{CV} \vec{q}\rho\, dV}_{\boxed{\mathrm{II}}}$$

II = Time rate of change of momentum inside CV is negligible

if tank area $\left(A_1\right)$ is large compared to the nozzle area $\left(A_2\right)$.

$$\mathrm{I} = \oint_{CS} \vec{q}\rho\left(\vec{q}\cdot d\vec{A}\right) = \int q_n \rho q_n dA_2 - \cancel{\int q_n \rho q_n dA_1} = V_2 \rho V_2 A_2$$

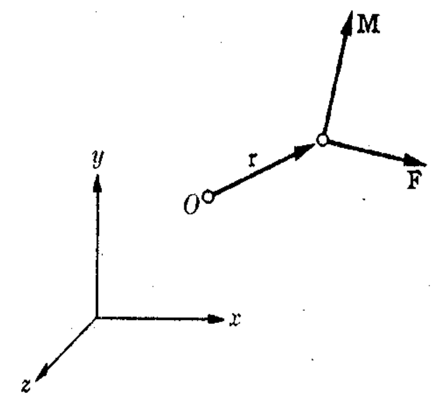
$$\therefore F_{px} = V_2 \rho V_2 A_2 = V_2 \rho Q_2$$

$$F_{px} = \left(24.05\right)\left(1.94\right)\left(2.405\right) = 112 \text{ lb}$$

4.4 The Moment of Momentum Equation for Finite Control Volumes

4.4.1 The Moment of momentum principle for inertial reference systems

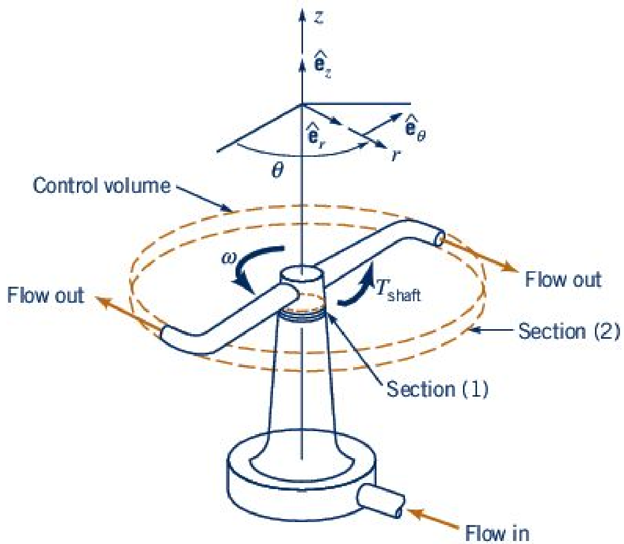
Apply Newton's 2nd law to rotating fluid masses



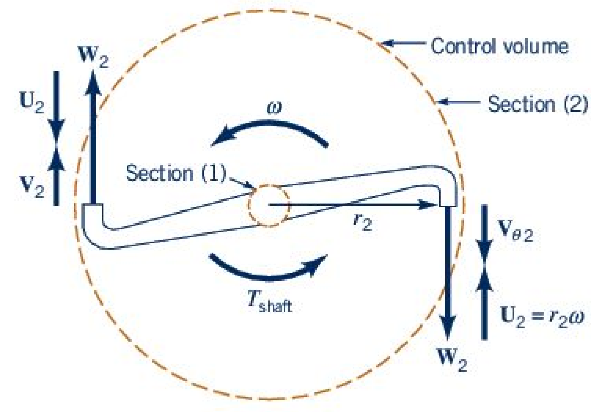
→ The vector sum of all the external moments acting on a fluid mass $\left(\vec{r}\times\vec{F}\right)$ equals the time rate of change of the moment of momentum (angular momentum) vector $\left(\vec{r}\times\vec{M}\right)$ of the fluid mass.

Example: rotary lawn sprinklers, ceiling fans, wind turbines

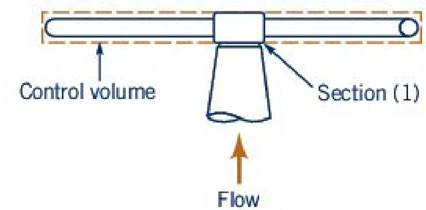
4.4 The Moment of Momentum Equation for Finite Control Volumes



(a)



(b)



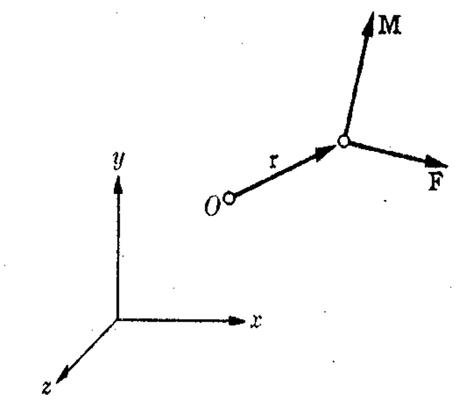
(c)

4.4 The Moment of Momentum Equation for Finite Control Volumes

$$T = \vec{r} \times \vec{F} = \frac{d}{dt}\left(\vec{r} \times \vec{M}\right) \tag{4.35}$$

where $\vec{r}$ = position vector of a mass in an arbitrary curvilinear motion

$\vec{M}$ = linear momentum



4.4 The Moment of Momentum Equation for Finite Control Volumes

[Re] Derivation of (4.35)

Eq. (4.27): $\vec{F} = \dfrac{d\vec{M}}{dt}$

Take the vector cross product of $\vec{r}$

$$\vec{r} \times \vec{F} = \vec{r} \times \frac{d\vec{M}}{dt}$$

By the way,

$$\frac{d}{dt}\left(\vec{r} \times \vec{M}\right) = \underbrace{\frac{d\vec{r}}{dt} \times \vec{M}}_{\boxed{\mathrm{I}}} + \vec{r} \times \frac{d\vec{M}}{dt}$$

4.4 The Moment of Momentum Equation for Finite Control Volumes

$$I = \frac{d\vec{r}}{dt} \times \vec{M} = \vec{q} \times m\,\vec{q} = 0 \quad \left(\because \frac{d\vec{r}}{dt} = \vec{q} \right)$$

$$\left(\therefore \ \vec{q} \times \vec{q} = |\vec{q}|\,|\vec{q}| \sin 0^\circ = 0 \right)$$

$$\therefore \quad \left(\vec{r} \times \frac{d\vec{M}}{dt} \right) = \frac{d}{dt}\left(\vec{r} \times \vec{M} \right)$$

$$\therefore\therefore \quad \boxed{\vec{r} \times \vec{F} = \frac{d}{dt}\left(\vec{r} \times \vec{M} \right)}$$

where $\vec{r} \times \vec{M}$ = angular momentum (moment of momentum)

4.4 The Moment of Momentum Equation for Finite Control Volumes

[Re] Torque $\vec{T} = \vec{r} \times \vec{F}$

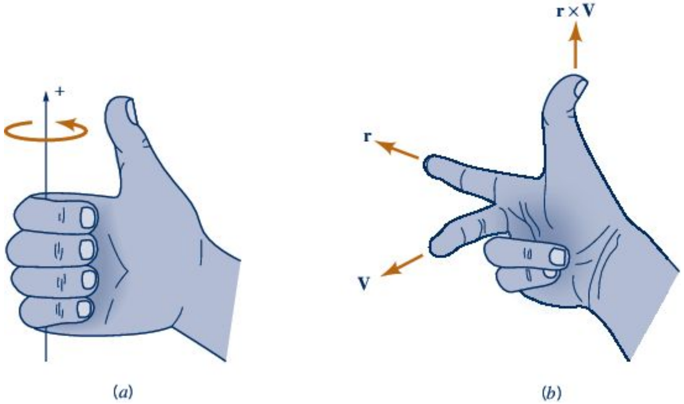
- translational motion →

Force – linear acceleration

- rotational motion →

Torque – angular acceleration

[Re] Vector Product



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$$\vec{V} = \vec{a} \times \vec{b}$$

Magnitude = $\left|\vec{V}\right| = \left|\vec{a}\right| \times \left|\vec{b}\right| \sin\gamma$ = area of parallelogram

direction = perpendicular to plane of $\vec{a}$ and $\vec{b}$ → right-handed triple

4.4 The Moment of Momentum Equation for Finite Control Volumes

$$\vec{b} \times \vec{a} = -\left(\vec{a} \times \vec{b}\right)$$

$$(k\vec{a}) \times \vec{b} = k\left(\vec{a} \times \vec{b}\right)$$

$$\vec{a} \times \left(\vec{b} + \vec{c}\right) = \left(\vec{a} \times \vec{b}\right) + \left(\vec{a} \times \vec{b}\right)$$

- External moments arise from external forces

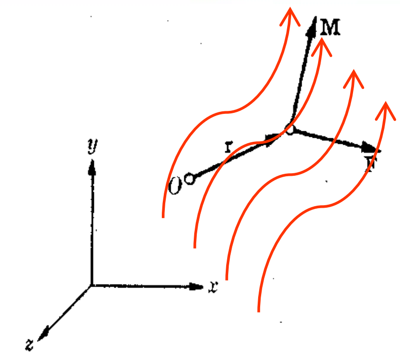
$$\underbrace{\left(\vec{r} \times \vec{F}_p\right)}_{\boxed{\vec{T}_b}} + \underbrace{\left(\vec{r} \times \vec{F}_s\right)}_{\boxed{\vec{T}_s}} + \underbrace{\left(\vec{r} \times \vec{F}_b\right)}_{\boxed{\vec{T}_p}} = \frac{d}{dt}\left(\vec{r} \times \vec{M}\right)$$

$$\vec{T}_p + \vec{T}_s + \vec{T}_b = \frac{d}{dt}\left(\vec{r} \times \vec{M}\right) \qquad (4.36)$$

where $\vec{T}_p$, $\vec{T}_s$, $\vec{T}_b$ = external torque

4.4 The Moment of Momentum Equation for Finite Control Volumes

4.4.2 The general moment of momentum equation



(4.29): $$\frac{d\vec{M}}{dt} = \oint_{CS} \vec{q}\rho\left(\vec{q}\cdot d\vec{A}\right) + \frac{\partial}{\partial t}\int_{CV} \vec{q}\rho\, dV$$

$$\therefore \quad \frac{d}{dt}\left(\vec{r}\times\vec{M}\right) = \oint_{CS}\left(\vec{r}\times\vec{q}\right)\rho\left(\vec{q}\cdot d\vec{A}\right) + \frac{\partial}{\partial t}\int_{CV}\left(\vec{r}\times\vec{q}\right)\rho\, dV$$

$$\vec{T}_p + \vec{T}_s + \vec{T}_b = \oint_{CS}\left(\vec{r}\times\vec{q}\right)\rho\left(\vec{q}\cdot d\vec{A}\right) + \frac{\partial}{\partial t}\int_{CV}\left(\vec{r}\times\vec{q}\right)\rho\, dV \qquad (4.37)$$

$$x-dir.: \quad \left|\left(\vec{r}\times\vec{q}\right)_{yz}\right| = r_{yz}\, q_{yz} \sin\left(\frac{\pi}{2}-\alpha_{yz}\right) = \left(rq\cos\alpha\right)$$

angle between q_{yz} and r_{yz}

4.4 The Moment of Momentum Equation for Finite Control Volumes

$$x-dir.: \quad \vec{T}_{px} + \vec{T}_{sx} + \vec{T}_{bx} = \oint_{CS} (rq\cos\alpha)_{yz}\, \rho\left(\vec{q}\cdot d\vec{A}\right) + \frac{\partial}{\partial t}\int_{CV} (rq\cos\alpha)_{yz}\, \rho\, dV$$

$$y-dir.: \quad \vec{T}_{py} + \vec{T}_{sy} + \vec{T}_{by} = \oint_{CS} (rq\cos\alpha)_{zx}\, \rho\left(\vec{q}\cdot d\vec{A}\right) + \frac{\partial}{\partial t}\int_{CV} (rq\cos\alpha)_{zx}\, \rho\, dV$$

$$z-dir.: \quad \vec{T}_{pz} + \vec{T}_{sz} + \vec{T}_{bz} = \oint_{CS} (rq\cos\alpha)_{xy}\, \rho\left(\vec{q}\cdot d\vec{A}\right) + \frac{\partial}{\partial t}\int_{CV} (rq\cos\alpha)_{xy}\, \rho\, dV$$

(4.38)

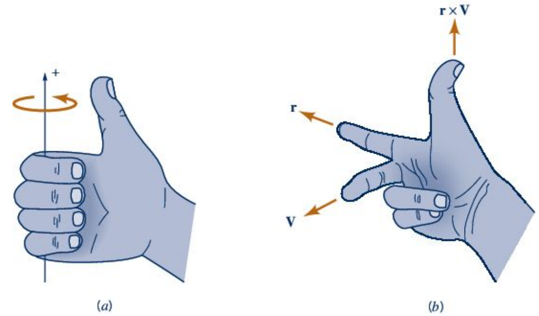


fig_05_05

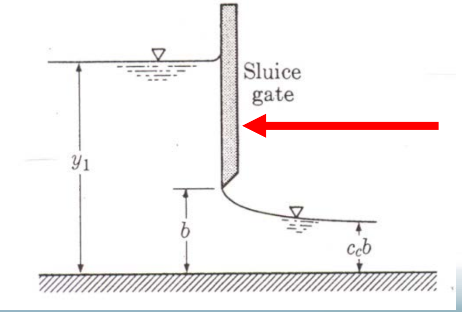
4.4 The Moment of Momentum Equation for Finite Control Volumes

Homework Assignment # 4

Due: 1 week from today

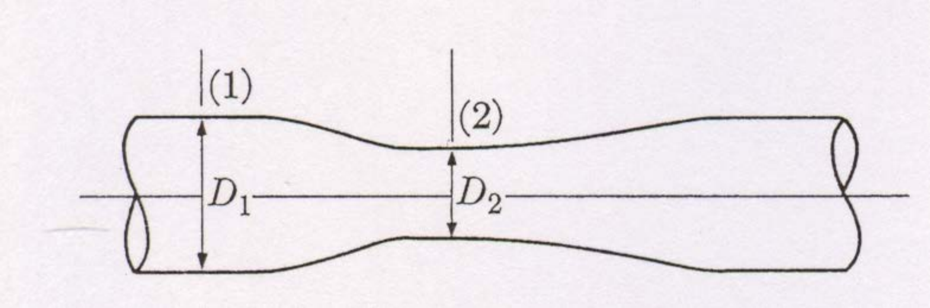
4-11. Derive the equation for the volume rate of flow per unit width for the sluice gate shown in Fig. 4-20 in terms of the geometric variable b, y_1, and C_C. Assume the pressure in hydrostatic at y_1 and $c_c b$ and the velocity is constant over the depth at each of these sections.

4-12. Derive the expression for the total force per unit width exerted by the sluice gate on the fluid in terms of vertical distances shown in Fig. 4-20.



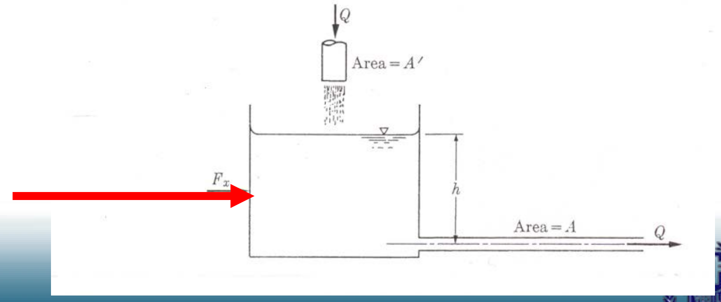
4.4 The Moment of Momentum Equation for Finite Control Volumes

4-14. Consider the flow of an incompressible fluid through the Venturi meter shown in Fig. 4-22. Assuming uniform flow at sections (1) and (2) neglecting all losses, find the pressure difference between these sections as a function of the flow rate *Q*, the diameters of the sections, and the density of the fluid, *P*. Note that for a given configuration, *Q* is a function of only the pressure drop and fluid density.



4.4 The Moment of Momentum Equation for Finite Control Volumes

4-15. Water flows into a tank from a supply line and out of the tank through a horizontal pipe as shown in Fig. 4-23. The rates of inflow and outflow are the same, and the water surface in the tank remains a distance *h* above the discharge pipe centerline. All velocities in the tank are negligible compared to those in the pipe. The head loss between the tank and the pipe exit is H_L (a) Find the discharge Q in terms of $h, A,$ and H_L. (b) What is the horizontal force, F_X required to keep the tank from moving? (c) If the supply line has an area A', what is the vertical force exerted on the water in the tank by the vertical jet?



4.4 The Moment of Momentum Equation for Finite Control Volumes

4-28. Derive the one-dimensional continuity equation for the unsteady, non-uniform flow of an incompressible liquid in a horizontal open channel as shown in Fig. 4-29. The channel has a rectangular cross section of a constant width, b. Both the depth, y_0 and the mean velocity, V are functions of x and t.

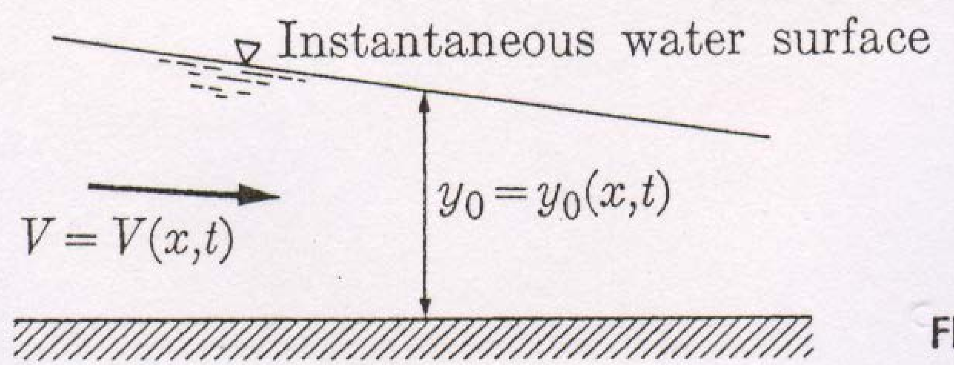


FIGURE 4–29