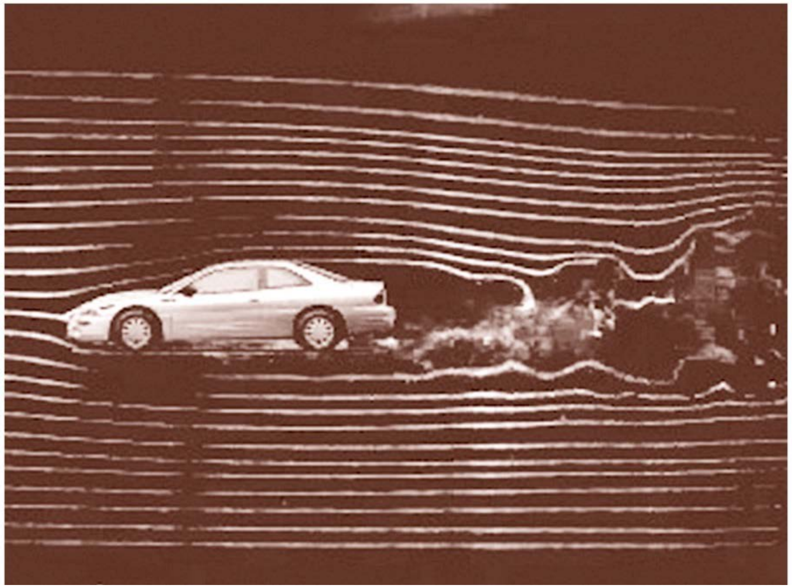
Chapter 8

Origin of Turbulence and Turbulent Shear Stress



Chapter 8 Origin of Turbulence and Turbulent Shear Stress

Contents

8.1 Introduction

8.1.3 Length scales of turbulent flows

Motions in a turbulent flow exist over a broad range of length and time scales.

(1) Integral length scale

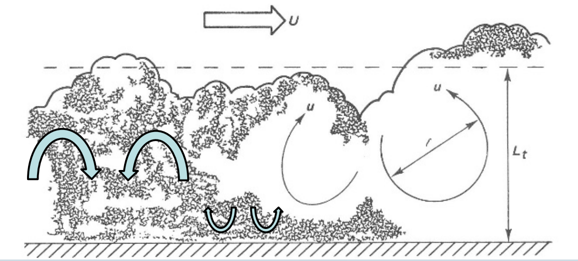
The largest scales are bounded by the geometric dimensions of the flow, for instance the diameter of a pipe or the depth of an open channel. → Integral length scale

Eddies lose the most of their energy after one or two overturns.

→ Thus, because the rate of energy transferred from the largest eddies is proportional to their energy times their rotational frequency.

The rate of energy dissipation, ε, is of the order:

$$\varepsilon \propto \tilde{u}^2 \cdot \frac{\tilde{u}}{l} \propto \frac{\tilde{u}^3}{l} \qquad \tilde{u} = \text{tubulence intensity}$$



8.1 Introduction

The rate of dissipation is independent of the fluid viscosity and only depends on large-scale motions, even though the scale at which the dissipation occurs is strongly dependent on the fluid viscosity.

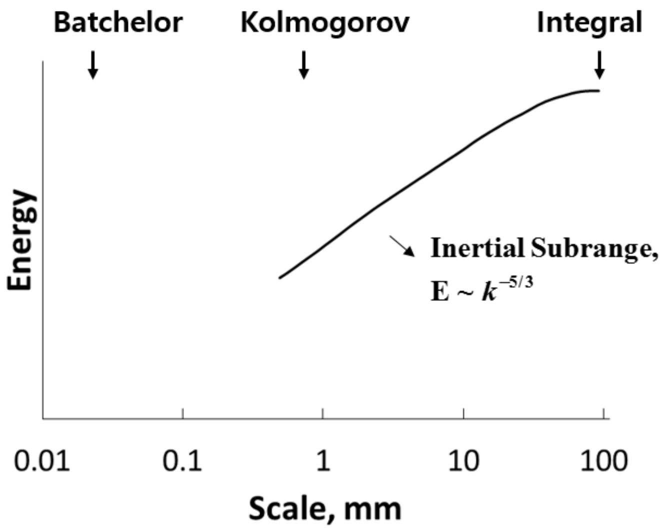
(2) Kolmogorov microscale

~ turbulent velocity field

Dissipation length scale of eddy is

$$\eta \propto \left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}}$$

ν = kinematic viscosity



8.1 Introduction

Time scale of the smallest eddies is

$$\tau \propto \left(\frac{\nu}{\varepsilon}\right)^{\frac{1}{2}}$$

Velocity scale is

$$u \propto (\nu\varepsilon)^{\frac{1}{4}}$$

(3) Batchelor scale (molecular scale)

~ turbulent concentration field

$$L_B \propto \left(\frac{D}{\gamma}\right)^{\frac{1}{2}}$$

8.1 Introduction

where D = molecular diffusivity (m^2/s);

γ = the strain rate of the smallest velocity scale which is given as

$$\gamma = \frac{u}{\eta} = \frac{(\nu\varepsilon)^{1/4}}{\left(\dfrac{\nu^3}{\varepsilon}\right)^{1/4}} = \left(\frac{\varepsilon}{\nu}\right)^{1/2}$$

Therefore the Batchlor's length scale can be recast into a form that include both the molecular diffusivity and the kinematic viscosity.

$$L_B \propto \left(\frac{\nu D^2}{\varepsilon}\right)^{1/4}$$

8.1 Introduction

- **Schmidt number**

~ is defined as the ratio of the Kolmogorov and Batchelor length scales

$$S_c \approx \left(\frac{\eta}{L_B} \right)^2$$

[Ex 1] For the <u>open channel flow</u>,

$$\bar{u} = 50\, mm/s,\ \tilde{u} \approx 5\, mm/s;\ \text{tubulence intensity}$$

· Integral length scale, $l \approx \frac{1}{2} h \approx 100 mm$

Water @ 20℃: $\nu_m = 1\times 10^{-6}\, m^2/s = 1\times 10^{0}\, mm^2/s;\quad D = 1\times 10^{-10} m^2 / s$

8.1 Introduction

Solution:

· Kolmogorov scales

$$\varepsilon \approx \frac{(5)^3 (mm/s)^3}{100(mm)} = 1.25 mm^2/s^3 = 1.25 \times 10^{-6}\, m^2/s^3$$

$$\eta = \left[\frac{\left(1 \times 10^0\, mm^2/s\right)^3}{1.25} \right]^{1/4} = 0.7\ mm < 100\, mm$$

$$\tau = \left[\frac{1\left(mm^2/s\right)}{1.25\left(mm^2/s^3\right)} \right]^{\frac{1}{2}} = 0.5\ s$$

8.1 Introduction

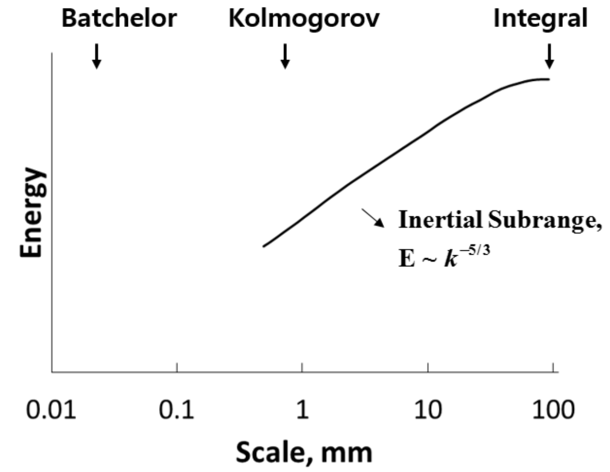
· Batchelor scale

$$L_B = 0.02mm$$

~ Batchelor scale is 35 times smaller than the Kolmogorov scale.

→ We would expect a much finer structure of the concentration field than the velocity field.

$$l = 100\,mm > \eta = 0.7\,mm > L_B = 0.02\,mm$$



8.1 Introduction

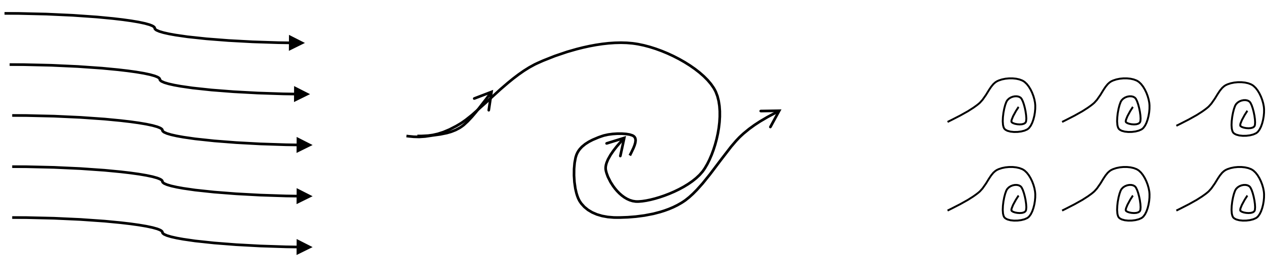
8.1.4 Energy cascade

- Spreading of kinetic energy of the fluid motion over a range of eddy sizes through the non-linear interaction of the large and smaller scales of motion
- Large eddies draw energy from the mean flow, then transfer the energy to smaller scales until it is dissipated at the Kolmogorov microscales.

  → energy cascade
- To maintain turbulence, a constant supply of energy must be fed to the turbulent fluctuations at the largest scales from the mean motion.
- At the smallest scales, the energy is dissipated into heat by viscous effects.

8.1 Introduction

· Energy transfer



mean flow → large eddy → small eddy → heat

generation of turbulence	energy cascade	dissipation by viscosity

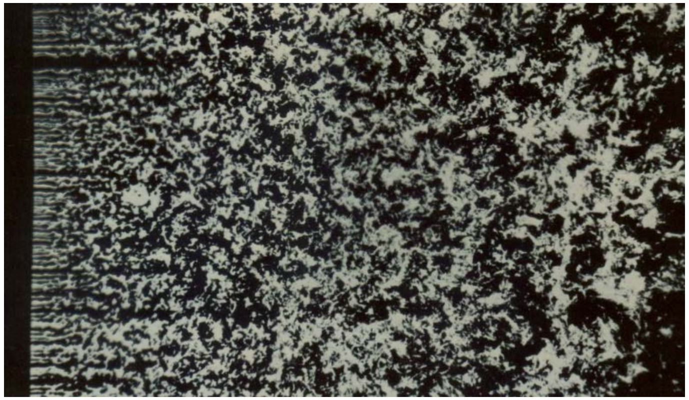
Kolmogorov (1941): In equilibrium, transfer rate = dissipation rate

8.1 Introduction

- The small scale motions tend to have small time scales.
- These motions are statistically independent of the relatively slow, large-scale turbulence and of the mean flow.

  → isotropic turbulence

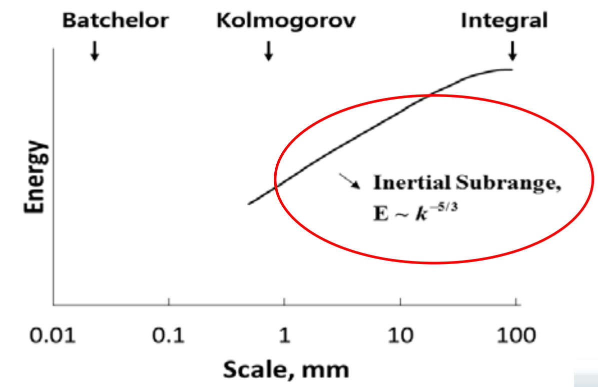
- The small scale motion should depend only on the rate of energy transfer from the larger scales and on the viscosity of the fluid.



8.1 Introduction

- **Kolmogorov's universal equilibrium theory of turbulence**

- The kinetic energy of the small and intermediate scale motions varies only at the rate at which the mean flow varies.
- The behavior of the intermediate scales (inertial subrange) is governed by the transfer of energy which, in turn, is exactly balanced by dissipation at the smallest scales.
- In equilibrium, the transfer process must be in equilibrium with the energy dissipation rate.



8.1 Introduction

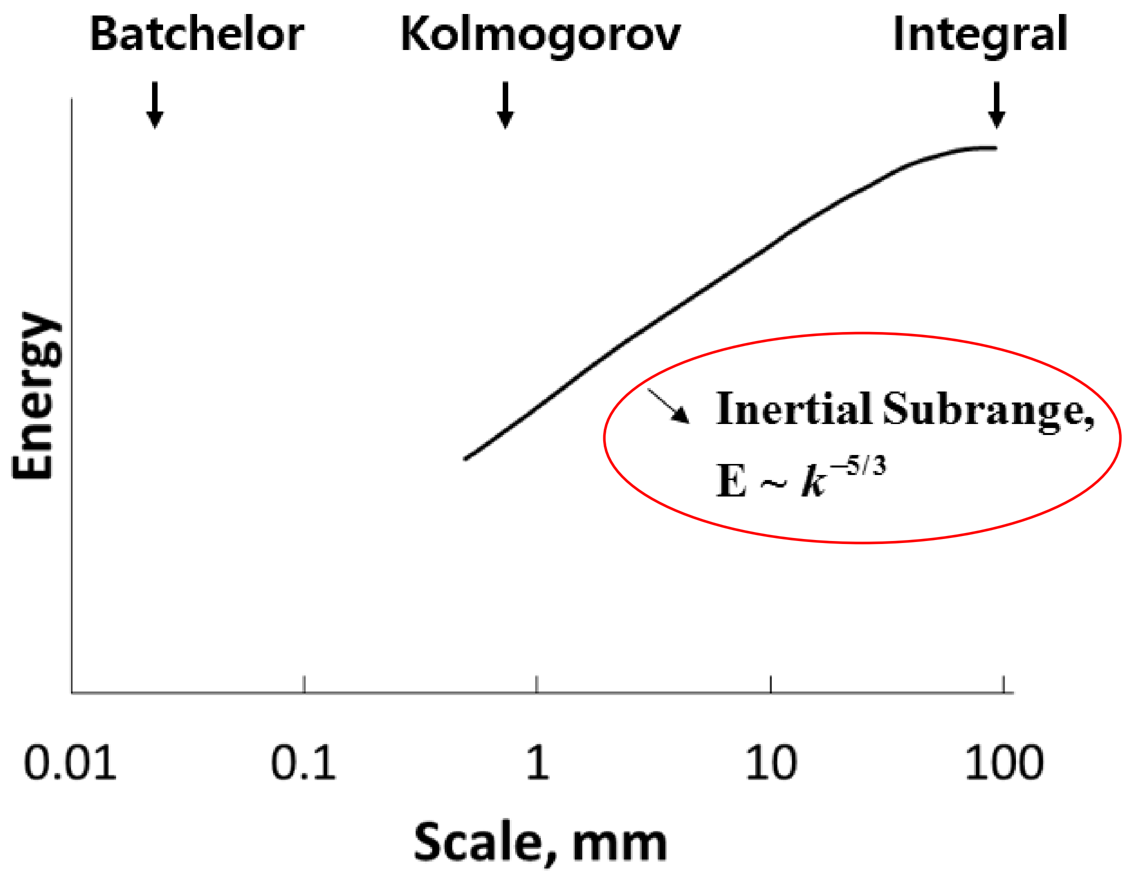
- **Energy spectrum**

- The energy spectrum characterizes the turbulent kinetic energy distribution as a function of length scale.
- The spectrum indicates the amount of turbulent kinetic energy contained at a specific length scale.
- For inertial subrange $(\eta \le L \le \ell)$, the energy spectrum will only be a function of the length scale and the dissipation rate. → five-third rule

$$\boxed{E = \alpha \varepsilon^{2/3} k^{-5/3}}$$

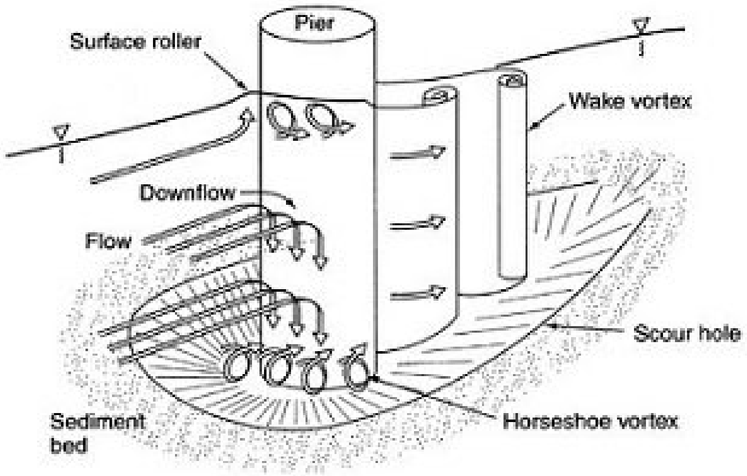
k = wave number ~ inverse of length

8.1 Introduction



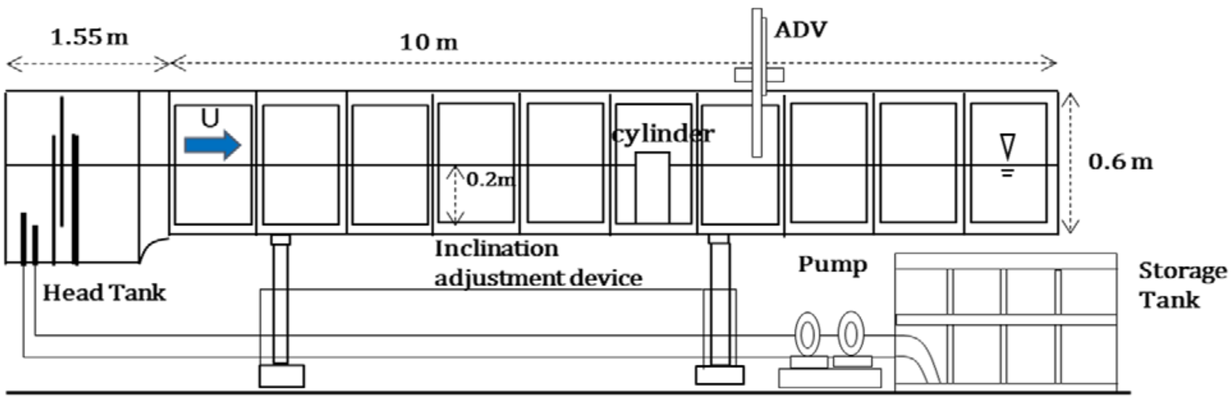
[Ex 2] Flow around the cylinder

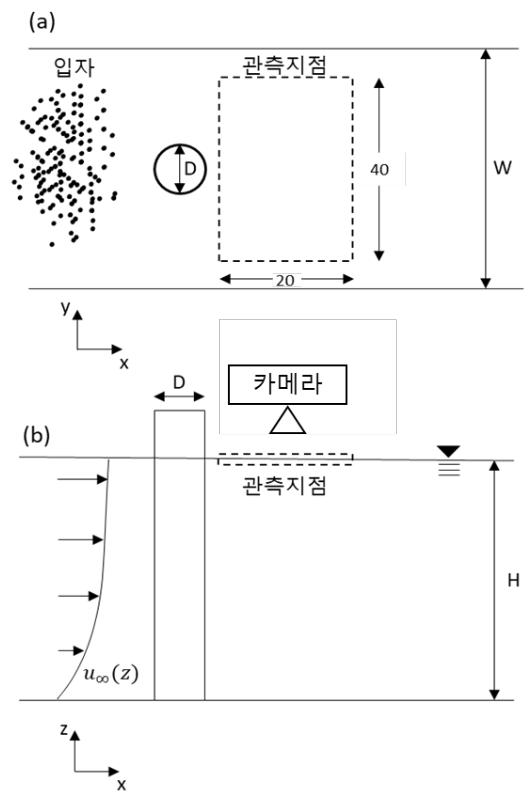
- ❐ Flow around the cylinder is classic and practical problems
- → Offshore structures, Bridge piers, Vortex Induced Vibration (VIV)
- ❐ Vortex: low frequency, narrow-band process
- ❐ Turbulence: high frequency



Experiments

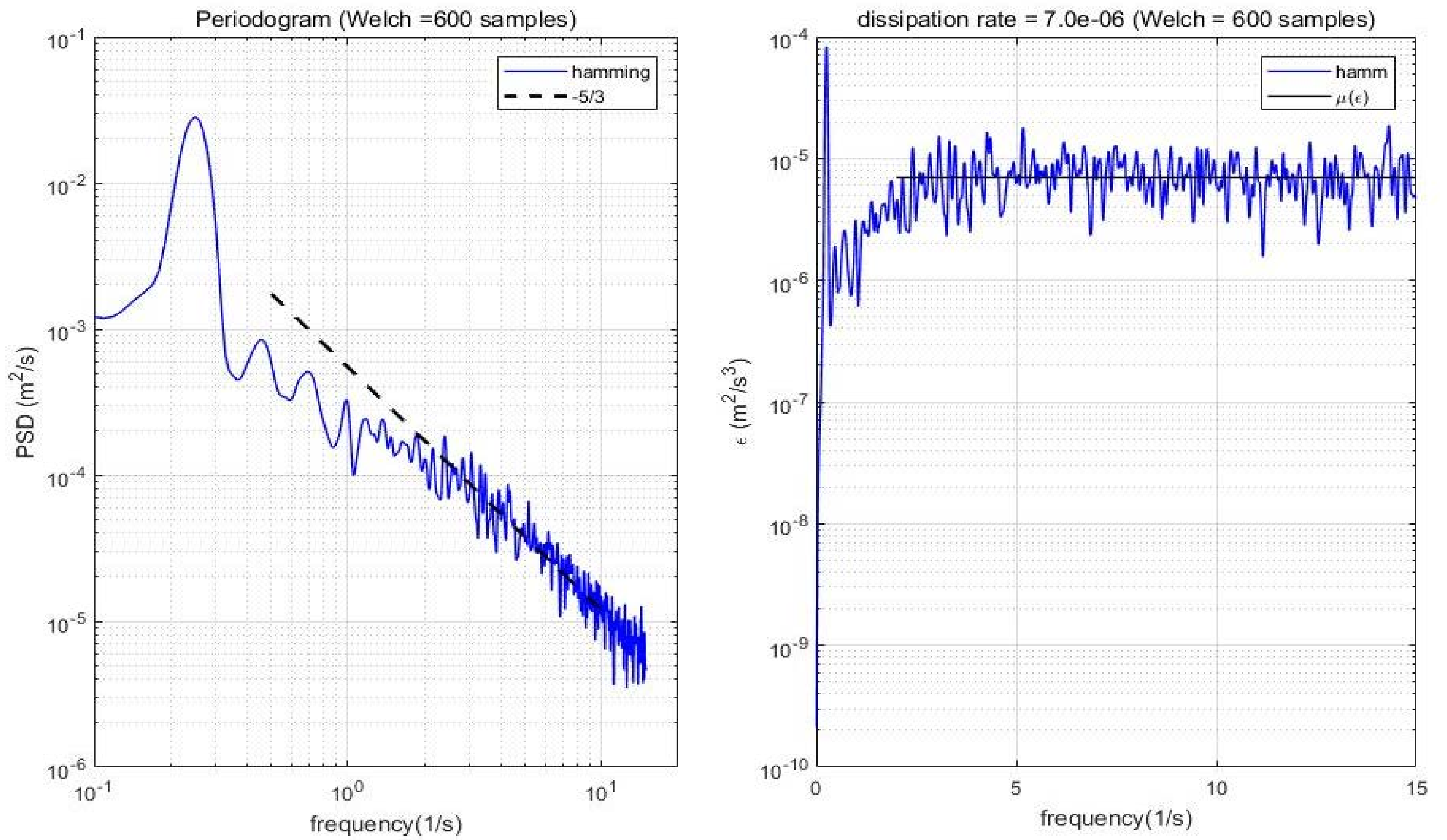
- 3D, 30Hz-3min velocity data was obtained by Acoustic Doppler Velocimeter (ADV).





Q=5L/s, H = 20cm B = 30cm, U=8.3cm/s, cylinder diameter = 9cm

9cm cylinder, h=10cm, x=3.5,6.5,9.5,12.5,16.5 ∞, y=0,$\pm$3,$\pm$6



Periodogram (Welch =600 samples)
hamming
-5/3
PSD (m^2/s)
frequency(1/s)
dissipation rate = 7.0e-06 (Welch = 600 samples)
hamm
$\mu(\epsilon)$
ϵ (m^2/s^3)
frequency(1/s)

- Smallest scale dissipated into heat

$\Phi_{vv}(f) = C\epsilon^{2/3} f^{-5/3}$ ($C = 1.53 \sim 1.68$) in inertial subrange.

$$\boldsymbol{\epsilon_x} \approx \boldsymbol{\epsilon_y} = O(10^{-5} \sim 10^{-6})\ (m^2/s^3)$$

- Kolmogorov microscale length (η), time (τ), velocity (υ)

$$\eta = (\nu^3/\epsilon)^{1/4} \approx 0.8(\mathrm{mm})$$
$$\tau = (\nu/\epsilon)^{1/2} \approx 0.4 - 0.8(\mathrm{sec})$$
$$\upsilon = (\nu\epsilon)^{1/4} \approx 1 - 1.5(\mathrm{mm/s})$$

$[Ex.1]$

$\varepsilon = 1.25 \times 10^{-6} m^2 / s^3$

$\eta = 0.7\ mm; \tau = 0.5s$

- Non-dimensional representation by the global system scale (D = 9cm, T = 4sec, U = 8.33cm/s)

$$\eta/D \approx \mathrm{O}(10^{-2})$$
$$\tau/\mathrm{T} \approx \mathrm{O}(10^{-1})$$
$$\upsilon/U \approx \mathrm{O}(10^{-2})$$

8.2 Sources of Turbulence

8.2.1 Source of turbulence

(1) Surfaces of flow discontinuity (velocity discontinuity)

1) tip of sharp projections – a), b)

2) trailing edges of air foils and guide vanes – c)

3) zones of boundary-layer separation – d)

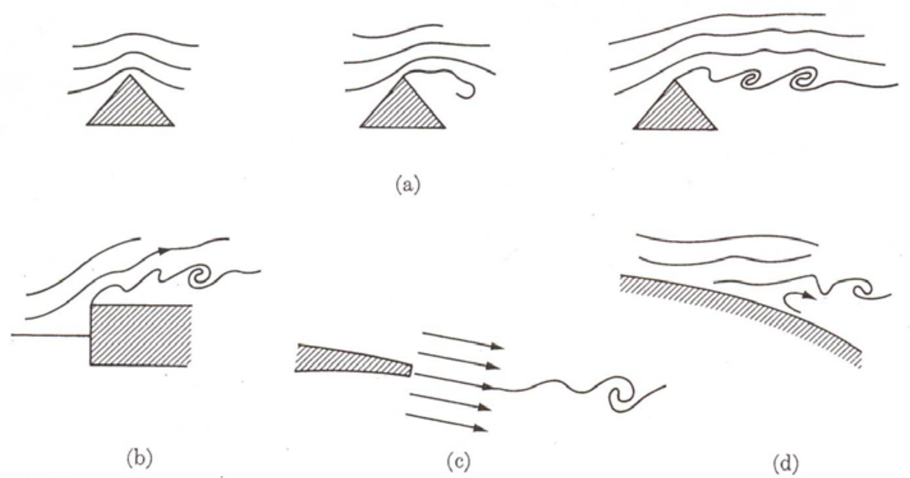


FIG. 11–1. Eddy formation at velocity discontinuity surfaces: (a) sharp projection; (b) bluff body; (c) trailing edge; (d) boundary-layer separation.

8.2 Sources of Turbulence

At surfaces of flow discontinuity,

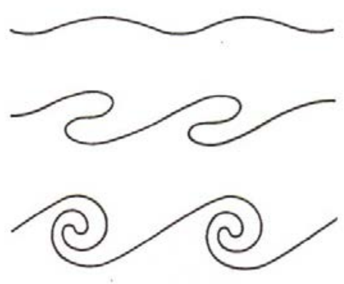
→ tendency for waviness to develop by accident from external cause or from disturbance transported by the fluid.

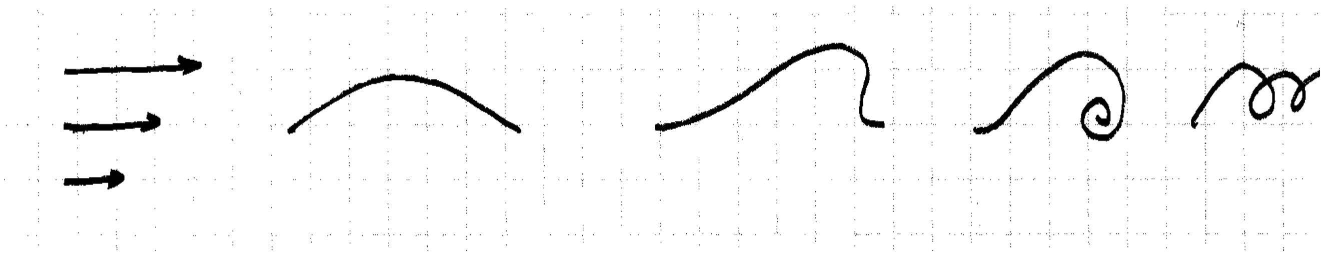
→ waviness tends to be unstable

→ amplify (grow in amplitude)

→ curl over

→ break into separate eddies



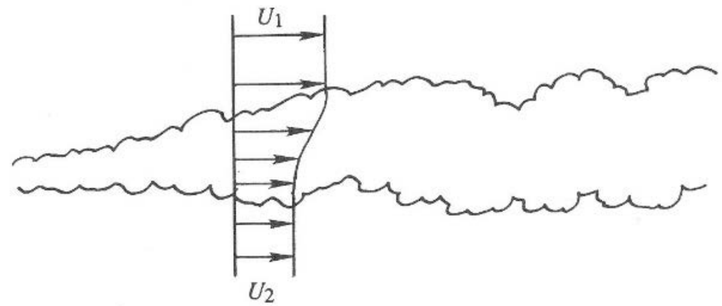


8.2 Sources of Turbulence

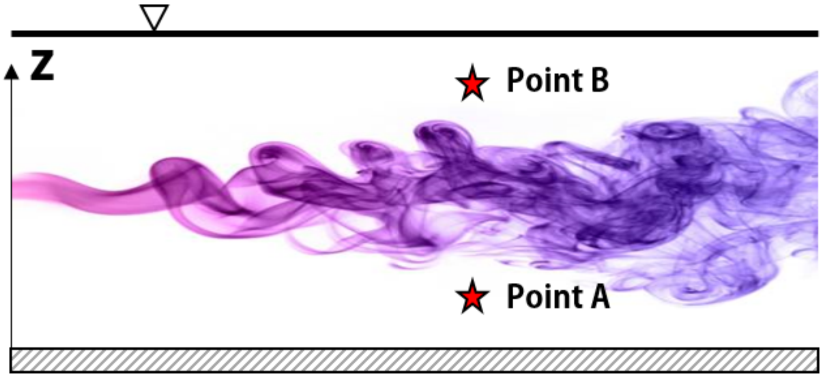
(2) Shear flows where velocity gradient occurs w/o an abrupt discontinuity

~ Shear flow is becoming unstable and degenerating into turbulence.

[Ex] Reynolds' experiment with a dye-streak in a glass tube



(c)



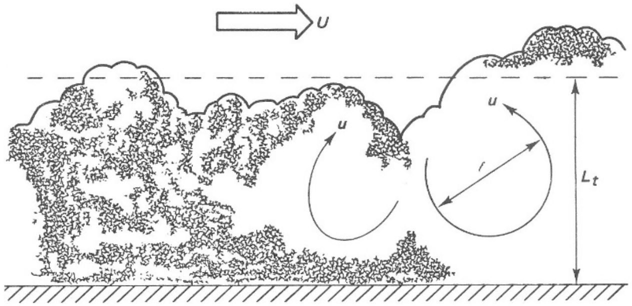
8.2 Sources of Turbulence

[Re] How turbulence arises in a flow

1) Presence of boundaries as obstacles creates vorticity inside a flow which was initially irrotational (vorticity, $\vec{\omega} = \nabla \times \vec{u}$).

2) Vorticity produced in the proximity of the boundary will diffuse throughout the flow which will become turbulent in the rotational regions.

3) Production of vorticity will then be increased due to vortex filaments stretching mechanism.



8.2 Sources of Turbulence

8.2.2 Mechanisms of instability

• **Tollmien-Schlichting's <u>small perturbation theory</u>**

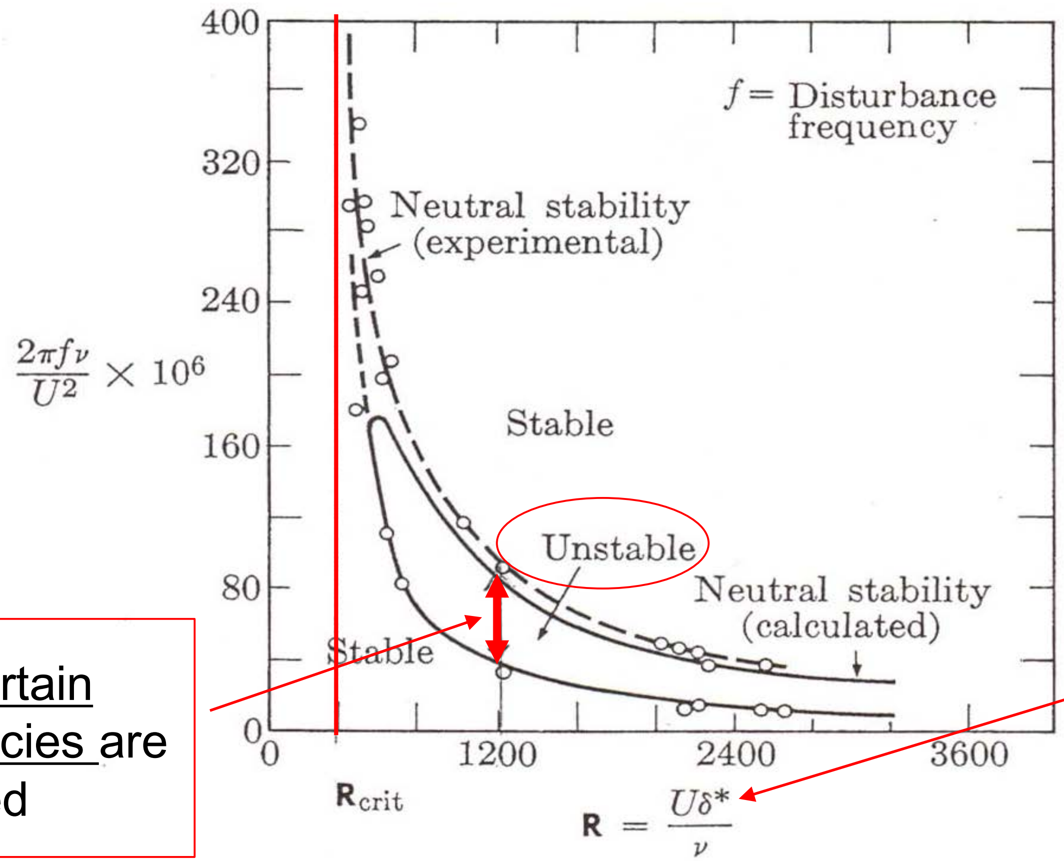
~ Disturbance are composed of oscillations of a range of frequencies which can be <u>selectively amplified</u> by the hydrodynamic flow field.

$\mathrm{Re} < \mathrm{Re}_{crit}$ → all disturbances will be damped

$\mathrm{Re} > \mathrm{Re}_{crit}$ → disturbances of <u>certain frequencies</u> will be amplified and others damped

8.2 Sources of Turbulence

Tollmien-Schlichting stability diagram



Only certain frequencies are amplified

mass displacement thickness

8.2 Sources of Turbulence

- **Wave characteristics**

wavelength (m), λ

wave period (sec), T

wave velocity (m/s), V

wave frequency (Hz), f

wave number (rad/m): k

$$T = \frac{\lambda}{V}$$

$$f = \frac{1}{T}$$

$$f = \frac{V}{\lambda}$$

$$k = \frac{2\pi}{\lambda}$$