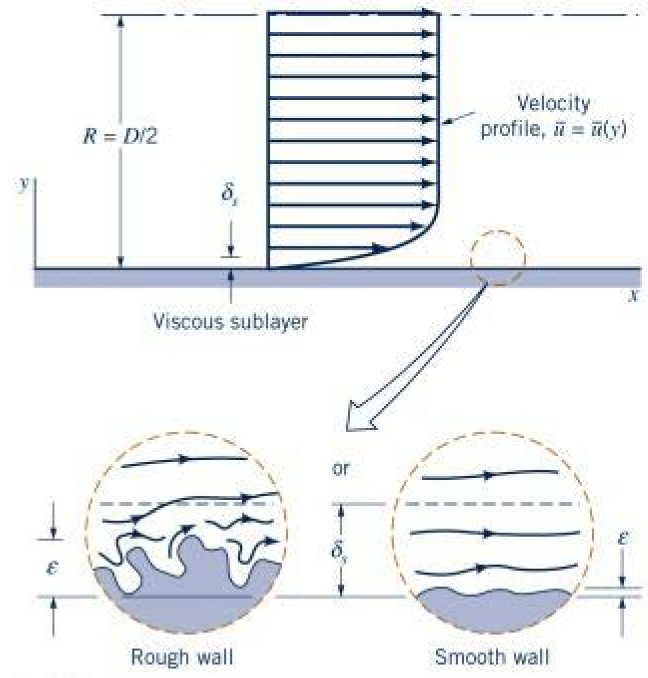
Ch. 4 Steady Flow in Pipes

4-2 Turbulent Flow in Pipes



Contents

Today's objectives

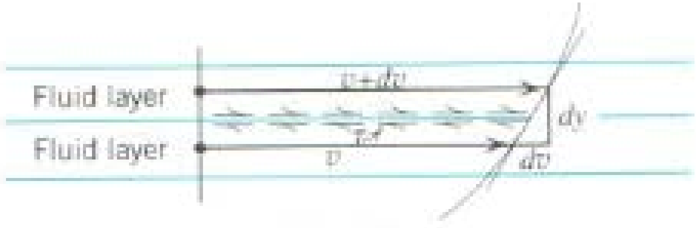
- Review the shear stress for laminar and turbulent flows
- Apply Prandtl's theory to the smooth pipe flows
- Figure out the wall law for flow in smooth pipe
- Similarly to the smooth pipe case, turbulent flow in rough pipe will be understood.
- Classifying the pipes whether they are smooth or rough
- Evaluating pipe friction factors in the given condition

4.3 Turbulent flow in smooth pipe

1. Shear stress

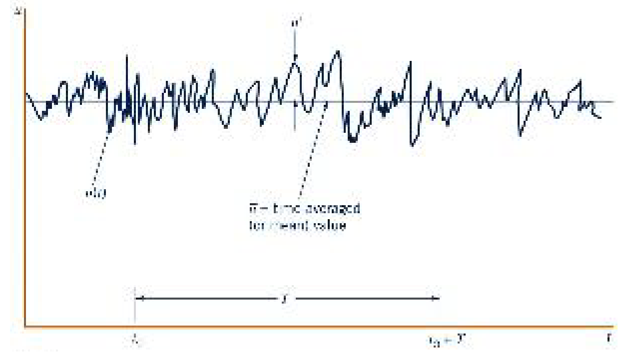
- Shear stress between adjacent layers in a simple parallel flow is determined by the viscosity and the velocity gradient

$$\tau = \mu \frac{dv}{dy}$$



- If laminar flow is disturbed by an obstacle or roughness, then disturbances must be damped by the molecular viscosity (stable)
- However, in turbulent flow, inertia overcomes the viscous force and disturbance grows and becomes random motion.

$$R_e = \frac{F_I}{F_V} = \frac{\rho V^2 l^2}{\mu V l} = \frac{\rho V l}{\mu} = \frac{V l}{\mu / \rho} = \frac{V l}{\nu}$$



- turbulent shear stress (**Reynolds** stress)

$$\tau = -\rho \overline{u'v'}$$

- **Boussinesq** modeled the Reynolds stress with gradient transport theorem (simply mimicking the molecular viscosity),and **Prandtl** suggested the mixing length model

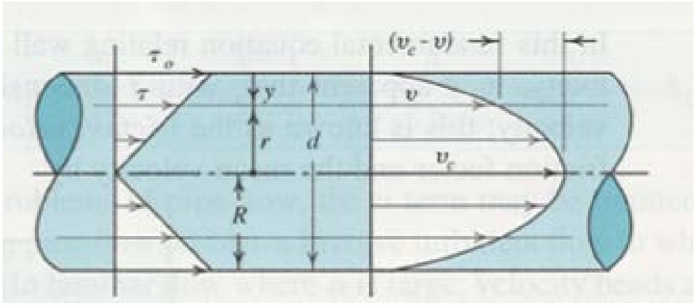
$$\tau = -\rho \overline{u'v'} = \epsilon \frac{dv}{dy} = \rho l^2 \left(\frac{dv}{dy} \right)^2 \qquad \epsilon = \rho l^2 \left(\frac{dv}{dy} \right)$$

- Finally, total shear stress is the sum of the viscous shear stress and the turbulent shear stress

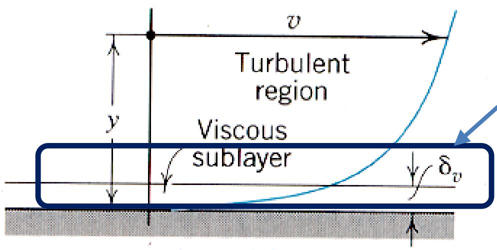
$$\tau = \tau_m + \tau_t = \mu \frac{dv}{dy} + \epsilon \frac{dv}{dy} = \mu \frac{dv}{dy} - \rho \overline{u'v'}$$

- Shear stress in turbulent pipe flow

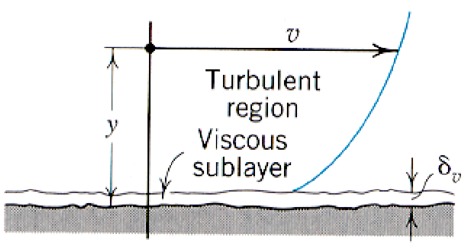
- *t* is maximum at the pipe wall due to viscous shear stress in the viscous sublayer.
- *t* is decreasing linearly with *y* from the wall.



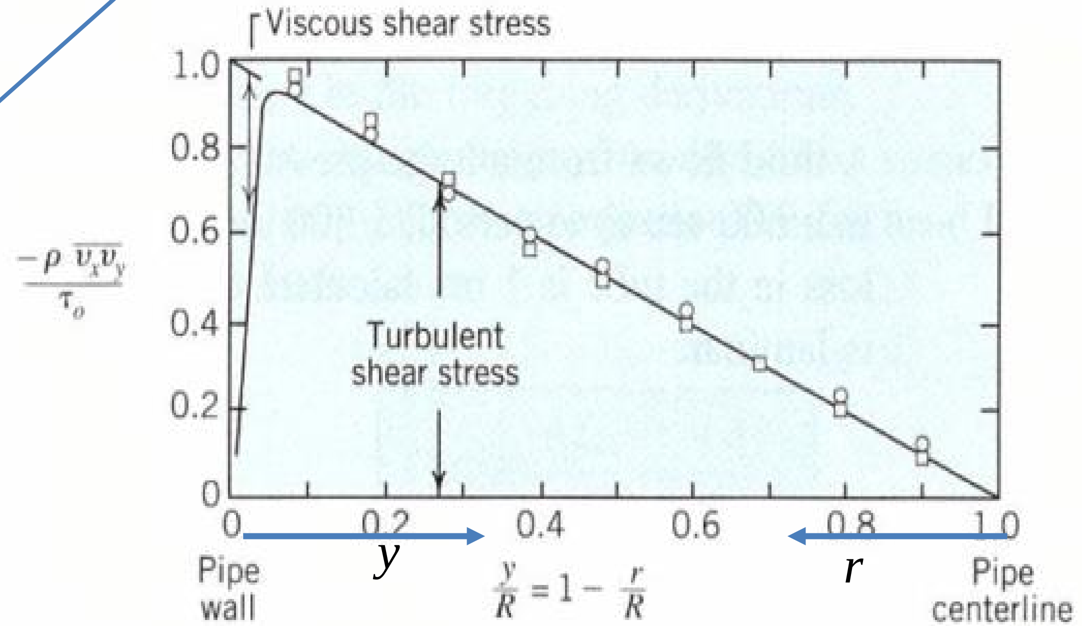
$$\tau = \tau_0\left(1 - \frac{y}{R}\right) = \frac{\tau_0 r}{R}$$



(a) **Smooth boundary**



(b) **Rough boundary**



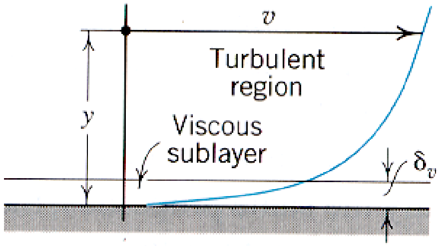
2. Prandtl's mixing length theory

- This figure shows linear variation of total shear stress in a turbulent pipe flow.
- For smooth pipes, discussion (Section 7.3) strongly suggests the existence of a viscous sublayer near the pipe walls.

- Employing the Prandtl relationship by assuming the viscous stress is negligible over most of the flow

$$\tau = \rho l^2 \left(\frac{dv}{dy} \right)^2 - \cancel{\mu \frac{dv}{dy}} = \tau_0 \left(1 - \frac{y}{R} \right) \Rightarrow \boxed{\rho l^2 \left(\frac{dv}{dy} \right)^2 = \tau_0 \left(1 - \frac{y}{R} \right)} \qquad (9.12)$$

where y is measured from the pipe wall



(a) Smooth boundary

3. Velocity profile of turbulent flow in pipe

3.1 Nikuradse's empirical equation

- From Nikuradse's measurement in experiments: smooth or rough but uniform sand grain, all velocity profiles could be represented by

$$\boxed{\frac{v_c - v}{v_*} = -2.5 \ln \frac{y}{R} \quad \text{(All pipes)}} \qquad (9.13)$$

Differentiate Eq. 9.13 wrt *y*, then $dv/dy \propto 1/y$, and from (9.12)

$$\rho l^2 \left(\frac{dv}{dy}\right)^2 \propto \rho l^2 \left(\frac{1}{y}\right)^2 \propto \tau_0 \left(1 - \frac{y}{R}\right) \rightarrow l \propto \sqrt{\frac{\tau_0}{\rho}}\, y \sqrt{1 - \frac{y}{R}} \propto v^* y \sqrt{1 - \frac{y}{R}}$$

Near wall, $l = \kappa y$ then in a pipe flow, length scale follows;

$$\boxed{l = \kappa y \left(1 - \frac{y}{R}\right)^{1/2}} \qquad (9.14)$$

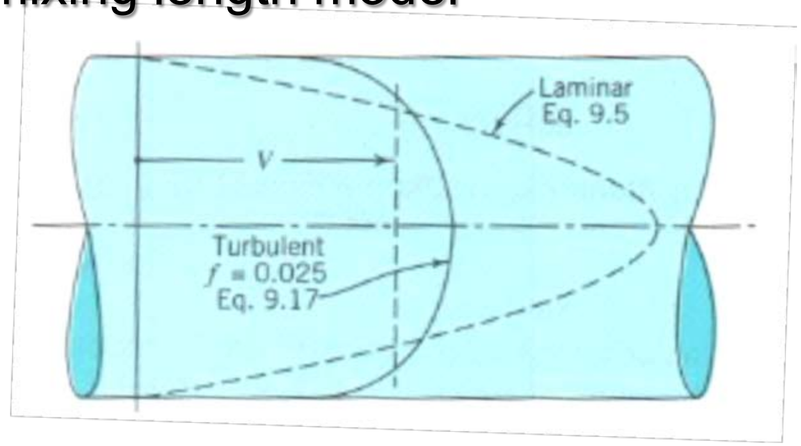
3.2 Theoretical equation from Prandtl's mixing length model

Eq. (9.12) now becomes

$$\left(\frac{dv}{dy}\right)^2 = \frac{\tau_0}{\rho\kappa^2 y^2}$$

$$\frac{dv}{dy} = \frac{\sqrt{\tau_0 / \rho}}{\kappa y} = \frac{v_*}{\kappa y}$$

(9.15)



Integrating Eq. 9.15 produces

$$v_c = \frac{v_*}{\kappa} \ln R + C$$

$$C = v_c - \frac{v_*}{\kappa} \ln R$$

$$\boxed{\frac{v_c - v}{v_*} = -\frac{1}{\kappa} \ln \frac{y}{R} \qquad (\kappa = 0.4)}$$

(9.13)

[Remark]

- Near wall $(y \ll R)$, $\boxed{l = \kappa y}$

  Then (9.12) can be written

$$\rho \kappa^2 y^2 \left(\frac{dv}{dy}\right)^2 = \tau_0 \left(1 - \frac{y}{R}\right)$$

$$\left(\frac{dv}{dy}\right)^2 = \frac{\tau_0}{\rho \kappa^2 y^2}$$

$$\frac{dv}{dy} = \frac{\sqrt{\tau_0 / \rho}}{\kappa y} = \frac{v_*}{\kappa y}$$

$$v = \frac{v_*}{\kappa} \ln y + C$$

- $v=v_c$ at $y=R$ (center)

$$v_c = \frac{v_*}{\kappa} \ln R + C, \quad C = v_c - \frac{v_*}{\kappa} \ln R$$

- So,

$$\boxed{\frac{v_c - v}{v_*} = -\frac{1}{\kappa} \ln \frac{y}{R} \qquad (\kappa = 0.4)}$$

In all pipes (no matter whether rough or smooth)

4. Wall law

- For smooth pipe, at very near wall (viscous sublayer), velocity profile is linear and to match with Nikuradse's experiments, then

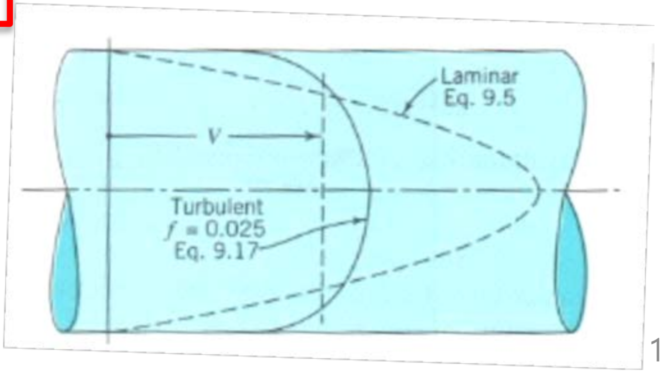
$$(9.13): \frac{v}{v_*} = 2.5\ln\frac{y}{R} + \frac{v_c}{v_*} \longrightarrow \frac{v}{v_*} = 2.5\ln\frac{v_* y}{\nu} + 5.5 \qquad (9.17)$$

$$\nu = \frac{\mu}{\rho}$$

- In terms of common logarithms,

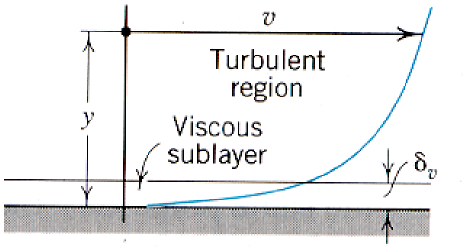
$$\boxed{\frac{v}{v_*} = 5.75\log\frac{v_* y}{\nu} + 5.5} \qquad (9.17)$$

- This is the general equation of the velocity profile for turbulent flow in smooth pipes.



- For smooth pipe, a viscous sublayer must exist near the smooth wall, and the velocity profile is given by

$$\boxed{\frac{v}{v_*} = \frac{v_* y}{\nu}} \qquad (9.7)$$



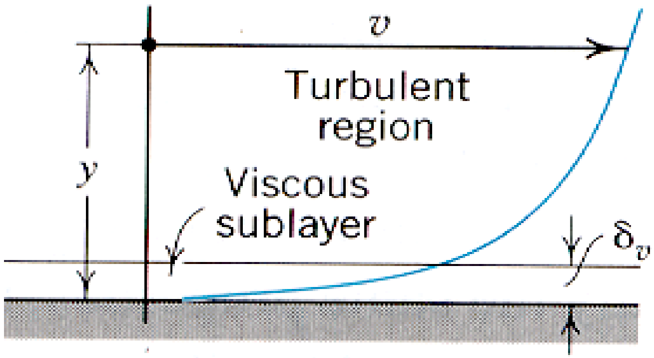
(a) Smooth boundary

- The nominal extent of the viscous sublayer, *y'*, is obtained by finding the intersection of the viscous profile of Eq. 9.7 with the turbulent profile given by Eq. 9.17. The sublayer thickness δ_ν is given as

$$\frac{v}{v_*} = \frac{v_* \delta_\nu}{\nu} = 5.75 \log \frac{v_* \delta_\nu}{\nu} + 5.5 \quad (at\ y = y')$$

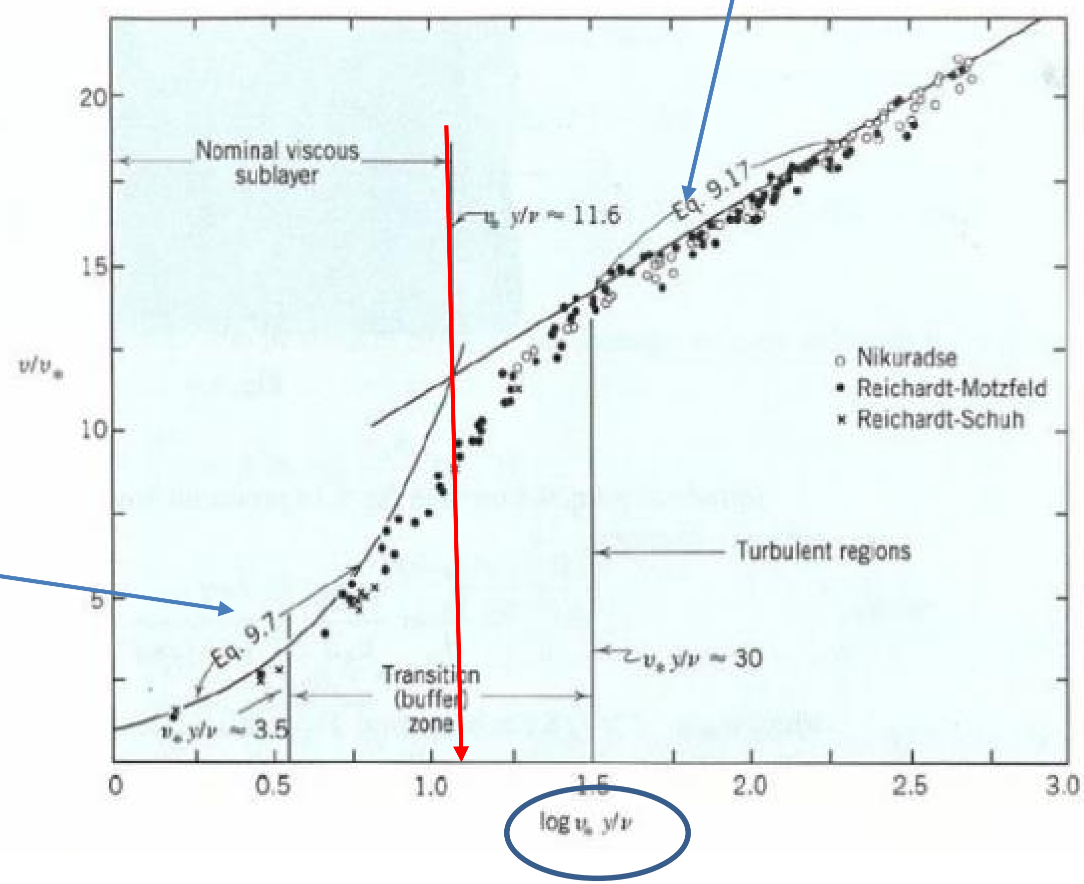
$$\frac{v_* \delta_\nu}{\nu} = 11.6 \qquad \boxed{\delta_\nu = 11.6 \frac{\nu}{v_*}} \qquad (9.18)$$

$$\frac{v}{v_*} = 5.75 \log \frac{v_* y}{\nu} + 5.5$$



(a) **Smooth boundary**

$$\frac{v}{v_*} = \frac{v_* y}{\nu}$$



Velocity distribution near a smooth wall

5. Mean velocity

- Mean velocity, *V*

$$Q=\int_0^R v(2\pi r dr)=2\pi v_* \int_0^R \left(5.75\log\frac{v_*(R-r)}{\nu}+5.5\right) r dr$$

$$\boxed{\frac{V}{v_*}=\frac{Q}{\pi R^2 v_*}=5.75\log\frac{v_* R}{\nu}+1.75} \qquad (9.19)$$

- Maximum velocity (velocity at the center)

$$\frac{v_c}{v_*}=5.75\log\frac{v_* R}{\nu}+5.5$$

$$\boxed{v_*=V\sqrt{f/8}}$$

- Subtract the mean from the maximum, then adjust to the experiment

$$\boxed{\frac{v_c}{V}=1+4.07\sqrt{f/8}} \qquad (9.20)$$

6. Friction factor in turbulent flow of smooth pipe

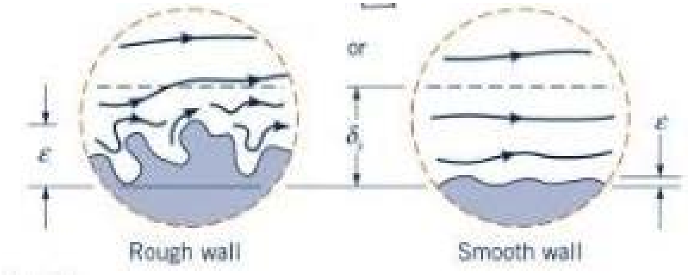
- Friction factor *f* can be obtained by introducing Eq. 9.4 into Eq. 9.19, and adjusting the result based on the experiments

$$\boxed{\frac{1}{\sqrt{f}} = 2.0\log\left(\mathrm{Re}\sqrt{f}\right) - 0.8} \qquad \mathrm{Re} = \frac{Vd}{\nu} \qquad (9.21)$$

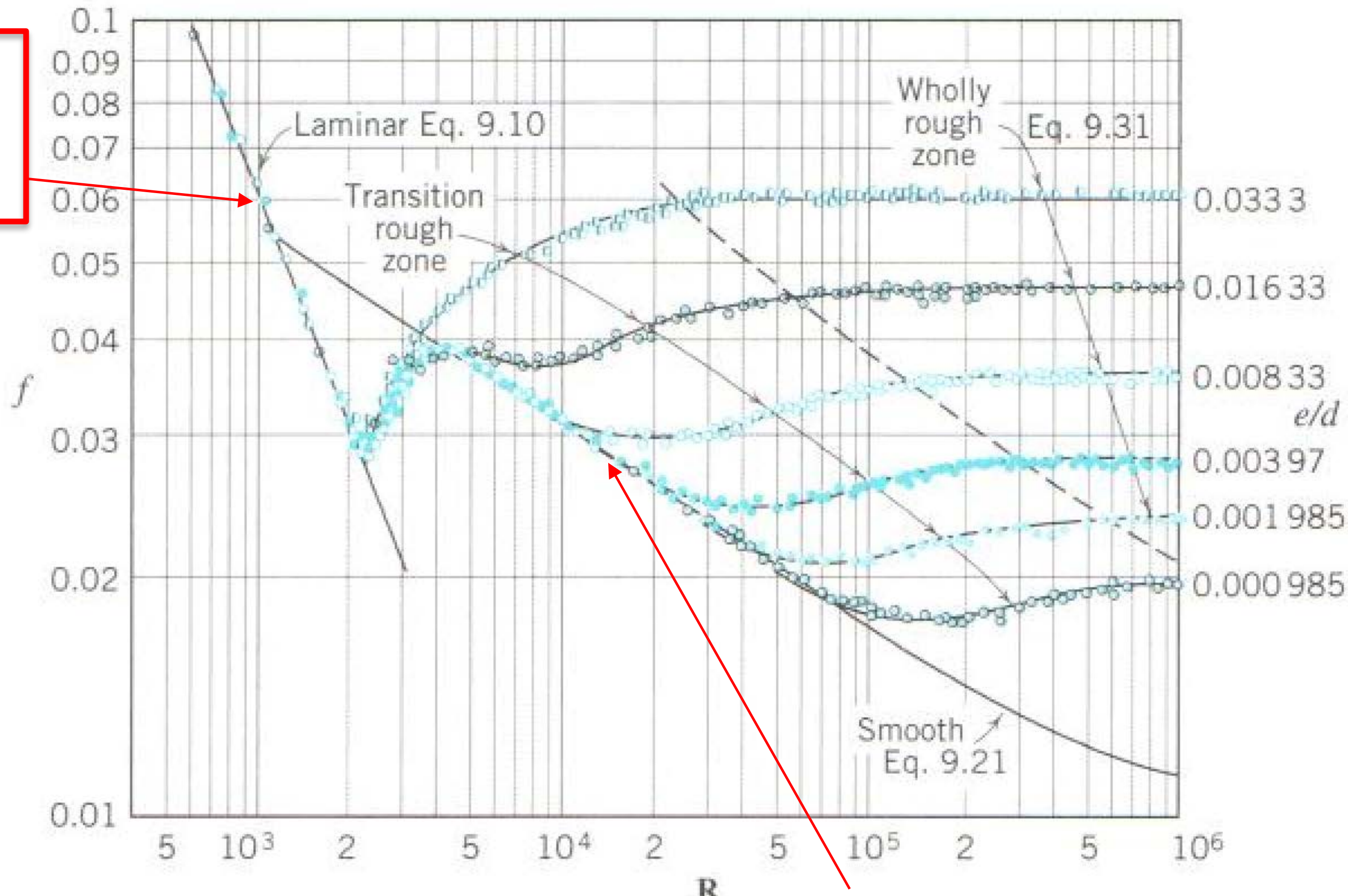
- Introducing Eq. 9.4 into Eq. 9.18 yields the expression for the <u>laminar sublayer thickness</u>

$$\frac{\delta_v}{d} = \frac{11.6\nu}{v_* d} = \frac{11.6\nu}{V\sqrt{f/8}\,d} = \frac{32.8}{\mathrm{Re}\sqrt{f}} \qquad (9.22)$$

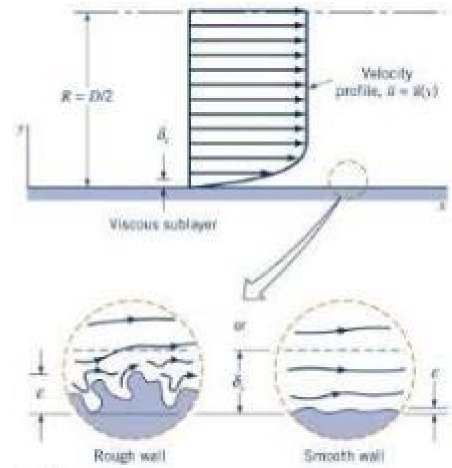
- It means that <u>laminar sublayer is decreasing with increase of Reynolds number</u>
→ rough wall



$$f = \frac{64\mu}{Vd\rho} = \frac{64}{\mathrm{Re}}$$



$$\frac{1}{\sqrt{f}} = 2.0\log\left(\mathrm{Re}\sqrt{f}\right) - 0.8$$



- Rewrite Eq. (9.22)

$$\mathrm{Re}\sqrt{f} = \frac{32.8}{\dfrac{\delta_v}{d}}$$

- Substituting this into Eq. 9.21 yields the equation below

$$\frac{1}{\sqrt{f}} = 2.0\log\left(\frac{32.8}{\boxed{(\delta_v / d)}}\right) - 0.8 \qquad (9.23)$$

- For turbulent flow over smooth walls, the friction factor is a function of the ratio of the sublayer thickness to the pipe diameter.

Example problem #1 (pp. 334-335)

- Water at 20°C flows in a 75 mm diameter smooth pipeline. According to a wall shear meter, $\tau_0 = 3.68\ N/m^2$. Calculate the thickness of the viscous sublayer, the friction factor, the mean velocity and flowrate, the centerline velocity, the shear stress and velocity 20 mm from the pipe centerline, and the head lost in 1,000 m of this pipeline.

  - Thickness of the viscous sublayer
  - Friction factor
  - Mean velocity
  - Flowrate
  - Centerline velocity
  - Shear stress
  - Head loss

Example problem 1

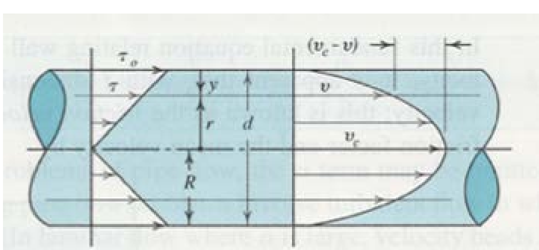
- Thickness of the viscous sublayer $\quad \mathrm{v}_* = \sqrt{\frac{\tau_0}{\rho}} = 0.061\,\mathrm{m/s};\ \delta_\nu = 11.6\frac{\nu}{\mathrm{v}_*} = 0.19\,\mathrm{mm}$
- Friction factor $\quad \frac{1}{\sqrt{f}} = 2.0\log\left[\frac{32.8}{\delta_\nu / d}\right] - 0.8 \rightarrow f = 0.018$
- Mean velocity $\quad \frac{\mathrm{V}}{\mathrm{v}_*} = 5.75\log\frac{\mathrm{v}_*\mathrm{R}}{\nu} + 1.75;\ \mathrm{V} = 1.29\,\mathrm{m/s}$
- Flowrate $\quad Q = VA = 0.0057\,m^3 / s$
- Centerline velocity $\quad \frac{v_c}{V} = 1 + 4.07\sqrt{\frac{f}{8}} \rightarrow v_c = 1.54\,m / s$
- Shear stress $\quad \tau_{y=25mm} = 2.45\,N / m^2$
- Head loss $\quad h_L = f\frac{l}{d}\frac{V^2}{2g} = \boxed{20.4\,m\,of\ water}$

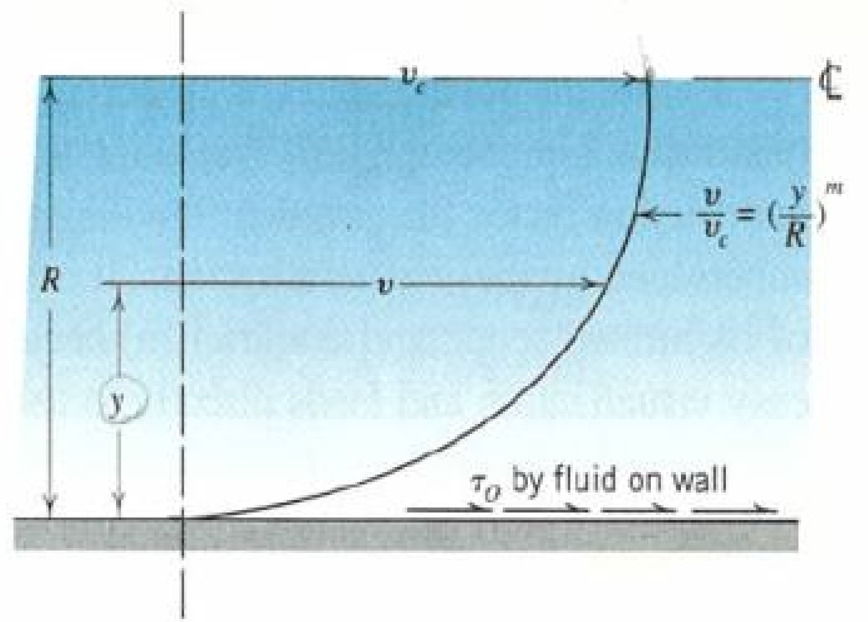
7. Blasius' equation

- Before the generalization by Prandtl, von Karman, and Nikuradse, **Blasius (1913)** developed simpler equations.

- Blasius work has limited scope and empiricism.
  - Valid only for 3,000<Re<100,000
  - Often called seventh-root law because the turbulent velocity profile is given by

$$\frac{v}{v_c} = \left(\frac{y}{R}\right)^{1/7}$$





- For 3,000<Re<100,000, Blasius showed that the friction factor could be closely approximated by the equation

$$\boxed{f = \frac{0.316}{\mathrm{Re}^{0.25}}} \tag{9.24}$$

- Derivation of Blasius velocity profile and shear stress.

$$\tau_0 = \frac{f\rho V^2}{8} = \frac{0.316}{\left(2RV\rho/\mu\right)^{0.25}}\frac{\rho V^2}{8} = 0.0332\mu^{1/4}R^{-1/4}V^{7/4}\rho^{3/4}$$

- Blasius then assumed the velocity profile as a power relationship

$$\boxed{\frac{\mathrm{v}}{\mathrm{v_c}} = \left(\frac{\mathrm{y}}{\mathrm{R}}\right)^{\mathrm{m}}}$$

$$\mathrm{Q} = \mathrm{V}\pi\mathrm{R}^2 = \int_{\mathrm{R}}^{0} \mathrm{v_c}\left(\frac{\mathrm{y}}{\mathrm{R}}\right)^{\mathrm{m}} 2\pi(\mathrm{R}\text{-}\mathrm{y})(\text{-}\mathrm{dy})$$

Therefore,

$$\frac{V}{v_c} = \frac{2}{(m+1)(m+2)}$$

$$V = \frac{2}{(m+1)(m+2)} v\left(\frac{R}{y}\right)^m$$

$$\tau_0 = 0.0332 \mu^{1/4} R^{-1/4} V^{7/4} \rho^{3/4}$$

$$= 0.0332 \left[\frac{2}{(m+1)(m+2)}\right] \mu^{1/4} R^{-1/4+(7m/4)} v^{7/4} y^{-(7m/2)} \rho^{3/4}$$

But, wall shear stress could not be affected by pipe radius (*R*), so *m* = 1/7.

Finally,

$$\boxed{\frac{v}{v_c} = \left(\frac{y}{R}\right)^{1/7}}$$

→ seventh-root law

At this moment,

$$\tau_0 = 0.0464\left(\frac{\mu}{v_c \rho R}\right)^{1/4} \frac{\rho v_c^2}{2}$$

Example problem #2 (pp. 337-338)

- For the conditions of previous examples, calculate the friction factor, wall shear stress, centerline velocity and the velocity 25 mm from the pipe centerline using the seventh root law.

Check whether we can use Blasius eq. The Reynolds number is

$$\mathrm{Re} = \frac{Vd}{\nu} = \frac{1.29 m/s \times 0.075 m}{1\times10^{-6} m^2/s} = 96,750 < 100,000$$

Then

$$f = \frac{0.316}{\mathrm{Re}^{0.25}} = \frac{0.316}{(96,750)^{0.25}} = 0.018$$

The centerline velocity.

$$\frac{V}{v_c} = \frac{2}{(m+1)(m+2)} = \frac{2}{(8/7)(15/7)} = \frac{49}{60}$$

1.54 m/s in
the previous example

$$v_c = \frac{60}{49}V = \frac{60}{49}\times 1.29(\mathit{from}\ \text{the previous example}) = 1.58 m/s$$

Example

- The shear stress

$$\tau_0 = 0.0464\left(\frac{\nu}{v_c R}\right)^{0.25}\frac{\rho v_c^2}{2} = 3.70\text{Pa}$$

3.68 Pa given in
the previous example

4.4 Turbulent flow in rough pipes

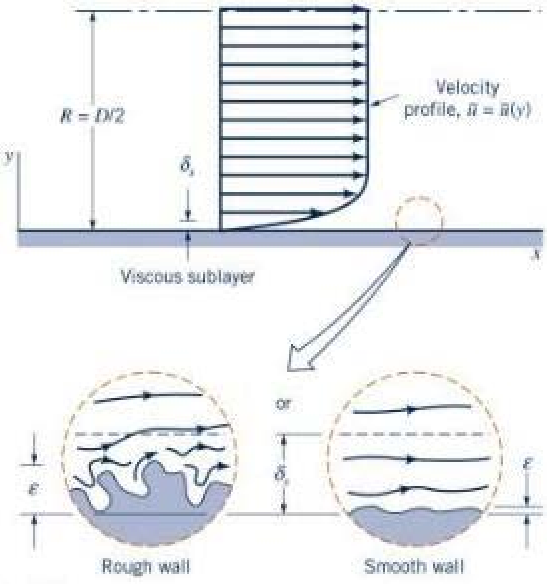
1. Velocity profile

- Pipe friction in rough pipe will be governed primarily by the size and patterns of the roughness
- Velocity profile follows logarithm when the flow is turbulent.
- When roughness is high, the viscous layer will be canceled and viscosity may not be the important parameter.
- Experiment by Nikuradse proved that the viscosity can be replaced by the roughness in the rough wall condition.

$$\boxed{\frac{v}{v_*} = 5.75\log\frac{y}{e} + 8.5}$$

e = sand grain diameter

$$\text{Smooth pipe: } \frac{v}{v_*} = 5.75\log\left(\frac{v_*}{\nu}y\right) + 5.5$$



Q (flow rate) for pipe flow can be determined as

$$Q = \int_0^R v(2\pi r dr) = 2\pi v_* \int_0^R \left[5.75\log\left(\frac{R-r}{e}\right) + 8.5 \right] r dr$$

The mean velocity is

$$\boxed{\frac{V}{v_*} = 5.75\log\frac{R}{e} + 4.75}$$

$$\frac{v_c}{v_*} = 5.75\log\frac{R}{e} + 8.5$$

Since

$$v_* = V\sqrt{\frac{f}{8}}$$

$$\frac{1}{\sqrt{f}} = 2.03\log\frac{R}{e} + 1.68$$

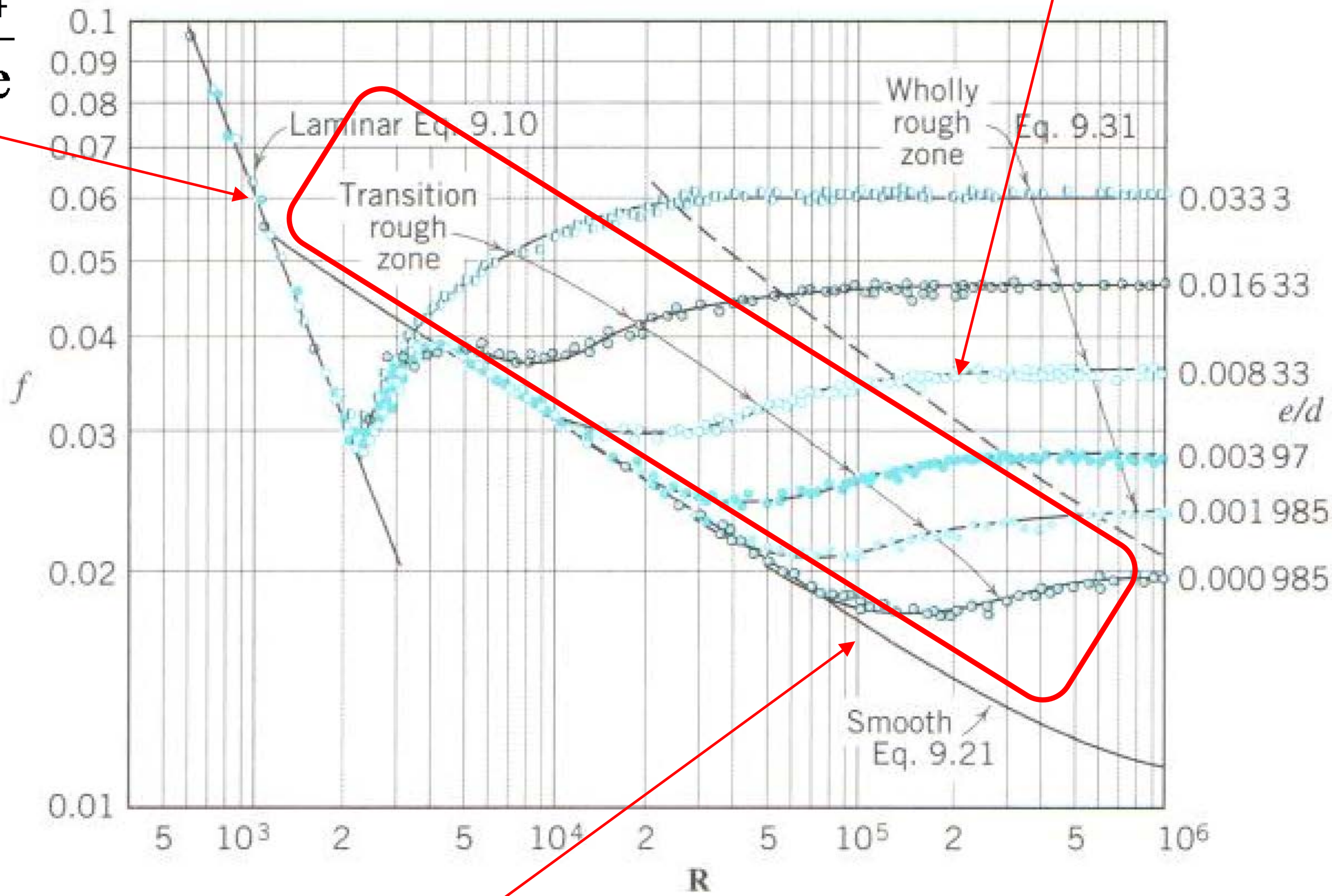
With adjustment by experimental data by Nikuradse,

$$\boxed{\frac{1}{\sqrt{f}} = 2.0\log\frac{d}{e} + 1.14}$$

Rough pipe's friction factor

$$\frac{1}{\sqrt{f}} = 2.0\log\frac{d}{e} + 1.14$$

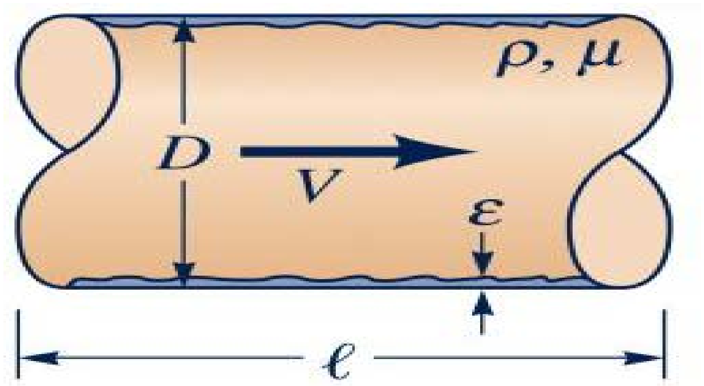
$$f = \frac{64\mu}{Vd\rho} = \frac{64}{\text{Re}}$$



$$\frac{1}{\sqrt{f}} = 2.0\log\left(\text{Re}\sqrt{f}\right) - 0.8$$

IP 9.6; pp. 339-340

- The mean velocity in a 300 mm pipeline is 3m/s. The relative roughness of the pipe is 0.002, and the kinematic viscosity of the water is 9×10^{-7} m^2/s. Determine the friction factor, the centerline velocity, the velocity 50 mm from the pipe wall, and the head lost in 300 m of this pipe under the assumption that the pipe is rough.
  - Friction factor
  - Centerline velocity
  - Velocity at y=50 mm from the wall
  - Head loss



Example

- Friction factor $$\frac{1}{\sqrt{f}} = 2.0\log\frac{d}{e} + 1.14 \rightarrow f = 0.0234$$
- Centerline velocity $$v_* = V\sqrt{\frac{f}{8}} = 0.162\, m/s$$

$$\frac{v_c}{v_*} = 5.75\log\frac{R}{e} + 8.5 \rightarrow v_c = 3.61\, m/s$$

- The velocity at 50 mm from the wall

$$\frac{v_{50}}{v_*} = 5.75\log\frac{y_{50}}{e} + 8.5 \rightarrow v_{50} = 3.17\, m/s$$

- Head loss

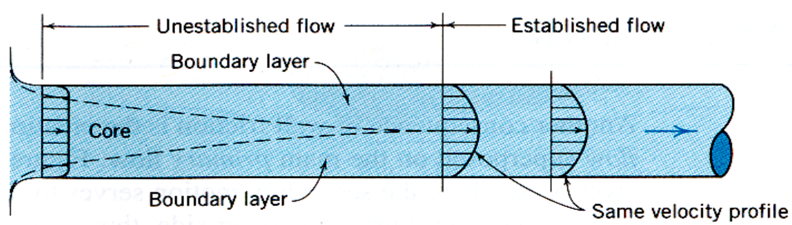
$$h_L = f\frac{l}{d}\frac{V^2}{2g_n} \rightarrow h_L = \boxed{10.7\, m\, of\ water}$$

4.5 Classification of smoothness and roughness

- For a turbulent flow in a smooth pipe, the characteristic length is the viscous sublayer thickness δ_v, and for a rough pipe it is the roughness height e.
- In case of transition, e/δ_v must be a significant parameter.
- In laminar flow, viscous sublayer thickness is the radius of pipe since viscosity governs the whole flow in pipe.

$$\delta_v = R$$

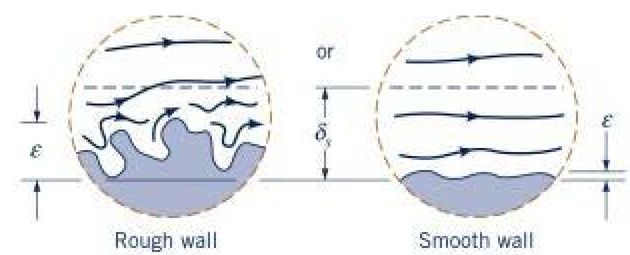
$$f = \frac{64}{\mathrm{Re}}$$



- In turbulent flow,

$$\frac{e}{\delta_v} = \frac{e/d}{\delta_v/d} = \frac{e/d}{32.8/\mathrm{Re}\sqrt{f}} = \left(\frac{e}{d}\right)\frac{\mathrm{Re}\sqrt{f}}{32.8}$$

$$\frac{e}{d}\mathrm{Re}\sqrt{f} = 32.8\frac{e}{\delta_v}$$



Eq. 9.22 $\delta_v / d = \dfrac{32.8}{\mathrm{Re}\sqrt{f}}$

2.1 Classification by friction factor

- Now we can plot the friction factor versus $\frac{e}{d}\mathrm{Re}\sqrt{f}$ or $\frac{e}{\delta_v}$
- For fully rough flow,

$$\boxed{\frac{1}{\sqrt{f}} - 2.0\log\frac{d}{e} = 1.14} \qquad (9.31)$$

Thus, it is convenient to plot $\frac{1}{\sqrt{f}} - 2.0\log\frac{d}{e}$ versus $\frac{e}{d}\mathrm{Re}\sqrt{f}$

- For smooth flow,

$$\frac{1}{\sqrt{f}} = 2.0\log\left(\mathrm{Re}\sqrt{f}\right) - 0.8$$

$$\boxed{\frac{1}{\sqrt{f}} - 2.0\log\frac{d}{e} = 2.0\log\left(\frac{e}{d}\mathrm{Re}\sqrt{f}\right) - 0.8} \qquad (9.33)$$

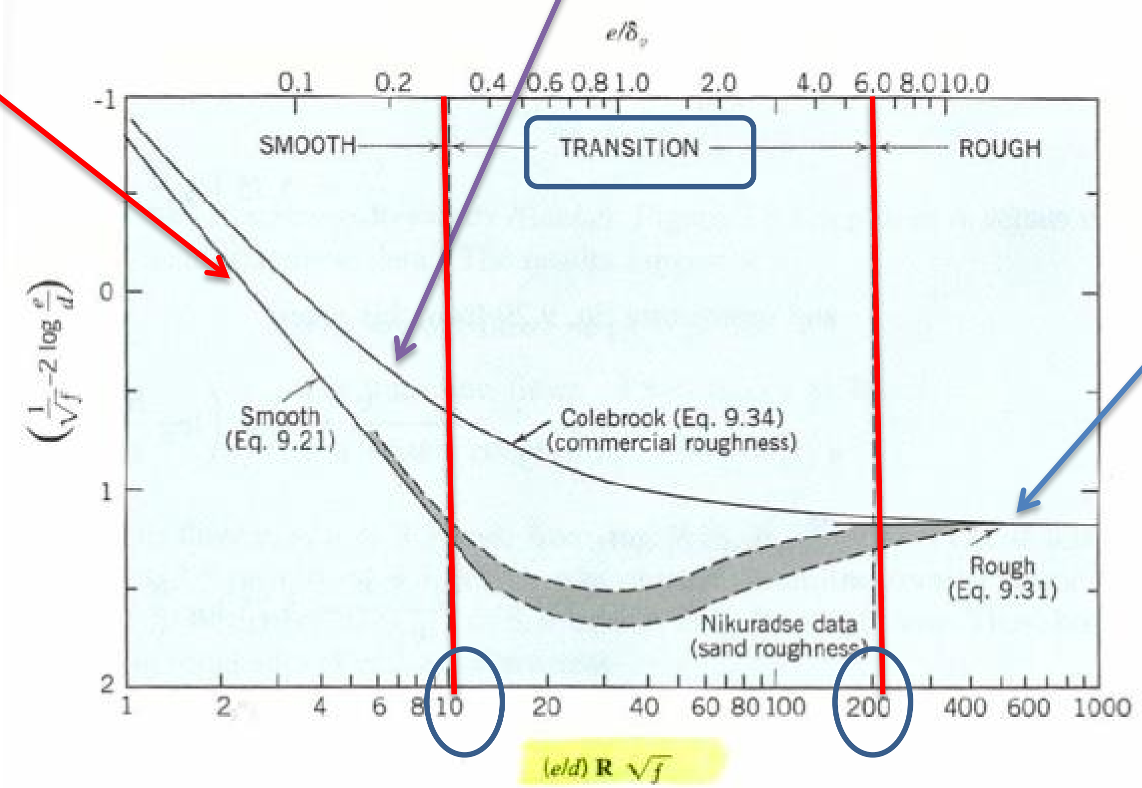
$$Colebrook: \frac{1}{\sqrt{f}} - 2.0\log\frac{d}{e} = 1.14 - 2\log\left[1 + \frac{9.28}{\mathrm{Re}(e/d)\sqrt{f}}\right] \text{ (For commertial roughness)}$$

$$\frac{1}{\sqrt{f}} - 2.0\log\frac{d}{e} = 2.0\log\left(\frac{e}{d}\mathrm{Re}\sqrt{f}\right) - 0.8$$

Smooth flow

$$\frac{1}{\sqrt{f}} - 2.0\log\frac{d}{e} = 1.14$$

Fully rough flow



- Classifications

For smooth flow: $\frac{e}{d}\mathrm{Re}\sqrt{f} \leq 10;\ \frac{e}{\delta_v} \leq 0.3$

For transition flow: $10 < \frac{e}{d}\mathrm{Re}\sqrt{f} < 200$

For rough flow: $200 \leq \frac{e}{d}\mathrm{Re}\sqrt{f}\ ;\ 6 \leq \frac{e}{\delta_v}$

Commercial pipe

- Colebrook Equation
  - Nikuradse experimental results cannot be applied directly to commercial pipes since the roughness patterns are entirely different, much more variable than the artificial roughness used by Nikuradse.
  - Colebrook suggested a single equation for a highly turbulent flow which can be applied to both smooth and rough commercial pipes.

$$\frac{1}{\sqrt{f}} - 2.0\log\frac{d}{e} = 1.14 - 2\log\left[1 + \frac{9.28}{\frac{e}{d}\mathrm{Re}\sqrt{f}}\right] \text{ (For commertial roughness)}$$

  - Distinctions between smooth, transition, and rough flow are not present.

2.2 Classification based on velocity profile

- In all pipes, use Eq. 9.13 by Nikuradse

$$\boxed{\frac{v_c - v}{v_*} = 5.75 \log \frac{R}{y}} \quad (1)$$

- In rough pipes, begin with Eq. 9.29

$$\frac{v}{v_*} = 5.75 \log \frac{y}{e} + 8.5 \quad (2)$$

$$\frac{v_c}{v_*} = 5.75 \log \frac{R}{e} + 8.5 \quad \text{(At center)} \quad (3)$$

Subtract (2) from (3)

$$\frac{v_c - v}{v_*} = 5.75 \left(\log \frac{R}{e} - \log \frac{y}{e} \right)$$

$$\boxed{= 5.75 \log \frac{R}{y}} \quad (4)$$

This means that Eq. 9.13 can be used for both smooth and rough pipes.

Classification based on velocity profile

- Now let's modify the equation

For both smooth and rough, Eq. 9.13

$$\frac{v}{v_*} = \frac{v_c}{v_*} + 5.75\log\frac{y}{R}$$

$$= \frac{v_c}{v_*} + 5.75\log\frac{e}{R} + 5.75\log\frac{y}{e}$$

$$= A + 5.75\log\frac{y}{e}$$

$$\boxed{A = \frac{v_c}{v_*} + 5.75\log\frac{e}{R}} \qquad (9.36)$$

For smooth flow, Eq. 9.17

$$\frac{v}{v_*} = 5.75\log\frac{v_* y}{\nu} + 5.5 \qquad \text{(Smooth pipe)}$$

$$= 5.5 + 5.75\log\frac{v_* e}{\nu} + 5.75\log\frac{y}{e}$$

$$= A + 5.75\log\frac{y}{e}$$

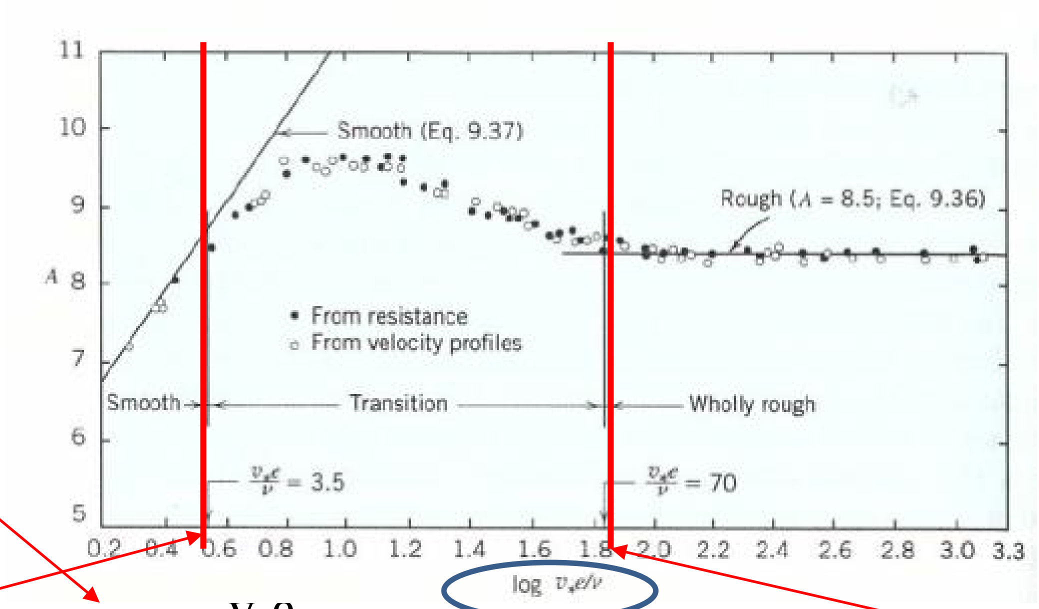
$$\boxed{A = 5.5 + 5.75\log\frac{v_* e}{\nu}} \qquad (9.37)$$

→ Thus, in rough flow, A=8.5 from experiment.

Classification based on velocity profile

- Plot *A* versus $\frac{v_* e}{\nu}$ (*Roughness Reynolds Number*) for Nikuradse's data

$$\delta_\nu = 11.6 \frac{\nu}{v_*}$$



0.544
(3.5)

1.845
(70)

$$\left(11.6/\delta_\nu\right)e = \frac{v_* e}{\nu} < 3.5 \quad \text{Smooth flow}$$

$$3.5 < \left(11.6/\delta_\nu\right)e = \frac{v_* e}{\nu} < 70 \quad \text{Transition flow}$$

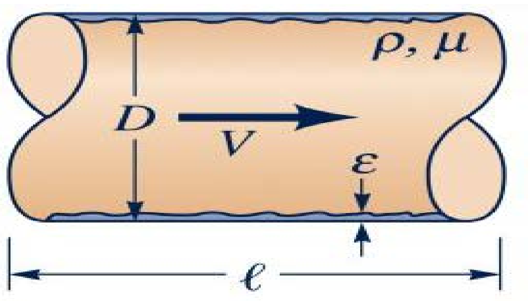
$$70 < \left(11.6/\delta_\nu\right)e = \frac{v_* e}{\nu} \quad \text{Wholly rough flow}$$

IP 9.7; p. 344

- Check Example #1 whether or not the flow is truly rough as assumed.
- The mean velocity in a 300 mm pipeline is 3m/s. The relative roughness of the pipe is 0.002 and the kinematic viscosity of the water is 9×10^{-7} m^2/s. Determine the friction factor, the centerline velocity, the velocity 50 mm from the pipe wall, and the head lost in 300 m of this pipe under the assumption that the pipe is rough.

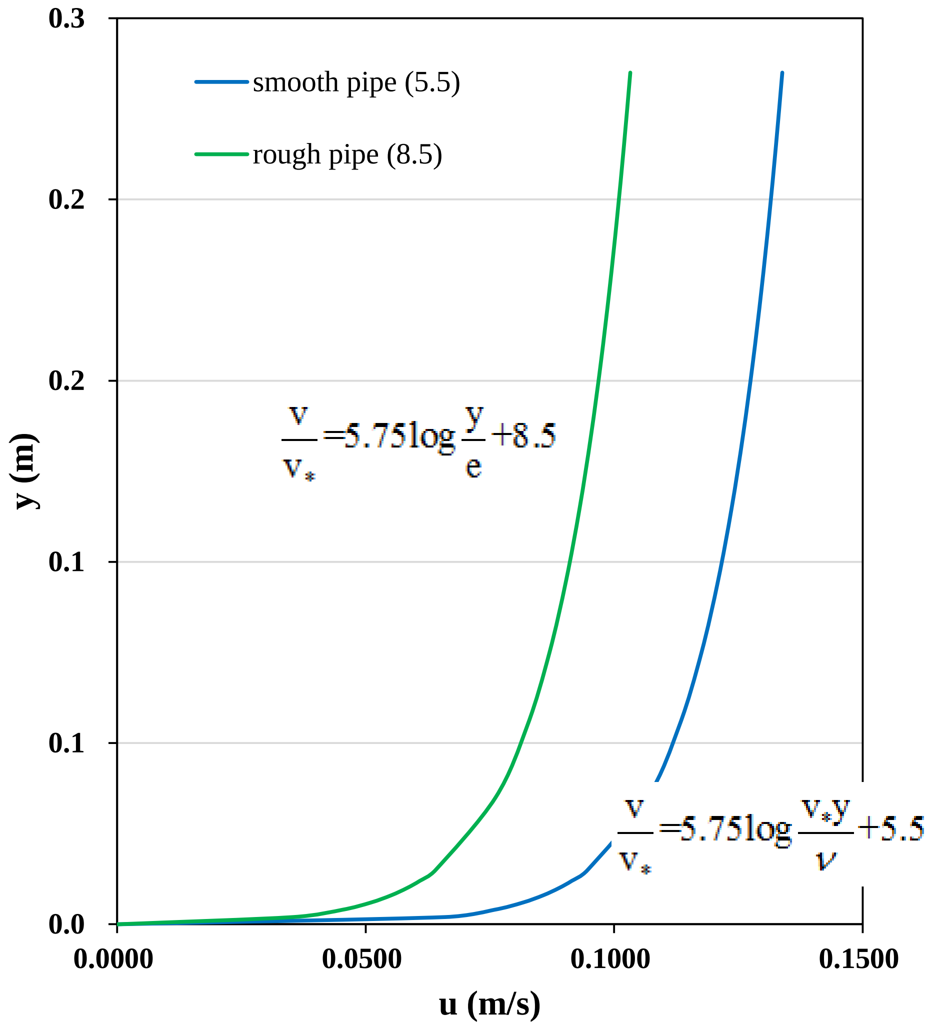
$$v_* = V\sqrt{\frac{f}{8}} = 0.162\, m/s$$

$$\frac{v_* e}{\nu} = \frac{0.162\,\mathrm{m/s} \times 0.0006\,\mathrm{m}}{9 \times 10^{-7}\,\mathrm{m^2/s}} = 108 > 70 \rightarrow \text{rough pipe}$$



Velocity profile equations

	Laminar flow	Turbulent flow: Smooth pipe	Turbulent flow: Rough pipe
Whole pipe	$\frac{v}{v_*}=\frac{v_*}{\nu}\left(y-\frac{y^2}{2R}\right)$	$\frac{v_c - v}{v_*}=-2.5\ln\frac{y}{R}$ (All pipes)	
		$\frac{v}{v_*}=5.75\log\frac{v_* y}{\nu}+5.5$	$\frac{v}{v_*}=5.75\log\frac{y}{e}+8.5$
		$Blasius: \frac{v}{v_*}=\left(\frac{y}{R}\right)^{1/7}$	
Near wall (Viscous sublayer)	$\frac{v}{v_*}=\frac{v_* y}{\nu}$	$\frac{v}{v_*}=\frac{v_* y}{\nu}$	-



$R(m): 0.01$
$\upsilon_*(m/s): 0.0057$

Friction factor equations

Laminar flow	Turbulent flow	
	Smooth pipe	Rough pipe
$f = \frac{64}{\mathrm{Re}}$	$\frac{1}{\sqrt{f}} = 2.0\log\left(\frac{32.8}{\frac{\delta_v}{d}}\right) - 0.8$ $\delta_v = 11.6\frac{\nu}{v_*}$ *Blasius* : $f = \frac{0.316}{\mathrm{Re}^{0.25}}$	$\frac{1}{\sqrt{f}} = 2.0\log\frac{d}{e} + 1.14$