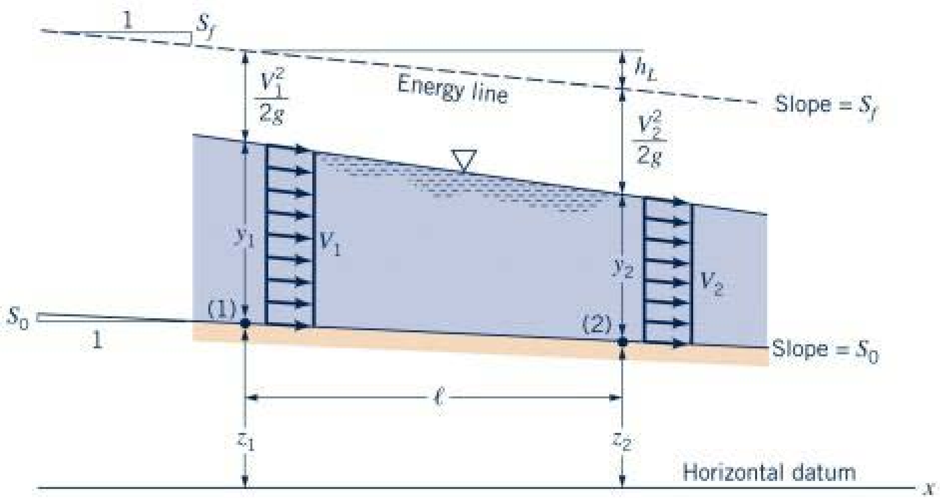
Ch. 8 Varied Flow in Open Channels

8-3 Numerical Methods



Contents

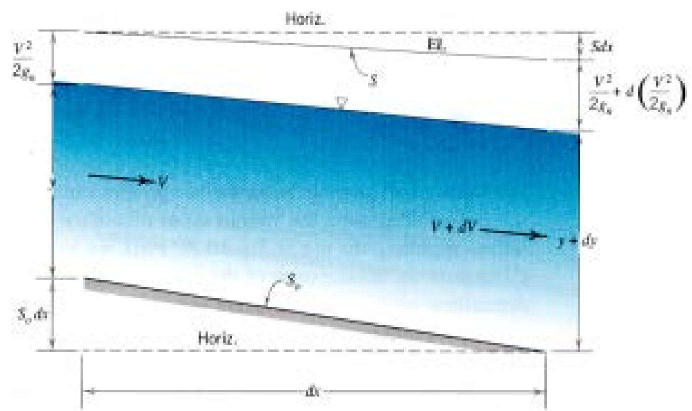
Objectives of Class

- Learn calculation methods for the gradually varied flow
- Solve example problems

8.6 Numerical Methods

- In real situation, Eq. (10.27) cannot be solved easily, especially in natural channels. Therefore, we need to consider some numerical method to solve varied flow's surface problem.

$$\frac{dy}{dx} = \frac{S_0 - S}{1 - F_r^2}$$



1) Direct step method (직접축차법)
2) Standard step method (표준축차법)
3) Prasad's method

1. Direct step method

Direct step method (직접축차법)

- The most straightforward technique to use in numerically solving the varied flow equation is referred to as the direct step method.
- From the total energy equation, (Eq. 10.26)

$$E + S_0 dx = E + dE + Sdx$$

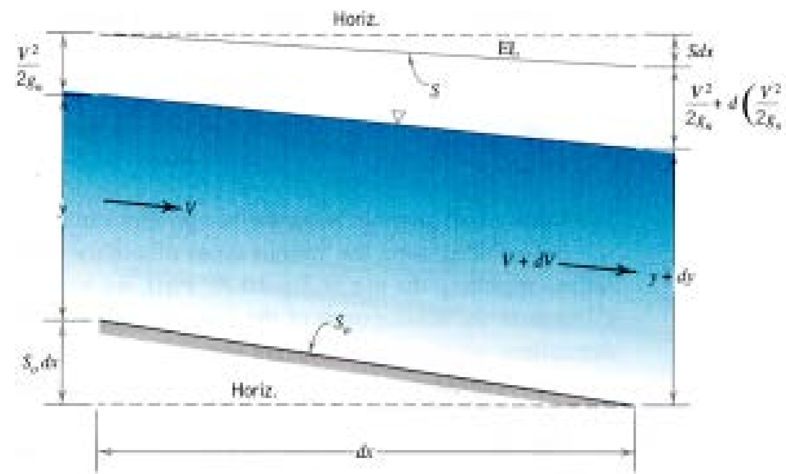
Energy loss

$$dE = (S_0 - S)\, dx$$

$$\boxed{dx = \frac{dE}{S_0 - S}}$$

where *E* is specific energy

$$E = y + \frac{V^2}{2g}$$

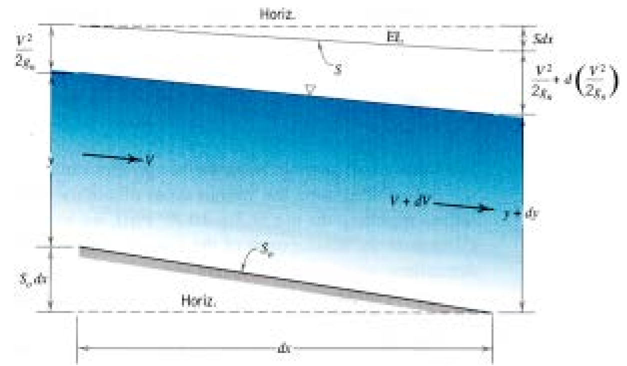


$$\frac{dy}{dx} = \frac{S_0 - S}{1 - F_r^2} \rightarrow dx = \frac{\left(1 - F_r^2\right)}{S_0 - S} dy$$

Direct step method

- In finite difference form

$$\boxed{\Delta x = \frac{E_{i+1} - E_i}{S_0 - \bar{S}}} \qquad (10.29)$$



where *i* and *i+1* represent locations along the channel separated by a distance *Δx*.

- $\bar{S}$ is average of *S*-values calculated at each end of the channel reach

$$\bar{S} = \frac{S_i + S_{i+1}}{2} = \frac{1}{2}\left(\frac{Qn}{uA_i R_{h_i}^{2/3}}\right)^2 + \frac{1}{2}\left(\frac{Qn}{uA_{i+1} R_{h_{i+1}}^{2/3}}\right)^2 \qquad (10.30)$$

Manning's uniform flow equation

- Computer program (DIRSTEP) may be used.

Direct step method

- **Computational procedure**

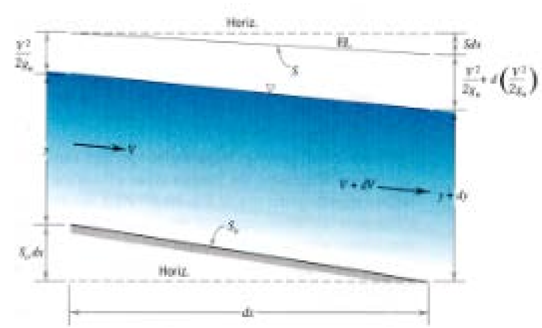
  1) Determine a point in the channel where the depth is known.

  Use that point as the starting point (기점수위) for the computations (It is often convenient to start at a control.

  2) Assume a depth which is somewhat larger or smaller than the initial depth

  3) Compute the distance upstream or downstream to the chosen depth using Eq. 10.29.

$$\Delta x = \frac{E_{i+1} - E_i}{S_0 - \bar{S}}$$



  4) Repeat this process

- The depth increment would be less than 10% of the difference the starting depth and the normal or critical depth, and the depth never crosses the normal depth or the critical depth.

IP 10.9 (pp. 469-470)

A flow rate of 10 m^3/s occurs in a rectangular channel 6 m wide, lined with concrete (troweled) and laid on a slope of 0.0001. If the depth at a point in the channel is 1.50 m, how far (upstream or downstream) from this point will the depth be 1.65 m?

- Solution: The first step is classifying the flow profile for a given Q and S_0.

→ Calculate the normal and critical depths.

To obtain the normal depth, use first Manning's equation

$$Q = \left(\frac{u}{n}\right) A R_h^{2/3} S_0^{1/2} = \left(\frac{1}{0.013}\right)(6y_0)\left(\frac{6y_0}{6+2y_0}\right)^{2/3} 0.0001^{1/2}$$

$$10 = 0.769(6y_0)\left(\frac{6y_0}{6+2y_0}\right)^{2/3} \quad then \quad \boxed{y_0 = 1.94m}$$

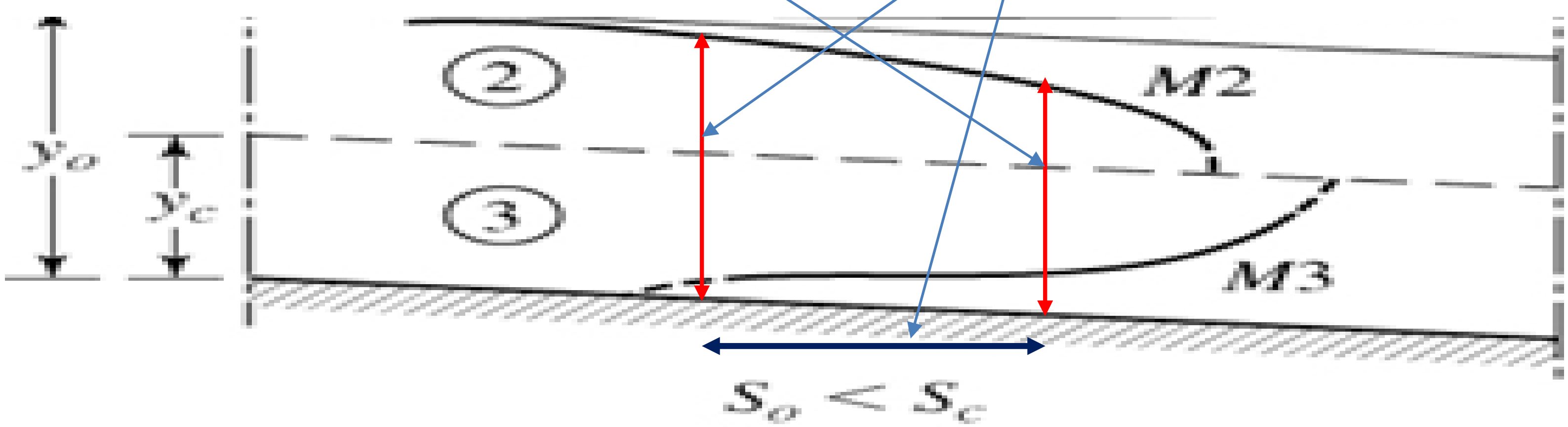
$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{(10/6)^2}{9.81}} \boxed{= 0.66m}$$

Because $y_0 > y_c$, the slope is mild, and because $y_c<y<y_0$ → M2 profile.

It means that depth of flow decreases in the downstream direction.

From this result, we now can conclude that the depth of 1.65m will occur upstream of the 1.50 m depth.

Thus, compute the distance upstream to find *L*.



(a) Mild slope *M*

- To perform the varied flow computations, we will set up a computation table. We will use depth increments of 0.05m.

$$E = y + \frac{V^2}{2g}$$

$$S_i = \left(\frac{Qn}{uA_i R_{h_i}^{2/3}} \right)^2$$

$$\Delta x = \frac{E_{i+1} - E_i}{S_0 - \bar{S}}$$

y	A	$V = Q/A$	$V^2/2g$	E	P	R_h	S	$\bar{S}$	Δx	$\sum x$
1.50	9.00	1.11	0.0628	1.5628	9.00	1.000	.000209			0
1.55	9.30	1.08	0.0594	1.6094	9.10	1.022	.000190	.000199	471	471
1.60	9.60	1.04	0.0551	1.6551	9.20	1.043	.000173	.000182	557	1,028
1.65	9.90	1.10	0.0520	1.7020	9.30	1.065	.000159	.000166	711	**1,739**

2. Standard step method

- Using the direct step method, the depth at a particular distance cannot be found directly.
- Use standard step method (Chow, 1959)
- This method is applicable to both prismatic and non-prismatic channels.
- In natural channels, it is necessary to conduct a field survey to collect the data required at all sections considered in the computation.
- The computation is carried on by steps from station to station where the hydraulic characteristics have been determined.

→ In such cases, the distance between stations is given, and the procedure is to determine the depth at the stations.

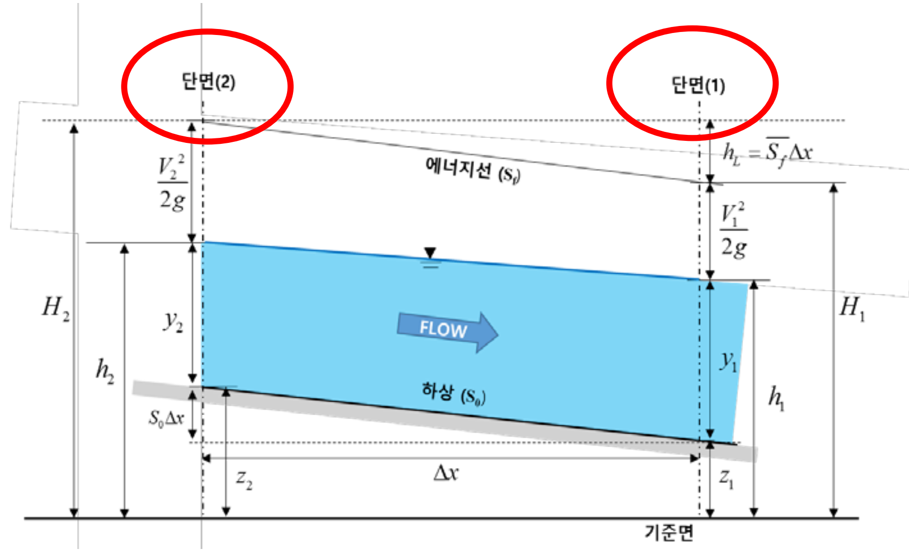
- Such a procedure is usually carried out by trial and error.

수준기준면: 인천만의 평균조위
영점표고: 하천수위표의 기준표고

The water-surface elevations (수위) above the datum are

$$h_1 = y_1 + z_1$$

$$h_2 = y_2 + z_2 = y_2 + z_1 + S_0 \Delta x$$



The friction loss is

$$\boxed{h_L = \overline{S_f}\Delta x = \frac{1}{2}\left(S_{f1} + S_{f2}\right)\Delta x} \qquad (1)$$

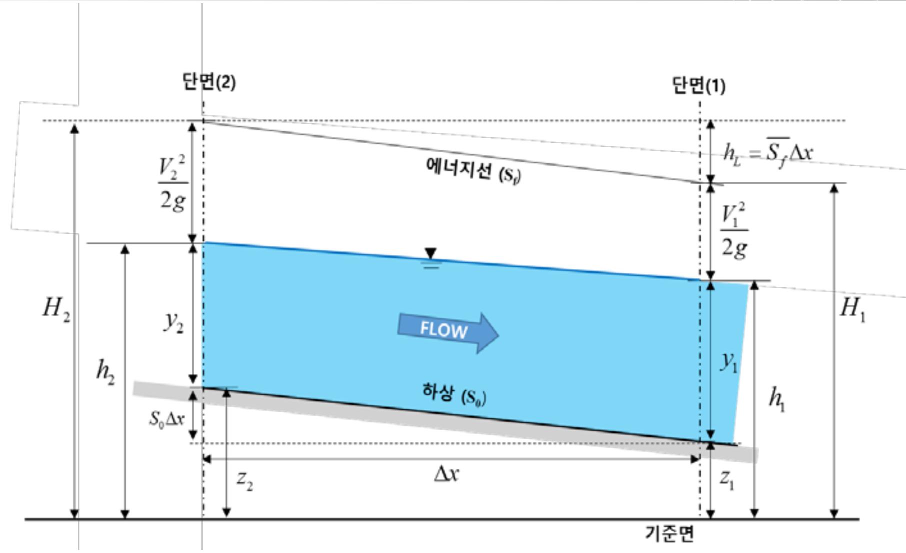
Equating the total heads at two sections yields

$$h_1 + \alpha_1 \frac{V_1^2}{2g} + h_L = h_2 + \alpha_2 \frac{V_2^2}{2g} \qquad (2)$$

$$H_1 = h_1 + \alpha_1 \frac{V_1^{\ 2}}{2g} \qquad (3)$$

$$H_2 = h_2 + \alpha_2 \frac{V_2^{\ 2}}{2g} \qquad (4)$$

$$H_2 = H_1 + h_L \qquad (5)$$



- **Computational procedure**

1) Determine a point in the channel where the depth is known.

→ Usually computation proceeds from starting point (Section 1) to upstream sections (mild curves)

2) Compute H_1 using Eq. (3)

3) Guess depth at Section 2 which is Δx apart from Section 1

4) Compute H_2 using Eq. (4) with guessed depth

5) Compute h_L using Eq. (1)

6) Compute H_2 using Eq. (5), and compare this with H_2 *obtained at Step 4*

3. Prasad's method

- Using the direct step method, the depth at a particular distance cannot be found directly.
- This means that the selection of *y*-values yields Δx values.

→ We cannot select a Δx and solve directly for a *y*-value.

- Prasad (1970) suggested a technique for determining depth for a give value of Δx in a prismatic and natural channels.
- Integration of varied flow equation, Eq. 10.27

$$y' = \frac{dy}{dx} = \frac{S_0 - S}{\left(1 - F_r^2\right)} = \frac{S_0 - \left(\dfrac{Qn}{uAR_h^{2/3}}\right)^2}{1 - \dfrac{Q^2 b}{gA^3}} \qquad (10.31)$$

- Integrate Eq. 10.31 by means of <u>trapezoidal method</u>

$$y_{i+1} = y_i + \frac{dy}{dx}\Delta x = y_i + y'\Delta x \qquad (10.32) \qquad y' = \frac{dy}{dx} = \frac{y_{i+1} - y_i}{\Delta x}$$

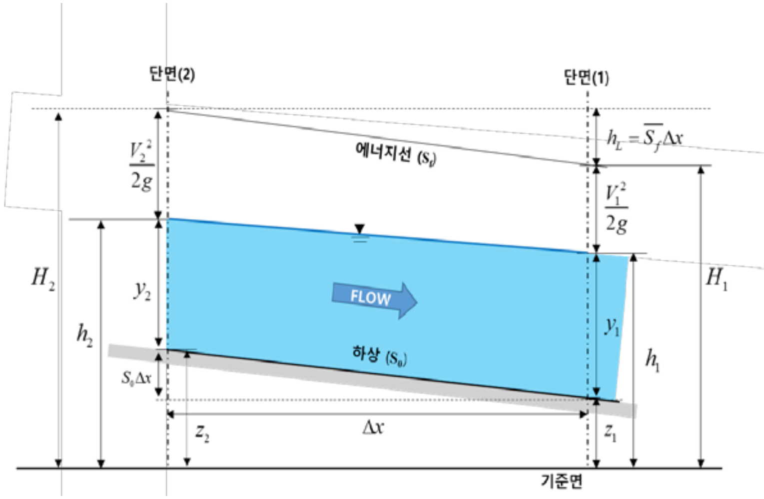
- For numerical integration, Δx is assumed to be very small so that

$$\boxed{y' = 0.5\left(y'_i + y'_{i+1}\right)}$$

- Then, Eq. 10.32 becomes

knowns

$$\boxed{y_{i+1}} = y_i + 0.5\left(y'_i + \boxed{y'_{i+1}}\right)\Delta x \qquad (10.33)$$



- This equation contains <u>two unknowns</u>, y_{i+1}, and y'_{i+1}
- Thus, trial-and-error *method needs to be applied*

Computational procedure

1) Use Eq. 10.31 to calculate y'_i where y_i is known as the <u>initial point</u>

2) Set $y'_{i+1} \cong y'_i$ as <u>a first estimate</u>

3) Use current values of $y'_i \; and \; y'_{i+1}$ to calculate y_{i+1} using Eq. 10.33 for a <u>selected Δx</u>

4) Calculate y'_{i+1} from Eq. 10.31 using y_{i+1} value from Step 3

5) If the new y'_{i+1} is not close enough to the previously calculated (or guessed) value, then repeat steps 3 ~4

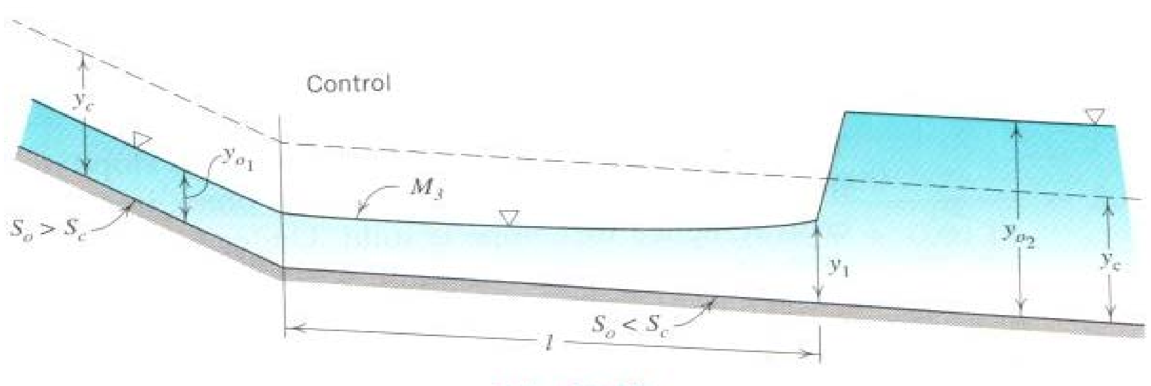
6) Once the iteration procedure has yielded successive estimate of y'_{i+1} and y_{i+1} within acceptable limits of accuracy, proceed to the next station of the channel.

$$y' = \frac{dy}{dx} = \frac{S_0 - S}{\left(1 - F_r^2\right)} = \frac{S_0 - \left(\dfrac{Qn}{uAR_h^{2/3}}\right)^2}{1 - \dfrac{Q^2 b}{gA^3}} \tag{10.31}$$

$$y_{i+1} = y_i + 0.5\left(y'_i + y'_{i+1}\right)\Delta x \tag{10.33}$$

8.7 Example Problems

- Hydraulic jump location
  - Where a steep channel slope changes to a mild slope, hydraulic jump can occur.

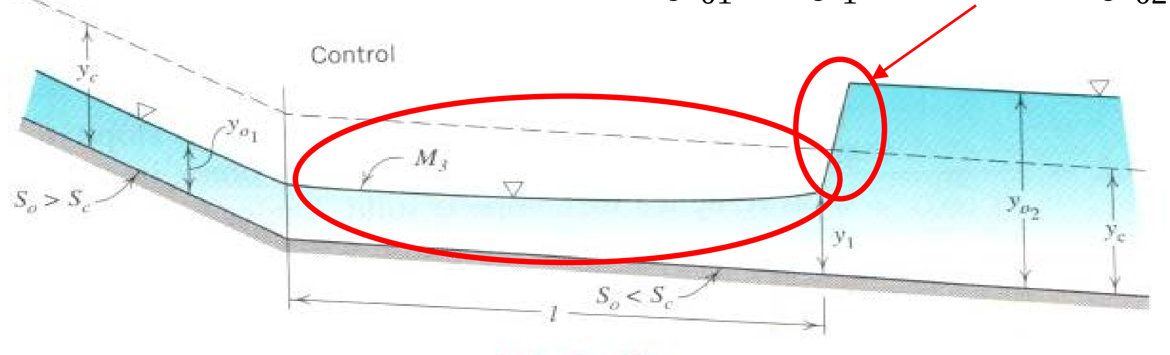


  - Both channels are long enough to have uniform flow at the end of channels.
  - Initially, we do not know whether the jump occur on the steep or mild slopes.

Case 1:

- If jump occur on the mild slope channel, then the flow remains at normal depth on the steep slope, and decelerates on the mild slope until a jump forms.
- Because the depth of flow on the mild sloped channel is less than both the normal and critical depth, flow is in M3 profiles
- The depth on the downstream side of the jump is the normal depth y_{02} in the mild sloped channel.

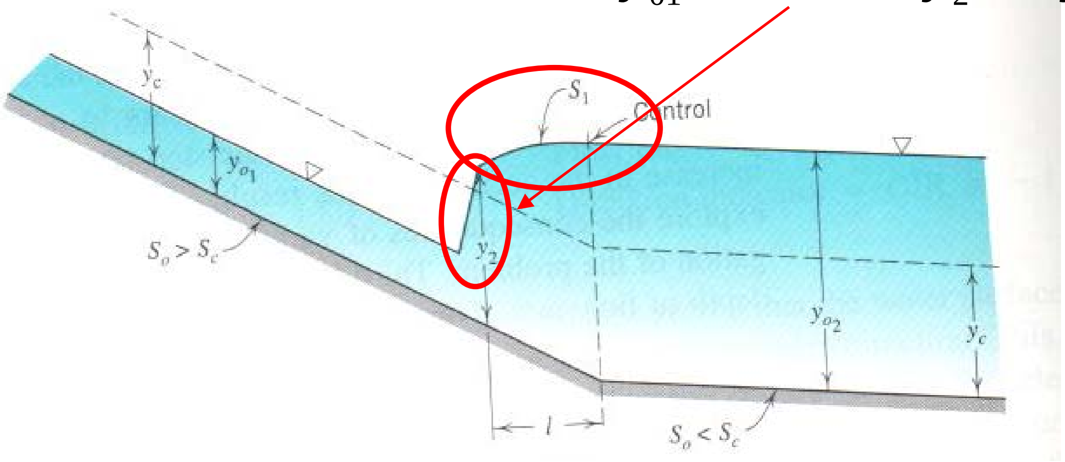
$$y_{01} \rightarrow y_1 \rightarrow HJ \rightarrow y_{02}$$



Case 2:

- If jump occurs on the steep slope, the flow jump from the normal depth on the steep slope to the conjugate depth y_2, decelerates until it reaches the normal depth y_{02} at the beginning of the mild sloped channel.
- Because the depth of flow downstream of the jump is greater than both the normal and critical depths, we have S1 profile.

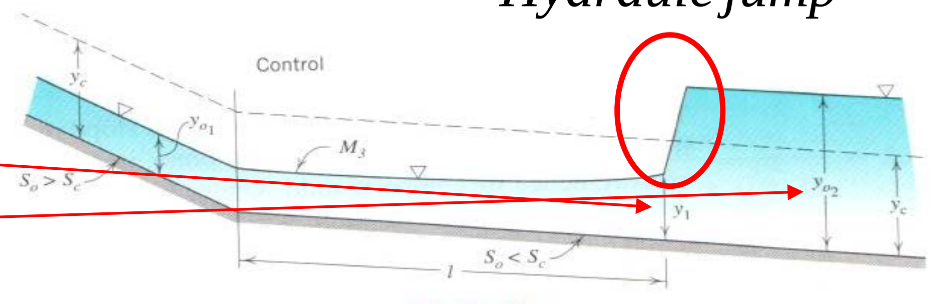
$$y_{01} \rightarrow HJ \rightarrow y_2 \rightarrow y_{02}$$



- In order to position the jump we need to determine which scenario applies.
- The conjugate depths across the jump corresponding to the two normal depths is given as

Hydrauic jump

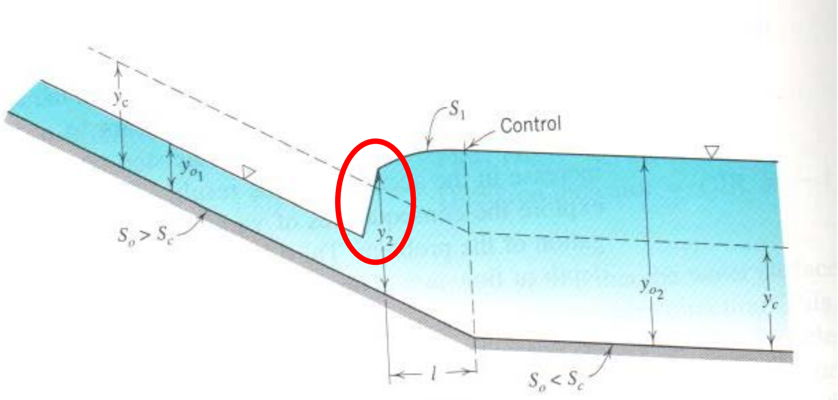
$$\frac{y_1}{y_2} = \frac{1}{2}\left[-1+\sqrt{1+\frac{8V_2^2}{gy_2}}\right]$$



Case 1: If the y_1 is greater than the normal depth in the steep channel, and ***M3* profile** will be needed to decelerate the flow and increases the depth so that the jump can occur.

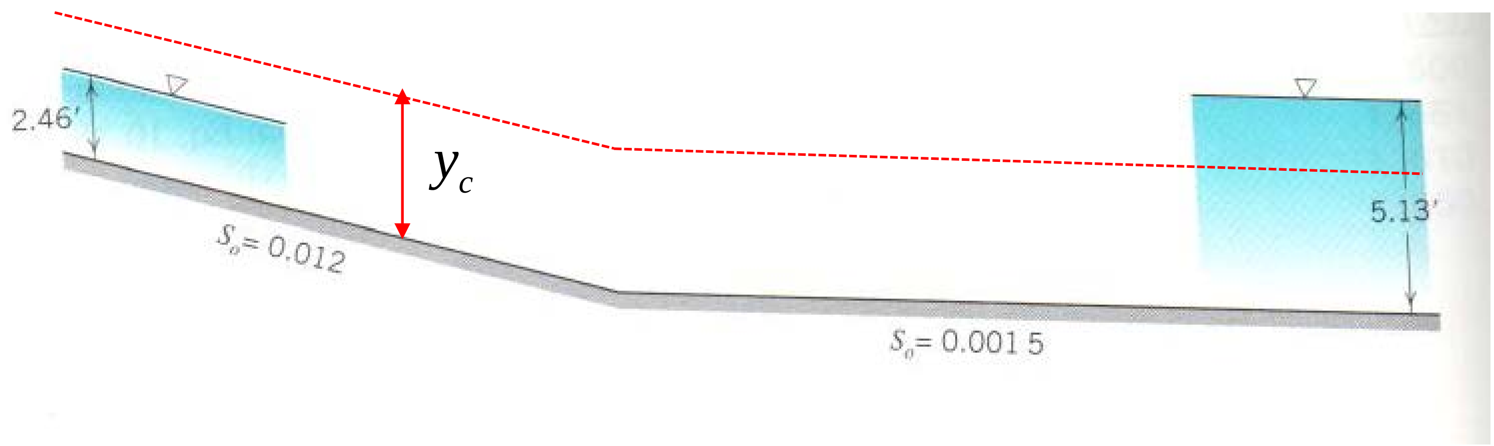
Case 2: If, on the other hand, the conjugate depth computed from the above equation, is less than the normal depth in the steep channel, *M3* profile cannot be possible, the depth cannot decreases in the downstream direction. Consequently, the jump must occur in the steep channel.

- We can now work with the varied flow equation to locate the jump.
- The channel roughness is very crucial for determining the location of jump.



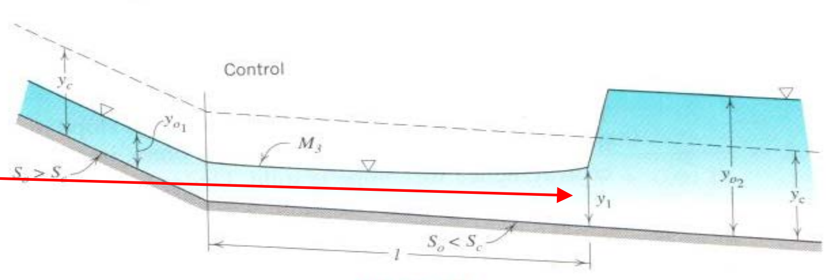
IP 10.11 (pp. 474-475)

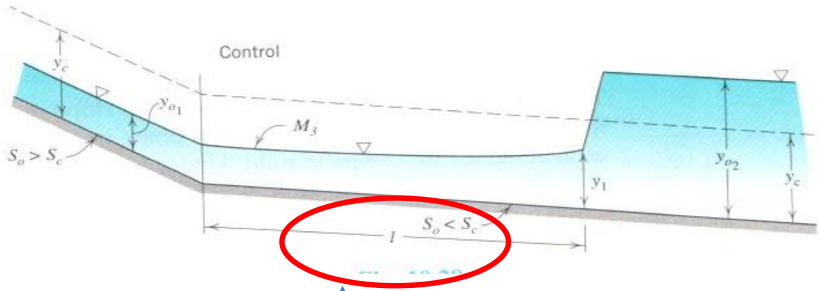
A flow of 500 ft^3/s occurs in a long rectangular channel 12 ft wide with an *n*-value of 0.014. The channel is shown below and has a break in slope. The channel slop upstream of the break in slope is 0.012 and downstream of the break is 0.0015. Uniform flow calculations determine that the normal depth upstream of the break is 2.46 ft and downstream of the break is 5.13 ft. The critical depth for this flow rate is 3.78 ft. Verify that a hydraulic jump must occur and locate its position in the channel.



- The channel slope upstream of the break in slope is steep because the normal depth is less than the critical. Downstream of the break in slope, the normal depth is greater than the critical, hence this slope is mild.
- Under these conditions, for a long channel downstream of the break, a hydraulic jump will occur.
- To determine whether the jump is upstream or downsteam of the channel break, we will assume it occurs downstream and check to see if the assumption is valid.
- To do this, we use a slightly modified form of equation to calculate the conjugate depth to the downstream normal depth.

$$y_1 = \frac{y_2}{2}\left[-1+\sqrt{1+\frac{8V_2^2}{gy_2}}\right]$$





- Velocity in the downstream is

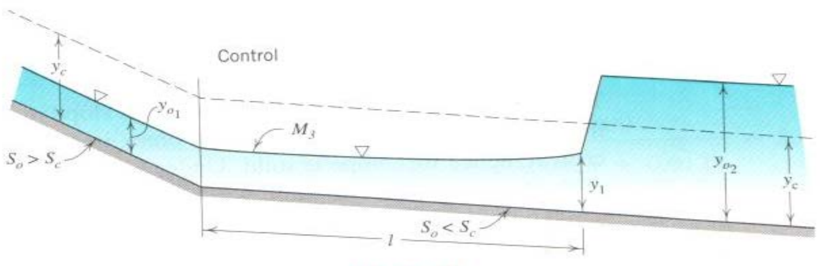
$$V_2 = \frac{Q}{A_2} = \frac{500}{12 \times 5.13} = 8.12\, ft/s$$

$$y_1 = \frac{5.13}{2}\left[-1 + \sqrt{1 + \frac{8 \times 8.12^2}{32.2 \times 5.13}}\right] = 2.69\, ft$$

- Because the conjugate depth of 2.69 ft is greater than the normal depth of 2.46 ft on the steep slope, the conditions are right for an M3 curve to form. Consequently, the jump will occur downstream of the break in channel slope.
- To locate the jump, we will compute the distance downstream from the break in slope to the point where the depth of 2.69 ft occurs.
- To calculate this, we use ***direct step method***

$$\Delta x = \frac{E_{i+1} - E_i}{S_0 - \bar{S}}$$

- But, for simplicity, we use only one computational step,
- Before using this equation, we determine the *E* at both depths and the average energy gradient slope of *S*.
- $$E_{2.69} = y + \frac{V^2}{2g} = 2.69 + \frac{\left(\frac{500}{12 \times 2.69}\right)^2}{2 \times 32.2} = 6.420\,ft;$$
  $$E_{2.46} = 2.46 + \frac{\left(\frac{500}{12 \times 2.46}\right)^2}{2 \times 32.2} = 6.90\,ft$$

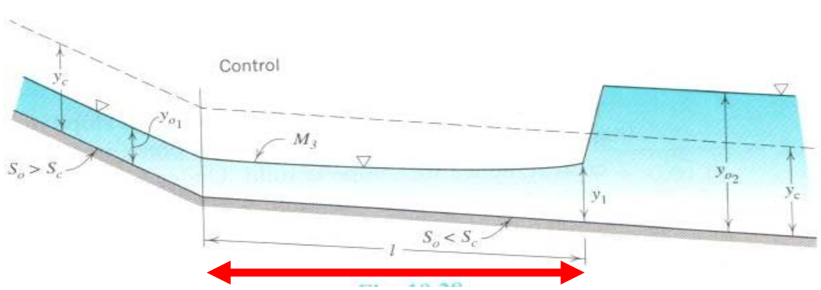


$$S_{2.69} = \left(\frac{Qn}{uAR^{2/3}}\right)^2 = \left(\frac{500 \times 0.014}{1.49 \times (12 \times 2.69)\left(\dfrac{12 \times 2.69}{12 + 2 \times 2.69}\right)^{2/3}}\right)^2 = 0.00928$$

$$S_{2.46} = 0.012$$

$$\Delta x = \frac{E_{i+1} - E_i}{S_0 - \bar{S}} = \frac{E_{2.69} - E_{2.46}}{0.0015 - \dfrac{S_{2.69} + S_{2.46}}{2}} = 53\,ft$$

The hydraulic jump is located 53 ft downstream of the break in the channel slope.



Homework Assignment No. 6
Due: 2 weeks from today
Answer questions in Korean or English

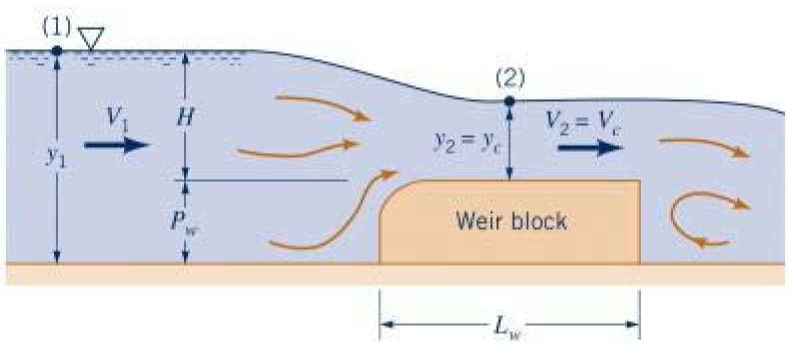
1. (10.47) At what depths may 0.85 m^3/s flow in a rectangular channel 1.8 m wide if the specific energy is 1.2 m?

2. (10.50) Flow occurs in a rectangular channel of 6 m width a has a specific energy of 3 m. Plot accurately the *q* - curve. Determine from the curve (*a*) the critical depth, (*b*) the maximum flowrate, (*c*) the flowrate at a depth of 2.4 m, and (d) the depths at which a flowrate of 28.3 m^3/s may exist, and the flow condition at these depths.

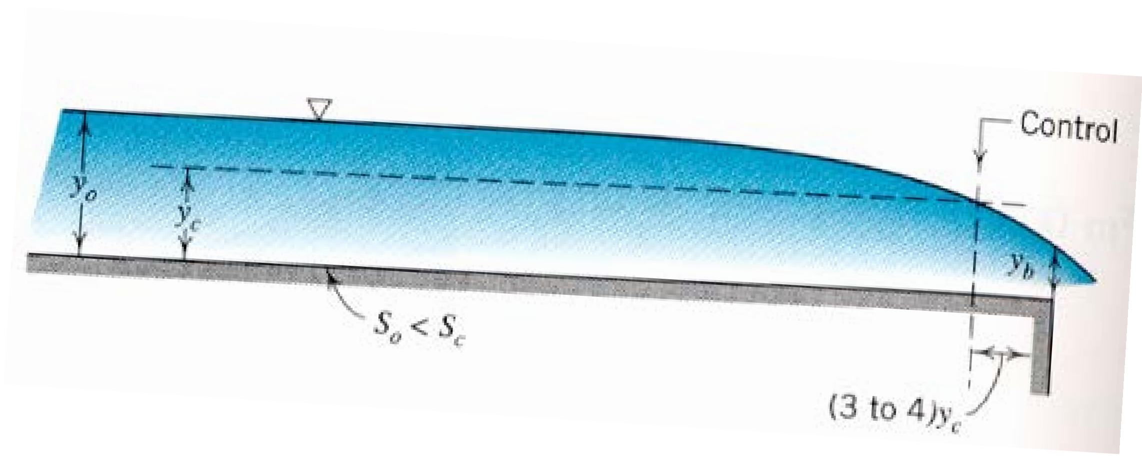
3. (10.53) If 8.5 m^3/s flow uniformly in a rectangular channel 3.6 m wide having $n = 0.015$ and laid on a slope of 0.005, is the flow subcritical or supercritical? What is the critical slope for this flowrate, assuming the channel to be of great width?

4. (10.66) Eleven cubic meters per second flow in a trapezoidal channel of base width 4.5 m and side slopes 1 (vert.) on 3(horiz.). Calculate the critical depth and the ratio of critical depth to minimum specific energy. If n = 0.020, what is the critical slope?

5. (10.80) A dam 1.2 m high and having a <u>broad horizontal crest</u> is built in a rectangular channel 4.5 m wide. For a depth of water on the crest of 0.6 m, calculate the flowrate and the <u>depth of water just upstream from the dam</u>.



6. (10.87) An open, rectangular channel 1.5 m wide and laid on a mild slope ends in a free outfall. If the brink depth is measured as 0.264 m, what flowrate exists in the channel?



7. (10.93) The critical depth is maintained at a point in a rectangular channel 1.8 m wide by building a gentle hump 0.3 m high in the bottom of the channel. When the depth over the hump is 0.66 m, what water depths are possible just upstream from the hump?

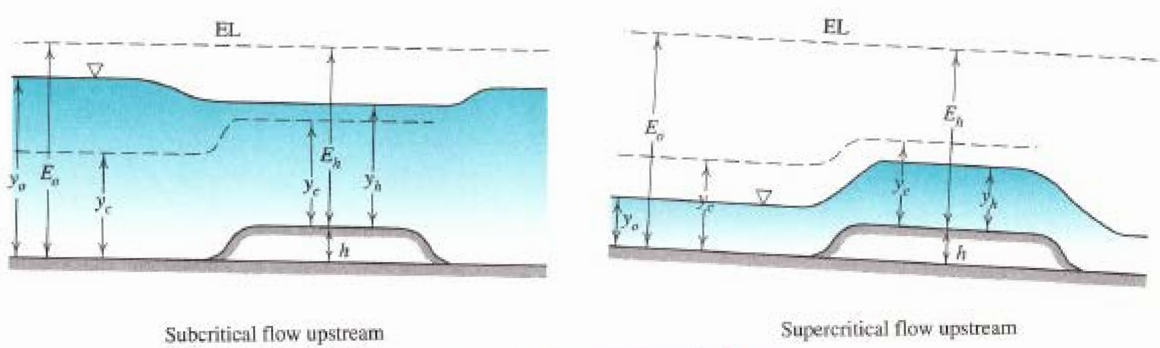
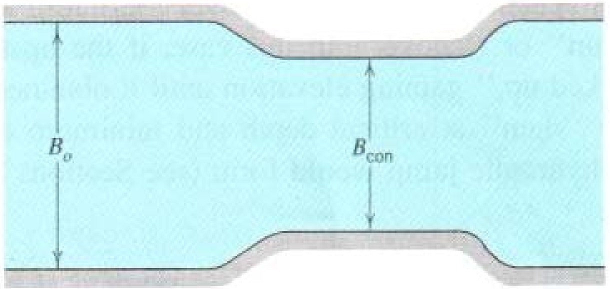


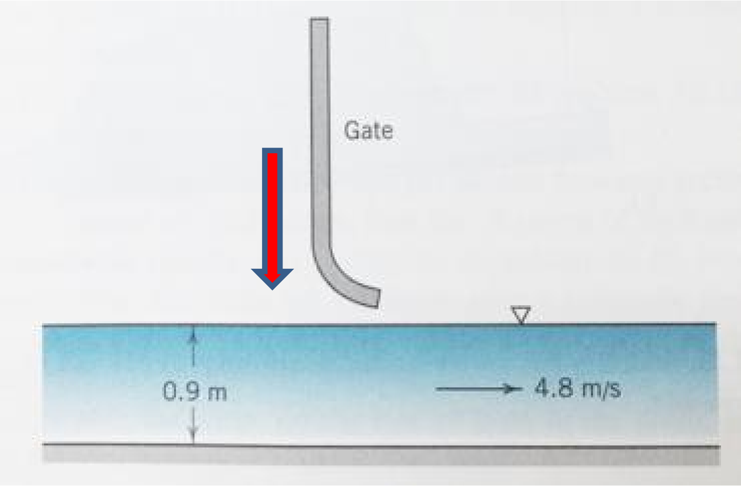
Fig. 10.12 Flow over a smooth hump.

8. (10.104) A rectangular channel 3.6 m wide is narrowed to a 1.8 m width to cause critical flow in the contracted section. If the depth in this section is 0.9 m, calculate the flowrate and the depth in the 3.6 m section, neglecting head losses in the transition. Sketch energy line and water surface, showing all pertinent vertical dimensions.



Channel constriction (top view)

9. (10.118) This uniform flow occurs in a very long, rectangular channel. The sluice gate is lowered to a position so that the opening is 0.6 m. Sketch the new water surface profiles to be expected and identify them by letter and number. Calculate and show all significant depths.



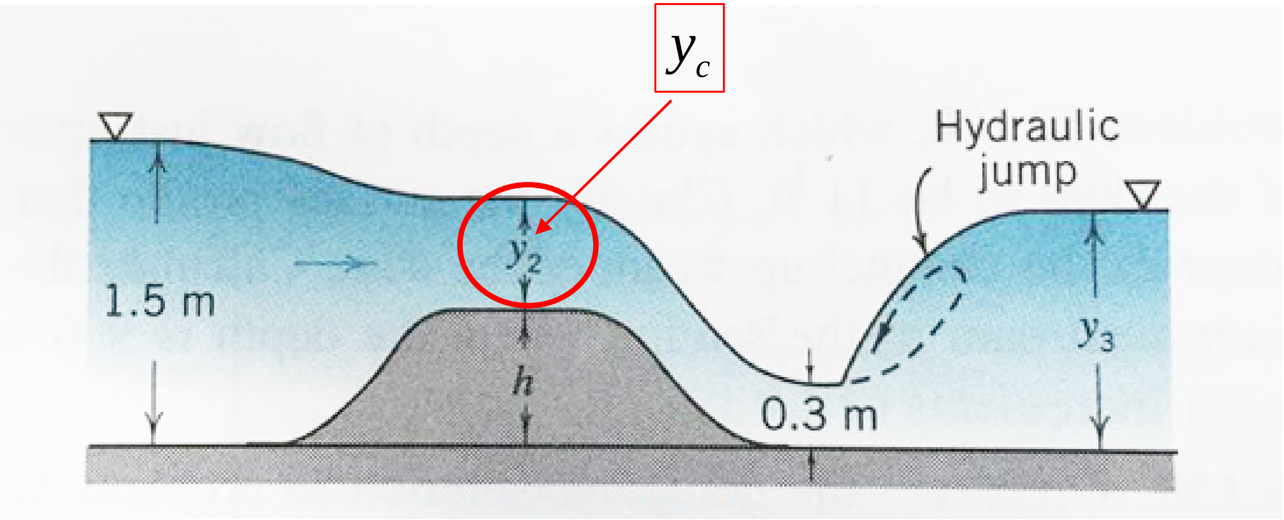
10. (10.119) Solve the preceding problem for the same flowrate but with 1.8 m depth of uniform flow.

11. (10.130) Compute the profile downstream from a point where y =1.9 m in a trapezoidal channel in which n = 0.02 z = 2, S_o = 0.002, b = 30 and Q = 211 m^3/s. How far can calculation proceed?

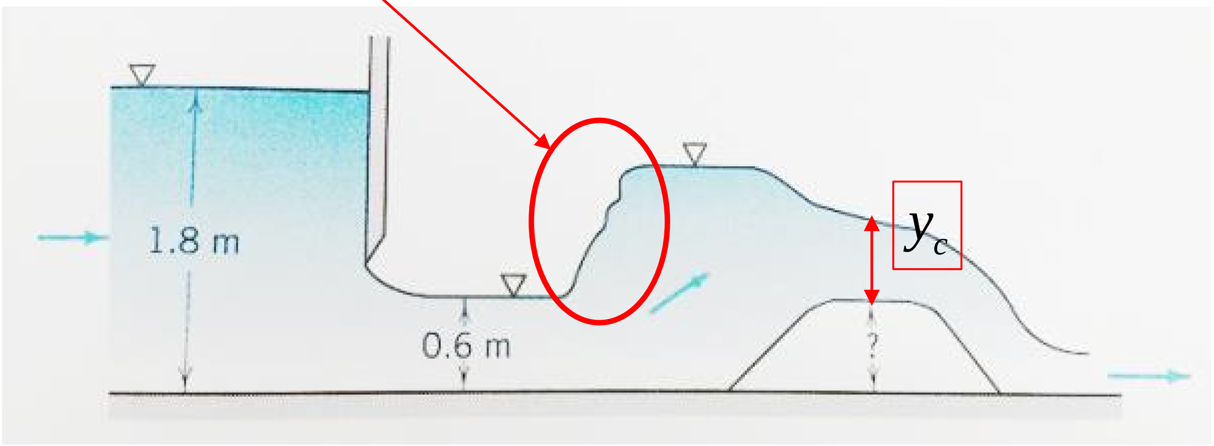
Classify water surface profile first (M2-profile).

Use direct step method.

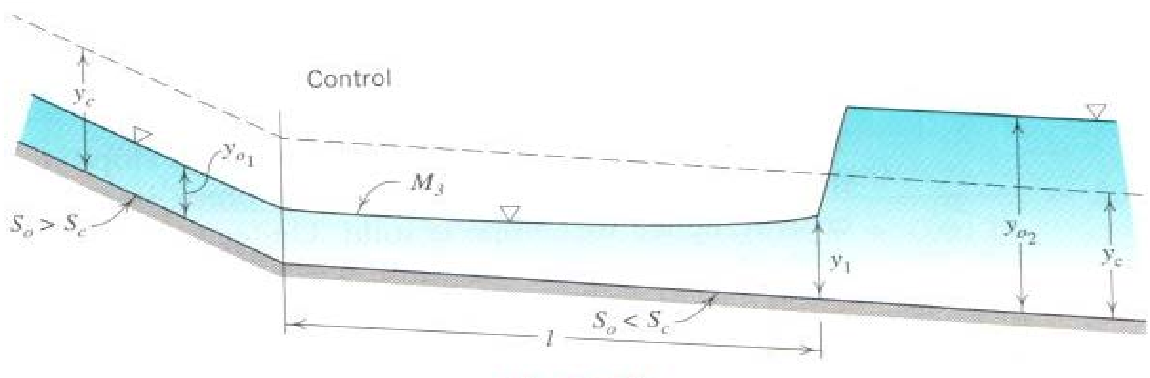
12. (10.145) Calculate y_2, h, and y_3 for this two-dimensional flow picture. State any assumptions clearly.



13. (10.159) Neglecting wall, bottom, and hump friction (but not <u>losses in the jump</u>), what height of hump will produce this flow picture?



14. (10.161) The flowrate in Fig. 10.28 is 15.4 m^3/s. If the channels are 3.6 m wide with n = 0.017 and the downstream channel is laid on a slope of 0.002 28, what depth must exist in this channel for a hydraulic jump to occur to uniform flow? Calculate the length l if y_{o1} = 0.80 m.



15. (10.162) The flowrate in Fig. 10.29 is 15.4 m^3/s, the channels are 3.67 m wide, and n = 0.017. The downstream channel is laid on a slope of 0.001 5. If y_{o1} = 0.79 m, calculate l (location of hydraulic jump).