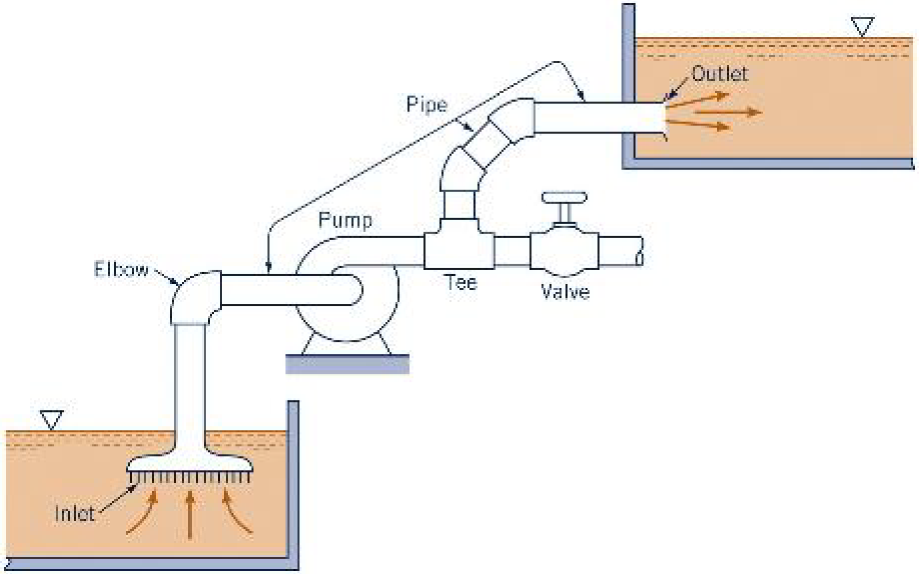
Ch. 5 Pipe Problems

5-1 Single Pipes



Contents

Today's objectives

- Analyze practical problems of pipe flow
- Extend the single pipe problem to the pumped pipeline

Review

- Fundamental equations
  - Based on impulse-momentum equation for incompressible flow, we got head loss, and shear velocity
- Laminar flow
  - We derived velocity profile and friction factor for laminar flow
- Turbulent flow
  - Firstly, we derived velocity profile for all pipe from Prandtl's theory.
  - Then, we learned smooth pipes and got velocity profiles near pipe wall
  - Based on this velocity profile we learned friction factor which only can be solved by trial-errors
  - Secondly, study rough pipe cases considering roughness height.
  - Here, we also knew that friction factor
  - Thirdly, we can figure out whether a pipe has smooth or rough wall.
- Engineering problem
  - What we learned so far, only works in laboratory, so generalized it for the more practical problem with commercial pipes.
  - Finally, we extended to non-circular pipes and empirical formula.

5.1 Classification of Pipeline Problems

- All steady-flow pipe problems may be solved by application of the work-energy equation and continuity equation.

→ Most effectively by the construction of the energy and hydraulic grade lines

- **Engineering pipe-flow problems** – commercial pipe
  1) Calculation of **head loss, h_L,** and pressure variation
  2) Calculation of **flow rate, Q**
  3) Calculation of **size of pipe, d**

→ Solution by trial-and-error is required for problems 2 and 3

→ using energy grade and hydraulic gradient lines is helpful to solve in real field.

Single pipe-flow problems

1. Type 1: Find h_L & Δp for given Q, d, e → IP. 9.10

   Explicit solution is possible

   Find f using Moody diagram and then use Darcy-Weisbach equation to calculate h_L

2. Type 2: Find Q for given d, e, H

   Explicit solution is not possible → trial & error method

   $$\text{friction factor } f = fn(\text{Re}) = fn(Q)$$

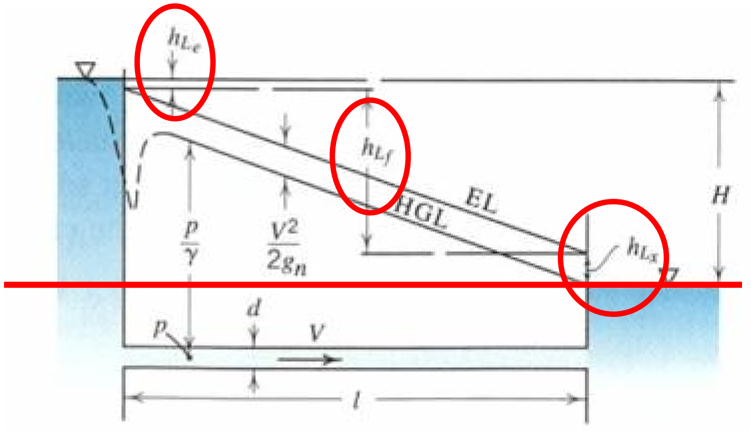
3. Type 3: Find d for given Q, Δp

   Explicit solution is not possible → trial & error method

   $$\text{friction factor } f = fn(\text{Re}) = fn(d)$$

5.2 Two Reservoirs Problems

Apply energy equation



$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g_n} = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g_n} + h_L$$

$$z_1 = H; z_2 = 0; p_1 = p_2 = 0; V_1 = V_2 \approx 0$$

$$h_L = h_{L_e} + h_{L_f} + h_{L_x}$$

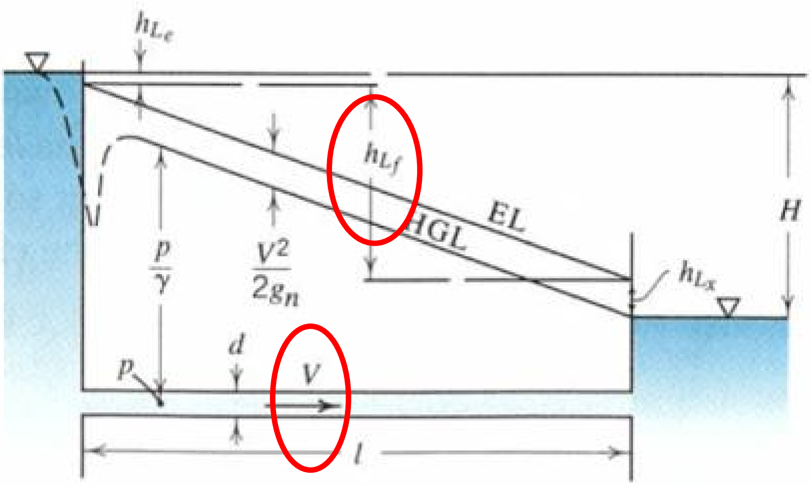
h_{L_e} =entrance loss; h_{L_x} =exit loss; h_{L_f} =pipe friction loss

$$\boxed{H = h_L = \left(0.5 + f\frac{l}{d} + 1\right)\frac{V^2}{2g_n}}$$

- Usually, *l* is much longer than *d*, *l/d* = 1,000, in this case, we can assume *f* to be 0.03. In such case, *f* · *l/d* ~30, and this is almost 30 times the sum of other terms.

- Therefore, as length of pipe increases, we can ignore the effect of local losses.
- Then we can simplify the equation

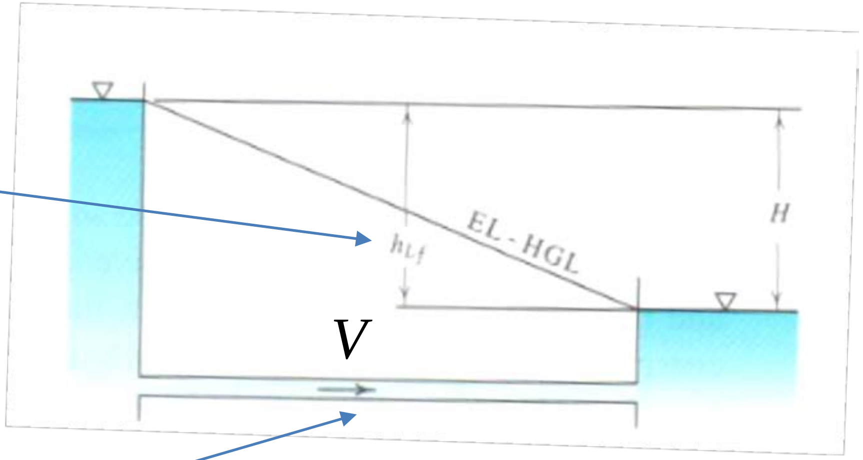
$$H = h_L \approx h_{Lf} = f\frac{l}{d}\frac{V^2}{2g_n}$$



- Another approximation, as length of pipe increases, is that we can <u>ignore the velocity head in the energy equation</u>.

→ Making energy and hydraulic grade lines coincident

$$h_L \approx h_{Lf} = f\frac{l}{d}\frac{V^2}{2g_n}$$

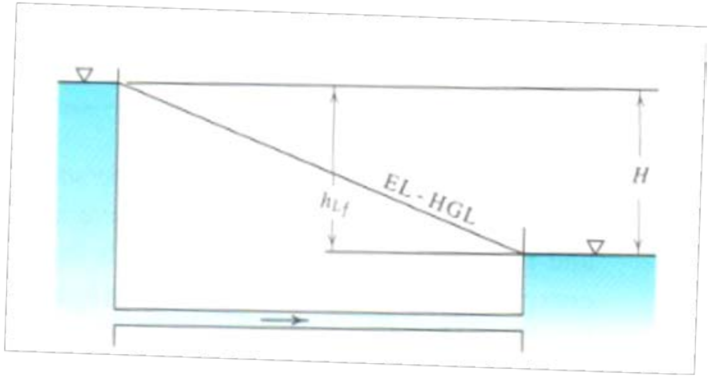


Ignore $\frac{V^2}{2g_n}$

IP 9.15 (p.368-9)

- A clean cast iron pipeline 0.30 m in diameter and 300 m long connects two reservoirs having surface elevations 60 m and 75 m. Calculate the flowrate through this pipe, assuming water at 10°C and a square-edged entrance.
- Calculation Procedure for Type 2:
  1. Calculate Reynolds number with assumed value of velocity in pipe (V_1)
  2. Determine *e/d* and friction factor (use **Moody diagram**)
  3. Calculate the local losses and whole pipe frictions
  4. Then apply work-energy eq.
  5. From this equation, determine pipe velocity (V_2).
  6. If ($V_2 - V_1$) is larger than error limit, recalculate Reynolds number.
  7. Then, obtain new friction factor from **Moody diagram**
  8. With new *f* value, apply work-energy eq.to calculate again velocity (V_3).
  9. Repeat this procedure until $\Delta V < \varepsilon$

$$\mathrm{Re} = \frac{Vd}{\nu} = \frac{V \times 0.3}{1.306 \times 10^{-6} m^2 / s} = 229{,}000V$$



1안: Guess $V \approx 2m/s,$

$$\mathrm{Re} = 458{,}000$$

2안 : $Guess\ f \rightarrow \left(0.5 + 0.025\frac{300}{0.3} + 1\right)\frac{V^2}{2g_n} = 26.5\frac{V^2}{2g_n} = 15$

$$V \approx 3.36m/s \rightarrow \mathrm{Re} = 769{,}440$$

- From Fig. 9.11, for clean iron casting pipe you can get *e/d* = 0.00083.
- *e/d* and Reynolds number give friction factor as 0.02 (Moody diagram).
- Now apply work-energy eq. with energy loss term consisting of friction loss and minor losses

$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g_n} = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g_n} + h_L$$

$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g_n} = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g_n} + \left(0.5 + f\frac{l}{d} + 1\right)\frac{V^2}{2g_n}$$

$$75 + 0 + 0 = 60 + 0 + 0 + \left(0.5 + 0.02\frac{300}{0.3} + 1\right)\frac{V^2}{2 \times 9.81} = 1.096V^2$$

$$V = 3.70m/s$$

- Now with this velocity, recalculate Reynolds number (847,250) and *f* (0.0193). Then we get *V=3.76m/s*.

$$\Delta V = 3.76 - 3.70 = 0.06 \sim small\, enough$$

$$Q = AV = \frac{\pi}{4}(0.3)^2(3.76) = 0.266\, m^3/s$$

IP 9.16 (p. 370)

- A smooth PVC pipeline 200 ft long is to carry a flowrate of 0.1 ft^3/s between two water tanks whose difference in surface elevation is 5 ft. If a square-edged entrance and water 50°F are assumed, what diameter of pipe is required.

  - Type 3 problem represents the most difficult to solve because the unknown diameter renders both the Reynolds number and the relative roughness as unknown.
  - But, in case of smooth pipe, the relative roughness is irrelevant, thus we can use the smooth pipe line on the Moody diagram.

- Sol)

$$\mathrm{Re} = \frac{Vd}{\nu} = \frac{Qd}{A\nu} = \frac{0.1d}{\frac{\pi}{4} d^2 (1.41 \times 10^{-5}\ ft^2/s)} = \frac{9,020}{d} \qquad \text{(A)}$$

- Apply work-energy eq. with energy loss term consisting of friction loss and minor losses

$$5 = \left(0.5 + f\,\frac{200}{d} + 1 \right) \frac{V^2}{2g_n} \qquad \text{(B)}$$

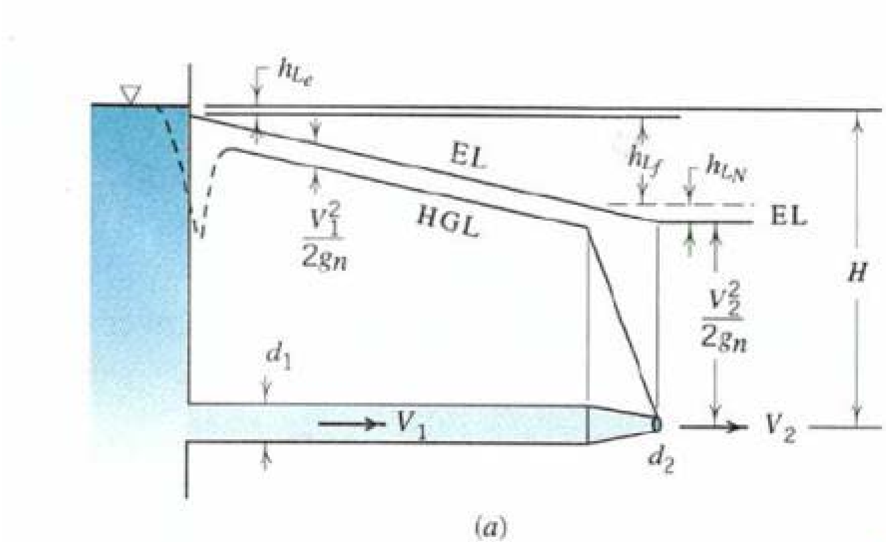
- Assume d = 0.25 ft, then from (A) we get Re = 36,080
- Use Moody diagram to get f = 0.022
- And, use $V = Q/A$ to get V = 2.04; $V^2/2g$ = 0.0644
- Then, RHS of (B) is 1.23 which is not close to LHS of (B).

- Repeat this process

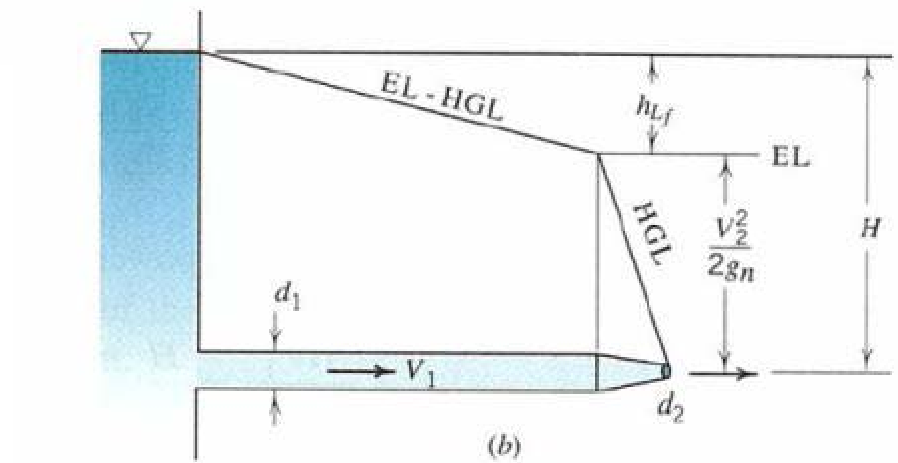
Moody diagram

d	*Re*	*f*	*V*	$V^2/2g$	*RHS of (B)*
0.25	36,000	0.0220	2.04	0.0644	1.23
0.20	45,100	0.0212	3.18	0.157	3.56
0.18	50,100	0.0208	3.93	0.240	5.90
0.187	48,200	0.0210	3.64	0.206	4.94

- *Pipeline from a reservoir terminating in a nozzle*



Real problem

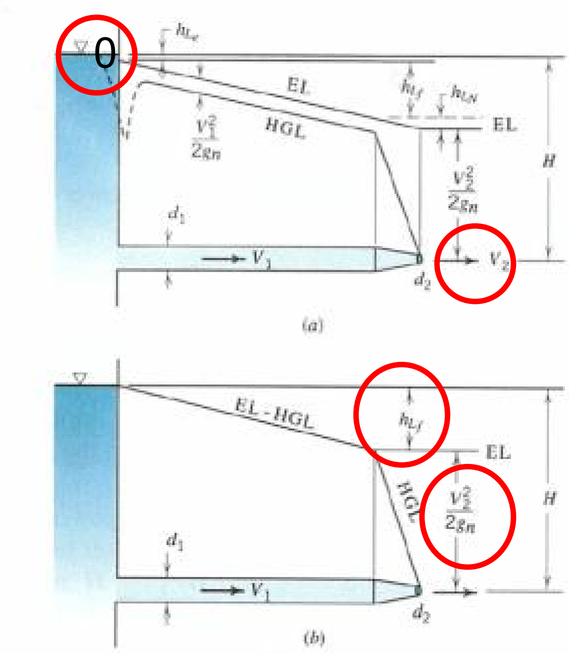


Simplified problem

- In this case, mainline velocity head is either significant or negligible but, nozzle's velocity need to be considered.

$$z_0 + \frac{p_0}{\gamma} + \frac{V_0^2}{2g_n} = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g_n} + h_L$$

$$p_0 = 0; V_0 \approx 0; z_2 = 0; p_2 = 0$$



- Exit loss can be ignored.

Nozzle velocity

$$h_L = \left(K_{L_e} + f\frac{l}{d_1}\right)\frac{V_1^2}{2g_n}$$

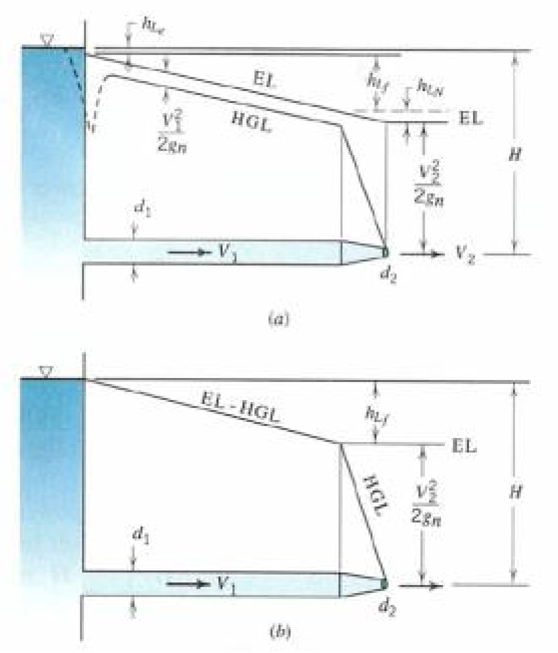
Pipe velocity

$$H = \frac{V_2^2}{2g_n} + \left(K_{L_e} + f\frac{l}{d_1}\right)\frac{V_1^2}{2g_n} \qquad (1)$$

- Using continuity equation

$$A_1V_1 = A_2V_2 \qquad V_2 = \left(\frac{d_1}{d_2}\right)^2 V_1$$

$$\boxed{H = \left(\frac{d_1^4}{d_2^4} + K_{L_e} + f\frac{l}{d_1}\right)\frac{V_1^2}{2g_n}} \qquad (2)$$



- Now think about the case of **generating electricity by jet**
- Power is work done in given time or product of pressure and flow rate.
- Here actual pressure is head (with specific weight)

$$Power = Energy\ \ flow\,rate = weight\ \ flow\,rate \times energy\ \ per\ unit\ weight$$

$$\boxed{P = (Q\gamma)\left(\frac{V_2^2}{2g_n}\right)} \qquad (3)$$

- Rearrange (1) neglecting local loss

$$\boxed{H_e = \frac{V_2^2}{2g_n} = H - f\frac{l}{d_1}\frac{V_1^2}{2g_n} = H - f\frac{l}{d_1}\frac{Q^2}{2g_n A_1^2}}$$

H_e = effective head

- Substitute this into (3)

$$P = \gamma Q\left(H - \frac{flQ^2}{2g_n d_1 A_1^2}\right)$$

- Find <u>maximum jet power</u> by differentiating; $\dfrac{\partial P}{\partial Q} = 0$

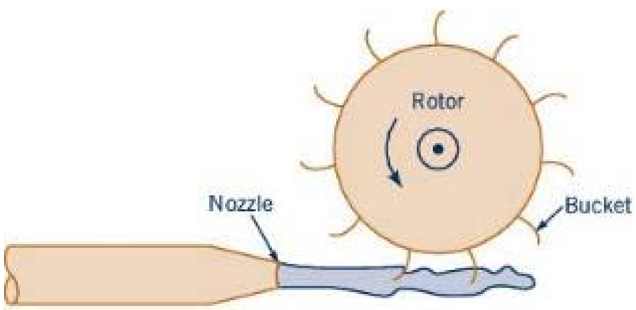
$$\frac{flQ^2}{2g_n d_1 A_1^2} = \frac{H}{3}$$

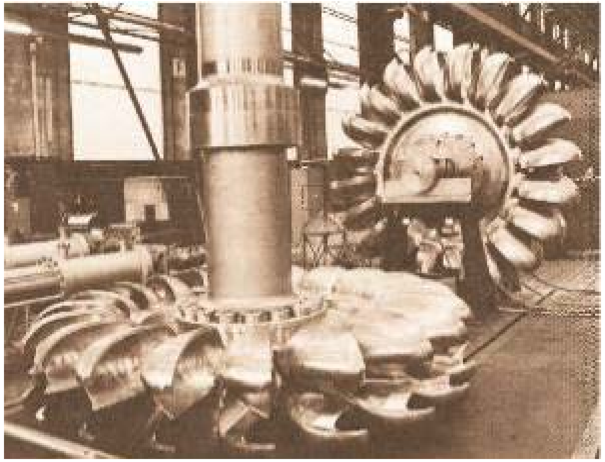
$$\Rightarrow \boxed{\frac{V_2^2}{2g_n} = \frac{2}{3}H \quad and \quad f\frac{l}{d_1}\frac{V_1^2}{2g_n} = \frac{1}{3}H}$$

→ <u>Actual limitation</u> of the flow rate.

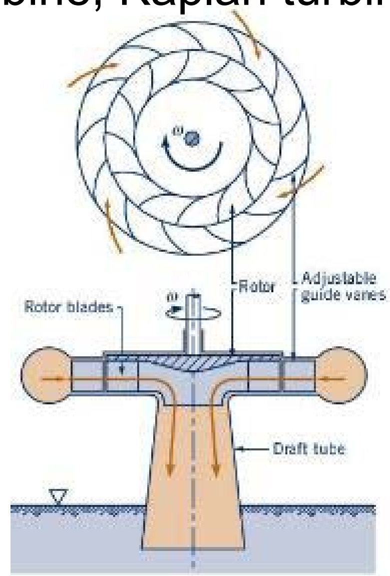
▪ Turbomachine

High head (impulse turbine): Pelton turbine
Low head (reaction turbine): Francis turbine, Kaplan turbine

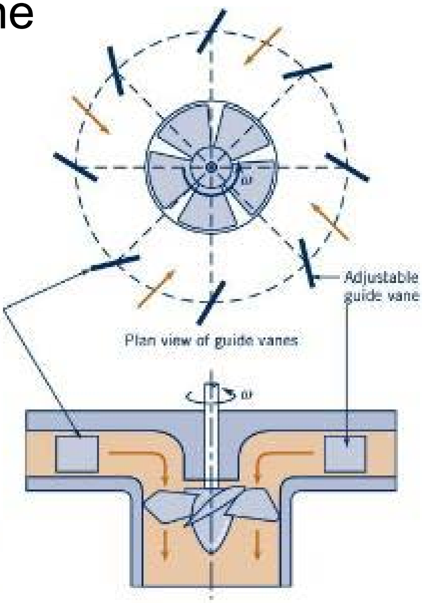




Pelton turbine



Francis turbine
(radial flow)



Kaplan turbine
(axial flow)

Cavitation

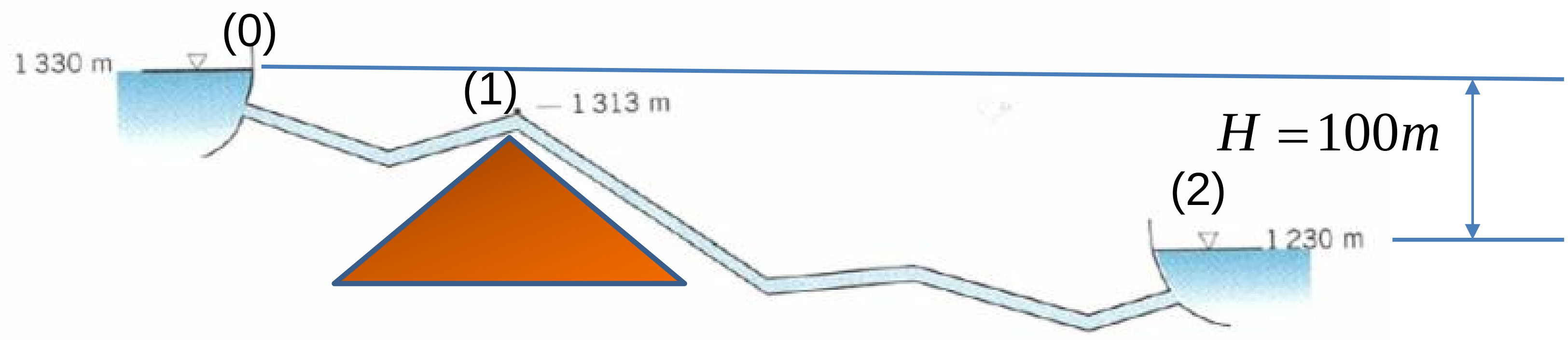


- When flow is too fast, the absolute pressure may fall to the vapor pressure of the liquid, at which point cavitation sets in.
- Large negative pressure in pipe should be avoided, if happens, then prevent from exceeding about two thirds of the difference between barometric (101.3 kPa) and vapor pressures (2.34 kPa for water at 20℃).
- Most engineering liquids contain dissolved gases which will come out of solution well before the cavitation point is reached.
- Such gases become bubbles which collect in the high points of the line, reduce the flow cross section and tend to disrupt flow.

IP 9.17 (p. 372~3)

A pipeline is being designed to convey water between two reservoirs whose elevations are shown below. The pipeline is 20km long and the preliminary pipeline profile has the line passing over a ridge where the pipeline elevation is 1,313 m at a distance of 4km from the upstream reservoir.

There is concern that the ridge is too high and will create an unacceptably low pressure in the pipeline. What is your recommendation on as to the feasibility of the proposed location of the pipeline?



Solution:

- Let's neglect the local losses and consider the <u>energy line and the hydraulic grade line to be coincident</u> (**neglect velocity head**)
- Then EL-HGL will fall uniformly 100 m over 20km. $\rightarrow h_L = 100$
- Since the ridge is 4 km, then EL-HGL elevation at the ridge will be

$$EL - HGL = 1330 - 1/5 \times 100 = 1310$$

- Since we assume EL=HGL, it means that <u>velocity head at the pipe is negligible</u>. Therefore, the difference of elevation and EL height will result in pressure drop.

$$z_0 = z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g_n}\left(\approx 0 \text{ EL=HGL}\right) + h_{f1}$$

$$1330 = 1313 + \frac{p_1}{\gamma} + h_{f1}(=20)$$

$$\frac{p_1}{\gamma} = -3m$$

- This is smaller than 2/3 of water vaporized pressure (-10 m of water), so seems to be OK.
- But, in practical, the air entrained and dissolved gas will be vaporized and it will reduce the capacity of pipe.
- We may need to think another route for this pipe.

5.3 Pumped Pipeline

- Gravity-flow pipelines: use gravity
- Pumped pipeline
  - Source pump: located at the upstream end of the pipeline
  - Booster pump: located at intermediate point in the pipeline
- To determine the power required to meet flow rate (Q) and pressure demands (H), use work-energy equation

$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g_n} + E_p = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g_n} + h_L \qquad (9.48)$$

- E_p is work per unit weight added to the fluid by the pump,
- For power

$$P(KW) = \frac{Q\gamma E_P}{1{,}000} \qquad (9.49)$$

- Waste water treatment plant: gravity-driven flows + booster pump

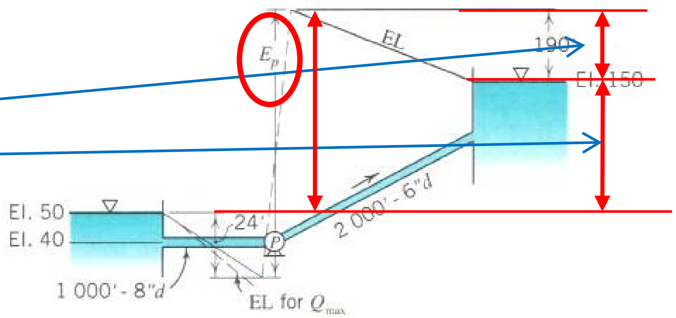
- Waste water treatment plant:

침사지 → 제1침전지 → 반응조 → 제2침전지 → 염소접촉조 → 하천 방류

Designing pump

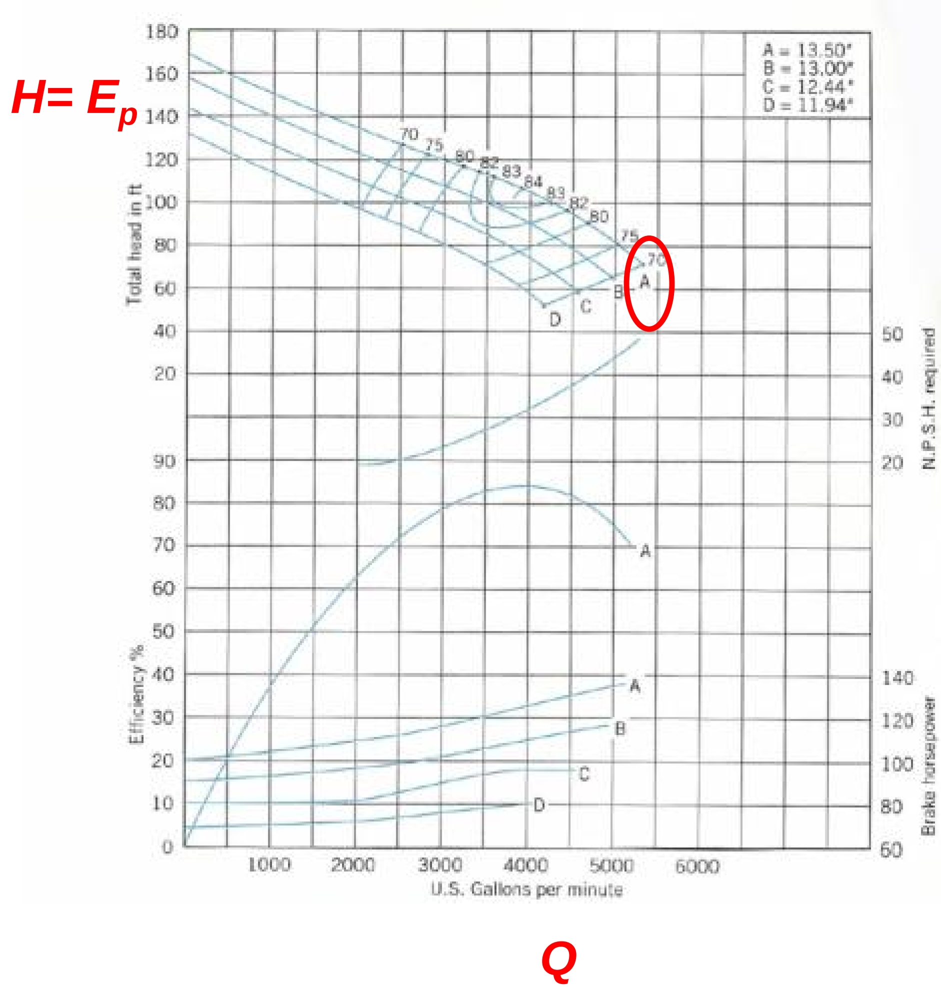
- To determine power requirements for the motor driven pump, the efficiency of the pump should be known.
- A pump is a reactive element in a pipe system.
- → The flowrate (Q) through the pump and head increase (H) across the pump depend on the pipeline system in which the pump is installed. The pump will respond to the demand placed on it by the system (pump + pipeline).
- The demand (E_p) the system makes on the pump depends on the friction losses (h_L) in the system as functions of flowrate (Q) and the vertical lift (head increase, Δz) required between the two ends of the pipeline.

$$E_p = H = h_L(Q) + \Delta z$$



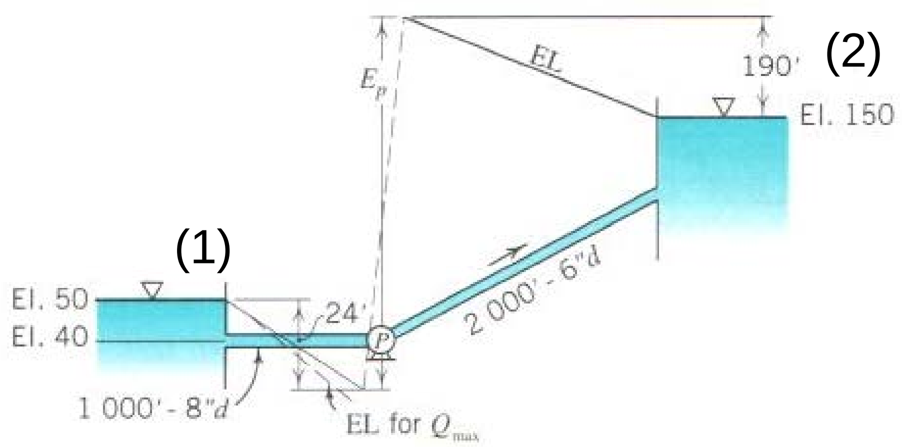
Typical pump characteristic diagram

- Graphical representation of both system demand and pump performance
- Fig. 9.25 depicts the <u>total head increase (*H*)</u> the pump will supply for a given <u>flowrate</u> for the <u>four different impeller sizes</u> (curves A, B, C, and D).
- Fig. 9.25 depicts the <u>power requirements</u> necessary to drive the various-sized impellers along with efficiency contours.



IP 9.18 (p. 375~6)

1) Calculate the horsepower that the pump must supply to the water (50°F) in order to pump 2.5 ft^3/s through a clean cast iron pipe from the lower reservoir to the upper reservoir. Neglect local losses and velocity heads. → find head loss
2) Using the criteria for minimum allowable pressure suggested earlier in this section, compute the maximum dependable flow which can be pumped through this system. → find velocity



Solution:

- To get the pump power, apply work-energy eq. between 1 & 2

$$z_1 + \cancel{\frac{p_1}{\gamma}} + \cancel{\frac{V_1^2}{2g_n}} + E_p = z_2 + \cancel{\frac{p_2}{\gamma}} + \cancel{\frac{V_2^2}{2g_n}} + h_L \rightarrow E_p = H + h_L$$

- Power can be determined by

$$WHP = \frac{Q\gamma E_p}{550}$$ (U.S. customoary)

$$V = \frac{Q}{A}; \quad V_8 = 7.16\ ft/s \quad and \quad V_6 = 12.72\ ft/s$$

$$\frac{V_8^{\ 2}}{2g} = 0.796\ ft; \quad \frac{V_6^{\ 2}}{2g} = 2.51\ ft$$

- Reynolds numbers for each pipe are

$$\mathrm{Re}_8 = 338{,}500 \qquad \mathrm{Re}_6 = 451{,}000$$

- Since we already know this pipe is <u>cast iron pipe</u> then we can use Fig. 9.11 for determining the roughness height, and with this and Reynolds numbers, we can find the friction factor.

$$\left(\frac{e}{d}\right)_8 = 0.00128 \qquad f = 0.021$$

$$\left(\frac{e}{d}\right)_6 = 0.00171 \qquad f = 0.022$$

- The head loss in each of the pipes can now be calculated.

$$h_{L_8} = f\frac{l}{d}\frac{V^2}{2g_n} = 0.021\frac{1,000\,ft}{8in./12}\frac{(7.16\,ft/s)^2}{2\times 32.2} = 25\,ft$$

$$h_{L_6} = f\frac{l}{d}\frac{V^2}{2g_n} = 0.022\frac{2,000\,ft}{6in./12}\frac{(12.72\,ft/s)^2}{2\times 32.2} = 221\,ft$$

- Consider the total losses from the both pipes

$$50 + 0 + \cancel{0} + E_p = 150 + 0 + \cancel{0} + 25 + 221$$

$$E_p = 346\,ft$$

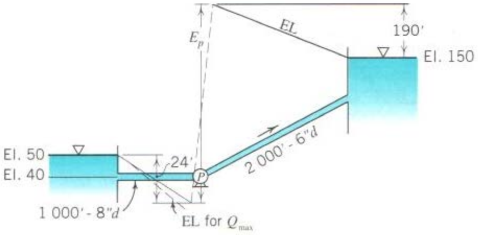
$$WHP = \frac{Q\gamma E_P}{550} = \frac{2.5\,ft^3/s \times 62.4\,lb/ft^3 \times 346\,ft}{550} = 98\,hp$$

2) We need to check the **pressure drop**, which should not be lower than 20 ft (cavitation occurs).

Apply work-energy eq. between 1 and suction side (neglecting the local loss)

$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g_n} = z_s + \frac{p_s}{\gamma} + \frac{V_s^2}{2g_n} + h_{L_8}$$

$$50 + 0 + 0 = 40 + \boxed{(-20)} + 0 + h_{L_8}$$



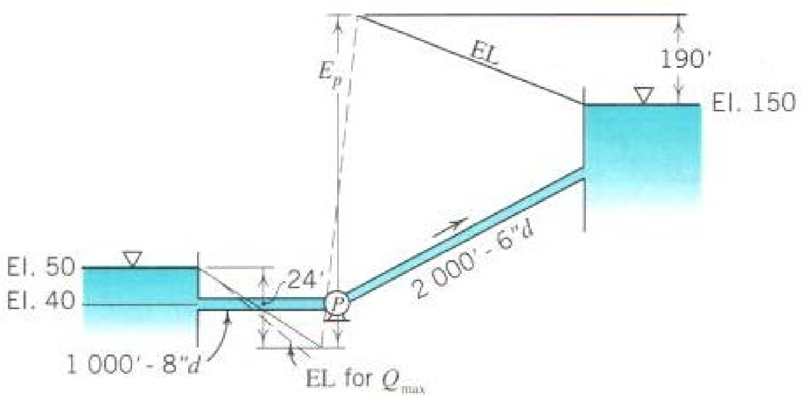
Maximum allowable pressure drop

- Therefore, $$h_{L_8} = 30\,ft = f\,\frac{l}{d}\frac{V_8^2}{2g_n}$$

$$V_8 = 7.8\,ft/s \qquad \longleftarrow \qquad > 7.16\,ft/s$$

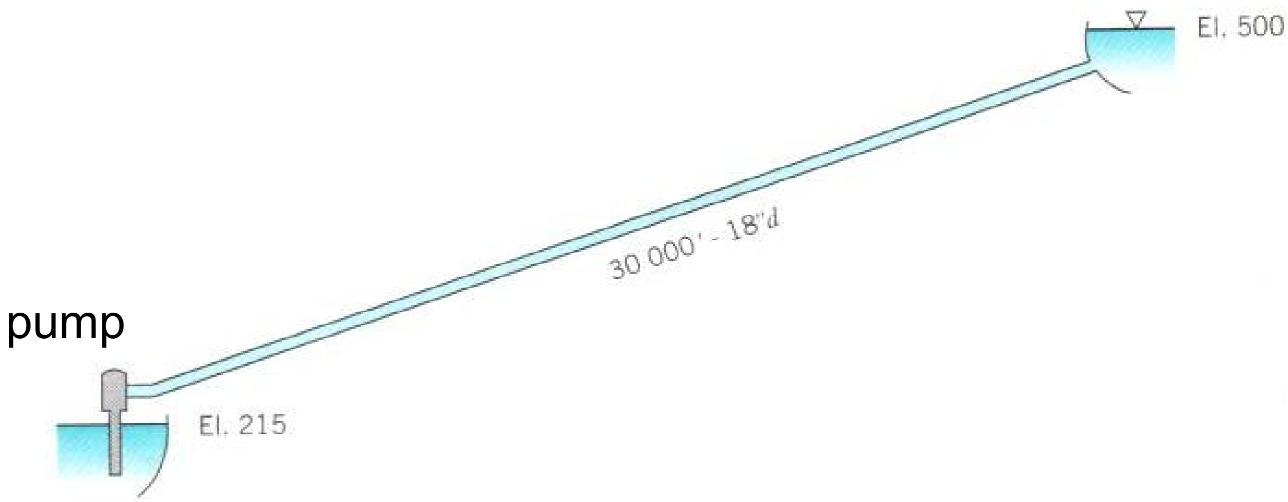
$$Q_{\max} = V_8 A = 2.7\ ft^2/s$$

- So this is the maximum flow speed that can be pumped through this pipe system.



IP 9.19 (p. 378~9)

- The pump whose characteristics are given in Fig. 9.25 (p. 374) is proposed for use in the pipe system shown below. In order to provide the total head (H) necessary to deliver the required flowrate (Q) to the upper reservoir, a four-stage pump is planned. Using the graphical technique determine the flowrate (Q) produced by the proposed pumping configuration and estimate the efficiency of the pump. Neglect local losses and use a Darcy-Weisbach *f*-value of 0.018.



I. System demand curve: Apply work-energy eq. between 1 and 2

$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g_n} + E_p = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g_n} + f\frac{L}{d}\frac{V^2}{2g_n}$$

$$E_p = H = \Delta z + h_L$$

Δz

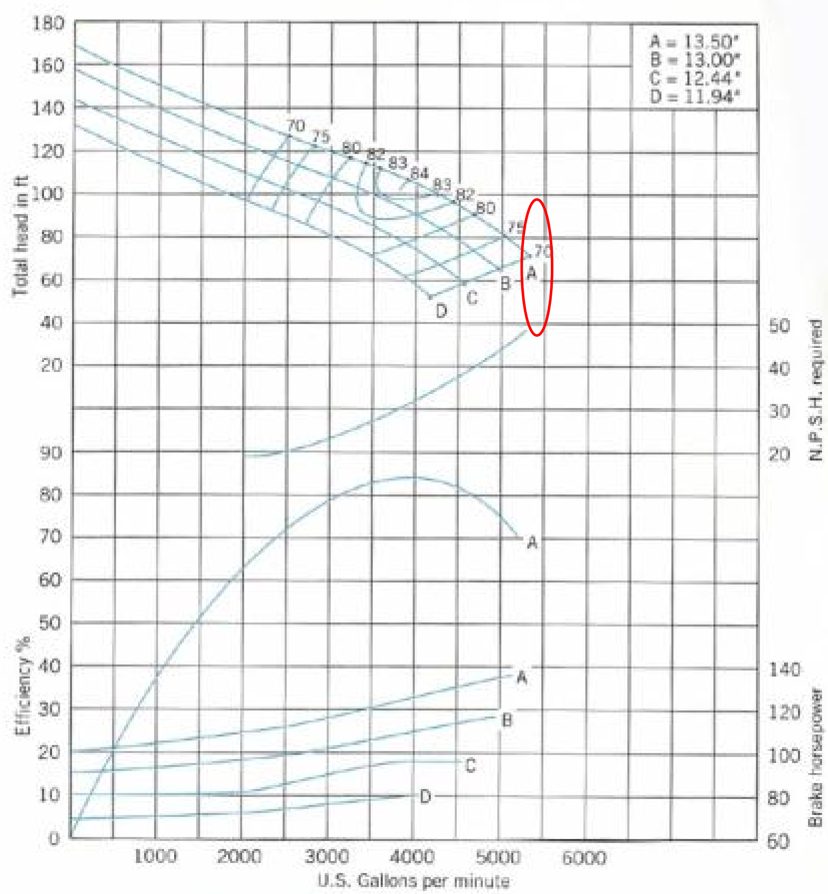
$$215 + 0 + 0 + E_p = 500 + 0 + 0 + 0.018\frac{3000\,ft}{18in/12}\frac{V^2}{2\times 32.2}$$

$$\boxed{E_p = 285 + 5.59V^2}$$

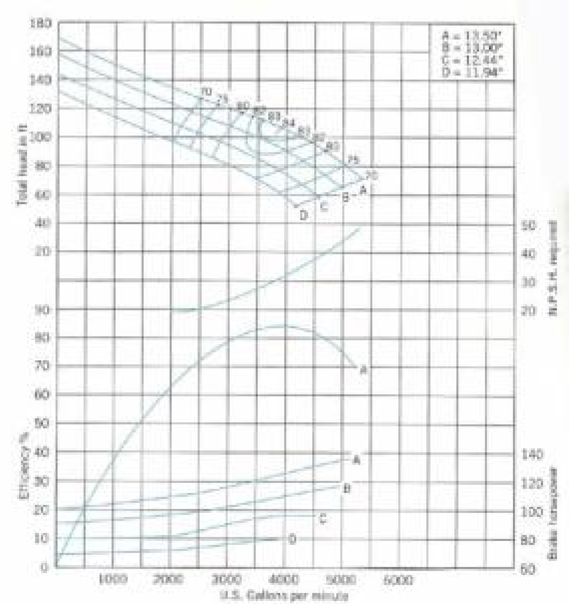
Q (ft^3/s)	V	$5.59V^2$	$E_p=285+5.59V^2$
0	0	0	285
2	1.13	7.1	292
4	2.26	28.6	314
6	3.40	64.6	350
8	4.53	114.7	400
10	5.66	179.1	464

II. Pump supply curve

- The flowrate through the pump and the head increase across the pump <u>depend on the pipeline system.</u>
- Fig. 9.25 depicts the head increase the pump will supply for a given flowrate for the four different impeller sizes (curves A, B, C, and D).
- <u>Use curve A</u>

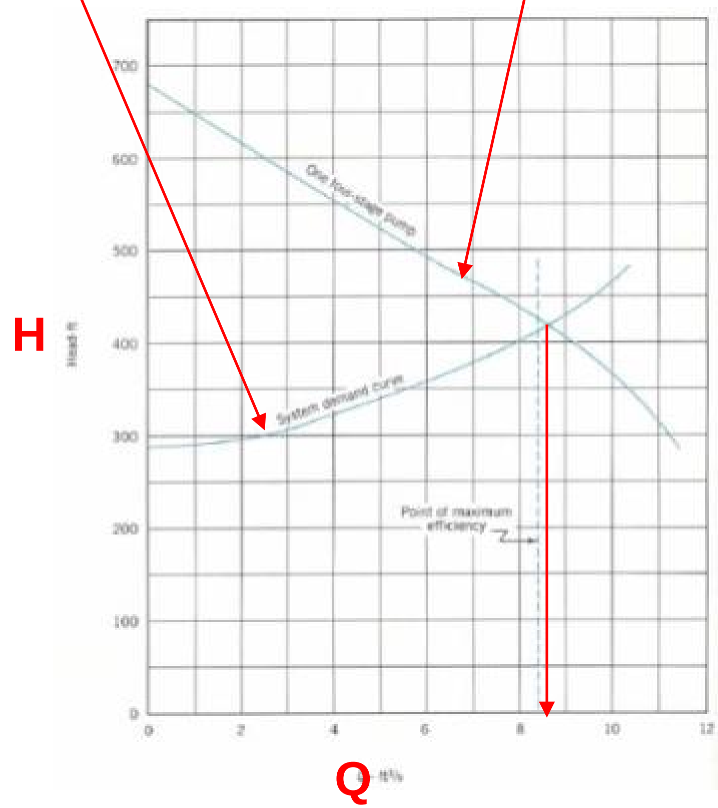


- Pump Supply Curve
- Use curve A for 1 stage pump



Q (gal/min)	Q(ft^3/sec)	1 stage head, H	4 stage head, H
0	0	168	672
1,000	2.23	150	600
2,000	4.45	133	532
3,000	6.68	119	476
4,000	8.91	103	412
5,000	11.14	78	312

- Plot the system demand curve and pump supply curve and find the flowrate from their intersection, which is 8.7 ft^3/s (3,900 gal/min).
- Pump's maximum efficiency is found at the similar value to the cross point of two curves.



- Pump Supply Curve (use figure 9.25)