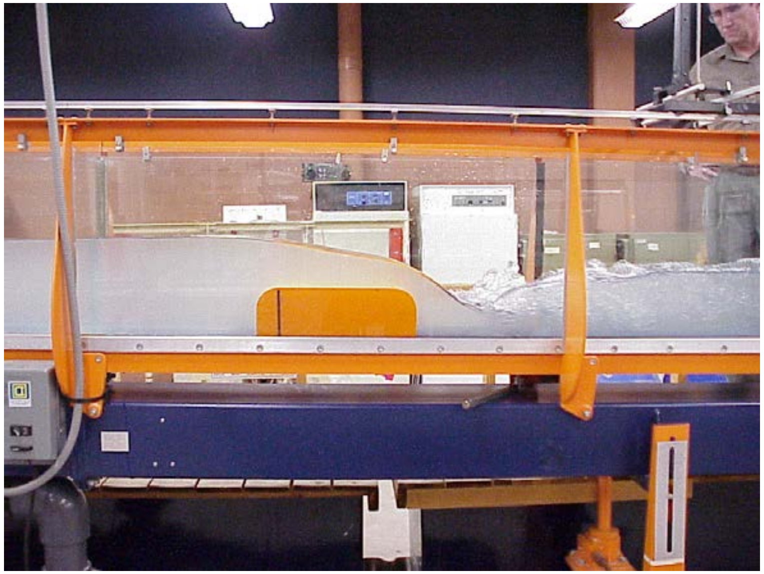
Ch. 7 Uniform Flow in Open Channels

7-1 Fundamentals



Contents

Class objectives

- Identify the open channel flows and types of open channels
- Review the momentum equations

Outline of Open Channel Flow

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- Open channels

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- Impulse-momentum Eq.

Uniform flow
- Chezy Eq.
- Manning Eq.
- Hydraulic radius and channel efficiency

Steady non-uniform flow
- Specific energy
- Critical flow
- Hydraulic jump
- Specific force

Gradually varied flow
- GVF Eq.
- Classification of GVF
- Numerical method

7.0 Open Channels

Open channels can be defined as <u>conveyance structures or flow passages</u> which transfer water from one location to the other under <u>gravity forces</u>

The characteristics of flow are greatly influenced by the <u>geometric characteristics of conduits</u>.

☞ A hydraulic study of flow requires consideration of these characteristics

Types of open channels

- Natural channels vs. Artificial channel
- Prismatic vs. Non-prismatic (uniform cross section and constant bed slope)
- Flumes
- Drops
- Culverts

Restored streams

Roman aqueduct

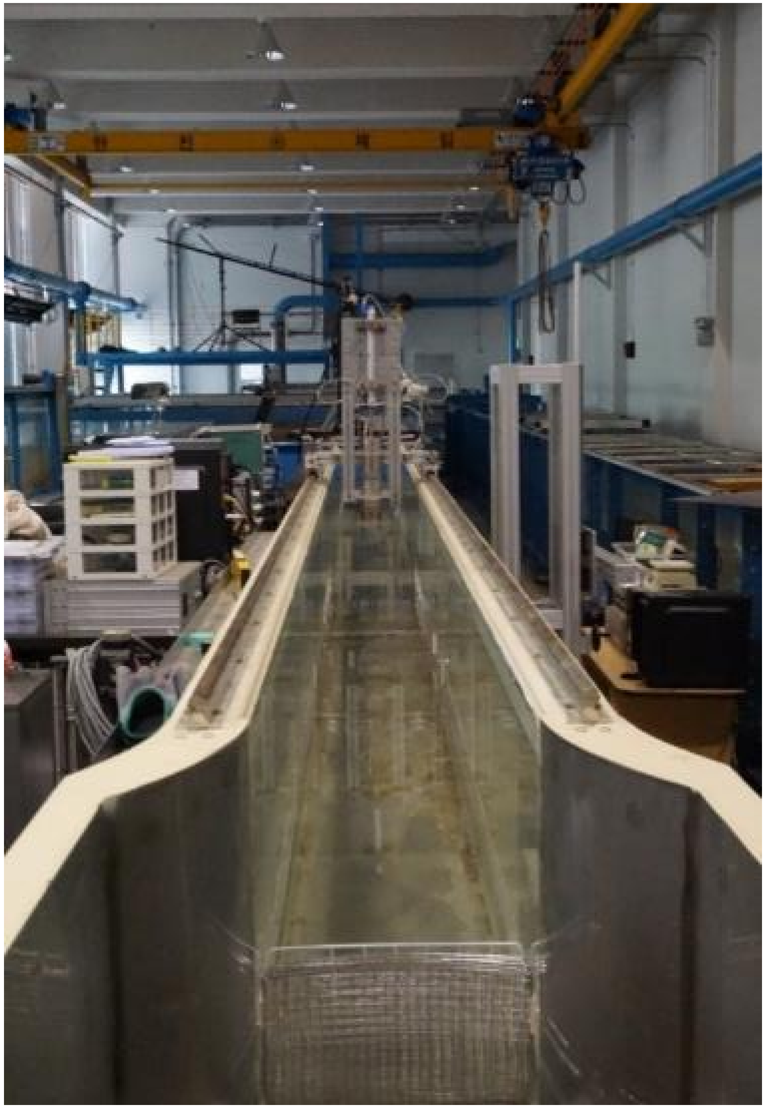
Israel's water irrigation canal

Old Korean Examples

Byunkgol Jae, Gimjae, built in 330.

Uelim Ji, Jaechen. (built firstly between B.C 300 A.D 300)

Culvert



7.1 Fundamentals

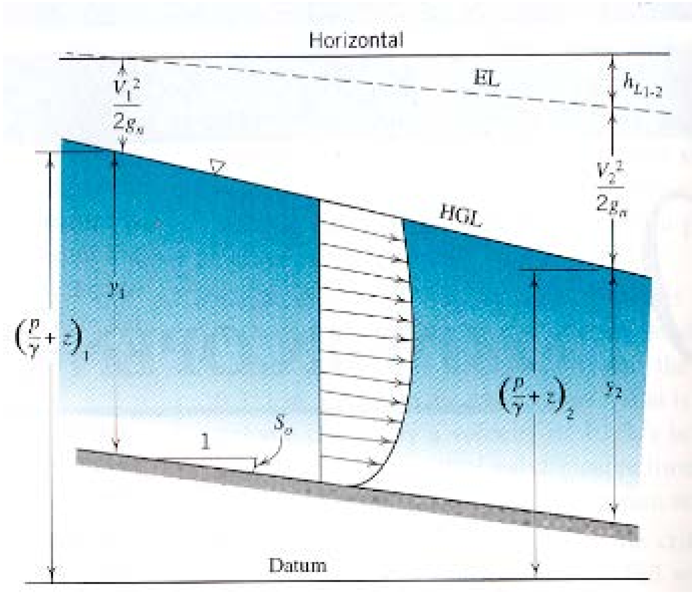
- Pipe flow vs. open channel flow

- In pipe flow, the pressure in the pipe can vary along the pipe

- In open-channel flow, the pressure is constant, usually atmospheric on the entire surface
- The pressure variations within the open-channel flow can be determined by the principle of hydrostatics since the streamlines are straight, and parallel
- Not pressure driven flow, but <u>gravity driven</u> flow

Apply work-energy equation to open-channel flows

$$\left(\frac{p}{\gamma}+z\right)_1+\frac{V_1^2}{2g_n}=\left(\frac{p}{\gamma}+z\right)_2+\frac{V_2^2}{2g_n}+h_L$$



- Classifications
  - Laminar (층류) or turbulent (난류) → Reynolds number
  - Steady (정상류) or unsteady (부정류)
  - Uniform (등류) of varied (non-uniform; 부등류)
  - Subcritical (상류) or supercritical (사류) → Froude number

- Prismatic channel (균일수로)
  - Has unvarying cross-sectional shape
  - Constant bottom slope

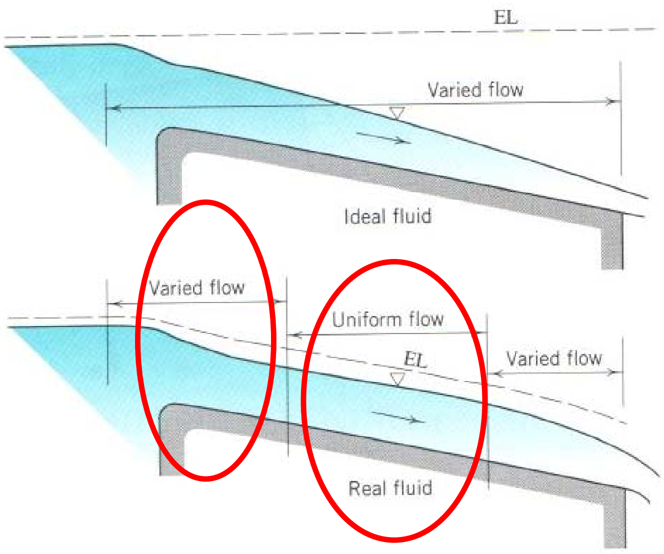
Fundamentals

- In ideal flow,
  - resistance is not encountered as it flows down the channel.
  - Because of this lack of resistance, the fluid continually accelerates under the influence of gravity
  - Thus, the mean velocity increases
  - Then cross section decreases by continuity.
  - So <u>depth continually varies</u>
- *In real fluid flow*
  - <u>Resistance forces exist</u>

  At upstream ~ flow is accelerated until gravitation acceleration and frictions are balanced.

  <u>Then, velocity becomes constant</u>

  → **Uniform flow** achieved



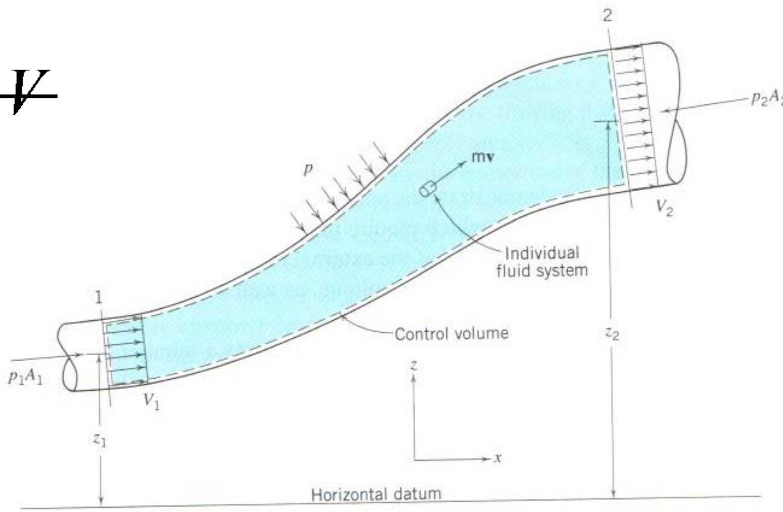
7.1.1 The impulse momentum principle

- Apply momentum principle to steady non-uniform flows
- The force is equal to rate of momentum change in a given time.
- For individual fluid system in the control volume

$$\sum \mathbf{F} = m\mathbf{a} = \frac{d}{dt} m\mathbf{v} = \frac{d}{dt} \rho \mathbf{v} d\forall$$

- For every individual fluid system

$$\sum \mathbf{F}_{ext} = \iiint_{sys} \frac{d}{dt}(\rho \mathbf{v} d\forall) = \frac{d}{dt} \iiint_{sys} \rho \mathbf{v} d\forall$$



The impulse momentum principle

- Right hand side can be evaluated by the Reynolds transport theorem for steady flow (Ch. 4)

$$\frac{dE}{dt}=\frac{d}{dt}\iiint_{sys}\rho\mathbf{v}\,d\cancel{V}=\cancel{\frac{\partial}{\partial t}\iiint_{CV} i\rho\, d\cancel{V}}+\iint_{c.s.out} i\rho\left(\mathbf{v}\cdot d\mathbf{A}\right)+\iint_{c.s.in} i\rho\left(\mathbf{v}\cdot d\mathbf{A}\right)$$

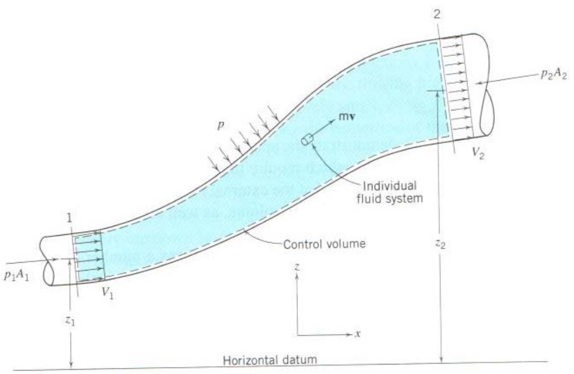
- Where *E* is momentum of the fluid system, *i* = velocity = *v*

$$\iint_{c.s.out}\rho\mathbf{v}\left(\mathbf{v}\cdot d\mathbf{A}\right)=\rho\mathbf{v}\left(\mathbf{v}\cdot\mathbf{A}\right)\Big|_{V_2}=\rho V_2 Q_2$$

($\mathbf{v}$ is constant, so only surface integral)

$$\iint_{c.s.in}\rho\mathbf{v}\left(\mathbf{v}\cdot d\mathbf{A}\right)=\rho\mathbf{v}\left(\mathbf{v}\cdot\mathbf{A}\right)\Big|_{V_1}=-\rho V_1 Q_1$$

$$\boxed{\frac{d}{dt}\iiint_{sys}\rho\mathbf{v}\,d\cancel{V}=Q\rho\left(\mathbf{V}_2-\mathbf{V}_1\right)}$$



The impulse momentum principle

- Substituting the previous results into the original equations, then

$$\boxed{\sum F_{ext} = Q\rho\left(\mathbf{V}_2 - \mathbf{V}_1\right)}$$

- In two dimensions,

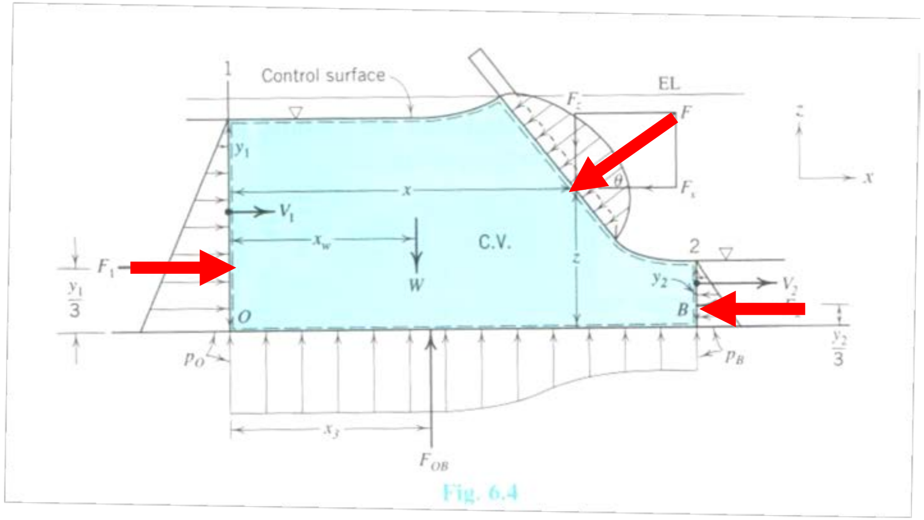
$$\left(\sum F_{ext}\right)_x = Q\rho\left(\mathbf{V}_{2_x} - \mathbf{V}_{1_x}\right)$$

$$\left(\sum F_{ext}\right)_y = Q\rho\left(\mathbf{V}_{2_y} - \mathbf{V}_{1_y}\right)$$

7.1.2 Open Channel Flow Applications

- Forces on open channel structure such as sluice gates and weirs.
- The hydraulic jump

1. Sluice gate problem



$$\left(\sum F_{ext}\right)_x = F_1 - F_2 - F_x$$

$$= Q\rho\left(\mathbf{V}_{2_x} - \mathbf{V}_{1_x}\right)$$

$$= Q\rho\left(V_2 - V_1\right)$$

$$\frac{\gamma y_1^2}{2} - \frac{\gamma y_2^2}{2} - F_x = q^2\rho\left(\frac{1}{y_2} - \frac{1}{y_1}\right)$$ (since $V = q/y$)

In hydrostatics $F = \gamma h_c A = \gamma \frac{y}{2}(1\times y) = \frac{1}{2}\gamma y^2$

- Finally, force on the sluice gate

$$F_x = \frac{\gamma}{2}\left(y_1^2 - y_2^2\right) + q^2 \rho \left(\frac{1}{y_1} - \frac{1}{y_2}\right)$$

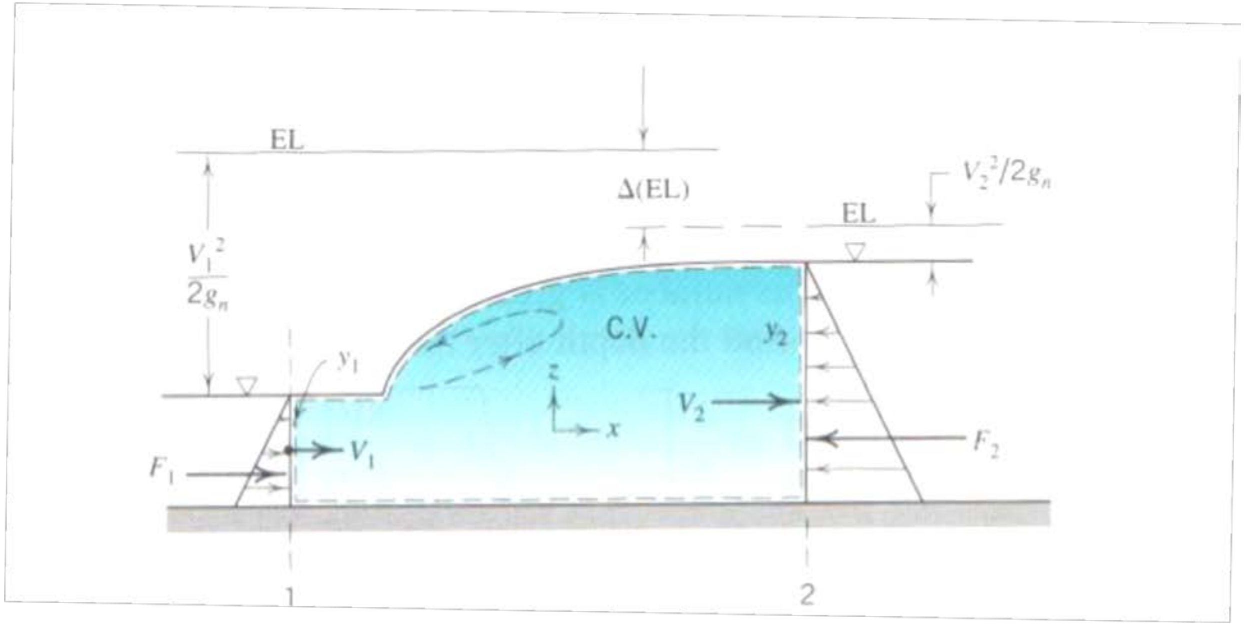
- Resultant force toward normal on the surface is

$$F = \frac{F_x}{\cos\theta}$$

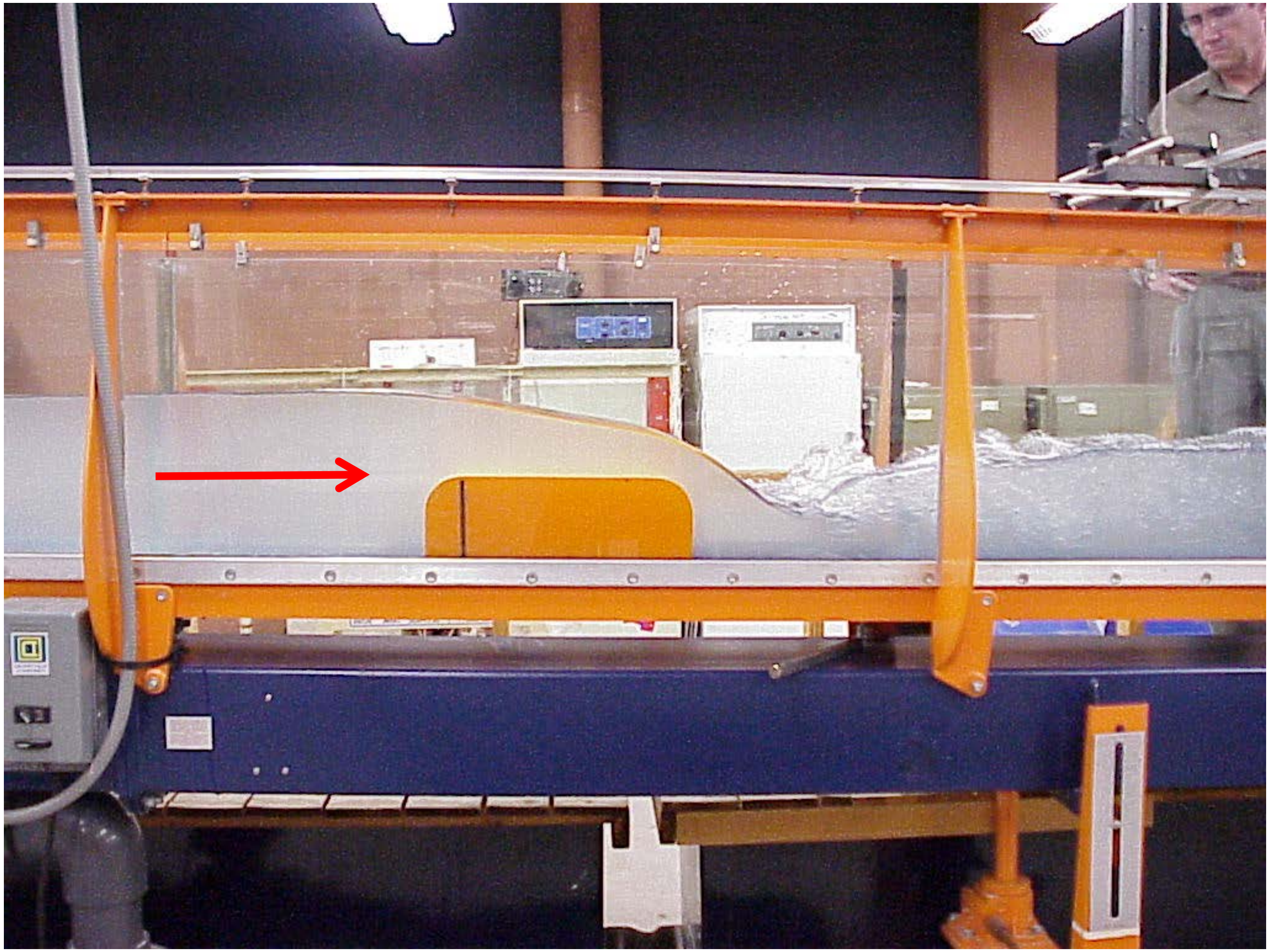
- We don’t need to calculate in *z* –direction.

2. Hydraulic jump

Frequently in open channel flow, when liquid at high velocity discharges into a zone of lower velocity, a rather abrupt rise (a standing wave) occurs in the liquid surface and is accompanied by violent turbulence, eddying, air entrainment, and surface undulations.→ ***hydraulic jump***



Flow over broad-crested weir



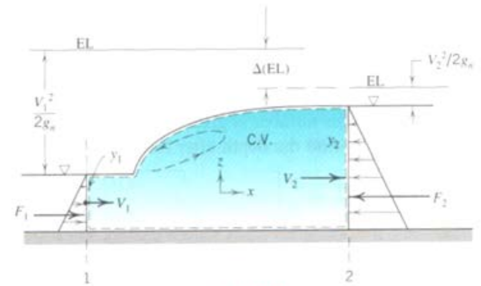


Fig. 6.5

- Hydraulic Jump calculations
- Force only 1 and 2
- Therefore

$$\left(\sum F_{ext}\right)_x = F_1 - F_2 = \frac{\gamma}{2}\left(y_1^2 - y_2^2\right) = q^2 \rho \left(\frac{1}{y_2} - \frac{1}{y_1}\right)$$

$$\frac{q^2}{g_n y_1} + \frac{y_1^2}{2} = \frac{q^2}{g_n y_2} + \frac{y_2^2}{2}$$

Solving for y_2 / y_1 yields,

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{8q^2}{g_n y_1^3}}\right] = \frac{1}{2}\left[-1 + \sqrt{1 + \frac{8V_1^2}{g_n y_1}}\right]$$

- In the results

$$\frac{y_2}{y_1}=\frac{1}{2}\left[-1+\sqrt{1+\frac{8q^2}{g_n y_1^3}}\right]=\frac{1}{2}\left[-1+\sqrt{1+\frac{8V_1^2}{g_n y_1}}\right]$$

When $\frac{V_1^2}{g_n y_1}=1,\ \sqrt{1+\frac{8V_1^2}{g_n y_1}}=3,\ \frac{1}{2}\left[-1+\sqrt{1+\frac{8V_1^2}{g_n y_1}}\right]=1=\frac{y_2}{y_1}$

So $Fr_1^{\,2}=\frac{V_1^2}{g_n y_1}=1$ (Critical Reynolds number)

If $\frac{V_1^2}{g_n y_1}<1,$ non-physical..

I.P. 6.3 (p. 199-200)

- Water flows in a horizontal open channel at a depth of 0.6m at a flowrate of 3.7m³/s/m of width. If a hydraulic jump is possible, calculate the depth just downstream from the jump and the power dissipated in the jump.

$$V_1 = \frac{q}{y} = \frac{3.7m^3/s/m}{0.6m} = 6.17m/s$$

$$Fr_1^2 = \frac{V_1^2}{g_n y_1} = \frac{(6.17m/s)^2}{9.81\times 0.6m} = 6.47 \text{ (so jump is possible)}$$

$$\frac{y_2}{0.6m} = \frac{1}{2}\left[-1+\sqrt{1+8\times 6.46}\right] = 3.13$$

$$y_2 = 3.13\times y_1 = 3.13\times 0.6 = 1.88m$$

$$V_2 = \frac{q}{y_2} = \frac{3.7m^3/s/m}{1.88m} = 1.97m/s$$

- The drop in the energy line across the jump is the power dissipated in the jump per weight of fluid flowing.

$$\Delta EL = \left(y_1 + \frac{V_1^2}{2g_n} \right) - \left(y_2 + \frac{V_2^2}{2g_n} \right) = 0.46m$$

- The power dissipated in the jump is

$$P = \frac{q\gamma \Delta EL}{1{,}000} = \frac{3.7m^3/s/m \times 9{,}800N/m^3 \times 0.46m}{1{,}000} = 16.7kW/metre\ of\ width$$