

# Ch. 7 Uniform Flow in Open Channels 7-2 Uniform Flow Equation





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### **Class objectives**

- Apply the Chezy and Manning's to the uniform open channel flow problems
- Understand how to determine the channel efficiency





## 7.2 Uniform Flow – Chezy Equation

- Apply impulse-momentum eq. (Antonine de Chezy, 1775)
- For uniform flow, there is no change in momentum



The force balance is given as

 $F_1 + W\sin\theta - F_2 - Pl\overline{\tau}_0 = 0$ 

(P is wetted perimeter and l is length of channel)



#### Uniform flow – The Chezy equation

#### Pressure forces on the cross sections are the same $(F_1 = F_2)$ Therefore, $W \sin \theta = P l \overline{\tau}_0$

 $W = A\gamma l; \sin \theta = h_L / l \approx \tan \theta = S_0$  (For unifrom flow and small slope)  $A\gamma lS_0 = Pl\overline{\tau}_0$ 

$$\overline{\tau}_0 = \gamma \frac{A}{P} S_0 = \gamma R_h S_0$$

- where  $R_h = \frac{A}{P}$  (hydraulic radius) In pipe flow,  $\tau_0 = \frac{f}{o} \rho V^2$  (the shear stresses are same to above)
- Combining two

$$V = \sqrt{\frac{8g_n}{f}} \sqrt{R_h S_0}$$

$$V = C\sqrt{R_h S_0}; \quad Q = CA\sqrt{R_h S_0} \quad \text{where} \quad C = \sqrt{\frac{8g_n}{f}}$$





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For established incompressible flow, g is constant;  $d(1/\gamma) = 0$ 

$$d\left(\frac{p}{\gamma} + \frac{V^2}{2g} + z\right) = -\left(\frac{\tau_0 dl}{\gamma R_h}\right)$$

Integrating this between points 1 and 2 yields

$$\left(\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1\right) - \left(\frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2\right) = \frac{\tau_0 \left(l_2 - l_1\right)}{\gamma R_h}$$
(7.8)

Now, note that the difference between total heads is the drop in the energy

line between points 1 and 2. Work-energy equation for real fluid flow is

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_{L_{1-2}}$$
(7.9)

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#### Comparing (7.8) and (7.9) gives

$$h_{L_{1-2}} = \frac{\tau_0 \left( l_2 - l_1 \right)}{\gamma R_h}$$

(7.10)

BTW, Darcy-Weisbach's head loss equation is

$$h_L = f \frac{l}{d} \frac{V^2}{2g}$$
(9.2)

Equations (7.10) and (9.2) are combined to give a basic relation between

friction stress and friction factor; this is

$$\tau_0 = \frac{f}{8}\rho V^2 \tag{9.3}$$



## 7.3 Chezy Coefficient and the Manning's n

- Many laboratory and field experiments have been done so far to determine the magnitude of the Chezy coefficient C.
- The simplest relation and the most widely used is the result of the work of <u>Manning (Robert Manning, UK engineer, 1890)</u>
- The results may be summarized by the empirical relation

$$C = \frac{R_h^{1/6}}{n}$$

Chezy-Manning equation (n's unit is t<sup>1</sup>L<sup>-1/3</sup>)

$$V = \left(\frac{1}{n}\right) R_h^{2/3} S_0^{1/2}$$

In general form (SI or US Customary)

$$C = \frac{uR_h^{1/6}}{n} \qquad Q = \left(\frac{u}{n}\right)AR_h^{2/3}S_0^{1/2} \qquad (u = 1 \ (SI), \ u = 1.49 \ (U.S.))$$



## The Chezy Coefficient and the Manning's n

- The Manning *n* is obtained from the <u>channel type and properties</u> and some typical values are given in Table 5 in text book, pp. 438-439.
- There are no clear physical explanation for Chezy C or Manning *n*.
- Related with the friction factor f with n

$$C = \sqrt{\frac{8g_n}{f}}$$
$$n = R_h^{1/6} \sqrt{\frac{f}{8g_n}}$$
$$n = fn(R_h, f) = fn(R_h, \text{Re}, \frac{e}{d})$$

 In the channel, *n* is not an absolute roughness coefficient because <u>it</u> <u>depends on the hydraulic radius</u>, and must exhibit the same characteristics as the friction factor, which in turn <u>depends on relative roughness</u> and <u>Reynolds number</u>.



#### The Chezy Coefficient and the Manning's n

- For wholly rough channel, as R<sub>h</sub> increases, f decreases, so that n may change gradually for a given boundary surface for increasing depths and flow rates.
- This coefficient is empirical to be determined with the theoretical explanation.





### The Manning's n

Cowan (1956)

 $n = m_5(n_0 + n_1 + n_2 + n_3 + n_4)$ 









#### A. Conduits



TABLE 5-6. VALUES OF THE ROUGHNESS COEFFICIENT n(Boldface figures are values generally recommended in design)

Type of channel and description	Minimum	Normal	Maximum
CLOSED CONDUITS FLOWING PARTLY FULL		D'harts i	a set and
A 1 Motel			
A-1. Metal	0.009	0.010	0.013
a. Drass, smooth			
0. Steel	0.010	0.012	0.014
1. Lockbar and spiral	0.013	0.016	0.017
2. Riveted and spiral		al de la composition de la com	
c. Cast iron	0.010	0.013	0.014
1. Coated	0.011	0.014	0.016
2. Uncoated			
d. Wrought iron	0.012	0.014	0.015
1. Black	0.013	0.016	0.017
2. Galvanized	0.010	0.010	0.011
e. Corrugated metal	0.017	0.010	0.021
1. Subdrain	0.017	0.019	0.021
2. Storm drain	0.021	0.024	0.030
A-2. Nonmetal	0.000	0.000	0.010
a. Lucite	0.008	0.009	0.010
b. Glass	0.009	0.010	0.013
c. Cement	A service by the	5 words	1 Section Section
1. Neat, surface	0.010	0.011	0.013
2. Mortar	0.011	0.013	0.015
d. Concrete		1.	0.000
1. Culvert, straight and free of debris	0.010	0.011	0.013
2. Culvert with bends, connections and some debris	, 0.011	0.013	0.014
3. Finished	0.011	0.012	0.014
4. Sewer with manholes, inlet, etc. straight	, 0.013	0.015	0.017
5. Unfinished, steel form	0.012	0.013	0.014
6. Unfinished, smooth wood form	0.012	0.014	0.016
7. Unfinished, rough wood form	0.015	0.017	0.020
e. Wood	0.010	0.0-1	
1 Stave	0 010	0.012	0 014
2 Laminated treated	0.015	0.017	0.020
f Clay	0.010	0.011	0.020
1 Common drainage tile	0.011	0.019	0.017
2. Vitrified comer	0.011	0.014	0.017
2. Vitrified sewer	0.011	0.014	0.017
etc.	, 0.013	0.015	0.017
4. Vitrified subdrain with open joint g. Brickwork	0.014	0.016	0.018
1. Glazed	0.011	0.013	0.015
2. Lined with cement mortar	0.012	0.015	0.017
h. Sanitary sewers coated with sewag slimes, with bends and connections	e 0.012	0.013	0.016
i. Paved invert, sewer, smooth bottom	0.016	0.019	0.020
i. Rubble masonry, cemented	0.018	0.025	0.030



#### B. Lined channels





#### C. Excavated channel



### D. Natural streams



Type of channel and description	Minimum	Normal	Maximum
XCAVATED OR DREDGED	P Creation	-autoria a	Cotter Co
a Earth straight and uniform		A Second	Ster bell
1 Clean recently completed	0.016	0.018	0.020
2 Clean after weathering	0.018	0.022	0.020
2. Gravel uniform section clean	0.018	0.025	0.025
4. With short grass for woods	0.022	0.020	0.030
b Forth winding and aluggish	0.022	0.021	0.033
1. No reportation	0.000	0.000	0.000
1. No vegetation	0.023	0.025	0.030
2. Grass, some weeds	0.025	0.030	0.033
3. Dense weeds or aquatic plants in deep channels	0.030	0.035	0.040
4. Earth bottom and rubble sides	0.028	0.030	0.035
5. Stony bottom and weedy banks	0.025	0.035	0.040
6. Cobble bottom and clean sides	0.030	0.040	0.050
c. Dragline-excavated or dredged			
1. No vegetation	0.025	0.028	0.033
2. Light brush on banks	0.035	0.050	0.060
d. Rock cuts	2002-0	No.	
1. Smooth and uniform	0.025	0.035	0.040
2. Jagged and irregular	0.035	0.040	0.050
e. Channels not maintained, weeds and	Contraction of Contract	and the second	
brush uncut	stays inva	1998.6	190.00
1. Dense weeds, high as flow depth	0.050	0.080	0.120
2. Clean bottom, brush on sides	0.040	0.050	0.080
3. Same, highest stage of flow	0.045	0 070	0 110
4. Dense brush, high stage	0.080	0 100	0 140
ATURAL STREAMS		10, 10,000	
-1. Minor streams (top width at flood stage	a sheep for a	I. Dyen	0.00
<100 ft)		1005	
a. Streams on plain	and station a	0751-8	
1. Clean, straight, full stage, no rifts or deen pools	0.025	0.030	0.033
2. Same as above, but more stones and weeds	0.030	0.035	0.040
3. Clean, winding, some pools and shoals	0.033	0.040	0.045
4. Same as above, but some weeds and stones	0.035	0.045	0.050
5. Same as above, lower stages, more ineffective slopes and sections	0.040	0.048	0.055
6. Same as 4, but more stones	0.045	0.050	0.060
7. Sluggish reaches, weedy, deep pools	0.050	0.070	0.080
8. Very weedy reaches, deep pools, or	0.075	0 100	0 150
the second	0.010	0.100	0.100



#### Natural streams





#### TABLE 5 Manning *n*-values<sup>9</sup>

Type of Conduit	Minimum	Normal	Maximu	
Pipes flowing partly full				
Welded steel	0.010	0.012	0.014	
Coated cast iron	0.010	0.013	0.014	
Corrugated metal	0.021	0.024	0.030	
Cement mortar lined (neat)	0.010	0.011	0.013	
Concrete culvert (finished)	0.011	0.012	0.014	
Concrete pipe (steel form)	0.012	0.013	0.014	
Vitrified clay	0.011	0.014	0.017	
Drainage tile	0.011	0.013	0.017	

#### TABLE 5 (Continued)

Type of Conduit	Minimum	Normal	Maximum
Lined open channels			
Smooth steel	0.011	0.012	0.014
Wood (planed)	0.010	0.012	0.014
Wood (unplaned)	0.011	0.013	0.015
Cement (neat)	0.010	0.011	0.013
Concrete (troweled)	0.011	0.013	0.015
Concrete (float finish)	0.013	0.015	0.016
Concrete (unfinished)	0.014	0.017	0.020
Gunite	0.016	0.019	0.023
Brick	0.012	0.015	0.018
Rubble masonry	0.017	0.025	0.030
Asphalt (smooth)	_	0.013	
Asphalt (rough)		0.016	
Unlined open channels			
Earth, straight and uniform, clean	0.016	0.018	0.020
Earth, straight and uniform, short vegetation	0.022	0.027	0.033
Earth, winding and sluggish, clean	0.023	0.025	0.030
Earth, winding, sluggish, short vegetation	0.025	0.030	0.033
Gravel, straight and uniform, clean	0.022	0.025	0.030
Dredged, clean	0.025	0.028	0.033
Rock cuts, smooth and uniform	0.025	0.035	0.040
Rock cuts, jagged and irregular	0.035	0.040	0.050
Natural channels			
Clean, straight, no riffles or pools	0.025	0.030	0.033
Clean, winding, some pools and shoals	0.033	0.040	0.045
Sluggish, weedy, deep pools	0.050	0.070	0.080
Mountain streams, gravel, cobbles	0.030	0.040	0.050
Mountain streams, cobbles, large boulders	0.040	0.050	0.070
Flood plains, pasture	0.025	0.030	0.035
Flood plains, light brush and trees	0.035	0.050	0.060
Flood plains, heavy stand of timber	0.080	0.100	0.120



## IP 10.1 (p. 440)

 Rectangular channel lined with asphalt is 20 ft wide and laid on a slope of 0.0001. Calculate the <u>depth of uniform flow (normal depth)</u> in this channel when the flowrate is 400 ft<sup>3</sup>/s.







A concrete-lined canal with an *n*-value of 0.014 is constructed on a slope 0.33 m/km and conveys 23 m<sup>3</sup>/s of water. Find the <u>uniform flow</u> <u>depth</u> if the canal is trapezoidal in cross section with a bottom with of *B*=6 m and side slopes of *z* =1.5.











## 7.4 Uniform Laminar Flow

- Laminar flow  $(\text{Re} = \frac{Vy_0}{v} < 500)$  in open channels occurs in <u>drainage from</u> streets, airport runaways, parking areas, and so forth.
- Infinite width with resistance only at the bottom of the <u>sheet flow</u>.
- The flow is a two-dimensional one.



- <u>Parabolic velocity profile</u> and <u>linear shear stress profile</u>.
- The <u>hydraulic radius consists only with bottom</u>.



 The limit of laminar flow is determined by an experimentally determined critical <u>Reynolds number (~500)</u>.

$$\operatorname{Re} = \frac{Vy_0}{v} = \frac{g_n y_0^3 S_0}{3v^2} \quad and \quad V = \underbrace{\begin{array}{c} g_n \sqrt{S_0} y_0^{3/2} \\ 3v \end{array}} \sqrt{y_0 S_0} \\ \text{Therefore,} \quad C = \sqrt{g_n \operatorname{Re}/3} \end{array} \quad \text{Laminar case}$$



## 7.5 Hydraulic Radius Considerations

- The <u>variation of hydraulic radius with depth</u> is important in open channel flow.
- 1) Rectangular section
  - In the case of narrow deep sections,

$$R_h = \frac{By}{B + 2y} \cong \frac{B}{2}$$

- In the case of wide shallow sections,

$$R_h = \frac{By}{B+2y} \cong y$$

The assumption of a wide section is valid only when





 $\frac{B}{-}>10$ 

V

i) Narrow deep section



ii) Wide shallow section  $\overrightarrow{P} = \overrightarrow{P} = 10y$ 



#### 2) Trapezoidal section

$$R_h = \frac{A}{P} = \frac{By + zy^2}{B + 2y\sqrt{1 + z^2}}$$



- The derivative of  $R_h$  with respect to y will allow the relationship between  $R_h$  and y

$$\frac{dR_{h}}{dy} = \frac{1 + 2z(y/B) \left[1 + (y/B)\sqrt{1 + z^{2}}\right]}{1 + 4\sqrt{1 + z^{2}}(y/B) \left[1 + (y/B)\sqrt{1 + z^{2}}\right]}$$

- The denominator is larger than the numerator  $\rightarrow \frac{dR_h}{dy} < 1$ 

-U

- $\frac{dR_h}{dy}$  diminishes with increasing y
- $\frac{dR_h}{dy}$  does not approach zero as y approaches infinity  $\rightarrow$  no asymptote









#### 3) Circular pipe flowing partially full

- Sewer pipes, storm drains, culverts
- $R_{h} = \frac{A}{P} = \frac{d}{4} (y=d/2 \text{ or completely full })$ For  $y = 0, R_{h} = 0$
- Continuous variation of  $R_h$  with y is expected to feature a maximum value - Variation of  $AR_h^{2/3}$  with y is also important  $\rightarrow$  maximum flowrate is achieved before pipe is full

$$Q = \left(\frac{1}{n}\right) A R_h^{2/3} S_0^{1/2}$$



#### Hydraulic characteristic curve

$$R_{h} = \frac{d}{4} \left( 1 - \frac{\sin \theta}{\theta} \right)$$
$$P = \frac{d}{2} \theta; A = \left( \theta - \sin \theta \right) \frac{d^{2}}{8}$$





### Best hydraulic section (channel efficiency)



- For open channels, another important engineering problem of section geometry is the <u>reduction of boundary resistance</u> by <u>minimizing of the</u> <u>wetted perimeter for a given area of flow section.</u>
- With A fixed,  $R_h = A/P$ , and, with P to be minimized, this may be considered a problem of maximization of the hydraulic radius.
- This reduces the cost of lining (most economical section), and maximizes the flowrate.
- For this reason, a section of <u>maximum hydraulic efficiency</u> is known as the most efficient section or the best hydraulic section.
- There is no generalized solution for all possible section shapes.



#### Friction in open channel

 Open channel flow has free surface (no friction) and walls (friction), this unbalance produces velocity distribution in channel









#### Friction in open channel



(a)



(b)





## Channel Efficiency

 The following channels have the same cross-sectional area of 20 m<sup>2</sup>. But, they have different wetted perimeter. Maybe the case of (c) has the minimum wetted perimeter and hence will encounter least energy loss.





### **Channel Efficiency**

#### Trapezoidal sections

- It is a simple matter to develop geometric relationships to minimize the wetted perimeter.



$$A = By + zy^{2} \rightarrow B = \frac{A - zy^{2}}{y} = \frac{A}{y} - zy \leftarrow \text{eleminateB}$$

$$P = B + 2y(1 + z^{2})^{1/2} = \frac{A}{y} - zy + 2y(1 + z^{2})^{1/2}$$

$$R_{h} = \frac{A}{P} = \frac{A}{\frac{A}{y} - zy + 2y(1 + z^{2})^{1/2}}$$
A is constant



JD

Differentiating  $R_h$  wrt y and equating the result to zero gives

$$\frac{d\kappa_h}{dy} = 0$$
  
$$A = y^2 \left( 2\sqrt{1+z^2} - z \right); B = 2y \left( \sqrt{1+z^2} - z \right)$$



 $\rightarrow$  For best hydraulic section of trapezoidal channel, its hydraulic radius is close to <u>one-half of the flow depth</u>.

In the case of **rectangular sections** (z = 0)

$$R_{h_{\text{max}}} = \frac{y}{2}$$
$$A = 2y^{2}$$
$$B = 2y$$

 $R_{h_{\text{max}}} = \frac{y}{2}$ 

 $\rightarrow$  For best hydraulic section of rectangular channel, its flow depth is one-half of the width of the channel.





#### IP 10.3 (pp. 445-446)

Find the dimensions of the <u>most-efficient cross section for a rectangular</u> <u>channel</u> that is to convey a <u>uniform flow</u> of 10m<sup>3</sup>/s if the channel is lined with Gunite concrete and is laid on a slope of 0.0001.

[Sol] For the most efficient cross section, the hydraulic radius  $R_h$  is one-half the depth, and, the area is given by  $2y_0^2$ . Substituting this information into Chezy and Manning equation

$$Q = \left(\frac{u}{n}\right) A R_h^{2/3} S_0^{1/2} = \left(\frac{u}{n}\right) \left(2y_0^2\right) \left(\frac{y_0}{2}\right)^{2/3} S_0^{1/2}$$

 From the Table 5 (in text book), the *n*-value for a Gunite-lined channel is 0.019 and for SI *u*=1.

$$10 = \left(\frac{1.0}{0.019}\right) \left(2y_0^2\right) \left(\frac{y_0}{2}\right)^{2/3} \left(0.0001\right)^{1/2}$$
$$y_0^{8/3} = 15.1 \qquad y_0 = 2.77 \qquad B = 2y_0 = 5.54$$







#### Compound channel

 Consider them composed of parallel channels separated by the vertical dashed line (with <u>zero resistance</u>)

$$Q = Q_{1} + Q_{2}$$

$$Q = \left(\frac{u}{n_{1}}\right) A_{1} R_{h_{1}}^{2/3} S_{0}^{1/2} + \left(\frac{u}{n_{2}}\right) A_{2} R_{h_{2}}^{2/3} S_{0}^{1/2}$$

$$A_{1} = B_{1} y_{0_{1}}$$

$$A_{2} = B_{2} y_{02}$$

$$P_{1} = B_{1} + y_{0_{1}}$$

$$P_{2} = B_{2} + 2 y_{0_{2}} - y_{0_{1}}$$











#### Homework Assignment No. 5 Due: 1 week from today Answer questions either in Korean or in English

1. (10-2) Water <u>flows uniformly</u> at a depth of 1.2 *m* in a <u>rectangular</u> <u>canal</u> 3 *m* wide, laid on a slope of 1 *m* per 1,000 *m*. What is the <u>mean shear stress</u> on the sides and bottom of the canal?

2. (10-8) Calculate the <u>uniform flowrate</u> in an earth-lined (n = 0.020) <u>trapezoidal canal</u> having bottom width 3 m, sides sloping 1 (vert.) on 2 (horiz.), laid on a slope of 0.0001, and having a depth of 1.8 m.



3. (10-15) At what depth will 4.25 *m<sup>3</sup>/s* flow uniformly in a <u>rectangular channel</u> 3.6 *m* wide lined with rubble masonry and laid on a slope of 1 in 4000?

4. (10-20) A <u>trapezoidal canal</u> of side slopes 1 (vert.) on 2 (horiz.) and having n = 0.017 is to carry a <u>uniform flow</u> of 37  $m^3/s$  on a slope of 0.005 at a depth of 1.5 *m*. What base width is required?

5. (10-25) Water (20 °C) flows uniformly in a channel at a depth of 0.009 *m*. Assuming a <u>critical Reynolds number</u> of 500, what is the largest slope on which <u>laminar flow</u> can be maintained? What mean velocity will occur on this slope?



6. (10-33) What <u>uniform flowrate</u> occurs in a 1.5 *m* <u>circular</u> brick conduit laid on a slope of 0.001 when the depth of flow is 1.05 *m*?
What is the mean velocity of this flow?



7. (10-36) A channel flow cross section has an area of 18 m<sup>2</sup>.
Calculate its <u>best dimensions</u> if (*a*) rectangular, and (*b*) trapezoidal with 1 (vert.) on 2 (horiz.) side slopes.

8. (10-42) What is the <u>minimum slope</u> at which 5.67  $m^3/s$  may be carried <u>uniformly in a rectangular channel</u> (having a value of *n* of 0.014) at a mean velocity of 0.9  $m^3/s$ .



9. (10-45) This flood channel has a Manning *n* of 0.017 and a slope of 0.0009. Estimate the <u>depth of uniform flow</u> for a flowrate of 1,200 *cfs*.

