Heat transfer

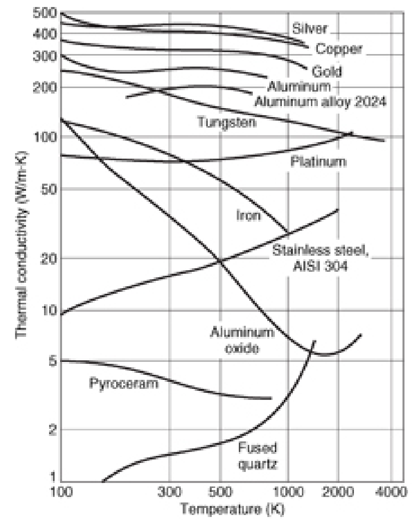
momentum transfer; result of molecular transfer of momentum
conduction; 1) by molecular interaction, 2) by free electrons
; primarily a molecular phenomenon

$$\tau = \mu \frac{dv}{dy}$$

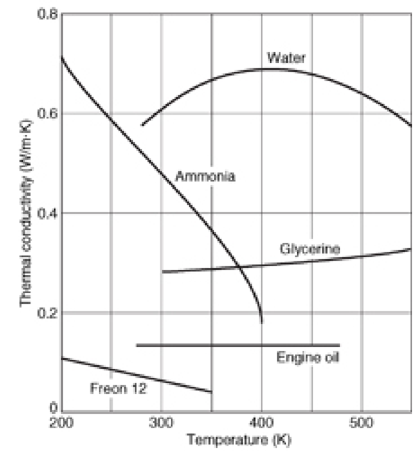
$$\boldsymbol{\tau} = \mu(\nabla \mathbf{v} + \nabla \mathbf{v}^T)$$

$$\frac{q_x}{A} = -k \frac{dT}{dx}$$

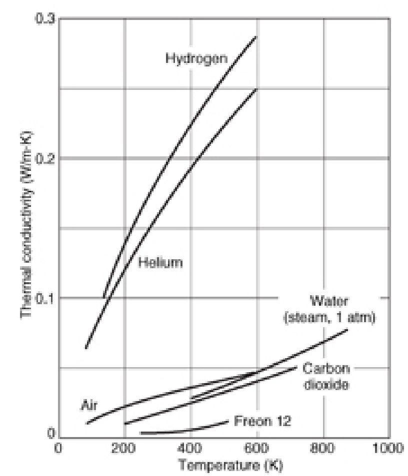
$$\frac{\mathbf{q}}{\mathrm{A}} = -\mathrm{k}\nabla\mathrm{T}$$



(a) Solid materials

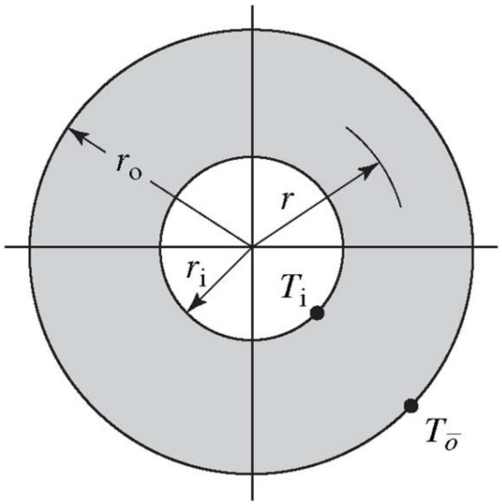


(b) Liquids



(c) Gases and vapors

heat conduction in a hollow cylinder



$$q_r = -kA\frac{dT}{dr} = -k(2\pi rL)\frac{dT}{dr}$$

$$q_r = \frac{2\pi kL}{\ln r_o / r_i}(T_i - T_o)$$

$$q_r = kA_{lm}\frac{\Delta t}{\Delta r} \qquad A_{lm} = \frac{(A_2 - A_1)}{\ln(A_2 / A_1)}$$

heat conduction in a hollow sphere

$$q_r = kA_{gm}\frac{\Delta t}{\Delta r} \qquad A_{gm} = \sqrt{A_1 A_2}$$

convection; involves energy exchange between a surface and an adjacent fluid ; forced convection, natural convection

$$q / A = h\Delta T$$

$$\nabla \bullet u = 0$$

$$\rho\left(\frac{\partial u}{\partial t} + u \bullet \nabla u\right) = -\nabla p + \nabla \bullet \tau$$

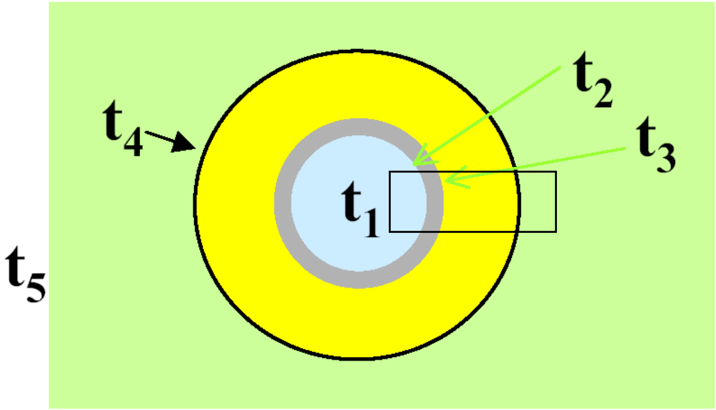
$$\rho c_p\left(\frac{\partial T}{\partial t} + u \bullet \nabla T\right) = k\nabla^2 T + \tau : \nabla u$$

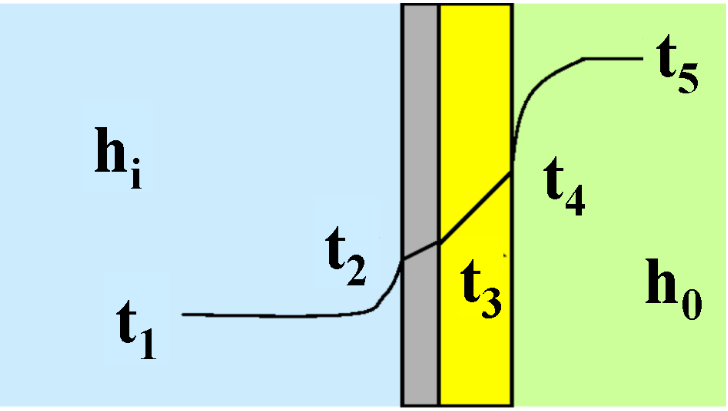
radiation; no medium is required, depends on temperature

$$\frac{q}{A} = \sigma T^4$$

Overall Heat-Transfer Coefficient

Heat is transfer by a series of conduction and convection mechanisms





$$q = h_i A_i (t_1 - t_2) = k_b A_{b,\,lm} \frac{t_2 - t_3}{\Delta r_b} = k_c A_{c,\,lm} \frac{t_3 - t_4}{\Delta r_c} = h_0 A_0 (t_4 - t_5)$$

(21-12)

Thermal resistance $\dfrac{1}{h_i A_i}$ $\dfrac{\Delta r_b}{k_b A_{b,\,lm}}$ $\dfrac{\Delta r_c}{k_c A_{c,\,lm}}$ $\dfrac{1}{h_0 A_0}$

Individual temperature drops

$$t_1 - t_2 = q \frac{1}{h_i A_i}$$

$$t_2 - t_3 = q \frac{\Delta r_b}{k_b A_{b,lm}}$$

$$t_3 - t_4 = q \frac{\Delta r_c}{k_c A_{c,lm}}$$

$$t_4 - t_5 = q \frac{1}{h_0 A_0}$$

Overall temperature drops

$$\Delta t_{overall} = t_1 - t_5$$

$$= q\frac{1}{h_i A_i} + q\frac{\Delta r_b}{k_b A_{b,lm}} + q\frac{\Delta r_c}{k_c A_{c,lm}} + q\frac{1}{h_0 A_0}$$

$$= q\left(\frac{1}{h_i A_i} + \frac{\Delta r_b}{k_b A_{b,lm}} + \frac{\Delta r_c}{k_c A_{c,lm}} + \frac{1}{h_0 A_0}\right)$$

$$q = \frac{t_1 - t_5}{\left(\frac{1}{h_i A_i} + \frac{\Delta r_b}{k_b A_{b,lm}} + \frac{\Delta r_c}{k_c A_{c,lm}} + \frac{1}{h_0 A_0}\right)}$$

$$q = \frac{\Delta t_{overall}}{\Sigma R}$$

Overall coefficients of heat transfer based on the outside area U_0

$$q = \frac{A_0 \Delta t_{overall}}{\left(\frac{A_0}{h_i A_i} + \frac{A_0 \Delta r_b}{k_b A_{b,lm}} + \frac{A_0 \Delta r_c}{k_c A_{c,lm}} + \frac{1}{h_0} \right)}$$

$$\frac{1}{U_0} = \frac{A_0}{h_i A_i} + \frac{A_0 \Delta r_b}{k_b A_{b,lm}} + \frac{A_0 \Delta r_c}{k_c A_{c,lm}} + \frac{1}{h_0}$$

$$q = U_0 A_0 \Delta t_{overall}$$

Overall coefficients of heat transfer based on the inside area U_i

$$q = \frac{A_i \Delta t_{overall}}{\left(\frac{1}{h_i} + \frac{A_i \Delta r_b}{k_b A_{b,lm}} + \frac{A_i \Delta r_c}{k_c A_{c,lm}} + \frac{A_i}{h_0 A_0} \right)}$$

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{A_i \Delta r_b}{k_b A_{b,lm}} + \frac{A_i \Delta r_c}{k_c A_{c,lm}} + \frac{A_i}{h_0 A_i}$$

$$q = U_i A_i \Delta t_{overall}$$

Overall coefficients of heat transfer

$$q = U_0 A_0 \Delta t_{overall}$$

$$= U_i A_i \Delta t_{overall}$$

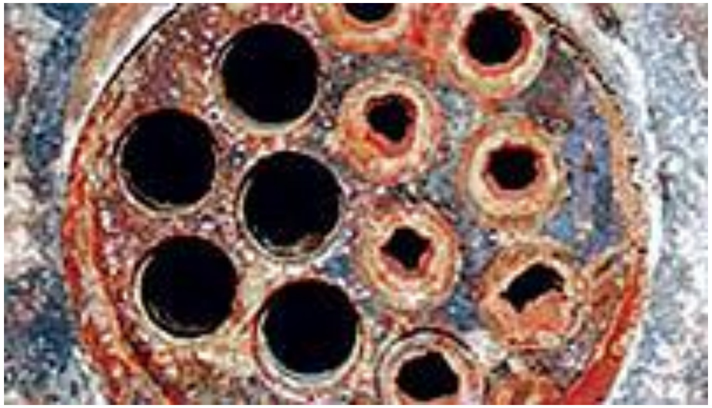
Overall coefficients of heat transfer

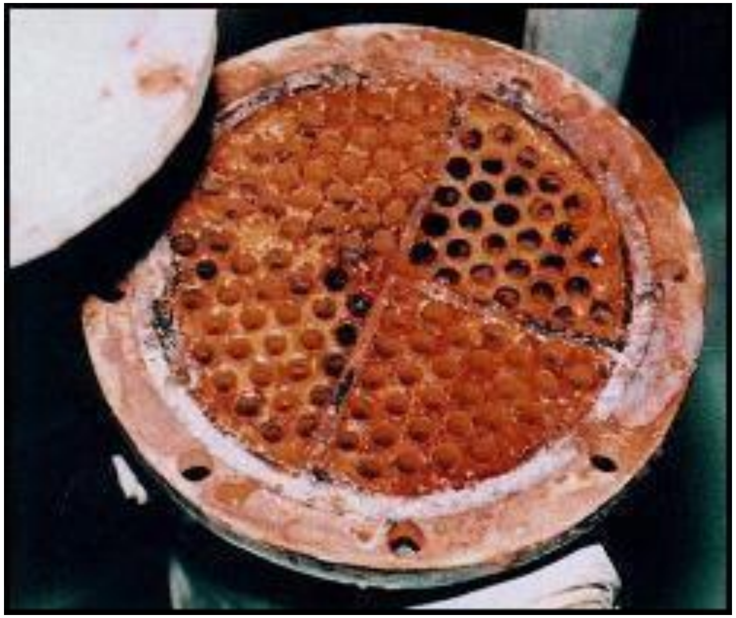
$$q = \frac{\Delta t_{overall}}{\Sigma R}$$

$$\Sigma R = \frac{1}{U_0 A_0} = \frac{1}{U_i A_i}$$

$$U_0 A_0 = U_i A_i$$

Fouling in heat exchanger



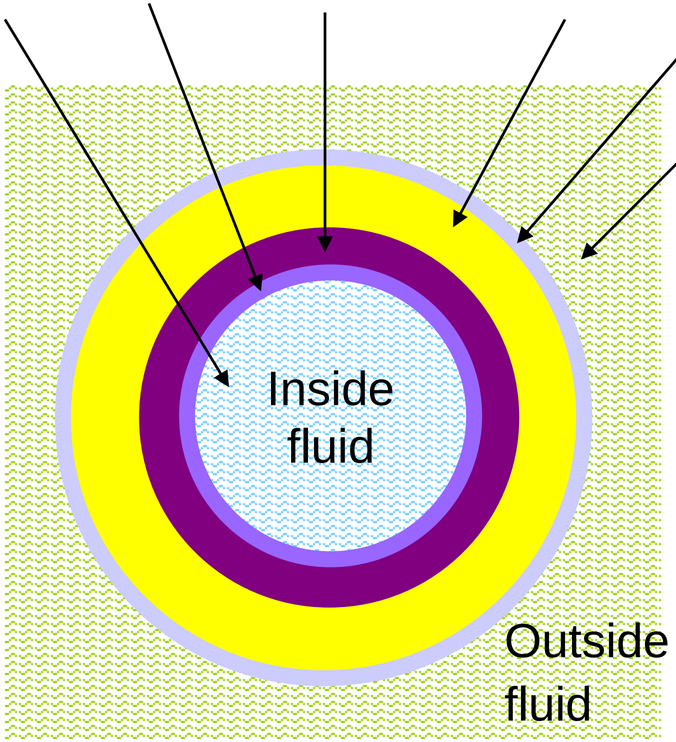


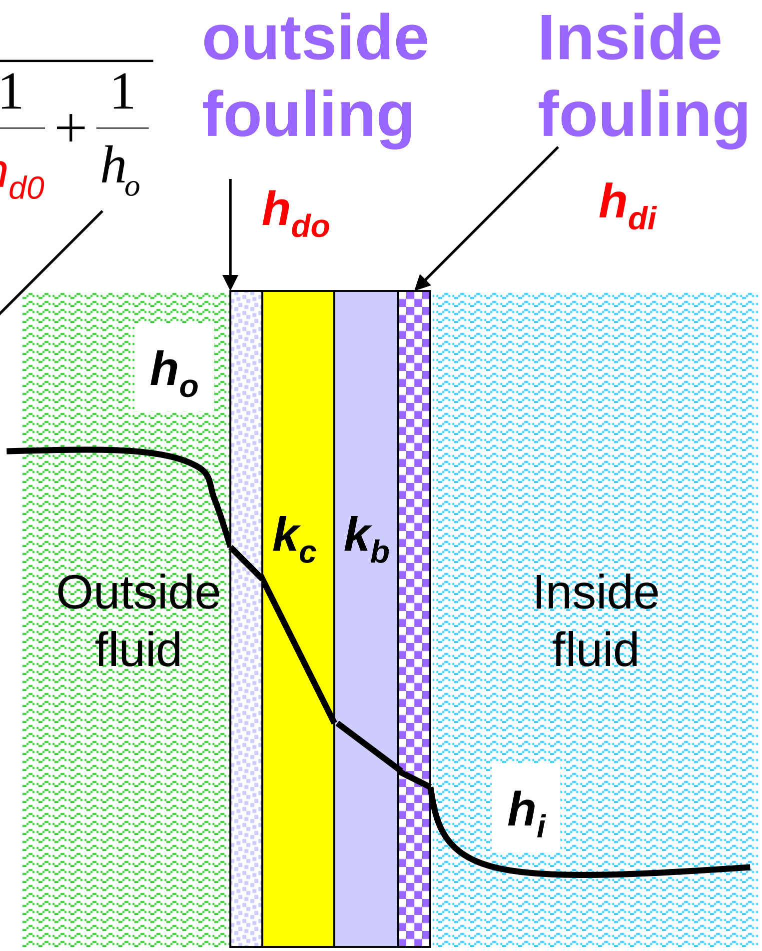
Fouling Coefficients

$$\frac{1}{U_0} = \frac{A_0}{h_i A_i} + \frac{A_0 \Delta r_b}{k_b A_{b,lm}} + \frac{A_0 \Delta r_c}{k_c A_{c,lm}} + \frac{1}{h_0}$$

$$q = h_d A \Delta t_{scale}$$

$$U_o = \frac{1}{\frac{A_o}{h_i A_i} + \frac{A_o}{h_{di} A_i} + \frac{\Delta r_b}{k_b}\frac{A_o}{A_{b,lm}} + \frac{\Delta r_c}{k_c}\frac{A_o}{A_{c,lm}} + \frac{1}{h_{d0}} + \frac{1}{h_o}}$$





Example

$$U_o = \frac{1}{\frac{A_o}{h_i A_i} + \frac{A_o}{h_{di} A_i} + \frac{\Delta r_b}{k_b}\frac{A_o}{A_{b,lm}} + \frac{1}{h_{do}} + \frac{1}{h_o}}$$

A reflux condenser contains ¾ in. 16-gauge copper tubes in which cooling water circulates. Hydrocarbon vapors condense on the exterior surfaces of the tubes. Find the overall heat-transfer coefficient U_o. The inside convective coefficient can be taken as 4500 W/m^2·K, and the outside coefficient as 1500 W/m^2·K.

Approximately fouling coefficients from Table are

	Btu/(h)(ft^2)(^{o}F)
Overhead vapors from crude-oil distillation	1000
Water, Great Lakes, over 125^{o}F	500

$\boldsymbol{h_{d0}}$ = 5700 W/m^2·K

$\boldsymbol{h_{di}}$ = 2840 W/m^2·K

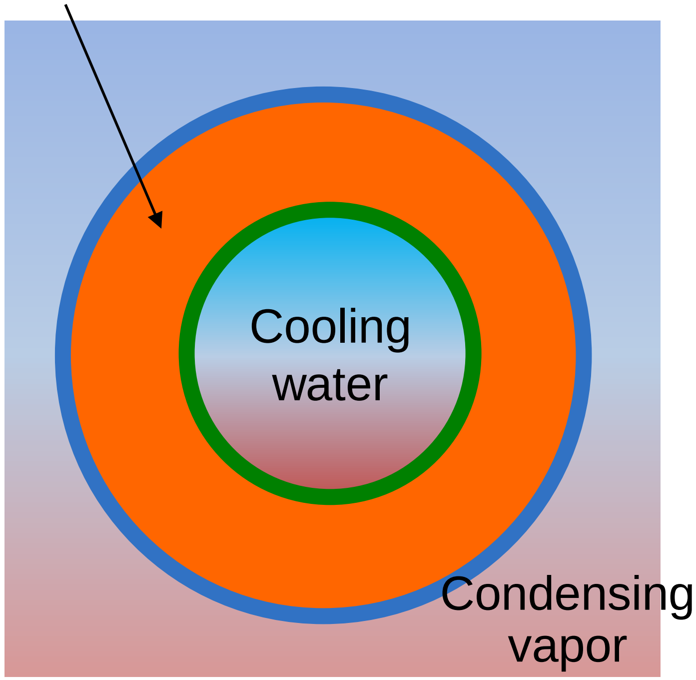
Example Find U_o

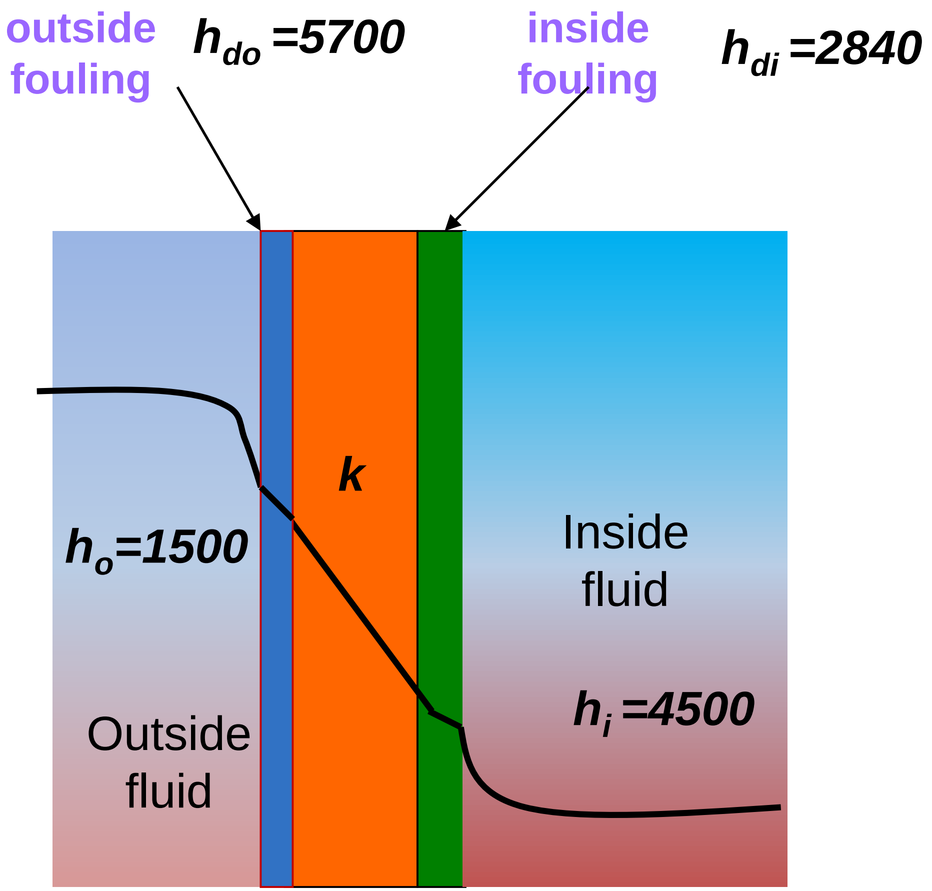
$$U_o = \frac{1}{\frac{A_o}{h_i A_i} + \frac{A_o}{h_{di} A_i} + \frac{\Delta r_b}{k_b}\frac{A_o}{A_{b,lm}} + \frac{1}{h_{do}} + \frac{1}{h_o}}$$

¾" 16-gauge copper tube

$r_i=0.0157$

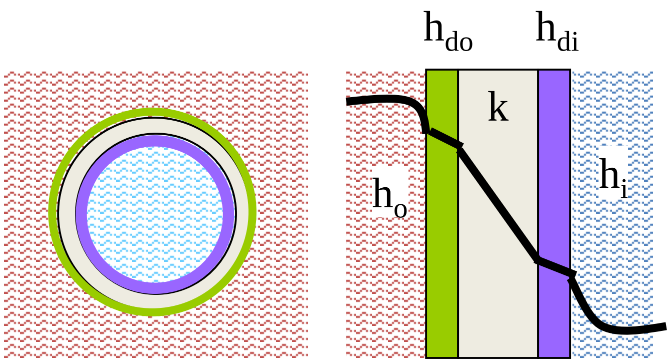
$r_o=0.0191$





unit: W/m²·K

Example



$$U_0 = \frac{1}{\frac{r_0}{h_i r_i} + \frac{r_0}{h_{d_i} r_i} + \frac{\Delta r \cdot r_0}{k_b r_{lm}} + \frac{1}{h_{d_0}} + \frac{1}{h_0}}$$

$$h_i = 4500 W/m^2 \cdot K, \quad h_0 = 1500 W/m^2 \cdot K$$

$$h_{d_0} = 1000 \times 5.678 = 5700 W/m^2 \cdot K$$

$$h_{d_i} = 500 \times 5.678 = 2840 W/m^2 \cdot K$$

$$k = 380\ W/m \cdot K \quad , \quad r_i = 0.0157 m \quad , \quad r_0 = 0.0191 m$$

$$r_{l_m} = \frac{r_0 - r_i}{\ln \frac{r_0}{r_i}} = \frac{0.0191 - 0.0157}{\ln \frac{0.0191}{0.0157}} = 0.0175 m$$

$$\frac{r_0}{h_i r_i} = \frac{0.0191}{(4500)(0.0157)} = 0.00027$$

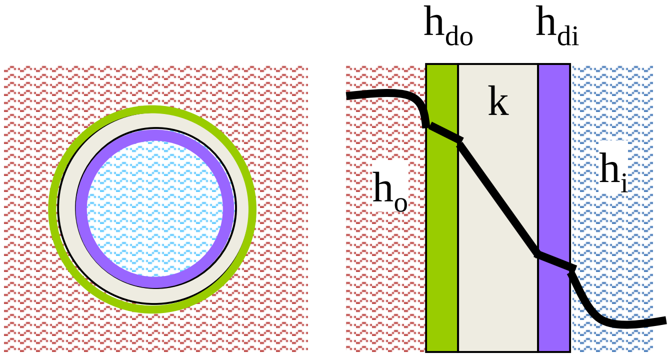
$$\frac{r_0}{h_{di} r_i} = \frac{0.0191}{(2840)(0.0157)} = 0.00043$$

$$\frac{\Delta r \cdot r_0}{k_b r_{lm}} = \frac{(0.00165)(0.0191)}{(380)(0.0175)} = 0.0000047$$

$$\frac{1}{h_{d_0}} = \frac{1}{5700} = 0.00018, \quad \frac{1}{h_0} = \frac{1}{1500} = 0.00067$$

Example

$$\Sigma R = \frac{1}{U_0 A_0} = \frac{1}{U_i A_i}$$



$$U_0 = \frac{1}{\frac{r_0}{h_i r_i} + \frac{r_0}{h_{d_i} r_i} + \frac{\Delta r \cdot r_0}{k_b r_{lm}} + \frac{1}{h_{d_0}} + \frac{1}{h_0}}$$

$$= \frac{1}{0.00027 + 0.00043 + 0.0000047 + 0.00018 + 0.00067}$$

Resistance of metal wall is negligible

Fouling Resistance is significant~39%

$$= \frac{1}{0.00155} = 645\, W/m^2 \cdot K$$

$$U_i A_i = U_o A_o$$

$$U_i = (645)(0.0191)/((0.0157) = 785\, W/m^2 \cdot K$$