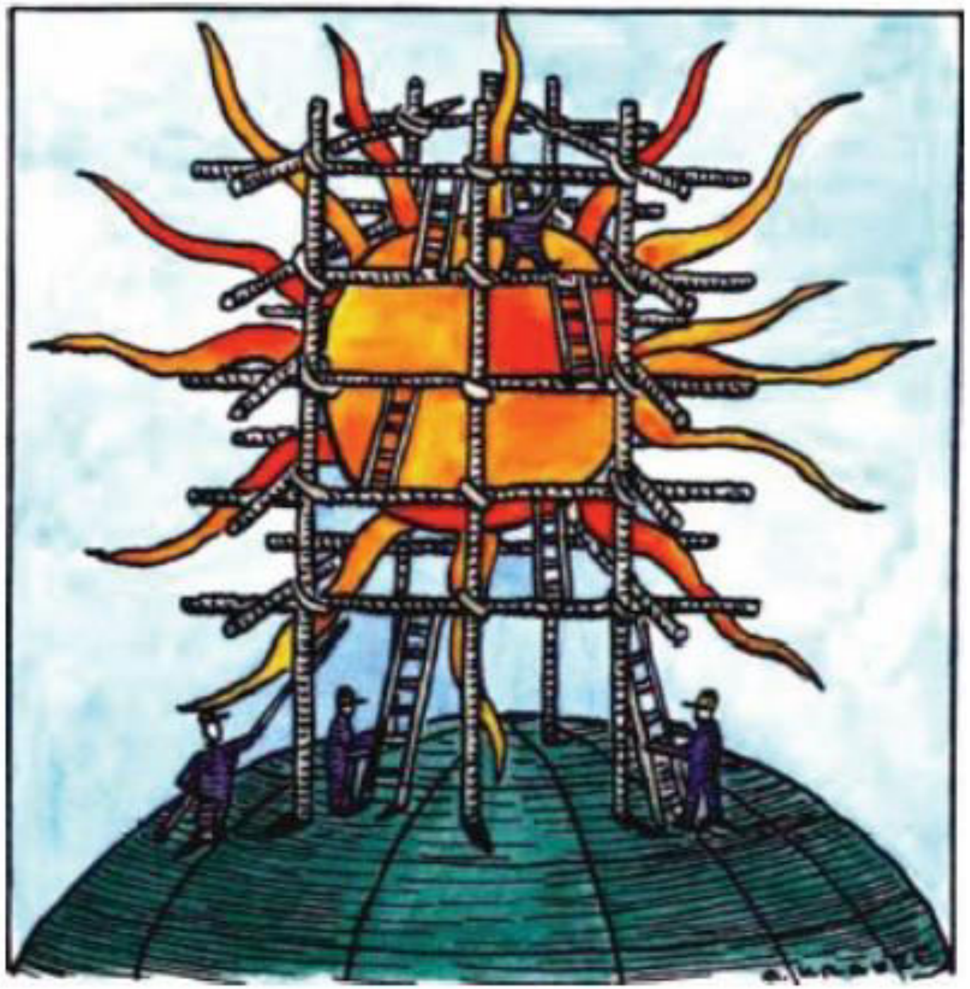
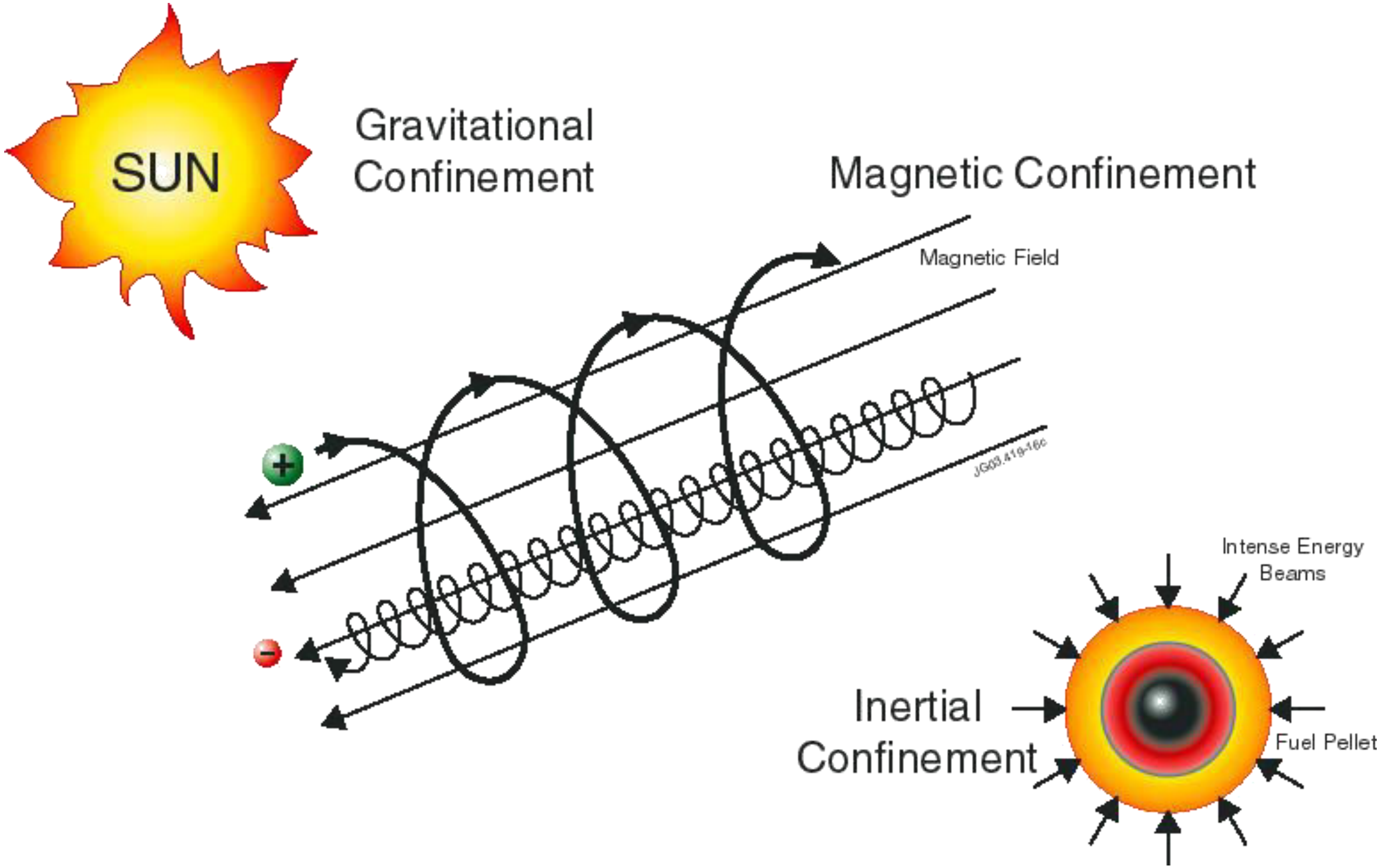
Introduction to Nuclear Fusion

Prof. Dr. Yong-Su Na

To build a sun on earth



To build a sun on earth



Magnetic Confinement

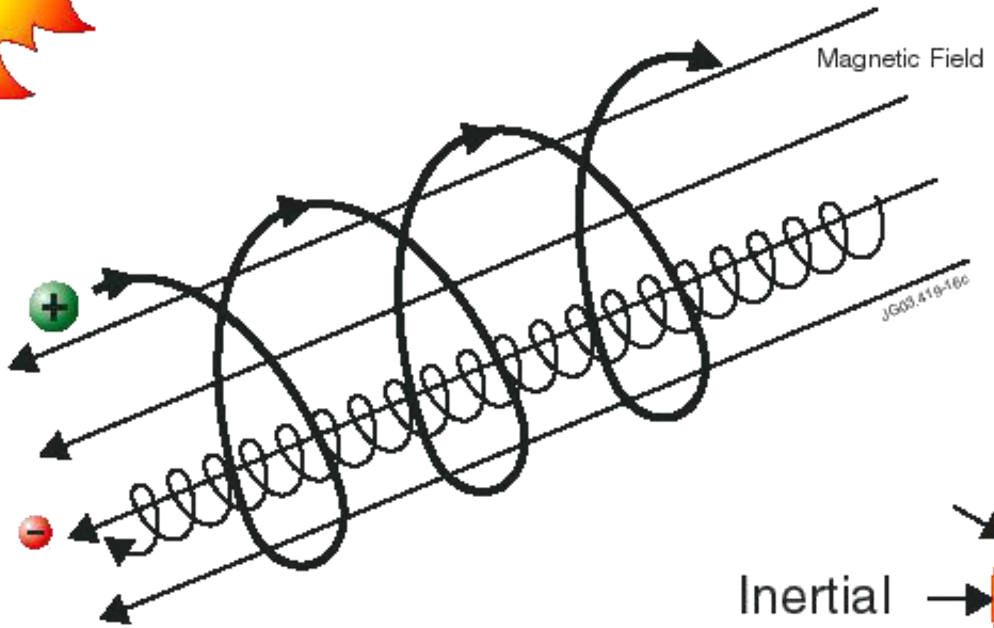
- **Bring the Sun on the Earth**

Quantity	ITER	Sun	Ratio
Diameter	16.4 m	140×10^4 km	$\sim 1/10^8$
Central temp.	200 Mdeg	15 Mdeg	10
Central density	$\sim 10^{20}/m^3$	$\sim 10^{32}/m^3$	$\sim 1/10^{12}$
Central press.	~5 atm	$\sim 10^{12}$ atm	$\sim 1/10^{11}$
Power density	$\sim 0.6\ MW/m^3$	$\sim 0.3\ W/m^3$	$\sim 2 \times 10^6$
Reaction	DT	pp	
Plasma mass	0.35 g	2×10^{30} kg	$\sim 1/10^{34}$
Burn time const.	200 s	10^{10} years	$\sim 1/10^{15}$

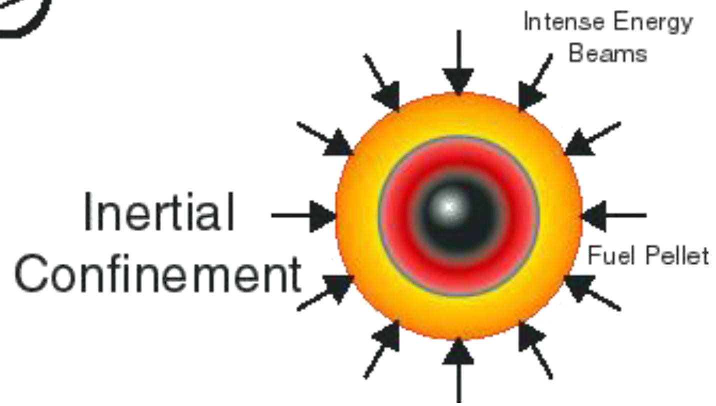
M. Kikuch, "steady state tokamak reactor and its physics issues", Talk at SNU, Korea, September 30, 2011

To build a sun on earth

Magnetic Confinement



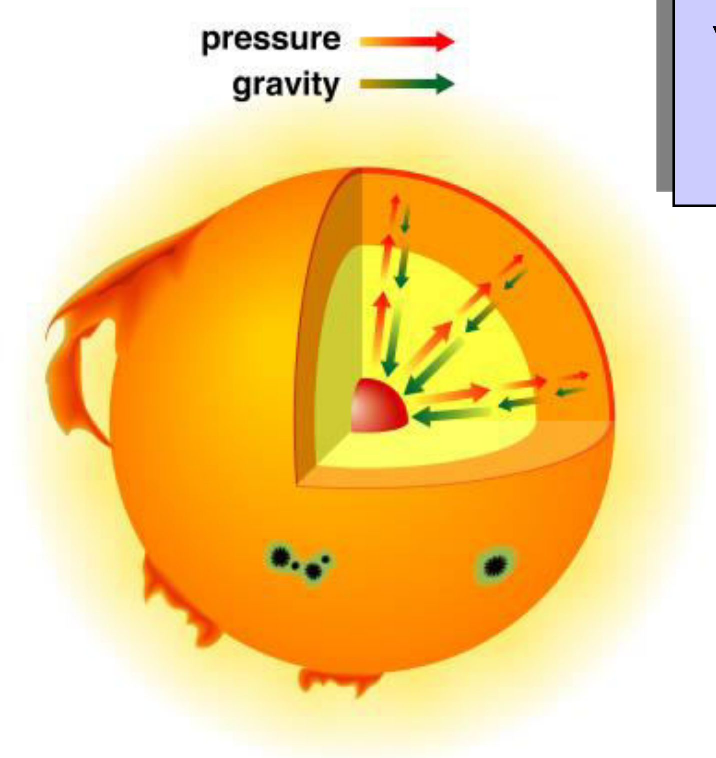
- Open magnetic confinement
- Closed magnetic confinement



What is open magnetic confinement?

Open Magnetic Confinement

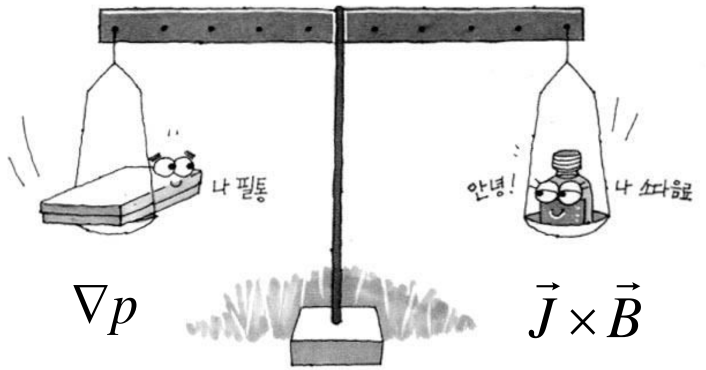
- **Equilibrium – Radial Force Balance**

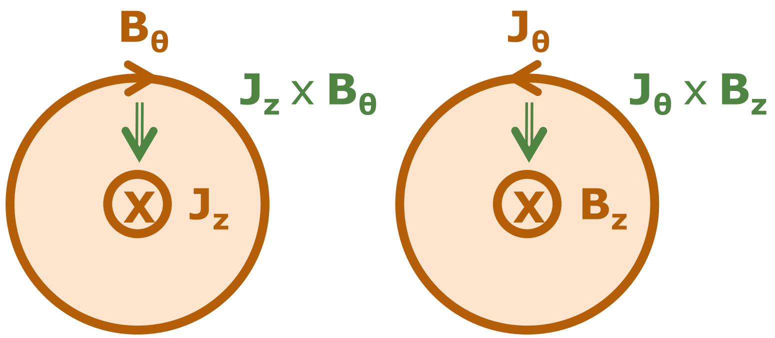


$$\nabla p = \vec{J} \times \vec{B}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

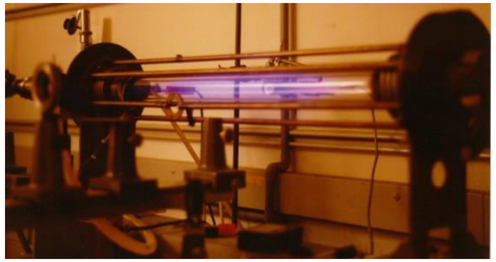
$$\nabla \cdot \vec{B} = 0$$





http://lasp.colorado.edu/education/outerplanets/solsys_star.php

Open Magnetic Confinement



Z pinch

θ pinch

screw pinch

Magnetic mirror

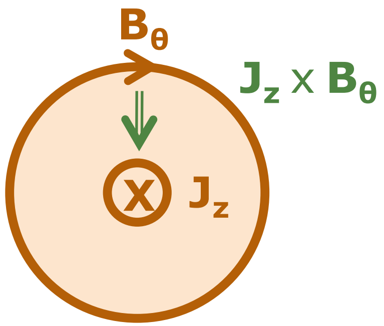
WIKIPEDIA
The Free Encyclopedia

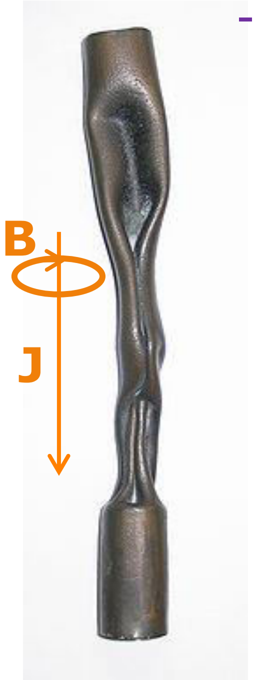
http://phys.strath.ac.uk/information/history/photos.php
http://www.frascati.enea.it/ProtoSphera/ProtoSphera%202001/6.%20Electrode%20experiment.htm

What is *Z*-pinch?

Magnetic Pinch

- **The *Z* Pinch**





- The Lorenz force created by a lightning strike crushed this hollow lightning rod and led to the discovery of the pinch.

G. Thomson and P. Thonemann

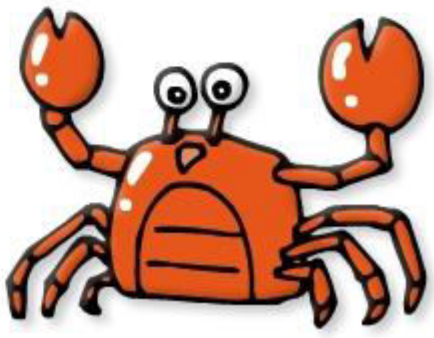
http://joiimg.tistory.com/416
http://www.thunderbolts.info/forum/phpBB3/viewtopic.php?p=731&sid=824cd77d679b252e92355fcbf212ed7a

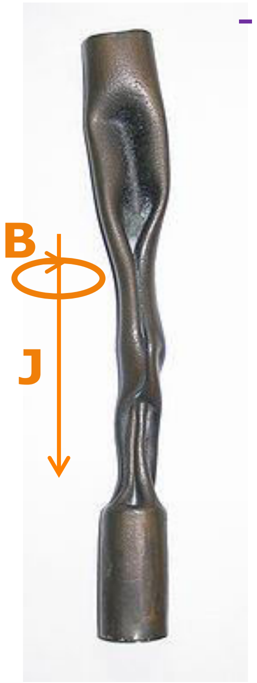
http://www.plasma-universe.com/Pinch_(plasma_physics)?title=Special:Booksources&isbn=0387975756
http://ask.nate.com/qna/view.html?n=6318216

Magnetic Pinch

- **The *Z* Pinch**

- The plasma carries an electric current and is confined by the magnetic field induced by this current.
- As the current is increased, the larger magnetic field compresses the plasma and also raises its temperature by Joule-heating.
- Confinement and heating is simultaneously provided.





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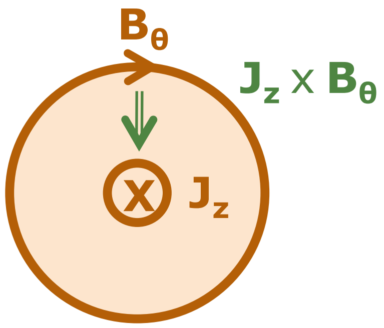
http://ask.nate.com/qna/view.html?n=6318216

Magnetic Pinch



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  - The plasma carries an electric current and is confined by the magnetic field induced by this current.
  - As the current is increased, the larger magnetic field compresses the plasma and also raises its temperature by Joule-heating.
  - Confinement and heating is simultaneously provided.

Where did the neutrons come from in ZETA?

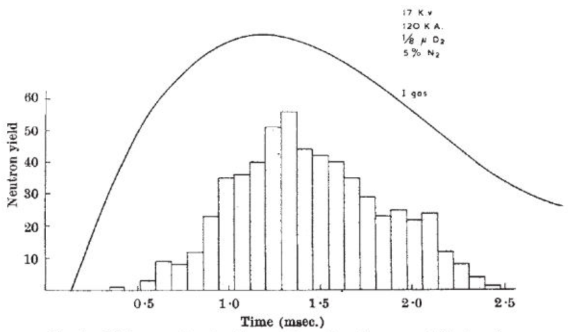


Fig. 4. Histogram showing the number of neutrons counted at various times during the current pulse

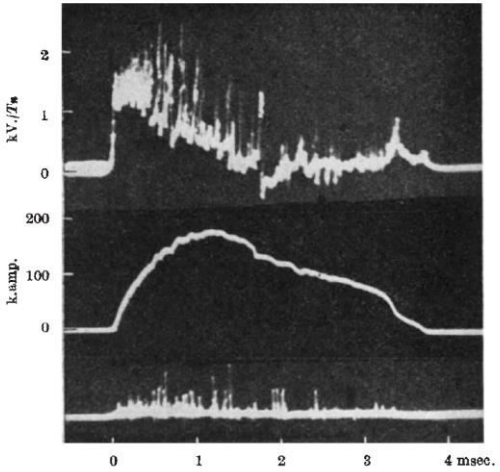
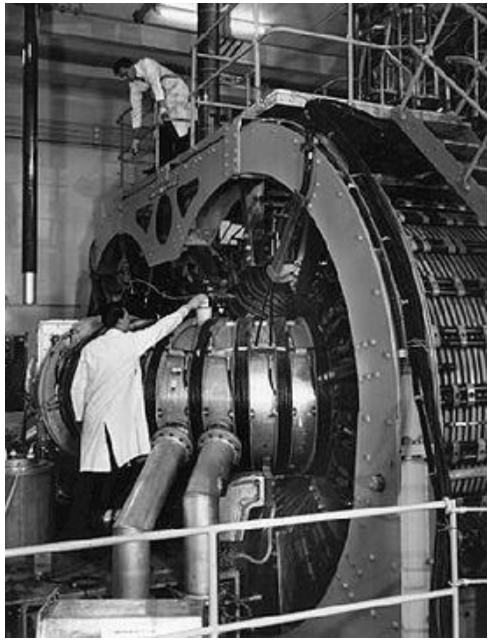


Fig. 2. Oscillograph recordings of the voltage per turn of the transformer, and the secondary current I_s. The lower trace shows the pulses produced by proton recoil in a scintillation neutron counter. Conditions: gas, deuterium + 5 per cent nitrogen + 10 per cent oxygen; pressure, 0·13 × 10^{-3} mm. mercury; axial field, 160 gauss

P. C. Thonemann et al, Nature ***181*** *217 (1958)*

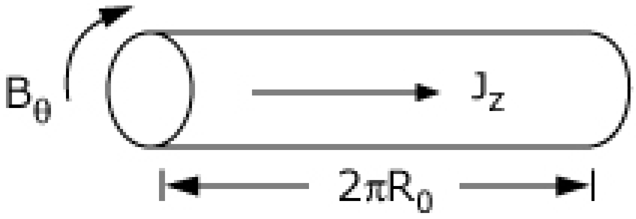


ZETA (1954-58, UK)

Magnetic Pinch

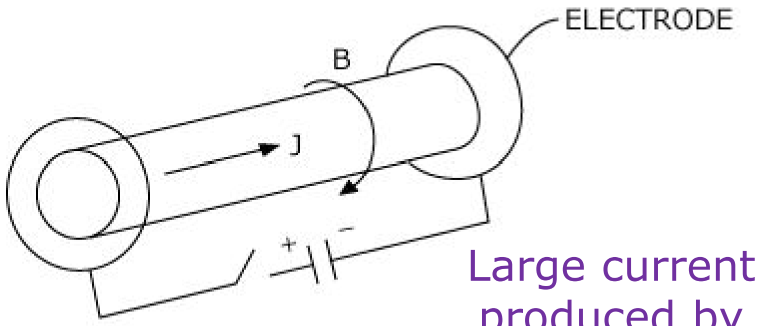
- **The *Z* Pinch**

How about the pinch effect in a electric cable/wire?



self field induced by J_Z

longitudinal plasma current



Large current produced by capacitor

- Equilibrium

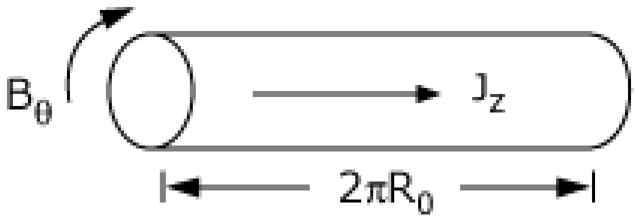
Sequence of solution of the MHD equilibrium equations

1. The $\nabla \cdot \mathbf{B} = 0$
2. Ampere's law: $\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$
3. The momentum equation: $\mathbf{J} \times \mathbf{B} = \nabla p$

$$\nabla p = \vec{J} \times \vec{B}$$
$$\nabla \times \vec{B} = \mu_0 \vec{J}$$
$$\nabla \cdot \vec{B} = 0$$

J.P. Freidberg, "Ideal Magneto-Hydro-Dynamics", lecture note

Magnetic Pinch



- **The *Z* Pinch**

Sequence of solution of the MHD equilibrium equations

1. The $\nabla \cdot \mathbf{B} = 0$

$$\frac{1}{r}\frac{\partial B_\theta}{\partial \theta} = 0$$

2. Ampere's law: $\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$

$$J_z = \frac{1}{\mu_0 r}\frac{d}{dr}(rB_\theta)$$

3. The momentum equation: $\mathbf{J} \times \mathbf{B} = \nabla p$

$$J_z B_\theta = -\frac{dp}{dr}$$

$$\frac{dp}{dr} + \frac{B_\theta}{\mu_0 r}\frac{d}{dr}(rB_\theta) = 0 \qquad \frac{d}{dr}\left(p + \frac{B_\theta^2}{2\mu_0}\right) + \frac{B_\theta^2}{\mu_0 r} = 0$$

↓ particle pressure + magnetic pressure force

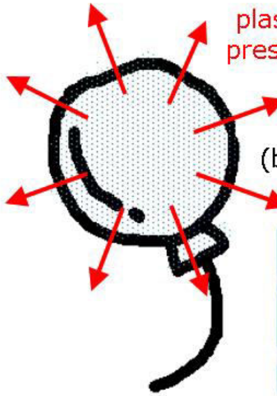
↓

tension force by the curvature of the magnetic field lines

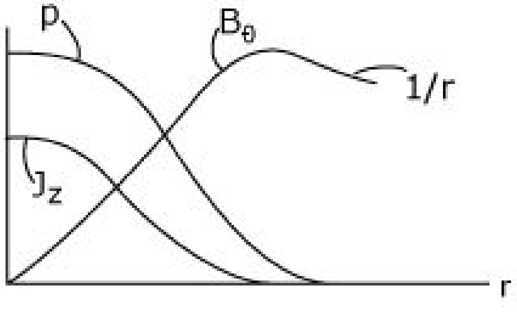
Magnetic Pinch

- **The *Z* Pinch**

$$\frac{d}{dr}\left(p+\frac{B_\theta^2}{2\mu_0}\right)+\frac{B_\theta^2}{\mu_0 r}=0$$



  - It is the tension force and not the magnetic pressure gradient that provides radial confinement of the plasma.

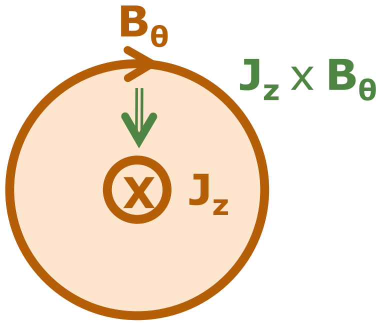


FIELDS

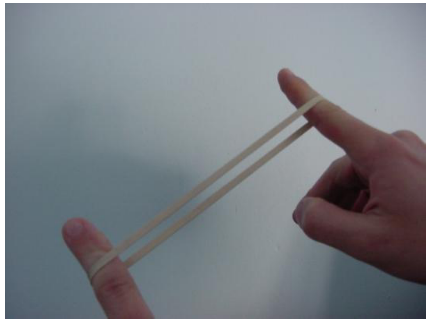
$$B_\theta=\frac{\mu_0 I_0}{2\pi}\frac{r}{r^2+r_0^2}$$

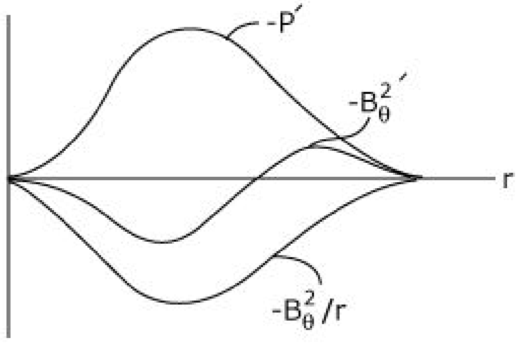
$$J_z=\frac{I_0}{\pi}\frac{r_0^2}{(r^2+r_0^2)^2}$$

$$p=\frac{\mu_0 I_0^2}{8\pi^2}\frac{r_0^2}{(r^2+r_0^2)^2}$$



Bennett profiles
(Bennett, 1934)





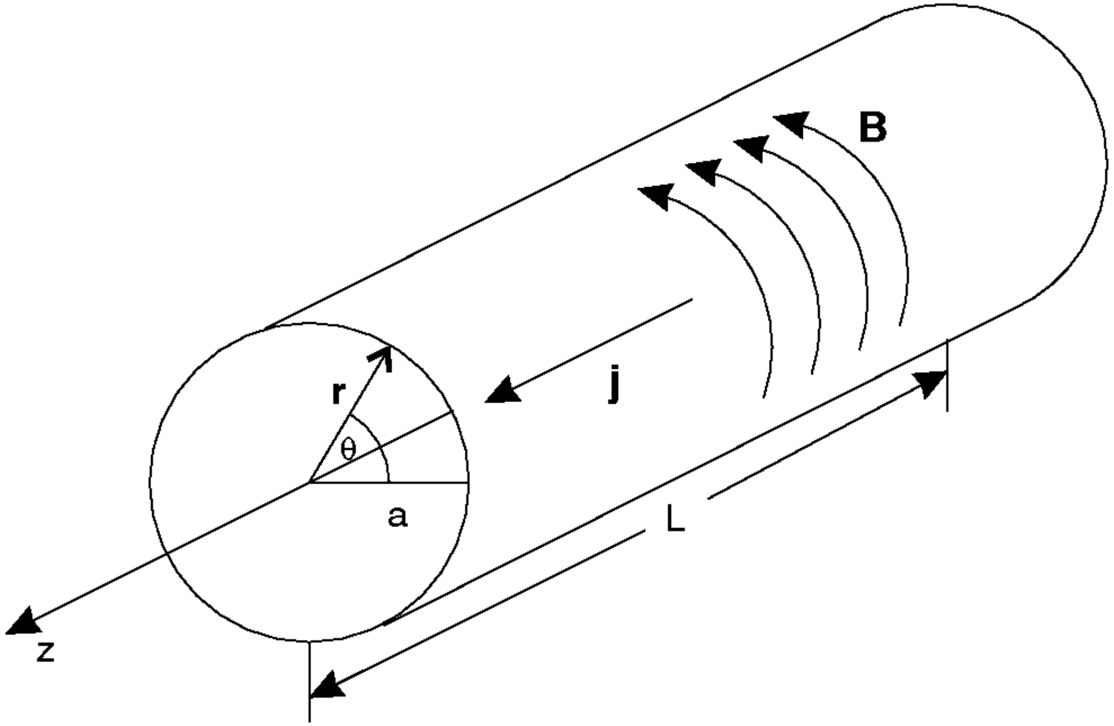
FORCES

  - The magnetic pressure and plasma pressure each produce positive (outward forces) near the outside of the plasma.
  - The plasma is confined by magnetic tension.

J.P. Freidberg, "Ideal Magneto-Hydro-Dynamics", lecture note
http://scienceray.com/technology/industry/where-do-rubber-bands-come-from/

Magnetic Pinch

- **The *Z* Pinch**



- Cylindrical co-ordinates: $j_z, B_\theta, dp/dr$

  Specify current profile $j_z = j_0 = I_P/(\pi a^2)$ for $r < a$

Magnetic Pinch

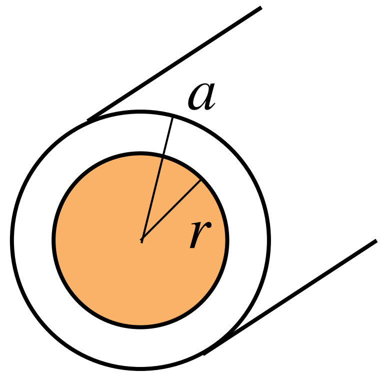
$j_z = j_0 = I_P/(\pi a^2)$ for $r < a$

- **The *Z* Pinch**
  - Ampere` law: $\dfrac{1}{r}\dfrac{\partial}{\partial r}(rB_\theta) = \mu_0 j$

$$\rightarrow B_\theta = \frac{\mu_0}{r}\int_0^r r' j\,dr' = \frac{\mu_0}{r}\frac{1}{2\pi}\int_0^r 2\pi r' j\,dr' = \frac{\mu_0}{r}\frac{I(r)}{2\pi} = \frac{\mu_0}{2\pi r}\frac{I_P \pi r^2}{\pi a^2} = \frac{\mu_0 I_P r}{2\pi a^2}$$

$$B_\theta = \frac{\mu_0 I_P}{2\pi a^2} r \quad \text{for} \quad r < a$$

$$B_\theta = \frac{\mu_0 I_P}{2\pi r} \quad \text{for} \quad r > a$$



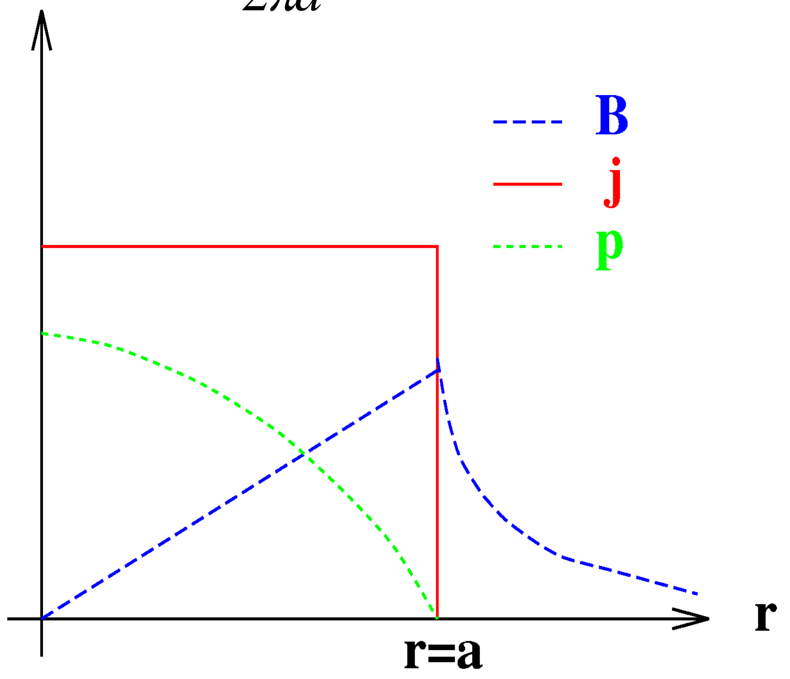
Magnetic Pinch

- **The *Z* Pinch**
  - Compute force balance:

$$\frac{dp}{dr} = -j_z B_\theta = -\frac{I_P}{\pi a^2}\frac{\mu_0 I_P}{2\pi a^2} r$$

$$j_z = j_0 = I_P/(\pi a^2) \text{ for } r < a$$

$$B_\theta = \frac{\mu_0 I_P}{2\pi a^2} r$$



Boundary condition *p(a)=0*:

$$p(r) = \frac{\mu_0 I_P^2}{2\pi^2 a^4}\left[1-\left(\frac{r}{a}\right)^2\right]$$

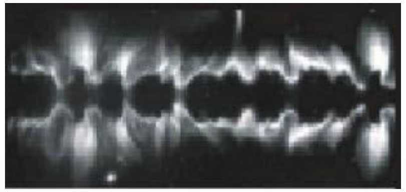
  - Calculate $\beta_p = \frac{2\mu_0 < p >}{B_\theta(a)^2}$, we find $\beta_p = 1$

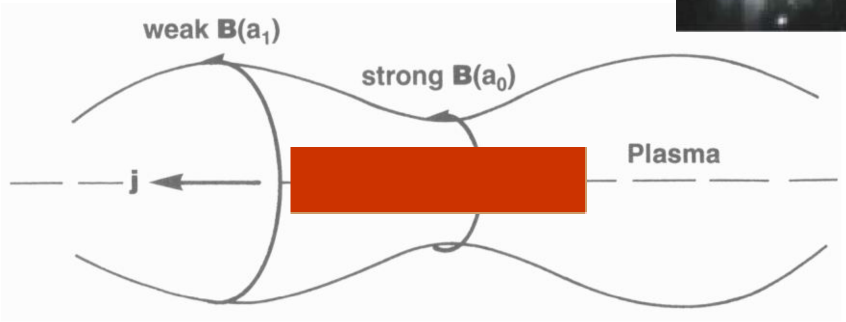
→ **general result for the *z*-pinch, not dependent on profiles.**

Magnetic Pinch

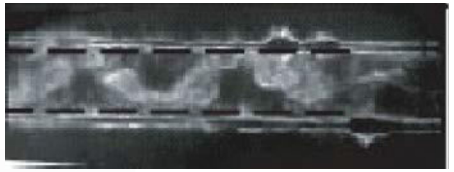
- **The *Z* Pinch**
  - A number of linear *Z*-pinch experiments were constructed during the early years of the fusion program.
  - Large currents of ~0.1 MA are needed thus rendering the *Z* pinch to operate only in short pulses.
  - Exhibiting disastrous instabilities, often leading to a complete quenching of the plasma after ~1-2 μs.

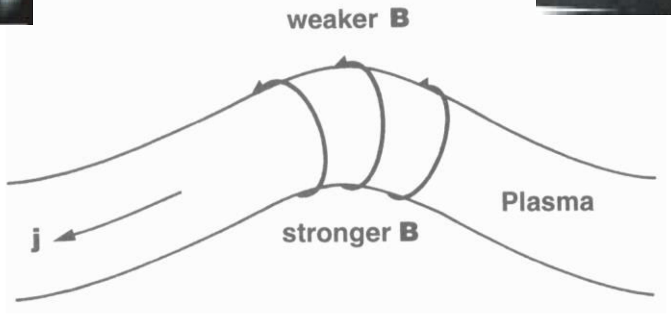
Sausage or pinch instability





Helical kink instability





A. A. Harms et al, "Principles of Fusion Energy", World Scientific (2000)

Magnetic Pinch

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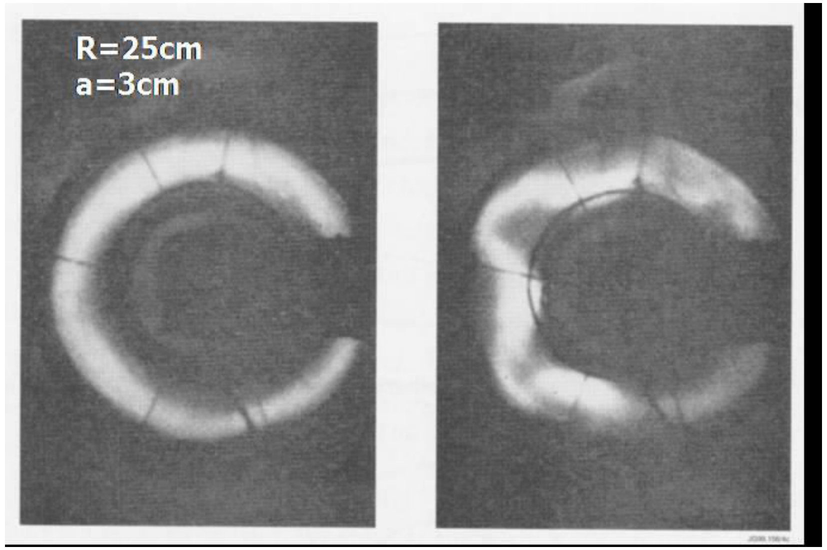

"kink instability" in one of the earliest *Z*-pinch devices, a pyrex tube used by the AEI team at Aldermaston, or earlier while still at Imperial College

Magnetic Pinch

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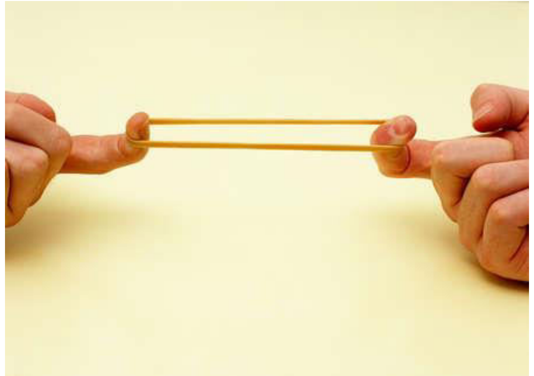
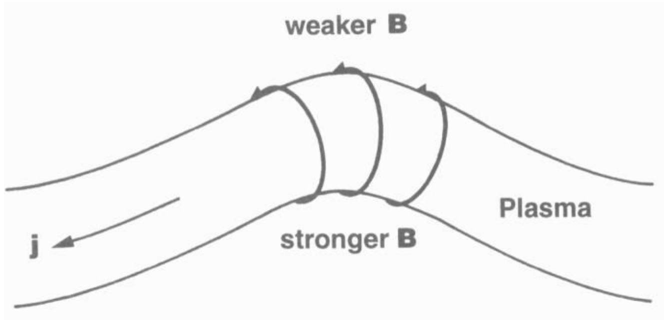
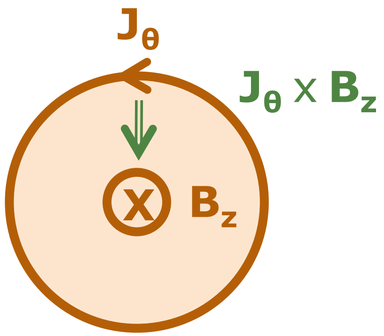




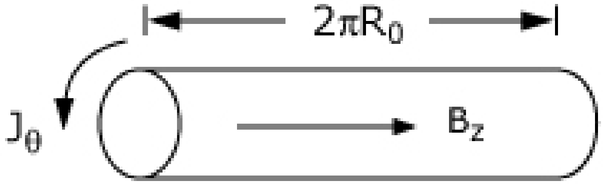
θ-direction tension for equilibrium, z-direction tension for stability

What is θ-pinch?

Magnetic Pinch

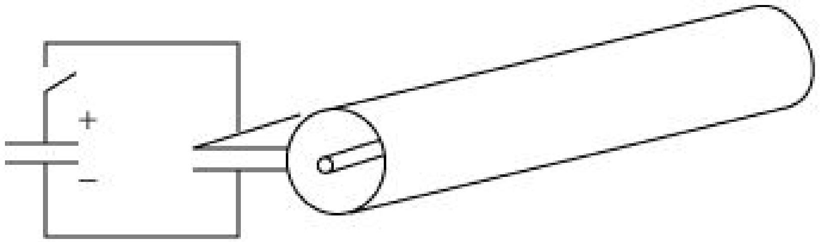


- **The θ Pinch**



induced
diamagnetic current

externally
driven



Magnetic and Kinetic Pressure

- **Plasma Equilibrium**

$$\nabla p = \vec{J} \times \vec{B}$$ → Force balance

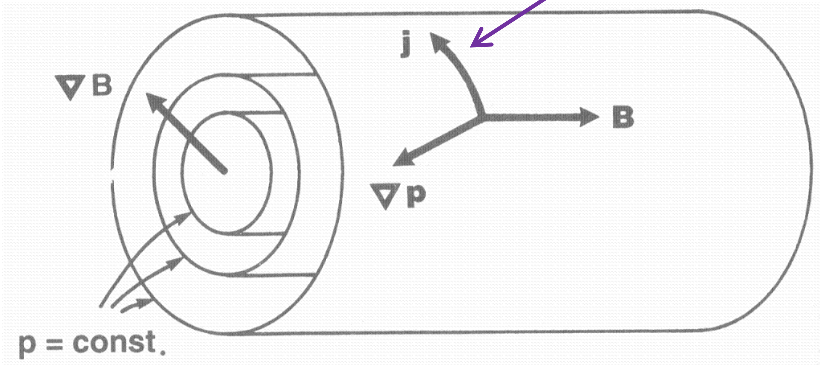
kinetic pressure balanced by **JxB** (Lorentz) force

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$ → Ampere`s law

$$\nabla \cdot \vec{B} = 0$$ → Closed magnetic field lines

$$\vec{B} \cdot \nabla p = 0 \qquad \vec{J} \cdot \nabla p = 0$$

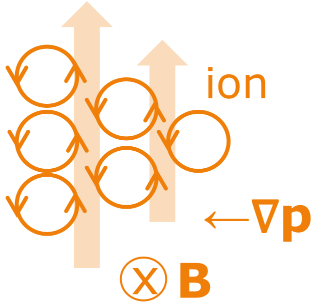
induced by the pressure gradient: causing a decrease in **B** → diamagnetism



Diamagnetic current

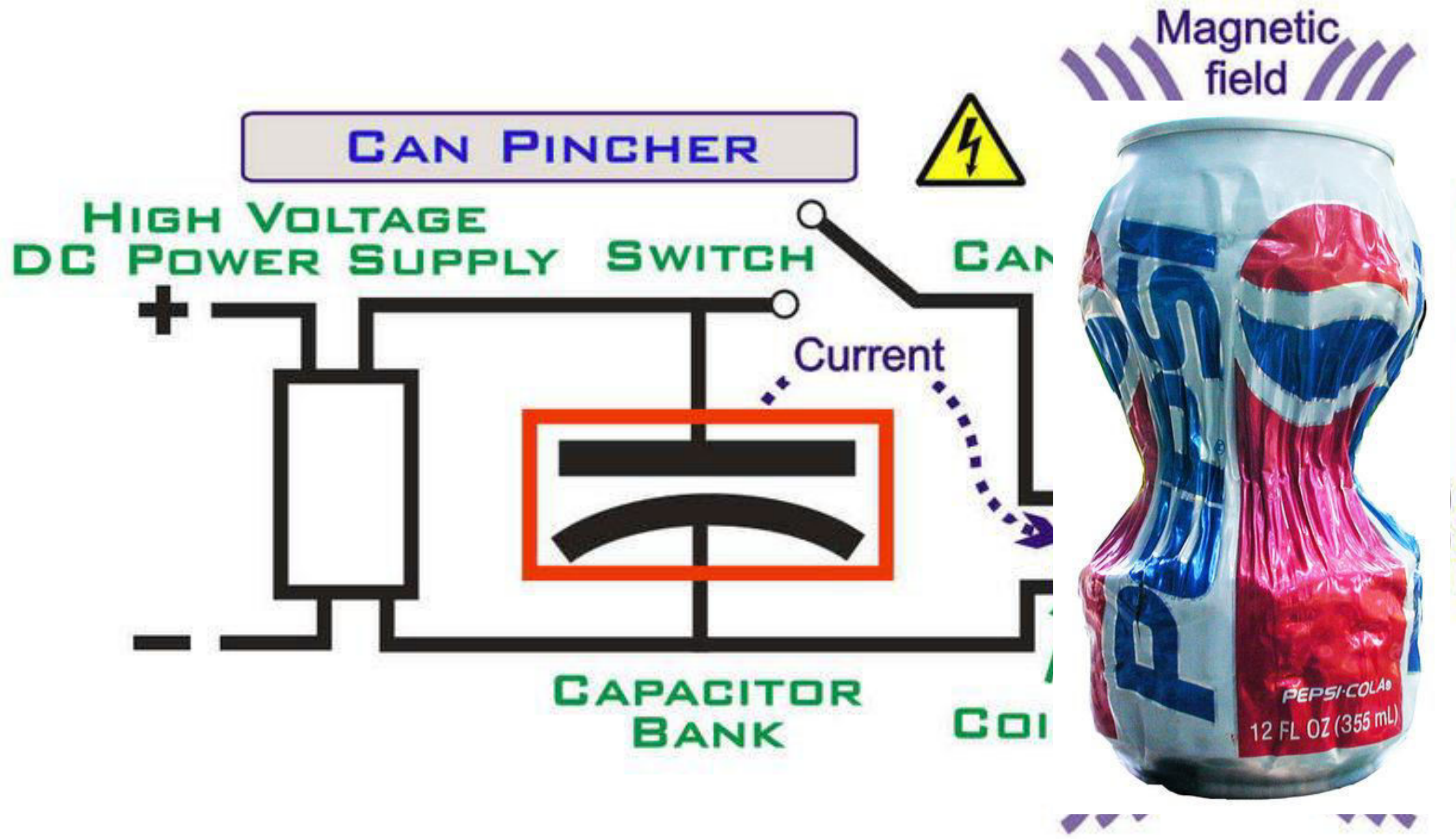
$$\vec{v}_{D,\nabla p} = -\frac{\nabla p \times \vec{B}}{nqB^2}$$

$$\vec{J} = n_i q_i \vec{v}_{D,i} + n_e q_e \vec{v}_{D,e} = \frac{\vec{B} \times \nabla p}{B^2}$$



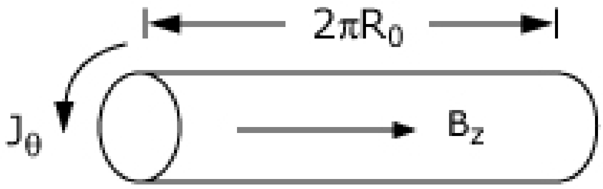
- If *B* is applied, plasma equilibrium can be built by itself due to induction of diamagnetic current. $\nabla p = \vec{J} \times \vec{B}$

Magnetic Pinch



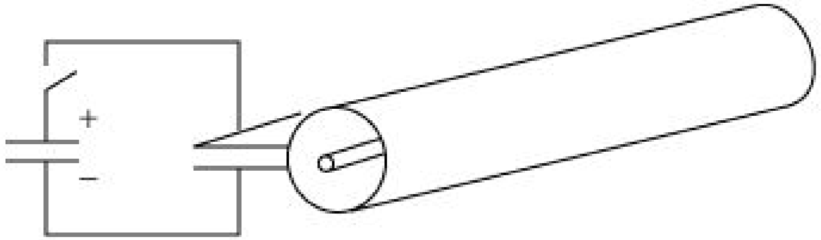
Magnetic Pinch

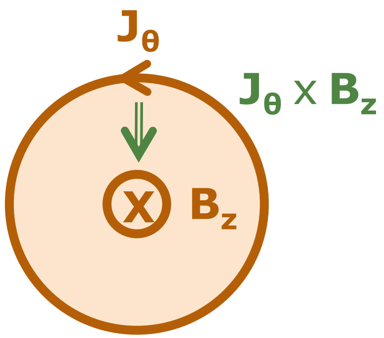
- **The θ Pinch**



induced diamagnetic current

externally driven





- Equilibrium

Sequence of solution of the MHD equilibrium equations

1. The $\nabla \cdot \mathbf{B} = 0$
2. Ampere's law: $\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$
3. The momentum equation: $\mathbf{J} \times \mathbf{B} = \nabla p$

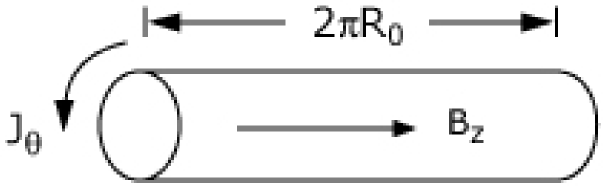
$$\nabla p = \vec{J} \times \vec{B}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\nabla \cdot \vec{B} = 0$$

J.P. Freidberg, "Ideal Magneto-Hydro-Dynamics", lecture note

Magnetic Pinch



- **The $\boldsymbol{\theta}$ Pinch**

Sequence of solution of the MHD equilibrium equations

1. The $\nabla\cdot\mathbf{B} = 0$

$$\frac{\partial B_z}{\partial z} = 0$$

2. Ampere's law: $\mu_0\mathbf{J} = \nabla\times\mathbf{B}$

$$J_\theta = -\frac{1}{\mu_0}\frac{\partial B_z}{\partial r}$$

3. The momentum equation: $\mathbf{J}\times\mathbf{B} = \nabla p$

$$J_\theta B_z = \frac{dp}{dr}$$

$$\frac{d}{dr}\left(p + \frac{B_z^2}{2\mu_0}\right) = 0 \qquad p + \frac{B_z^2}{2\mu_0} = \frac{B_0^2}{2\mu_0}$$

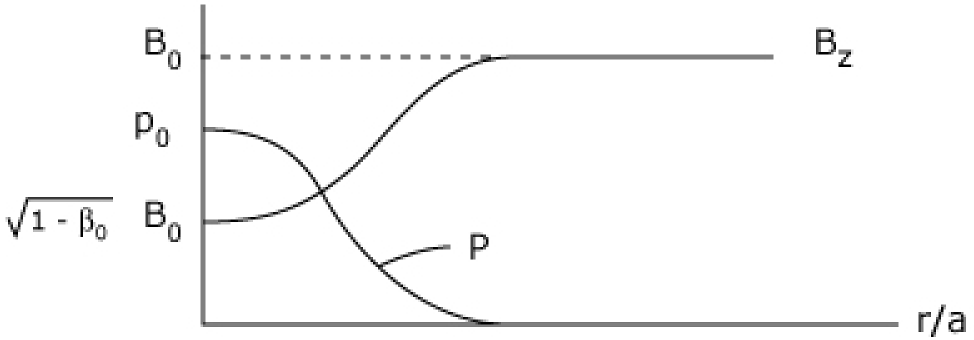
- At any local value of *r*, the sum of the particle pressure and the magnetic pressure is a constant, equal to the externally applied magnetic pressure.
- The plasma is confined radially by the pressure of the externally applied magnetic field.

Magnetic Pinch

- **The θ Pinch**

Plot the forces.

- Typical example



$$p = p_0 \exp(-r^2 / a^2)$$

$$B_z = B_0[1 - \beta_0 \exp(-r^2 / a^2)]^{1/2}$$

$$\beta_0 \equiv 2\mu_0 p / B_0^2$$

$$p + \frac{B_z^2}{2\mu_0} = \frac{B_0^2}{2\mu_0}$$

- The plasma is confined radially by the pressure of the externally applied magnetic field.

J.P. Freidberg, "Ideal Magneto-Hydro-Dynamics", lecture note

Magnetic Pinch

- **The θ Pinch**

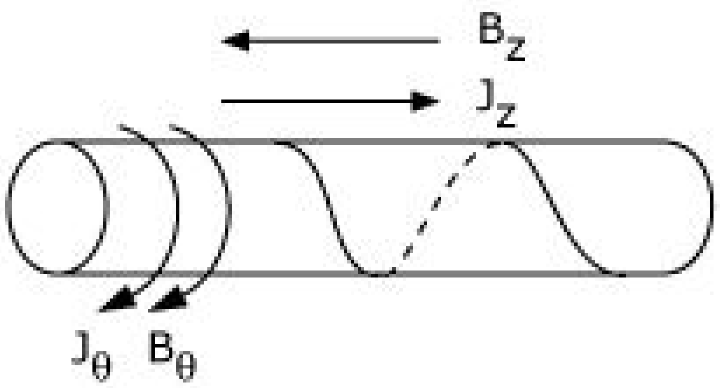
  - One of the early successes of the fusion program in terms of the performance.
  - T_i ~ 1-4 keV, n ~ 1-2x10^{22} m^{-3}, $\beta(0)$ ~ 0.7-0.9
  - High temperature using the implosion heating method: rapidly rising magnetic field acting like a piston
  - No indication of macroscopic instability (neutrally stable)
  - Severe end loss ($\tau \sim L/V_{Ti}$, e.g. 10 μs for a 5 m device)

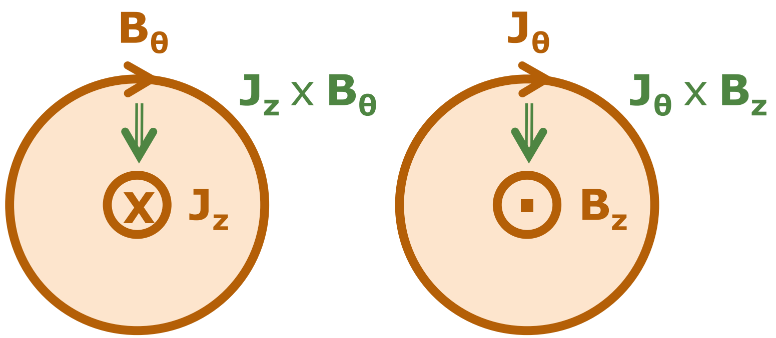
What is screw pinch?

Magnetic Pinch

- **The General Screw Pinch**

  - A hybrid combination of *Z* pinch and *θ* pinch
  - This combination of fields allows the flexibility to optimise configurations w.r.t. force balance and stability.





- Equilibrium

Sequence of solution of the MHD equilibrium equations

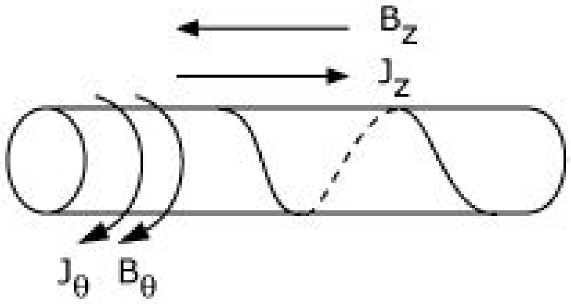
1. The $\nabla \cdot \mathbf{B} = 0$
2. Ampere's law: $\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$
3. The momentum equation: $\mathbf{J} \times \mathbf{B} = \nabla p$

$$\nabla p = \vec{J} \times \vec{B}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\nabla \cdot \vec{B} = 0$$

Magnetic Pinch



- **The General Screw Pinch**

  - Sequence of solution of the MHD equilibrium equations

  1. The $\nabla\cdot\mathbf{B} = 0$

$$\frac{1}{r}\frac{\partial B_\theta}{\partial \theta} + \frac{\partial B_z}{\partial z} = 0$$

  2. Ampere's law: $\mu_0\mathbf{J} = \nabla\times\mathbf{B}$

$$J_\theta = -\frac{1}{\mu_0}\frac{\partial B_z}{\partial r} \qquad J_z = \frac{1}{\mu_0 r}\frac{d}{dr}(rB_\theta)$$

  3. The momentum equation: $\mathbf{J}\times\mathbf{B} = \nabla p$

$$J_\theta B_z - J_z B_\theta = \frac{dp}{dr}$$

$$\frac{d}{dr}\left(p + \frac{B_\theta^2 + B_z^2}{2\mu_0}\right) + \frac{B_\theta^2}{\mu_0 r} = 0$$

- Even though the equations are nonlinear, the forces superpose because of symmetry.

Magnetic Pinch

- **The General Screw Pinch**

- Because of its flexibility the general screw-pinch relation describes a wide variety of configurations.