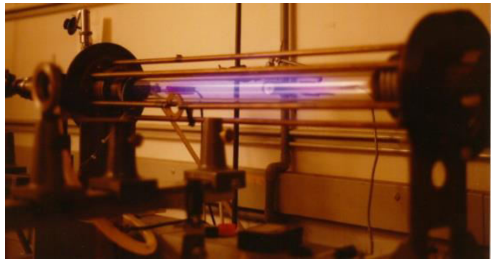
Introduction to Nuclear Fusion

Prof. Dr. Yong-Su Na

What is magnetic mirror?

Open Magnetic Confinement



Z pinch

θ pinch

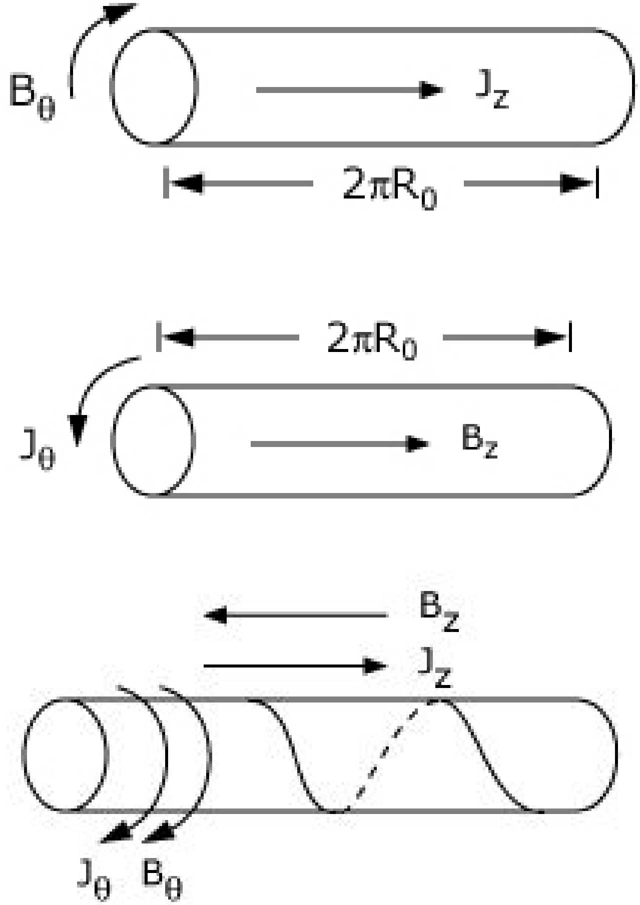
screw pinch

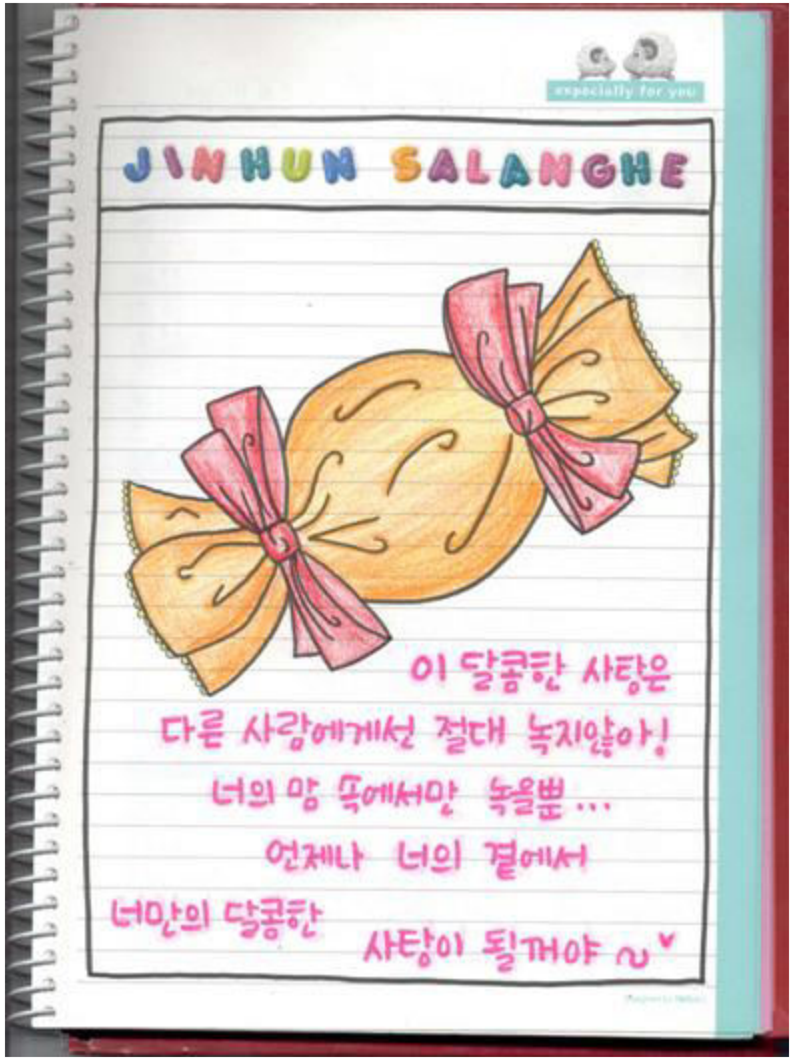
Magnetic mirror

WIKIPEDIA
The Free Encyclopedia

http://phys.strath.ac.uk/information/history/photos.php
http://www.frascati.enea.it/ProtoSphera/ProtoSphera%202001/6.%20Electrode%20experiment.htm

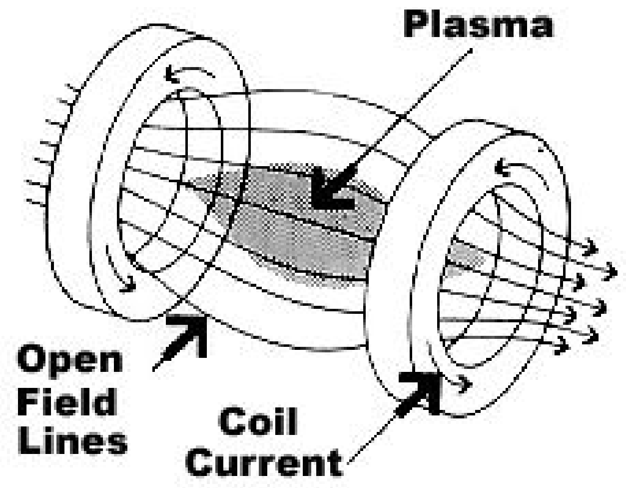
Magnetic Mirror

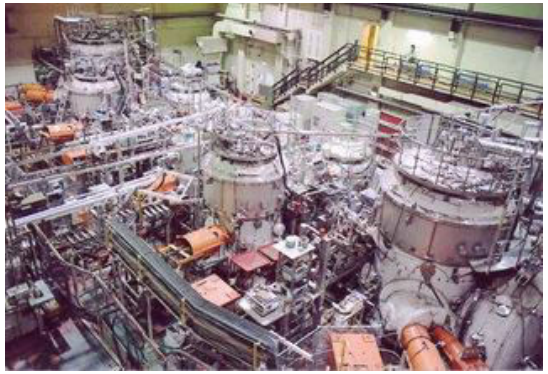


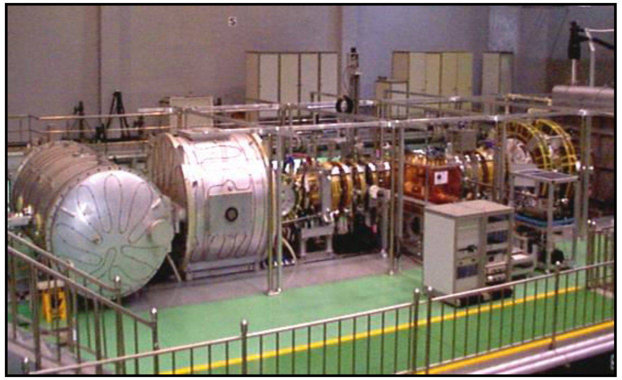


- Suffering from end losses

Magnetic Mirror



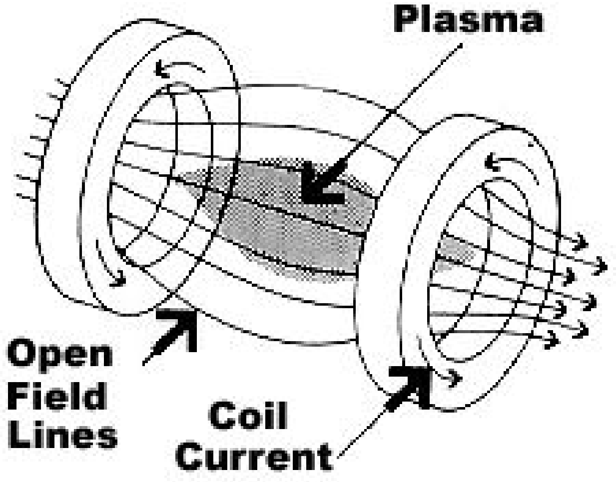




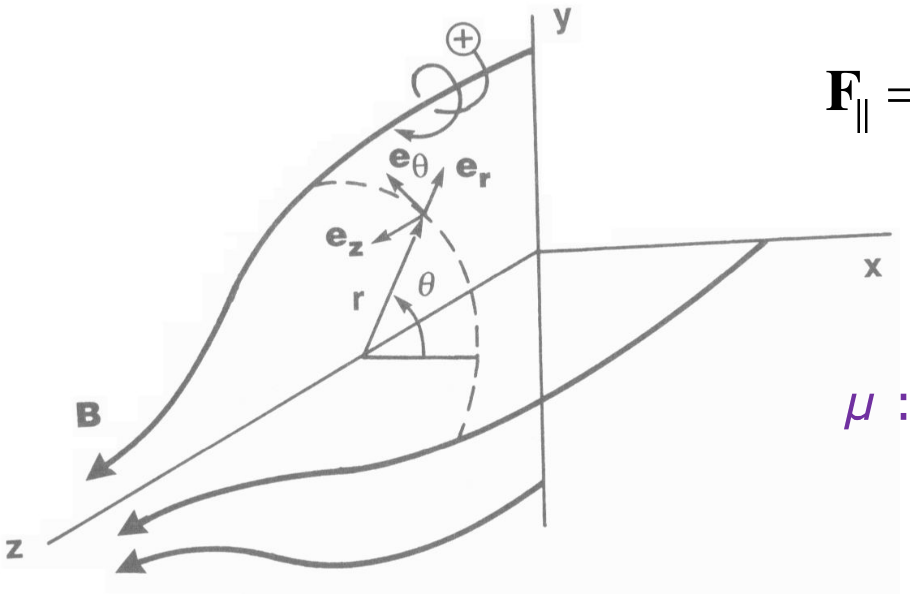
Gamma 10 and Hanbit

- To reduce end-leakage by establishing an increasing magnetic field at the two ends
- Many of charged particles are trapped due to the imposed constraints on particle motion with regards to conservation of energy and the magnetic moment.
- First proposed by Enrico Fermi as a mechanism for the acceleration of cosmic rays

Magnetic Mirror



What is the motion of particles?



$$\mathbf{F}_{\parallel} = -\frac{1}{2}\frac{mv_{\perp}^{2}}{B}\nabla_{\parallel}B = -\mu\nabla_{\parallel}B$$

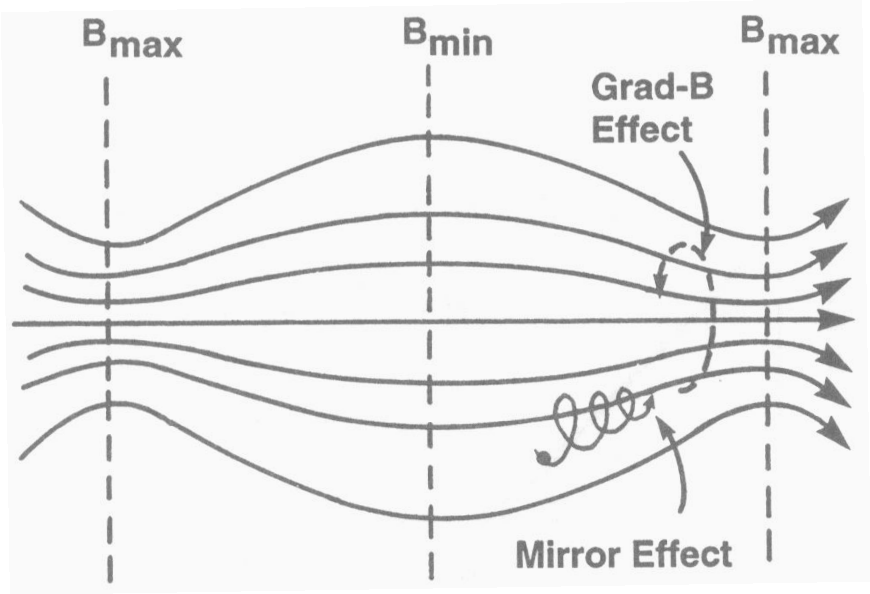
μ : magnetic moment of the gyrating particle

Magnetic Mirror

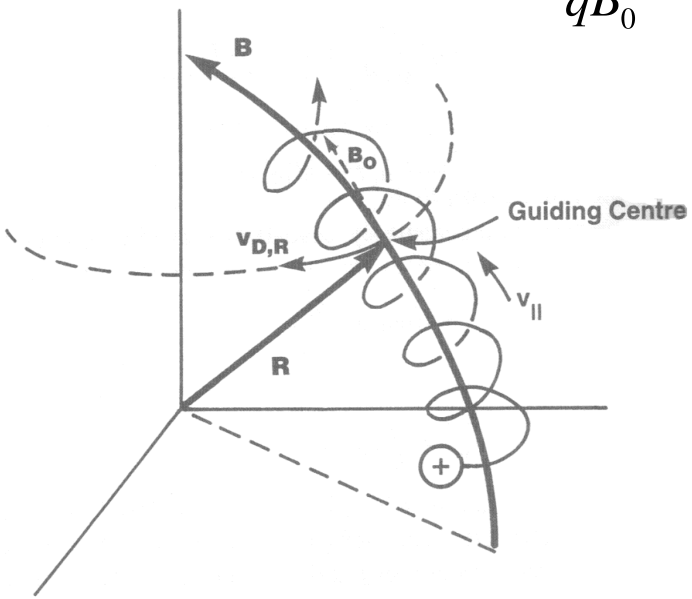
- **Single particle picture**

- As the ion approaches the ends, it is increasingly subjected to drifts due to the inhomogeneity of the mirror field.
- Drifts occur in azimuthal directions and the particles are still bound to their magnetic surfaces.

$$\mathbf{v}_{D,R} = \overline{\mathbf{v}}_{gc,x} = \frac{m v_{\parallel}^2}{q B_0^2} \frac{\mathbf{R}_0 \times \mathbf{B}_0}{R^2}$$



$$r_L = \frac{v_\perp m}{|q| B_z}$$



A. A. Harms et al, "Principles of Fusion Energy", World Scientific (2000)

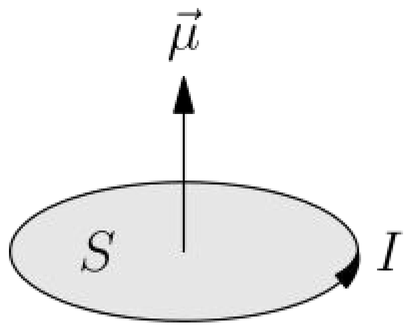
Magnetic Mirror

- **Conservation of kinetic energy and magnetic moment**

$$\frac{d}{dt}E_0 = \frac{d}{dt}\left(\frac{1}{2}mv_\perp^2 + \frac{1}{2}mv_\parallel^2\right) = 0$$

$$\frac{d}{dt}(\mu) = 0 \qquad \mu = \frac{mv_\perp^2/2}{B}$$

Magnetic moment



$$\vec{\mu} = \vec{M} = IS\hat{n} = \frac{|q|}{\tau}\pi r_L^2 \hat{n} = |q|\frac{\omega_c}{2\pi}\pi r_L^2 \hat{n}$$

$$= |q|\frac{B|q|}{2m}\left(\frac{mv_\perp}{B|q|}\right)^2 \hat{n}$$

$$= \frac{mv_\perp^2/2}{B}\hat{n}$$

Individual Charge Trajectories

- **Invariant of motion**

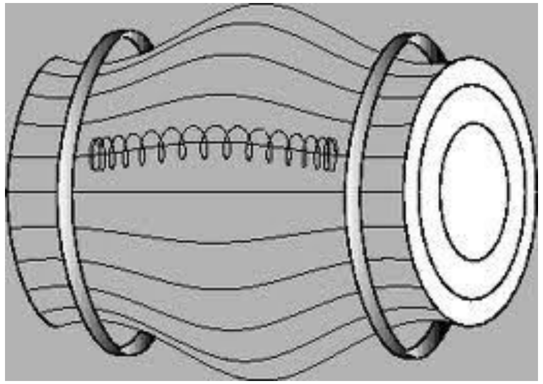
$$\frac{d}{dt}E_0=\frac{d}{dt}\left(\frac{1}{2}mv_\perp^2+\frac{1}{2}mv_\parallel^2\right)=0 \qquad \mu=\frac{mv_\perp^2/2}{B} \qquad m\frac{dv_\parallel}{dt}=-\frac{\mu}{v_\parallel}\frac{dB}{dt} \qquad mv_\parallel\frac{dv_\parallel}{dt}=-\mu\frac{dB}{dt}$$

$$\mathbf{F}_\parallel=m\frac{d\mathbf{v}_\parallel}{dt}=-\mu\nabla_\parallel\mathbf{B}=-\mu\frac{\partial\mathbf{B}}{\partial s}=-\mu\frac{\partial\mathbf{B}}{\partial s}\cdot\frac{ds}{dt}\cdot\frac{dt}{ds}=-\mu\frac{\partial\mathbf{B}}{\partial s}\cdot\frac{ds}{dt}\cdot\frac{1}{v_\parallel}=-\frac{\mu}{v_\parallel}\frac{d\mathbf{B}}{dt} \longrightarrow \frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2\right)=-\mu\frac{dB}{dt}$$

$$\longrightarrow \frac{d}{dt}\left(\frac{1}{2}mv_\perp^2+\frac{1}{2}mv_\parallel^2\right)=\frac{d}{dt}(\mu B)+\left(-\mu\frac{dB}{dt}\right)=0$$

$$\frac{d}{dt}(\mu B)=\mu\frac{dB}{dt}+B\frac{d\mu}{dt}$$

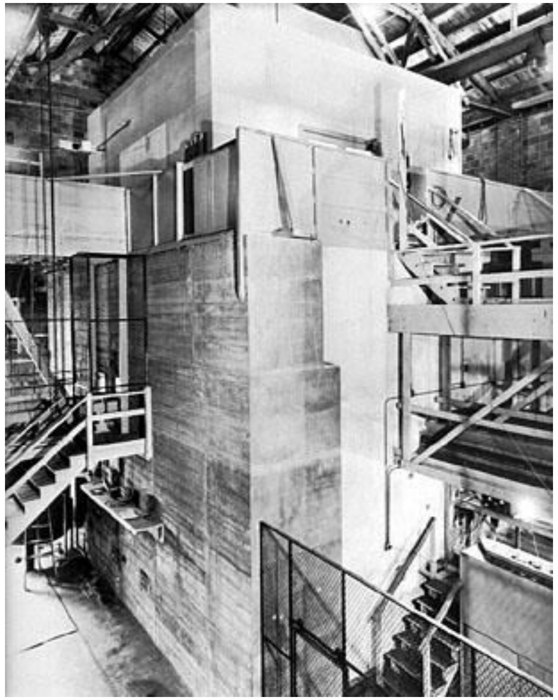
$\longrightarrow \dfrac{d\mu}{dt}=0$: adiabatic invariant

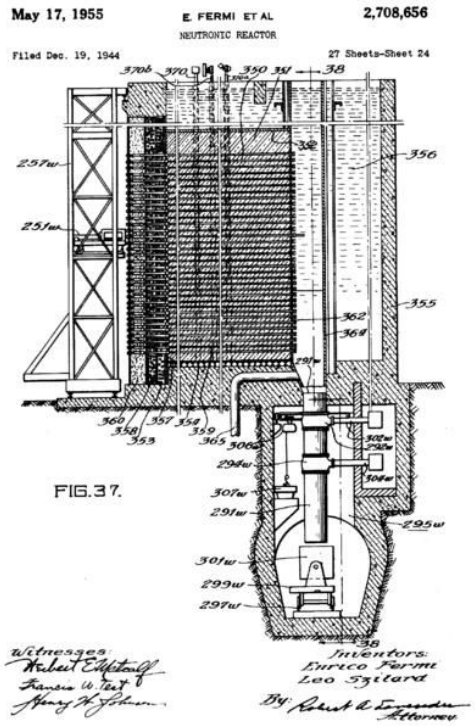


Magnetic Mirror

Enrico Fermi (1901-1954)

Nobel Laureate in physics in 1938
Cf. Marshall Rosenbluth (Doctoral student)





CP-1 (Chicago Pile-1, the world's first human-made nuclear reactor) and Drawings from the Fermi–Szilárd "neutronic reactor" patent

Magnetic Mirror

PHYSICAL REVIEW VOLUME 75, NUMBER 8 APRIL 15, 1949

On the Origin of the Cosmic Radiation

ENRICO FERMI
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois

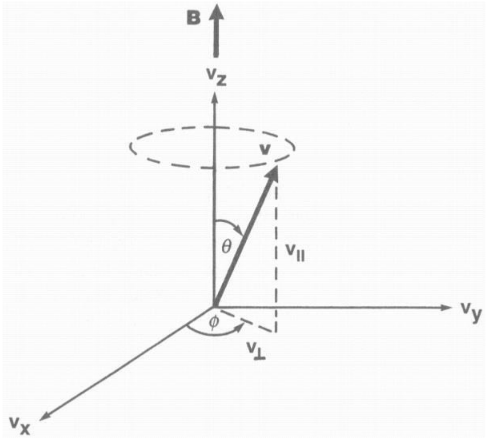
(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magmetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

Magnetic Mirror

$$E_0 = \frac{1}{2}mv^2 = \frac{1}{2}mv_\perp^2 + \frac{1}{2}mv_\parallel^2 = const.$$

$$\boldsymbol{\mu} = \frac{\frac{1}{2}mv_\perp^2}{B} = \frac{\frac{1}{2}mv^2\sin^2\boldsymbol{\theta}}{B} = const.$$



The path of a fast proton in an irregular magnetic field of the type that we have assumed will be represented very closely by a spiraling motion around a line of force. Since the radius of this spiral may be of the order of 10^{12} cm, and the irregularities in the field have dimensions of the order of 10^{18} cm, the cosmic ray will perform many turns on its spiraling path before encountering an appreciably different field intensity. One finds by an elementary discussion that as the particle approaches a region where the field intensity increases, the pitch of the spiral will decrease. One finds precisely that

$$\boxed{\sin^2\vartheta/H \approx \text{constant},} \qquad (12)$$

where ϑ is the angle between the direction of the line of force and the direction of the velocity of the particle, and H is the local field intensity. As the particle approaches a region where the field intensity is larger, one will expect, therefore, that the angle ϑ increases until $\sin\vartheta$ attains the maximum possible value of one. At this point the particle is reflected back along the same line of force and spirals backwards until the next region of high field intensity is encountered. This process will be called a "Type A" reflection. If the magnetic field were static, such a reflection would not produce any change in the kinetic energy of the particle. This is not so, however, if the magnetic field is slowly variable. It may happen that a region of high field intensity moves toward the cosmic-ray particle which collides against it. In this case, the particle will gain energy in the collision. Conversely, it may happen that the region of high field intensity moves away from the particle. Since the particle is much faster, it will overtake the irregularity of the field and be reflected backwards, in this case with loss of energy. The net result will be an average gain, primarily for the reason that head-on collisions are more frequent than overtaking collisions because the relative velocity is larger in the former case.

$$\frac{w'}{w} = \frac{1 + 2B\beta\cos\vartheta + B^2}{1 - B^2}, \qquad (13)$$

where βc is the velocity of the particle, ϑ is the angle of inclination of the spiral, and Bc is the velocity of the perturbation. It is assumed that the

Magnetic Mirror

- **Fermi as a genuine scientist**

spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

The present theory is incomplete because no satisfactory injection mechanism is proposed except for protons which apparently can be regenerated at least in part in the collision processes of the cosmic radiation itself with the diffuse interstellar matter. The most serious difficulty is in the injection process for the heavy nuclear component of the radiation. For these particles the injection energy is very high and the injection mechanism must be correspondingly efficient.

some equivalent mechanism. With respect to the injection of heavy nuclei I do not know a plausible answer to this point.

Magnetic Mirror

- **Condition for trapping of particles**

$$v_\parallel\Big|_{B_r \le B_{\max}} = 0$$

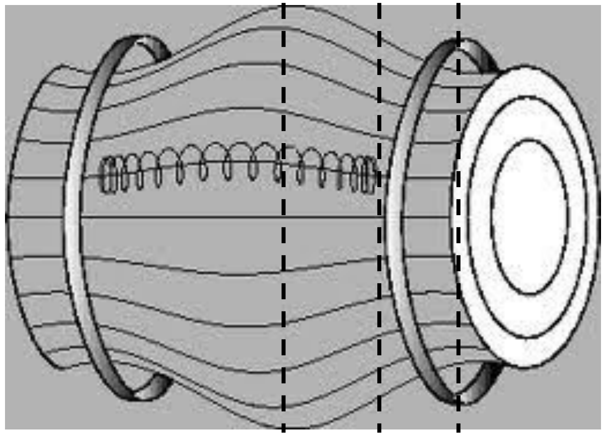
$$\mathbf{F}_\parallel = -\frac{1}{2}\frac{mv_\perp^2}{B}\nabla_\parallel B = -\mu\nabla_\parallel B$$

$$\mu = \frac{\frac{1}{2}mv_{\perp\min}^2}{B_{\min}} = \frac{\frac{1}{2}mv_{\perp r}^2}{B_r}$$

$$E = \frac{1}{2}mv_{\parallel\min}^2 + \frac{1}{2}mv_{\perp\min}^2 = \frac{1}{2}mv_{\parallel r}^2 + \frac{1}{2}mv_{\perp r}^2 = \frac{1}{2}mv_{\perp r}^2$$

$$\downarrow \qquad\qquad \downarrow$$

$$\frac{v_{\perp\min}^2}{v_{\perp r}^2} = \frac{B_{\min}}{B_r} \longrightarrow \frac{v_{\perp\min}^2}{v_{\parallel\min}^2 + v_{\perp\min}^2} = \frac{B_{\min}}{B_r}$$



Magnetic Mirror

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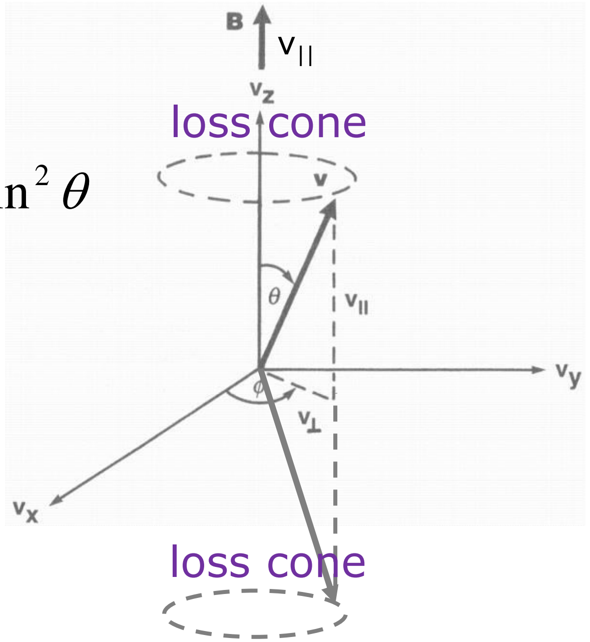
$$\downarrow \qquad\qquad \downarrow$$

$$\frac{v_{\perp\min}^2}{v_{\perp r}^2} = \frac{B_{\min}}{B_r} \longrightarrow \frac{v_{\perp\min}^2}{v_{\parallel\min}^2 + v_{\perp\min}^2} = \frac{B_{\min}}{B_r} = \frac{v_{\perp\min}^2}{v_{\min}^2} = \sin^2\theta$$



Condition for trapping of particles

$$B_r \le B_{\max} \qquad \frac{B_{\min}}{B_r} = \sin^2\theta \ge \frac{B_{\min}}{B_{\max}}$$

Magnetic Mirror

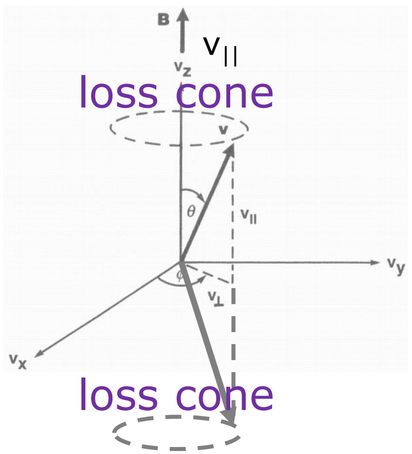
$$\frac{B_{\min}}{B_r} = \sin^2\theta \ge \frac{B_{\min}}{B_{\max}}$$

$$\theta_0 = \arcsin\sqrt{\frac{B_{\min}}{B_{\max}}}$$

- **Mirror ratio**

$$f_{loss} = \frac{\int\limits_{\text{double cone}} f(\vec{v})d^3v}{\int\limits_0^\infty f(\vec{v})d^3v} = \frac{\int\limits_0^{2\pi} d\phi\left[\int\limits_0^{\theta_0}\sin\theta d\theta + \int\limits_{\pi-\theta_0}^{\pi}\sin\theta d\theta\right]\int\limits_0^\infty \frac{f(v)}{4\pi v^2}v^2dv}{\int\limits_0^{2\pi} d\phi\int\limits_0^{\pi}\sin\theta d\theta\int\limits_0^\infty \frac{f(v)}{4\pi v^2}v^2dv}$$

$$= 1 - \cos\theta_0$$



$$f_{trap} = 1 - f_{loss} = \cos\theta_0 = \sqrt{1-\frac{B_{\min}}{B_{\max}}}$$

$$\frac{B_{\max}}{B_{\min}} \equiv R_m$$

mirror ratio:
Determining the effectiveness of confinement

$$f_{trap} = \sqrt{1-\frac{1}{R_m}}$$

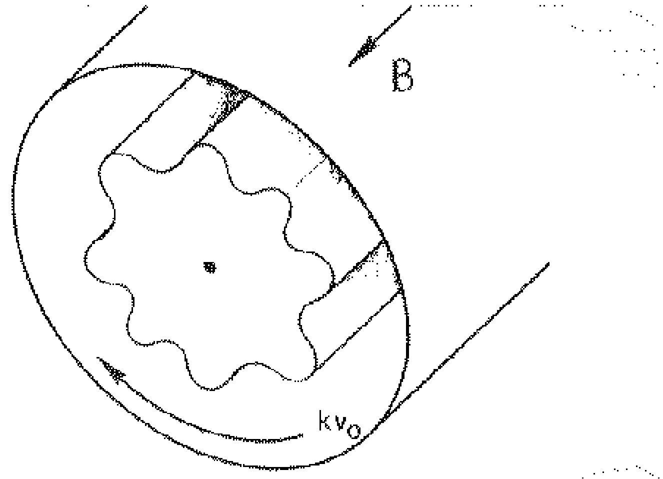
A. A. Harms et al, "Principles of Fusion Energy", World Scientific (2000)

Stability in mirror?

Magnetic Mirror

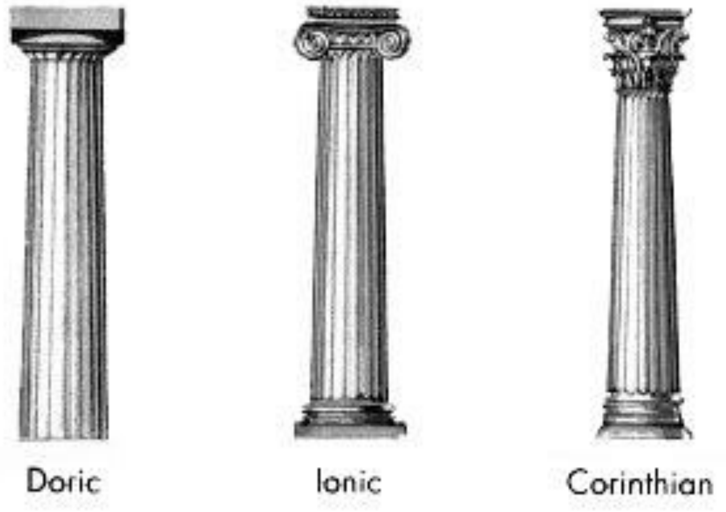
- **Instabilities**

- Flute instability: convex curvature of the magnetic field



A "flute" instability.

Figure 1: Flute instability — From F.F.Chen, 1974

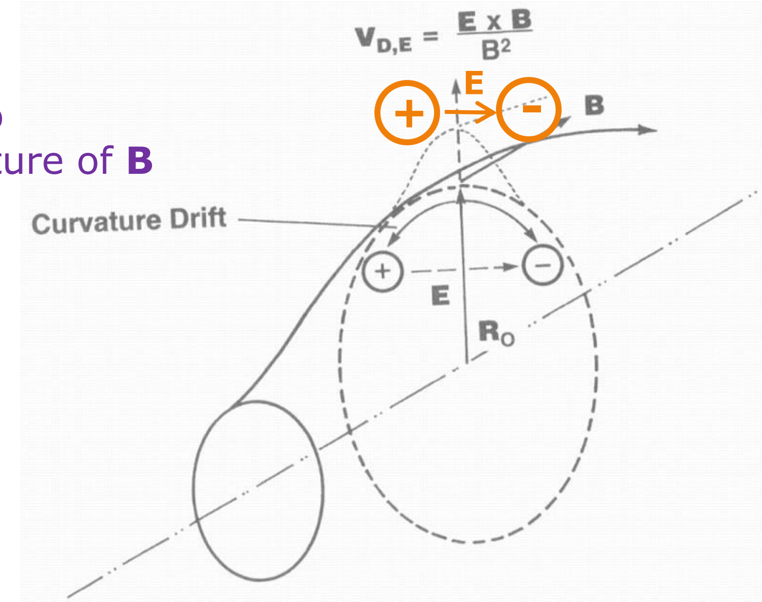


Magnetic Mirror

- **Instabilities**

- Flute instability: convex curvature of the magnetic field

due to
curvature of **B**
in *Z*



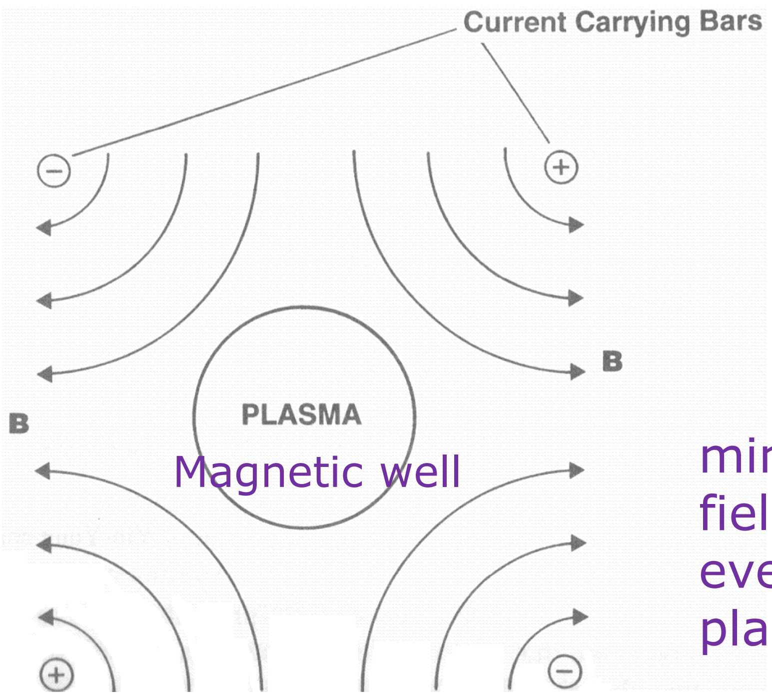
- Particle picture:
  The curvature drift leading to azimuthal polarisation to create the **E**-field resulting in the **E**x**B** drift displacing the plasma particles radially outward

$$\mathbf{v}_{D,\nabla B} = \pm \frac{1}{2} v_\perp r_L \frac{\mathbf{B} \times \nabla B}{B^2} \qquad \mathbf{v}_{D,R} = \frac{m v_\parallel^2}{q B_0^2} \frac{\mathbf{R}_0 \times \mathbf{B}_0}{R^2}$$

Magnetic Mirror

- **Instabilities**

- Flute instability: convex curvature of the magnetic field



minimum-B field configuration: field lines are (almost) everywhere concave into the plasma

Magnetic Mirror

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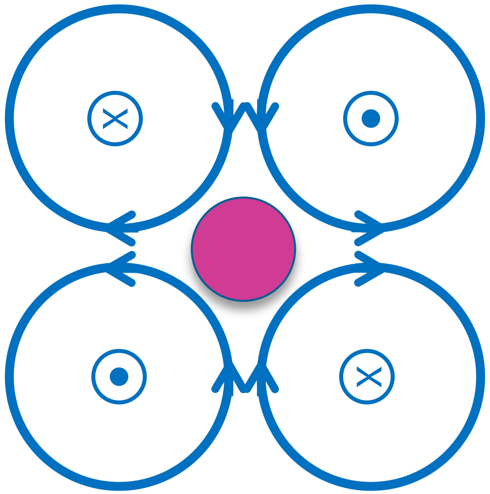

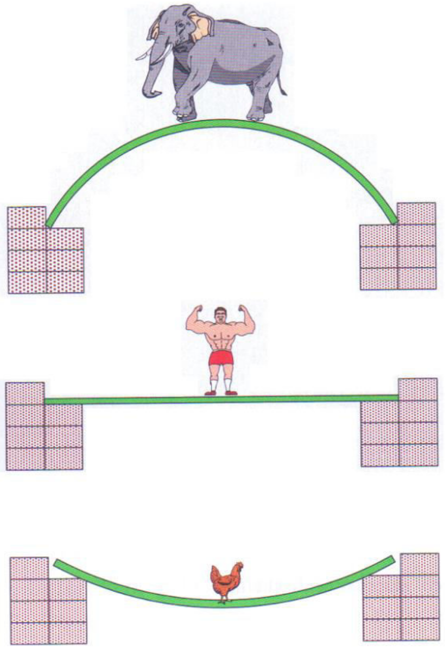

F. F. Chen, "An Indispensable Truth", Springer (2011)

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http://en.wikipedia.org/wiki/File:St_Louis_Gateway_Arch.jpg

Magnetic Mirror

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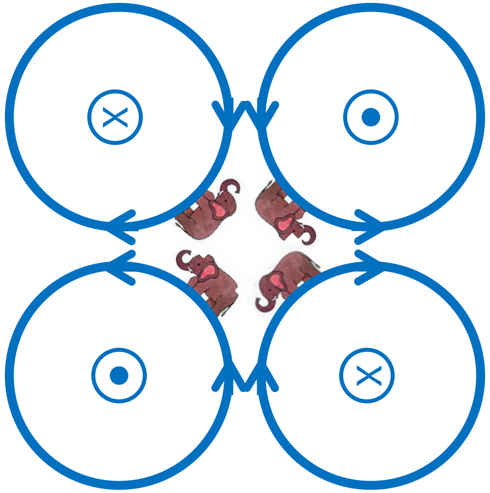

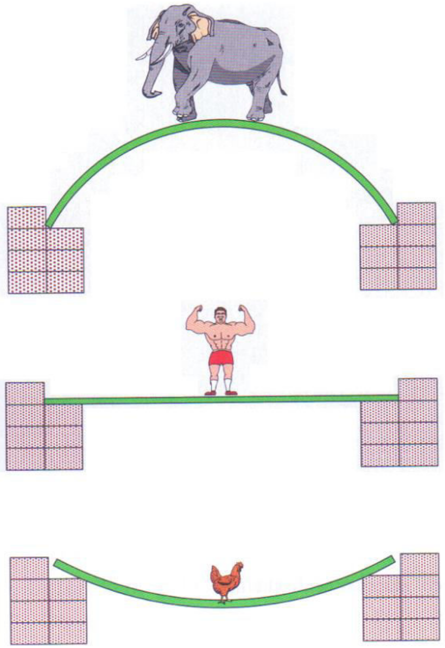

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Magnetic Mirror

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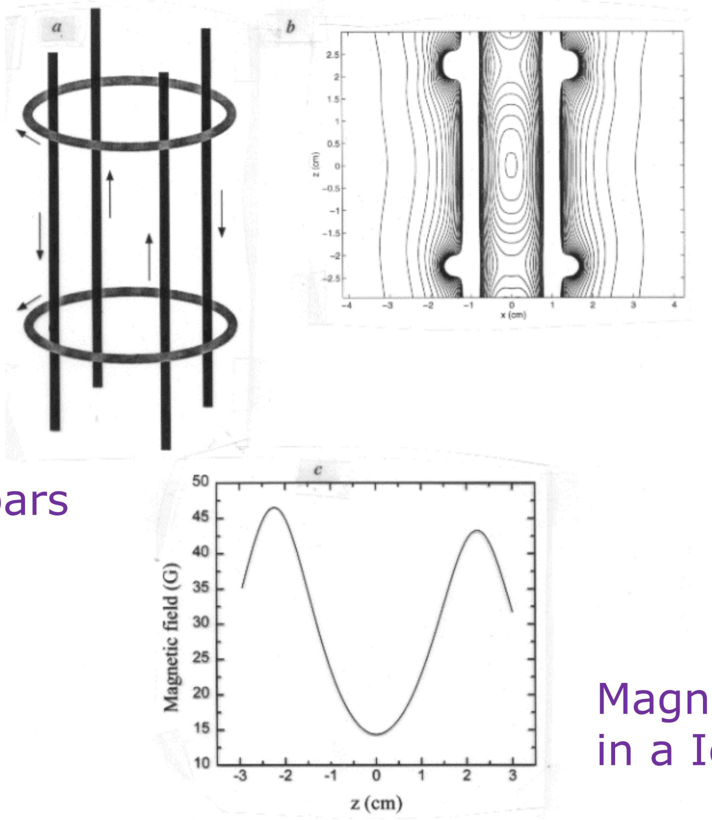
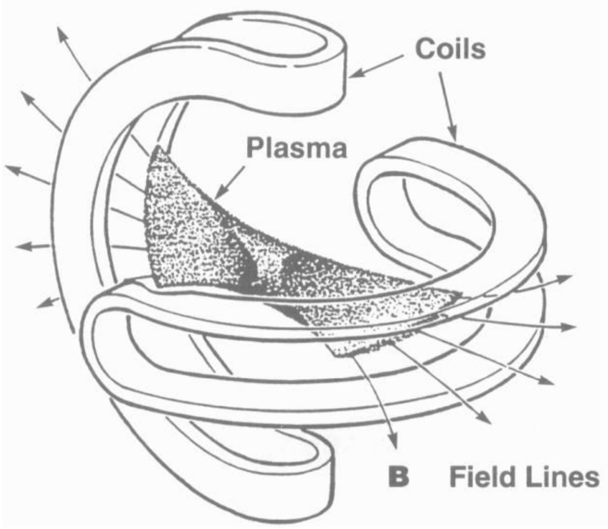


Ioffe bars

Magnetic field profiles
in a Ioffe-Pritchard trap



Yin-Yang coil

A. A. Harms et al, "Principles of Fusion Energy", World Scientific (2000)

Magnetic Mirror

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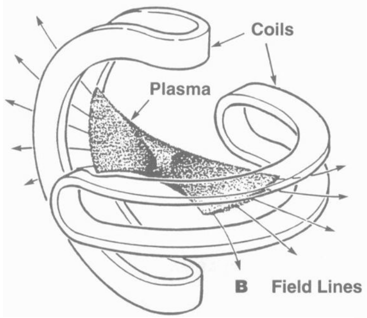
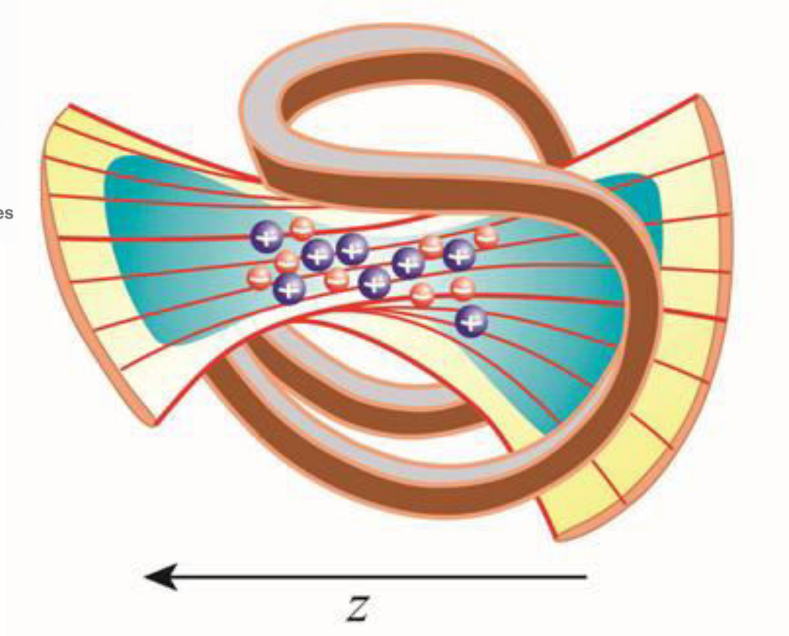




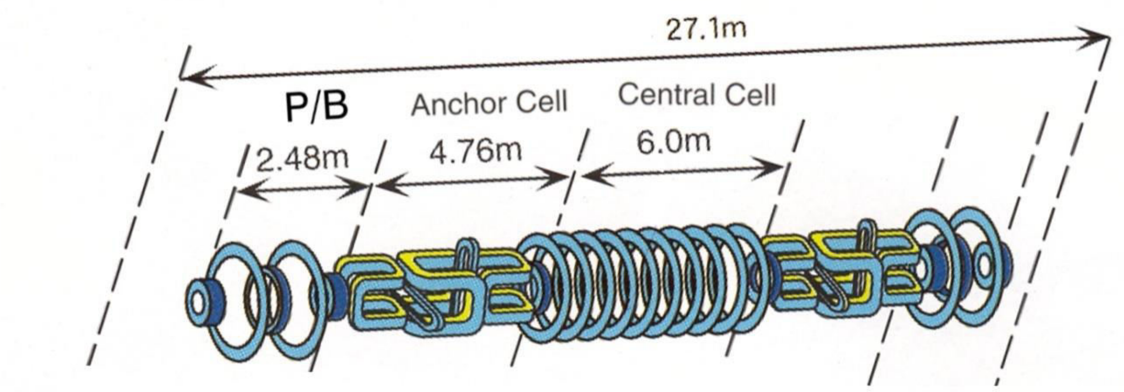
Baseball coil

http://www.wangnmr.com/Copper_magnet_catolog.htm

Magnetic Mirror

- **Instabilities**

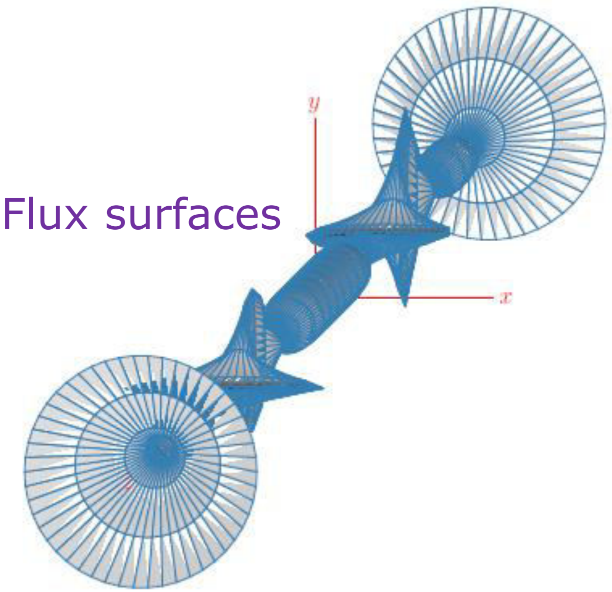
- Flute instability: convex curvature of the magnetic field



Plug/Barrier
(potential
plugging)
ECRH

Baseball coils
(MHD stabilising)
ICRF

Flux surfaces

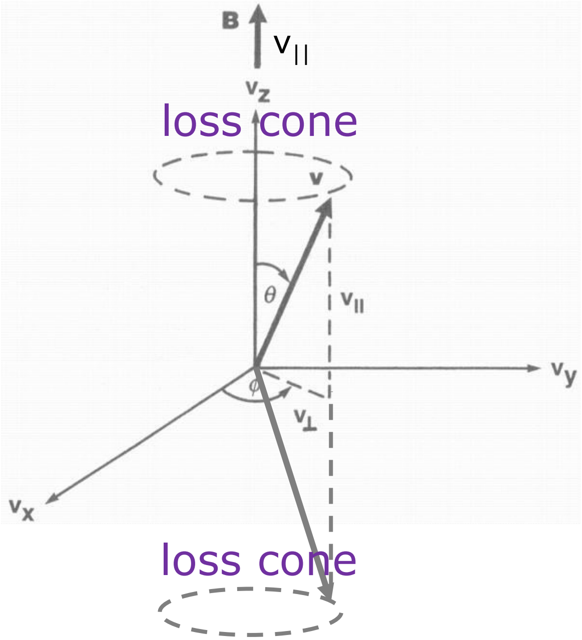


Gamma 10 Tandem mirror
(Univ. of Tsukuba, Japan)

Magnetic Mirror

- **Instabilities**

- Velocity-space instability: driven by the non-Maxwellian velocity distribution due to the preferred loss of particles with large $v_{||}/v_{\perp}$.
- Enhancing the velocity-space diffusion into the loss cone
- Observed that such it is less harmful to plasma confinement when the mirror device is short in dimension



Magnetic Mirror

- **Classical Mirror Confinement**

- If collisions are neglected, particles are trapped in a mirror when they do not appear in the loss cone.
- Collisions can bring them randomly from the confinement region into the loss cone.
- Due to their relatively small mass, electrons diffuse more rapidly.

→ a positive electrostatic potential built up in the confined plasma tending to retain the remaining electrons in the magnetic bottle

- Overall plasma confinement time is governed by the ion escape time.

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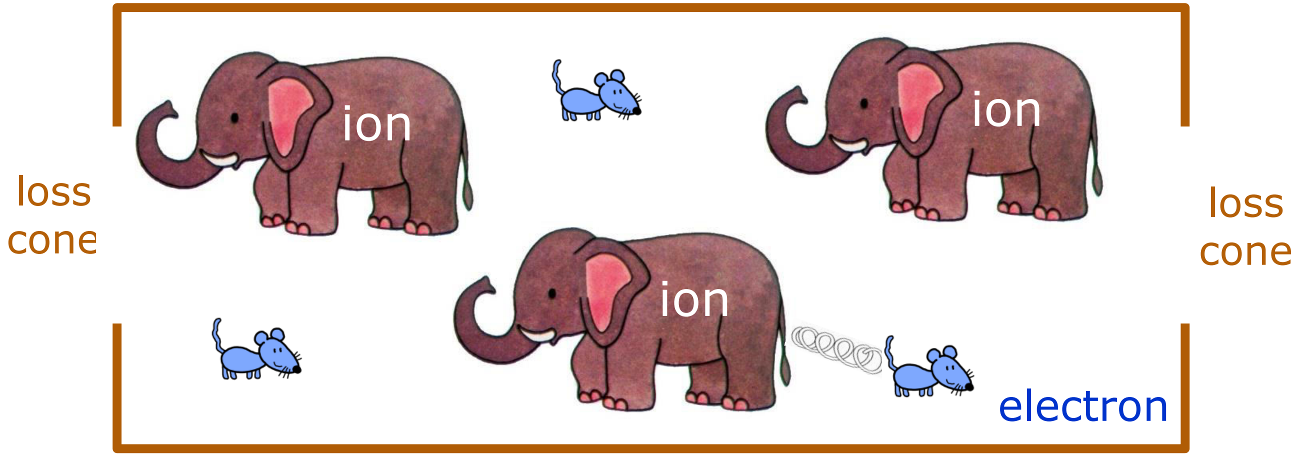

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$$\tau_M \approx \tau_{ii} \ln\left(\frac{B_{\max}}{B_{\min}}\right) \approx C \frac{A_i^{1/2} T_i^{3/2} \ln\left(\frac{B_{\max}}{B_{\min}}\right)}{n_i Z_i^4}$$: mirror confinement time

$$\frac{B_{\max}}{B_{\min}} \equiv R_m$$ mirror ratio

$$\tau_{ii} = \frac{1}{n_i \langle \sigma_s v_r \rangle} \propto \frac{A_i^{1/2} (kT_i)^{3/2}}{n_i q_i^4 \langle \sigma_s v_r \rangle}$$: ion-ion collision time

- Mirror confinement time does not depend on the actual magnitude of **B** or the plasma size but on the size of the loss cone.
- Higher density enhances the scattering into the loss cone.