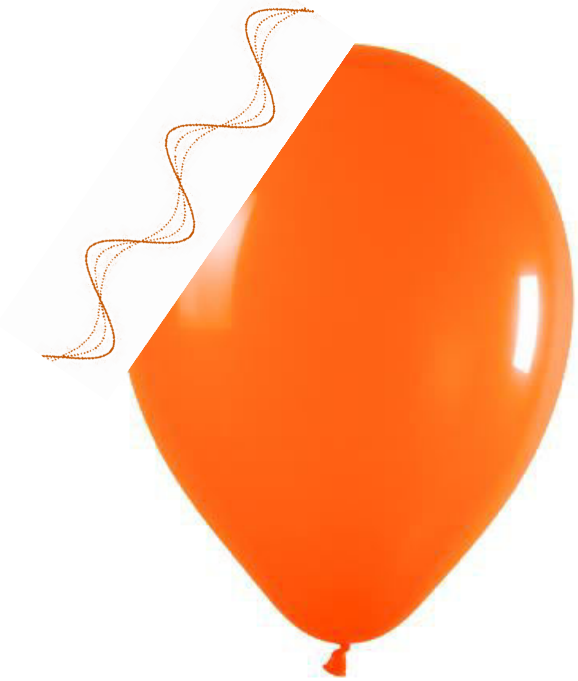
Introduction to Nuclear Fusion

Prof. Dr. Yong-Su Na

Tokamak stability



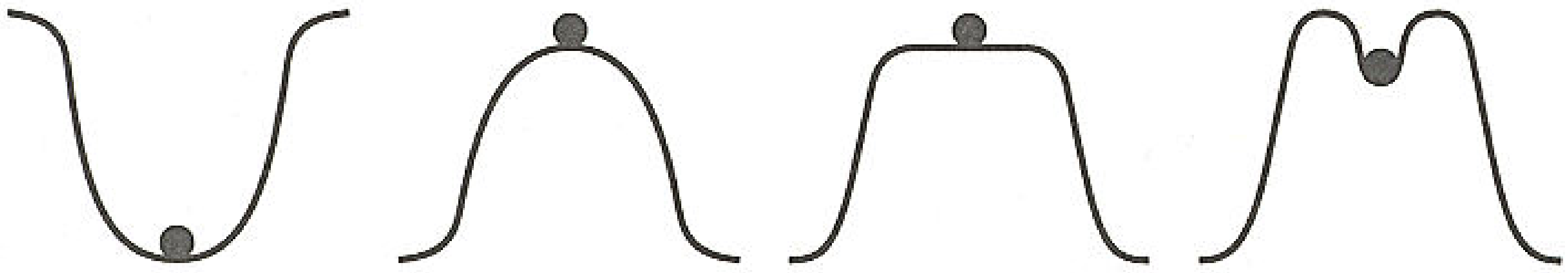
Tokamak Stability

- Considering plasma states which are not in perfect thermodynamic equilibrium (no exact Maxwellian distribution, e.g. non-uniform density), even though they represent equilibrium states in the sense that the force balance is equal to 0 and a stationary solution exists, means their entropy is not at the maximum possible and hence free energy appears available which can excite perturbations to grow: unstable equilibrium state

- The gradients of plasma current magnitude and pressure are the destabilising forces in connection with the bad magnetic field curvature: The ratio of these two free energies turns out to be β_p

Stability

- **Definition of Stability**



stable   linearly unstable   metastable   non-linearly unstable

- Assuming all quantities of interest linearised about their equilibrium values.

$$Q(\vec{r},t) = Q_0(\vec{r}) + \tilde{Q}_1(\vec{r},t) \quad \text{small 1st order perturbation} \quad \tilde{Q}_1 / |Q_0| << 1$$

$$\tilde{Q}_1(\vec{r},t) = Q_1(\vec{r}) e^{-i\omega t} = Q_1(\vec{r}) e^{-i(\omega_r + i\omega_i)t} = Q_1(\vec{r}) e^{-i\omega_r t} e^{\omega_i t} \qquad \omega = \omega_r + i\omega_i$$

Im $\omega > 0$ ($\omega_i > 0$): exponential instability

Im $\omega \leq 0$ ($\omega_i \leq 0$): exponential stability

Tokamak Stability

- **The Energy Principle**
  - Representing the most efficient and often the most intuitive method of determining plasma stability.

$$\omega^2 = \frac{\delta W}{K} \geq 0 \quad \text{stable}$$

$$\delta W \geq 0 \quad \text{stable}$$

- If the minimum value of potential energy is positive for all displacements, the system is stable.
- It it is negative for any displacement, the system is unstable.

$$\delta W = \delta W_F + \delta W_S + \delta W_V$$

$$\delta W_F = \frac{1}{2}\int_P d\vec{r}\left[\frac{\left|\vec{Q}\right|^2}{\mu_0} - \xi_\perp^* \cdot (\vec{J}\times\vec{Q}) + \gamma p\left|\nabla\cdot\xi\right|^2 + (\xi_\perp\cdot\nabla p)\nabla\cdot\xi_\perp^*\right]$$

$$\delta W_S = \frac{1}{2}\int_S d\vec{S}\left|\vec{n}\cdot\xi_\perp\right|^2 \vec{n}\cdot[[\nabla\left(p + B^2/2\mu_0\right)]]$$

$$\delta W_V = \frac{1}{2}\int_V d\vec{r}\,\frac{\left|\hat{B}_1\right|^2}{\mu_0}$$

Tokamak Stability

- **The Intuitive Form of δW_F**

$$\delta W_F = \frac{1}{2}\int_P d\vec{r}\left[\frac{|\vec{Q}_\perp|^2}{\mu_0} + \frac{B^2}{\mu_0}|\nabla\cdot\xi_\perp + 2\xi_\perp\cdot\vec{\kappa}|^2 + \gamma p|\nabla\cdot\xi|^2 - 2(\xi_\perp\cdot\nabla p)(\vec{\kappa}\cdot\xi_\perp^*) - J_\parallel(\xi_\perp^*\times\vec{b})\cdot\vec{Q}_\perp\right]$$

destabilising: $-2(\xi_\perp\cdot\nabla p)(\vec{\kappa}\cdot\xi_\perp^*) - J_\parallel(\xi_\perp^*\times\vec{b})\cdot\vec{Q}_\perp$

stabilising: $\frac{|\vec{Q}_\perp|^2}{\mu_0} + \frac{B^2}{\mu_0}|\nabla\cdot\xi_\perp + 2\xi_\perp\cdot\vec{\kappa}|^2 + \gamma p|\nabla\cdot\xi|^2$

Pressure-driven modes (+ or -)

current-driven (kinks) modes (+ or -)

Energy required to bend magnetic field lines: dominant potential energy contribution to the shear Alfvén wave

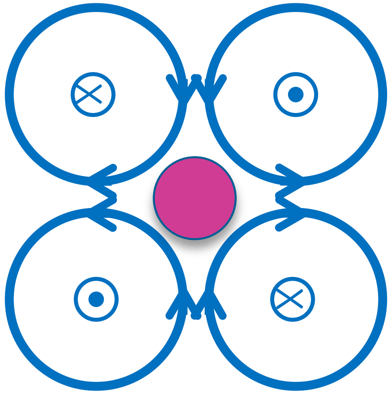
Energy necessary to compress the magnetic field: major potential energy contribution to the compressional Alfvén wave

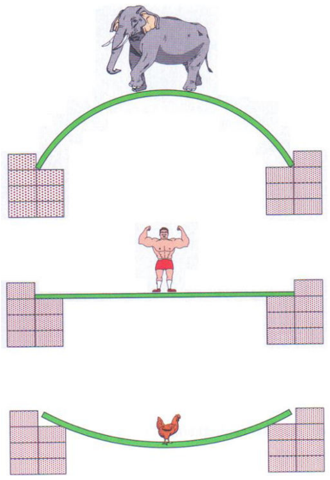
Energy required to compress the plasma: main source of potential energy for the sound wave

Tokamak Stability

- **Instabilities**

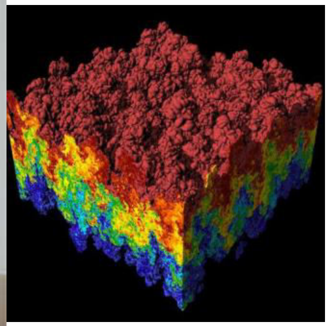
- Two sources of free energy available:
  plasma current
  pressure gradient of a plasma





current driven instabilities
(kink mode or sausage)

http://www.hosecouplingsuk.com/fabrication/fire-hose-assembly.htm



pressure driven instabilities
(Rayleigh-Taylor or
interchange instability)

Tokamak Stability

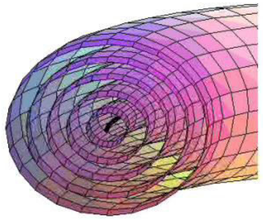
- **Ideal MHD instabilities**

- current driven (kink) instabilities
  - internal modes
  - external modes
- pressure driven instabilities
  - interchange modes
  - ballooning modes
- current+pressure driven: Edge Localised Modes (ELMs)
- vertical instability

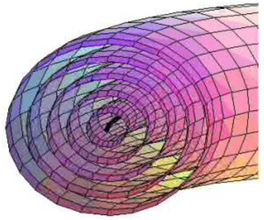
- **Resistive MHD instabilities**

- current driven instabilities
  - tearing modes
  - neoclassical tearing modes (NTMs)
- nonlinear modes
  - sawtooth
  - disruption

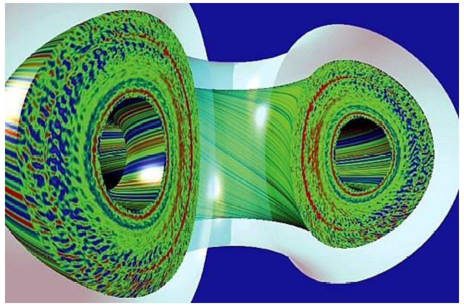
- **Microinstabilities - Turbulence**



Flux conservation
Topology unchanged



Reconnection of field lines
Topology changed



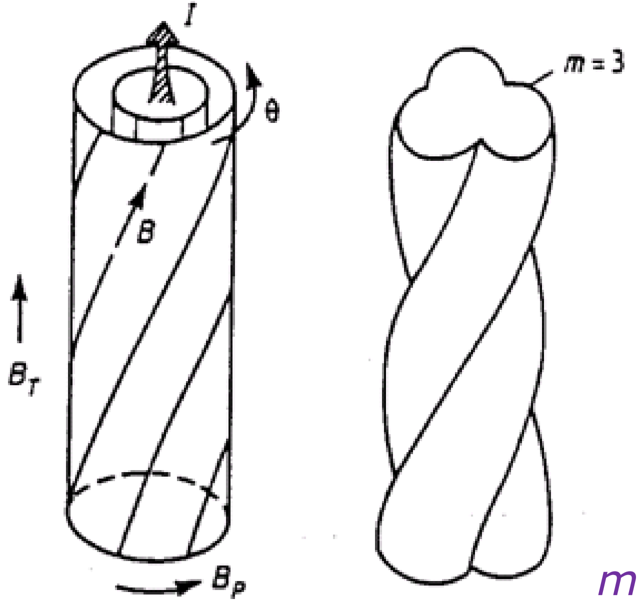
Ideal MHD instabilities in a Tokamak

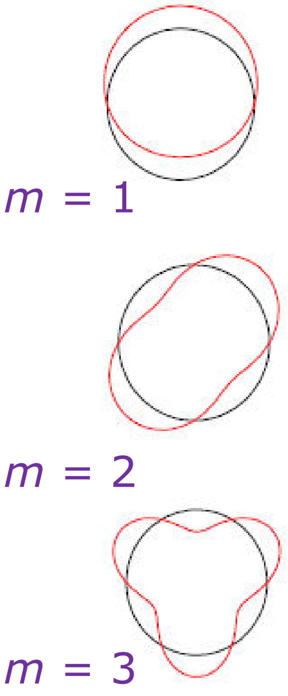
Ideal MHD Instabilities

- **The most Virulent Instabilities**
    - fast growth (microseconds)
    - the possible extension over the entire plasma

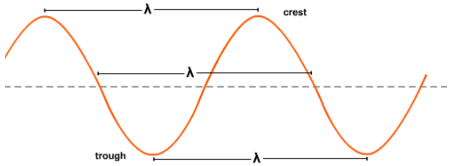
Ideal MHD Instabilities

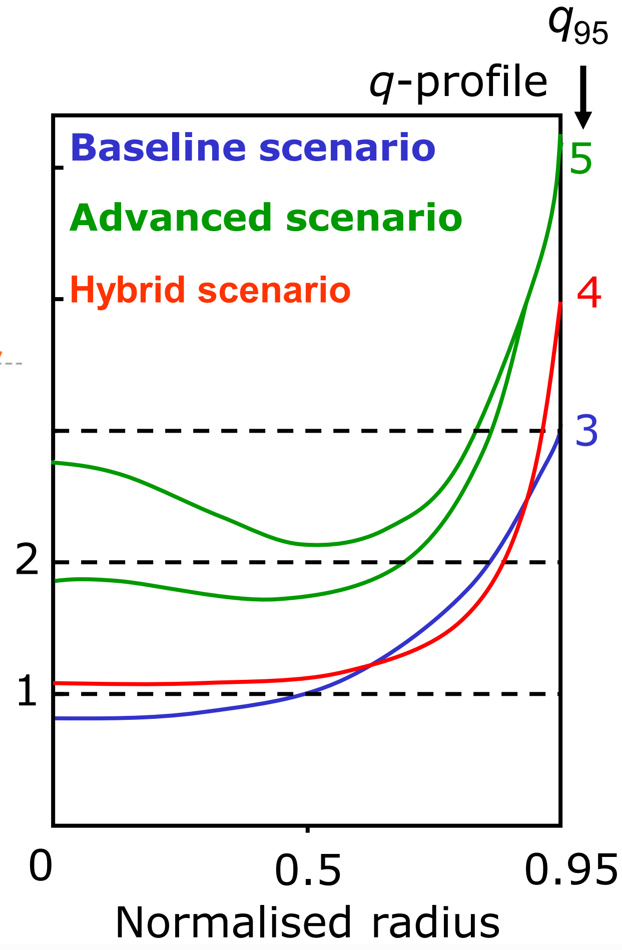
- **Kink modes**
  - Causing a contortion of the helical plasma column
  - Driven by the radial gradient of the toroidal current
  - External kind modes:
    Fastest and most dangerous
    Arising mainly when $q_a < 2$





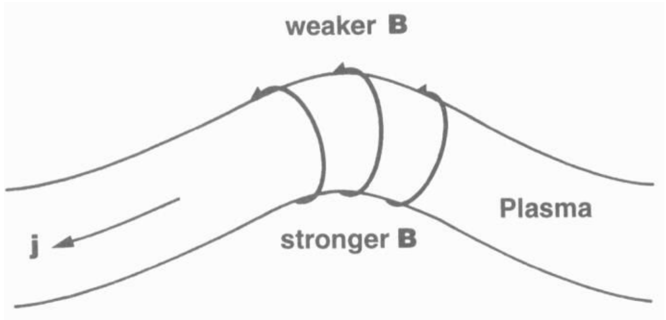
m: poloidal mode number





http://www.maysville-online.com/news/local/tollesboro-home-destroyed-in-fire/article_a5e0eb4e-235b-5c7d-afee-74bf98c4e738.html
http://www.bbc.co.uk/bitesize/higher/physics/radiation/waves/revision/1/

Ideal MHD Instabilities



- **Kink modes**
  - Stabilising effect by the conducting wall and strong toroidal magnetic field

$$q_a = \frac{aB_\phi}{R_0 B_\theta} = \frac{aB_\phi}{R_0 \mu_0 I_P / 2\pi a} \propto \frac{B_\phi}{I_p}$$

B_ϕ ↗ stabilising

I_p ↘ destabilising

Determining plasma current limit set by kink instabilities → **safety** factor

$$q_a > 1$$

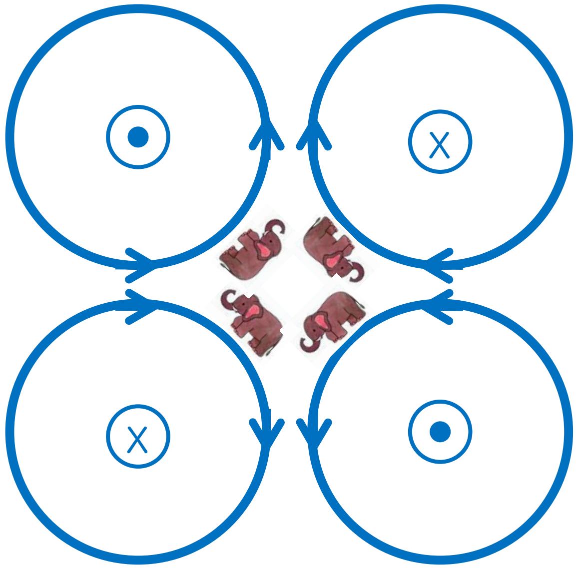
Kruskal-Shafranov criterion:
stability condition for external kink mode for the worst case

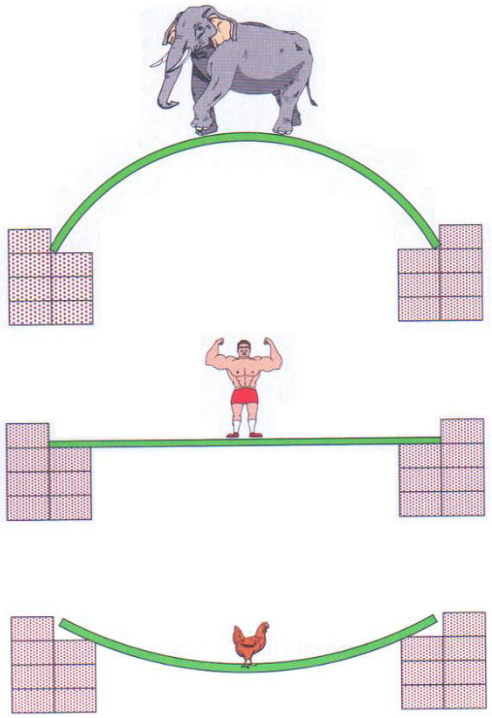
Imposing an important constraint on tokamak operation:
toroidal current upper limit: Kruskal-Shafranov current ($I < I_{KS}$)

$$q_a = \frac{aB_\phi}{R_0 B_p} = \frac{aB_\phi}{R_0 \mu_0 I_{KS} / 2\pi a} = 1 \qquad I_{KS} \equiv 2\pi a^2 B_\phi / \mu_0 R_0 = 5a^2 B_\phi / R_0 \; [\mathrm{MA}]$$

Ideal MHD Instabilities

- **Interchange modes**
  - A toroidally confined plasma sees 'bad' convex curvature of the helical magnetic field lines on the outboard side of the torus.



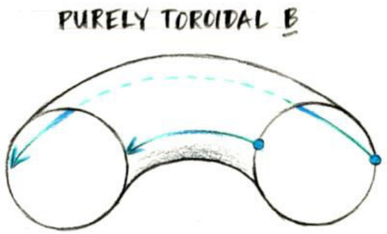


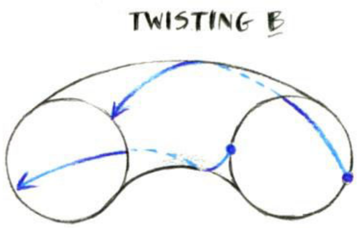
F. F. Chen, "An Indispensable Truth", Springer (2011)

http://blog.naver.com/PostView.nhn?blogId=ray0620&logNo=150112423635&parentCategoryNo=1&viewDate=&currentPage=1&listtype=0
http://en.wikipedia.org/wiki/File:St_Louis_Gateway_Arch.jpg

Ideal MHD Instabilities

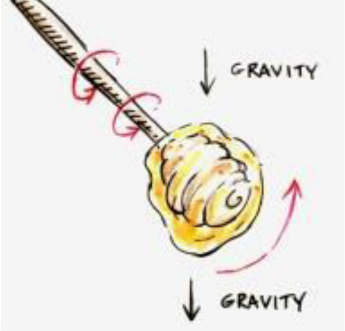
- **Interchange modes**
  - A toroidally confined plasma sees 'bad' convex curvature of the helical magnetic field lines on the outboard side of the torus.
  - The average curvature of **B**-field lines over a full poloidal rotation is 'good' for windings with a rotational transform $\iota \leq 2\pi$, i.e., $q \geq 1$.
  - Interchange perturbations do not grow in normal tokamaks if $q \geq 1$.





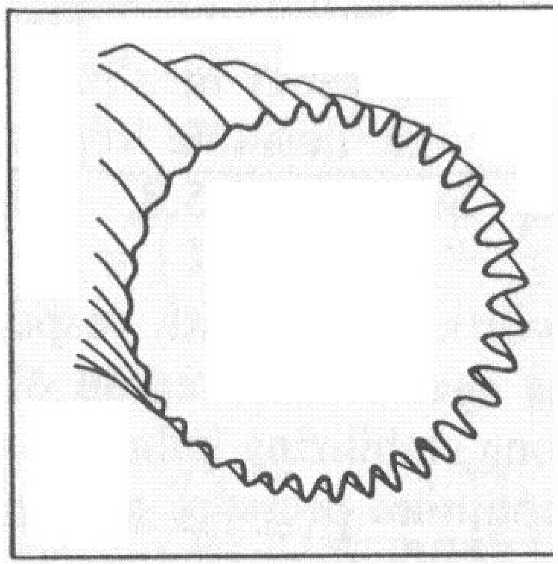
Unstable

Stable



Ideal MHD Instabilities

- **Ballooning modes**
  - locally grow in the outboard bad curvature region: ballooning modes
  - A high local pressure gradient is responsible for driving the ballooning instability.
  - Can be suppressed almost everywhere in the plasma by establishing appropriate pressure profiles and appropriate magnetic field line windings.

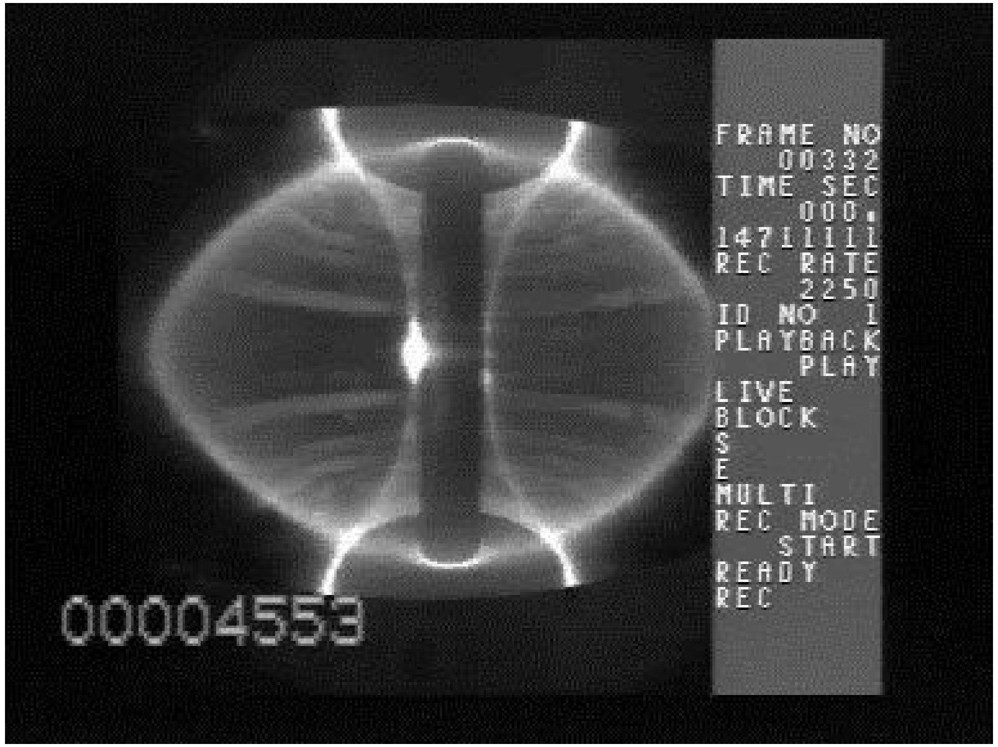


J.P. Freidberg, "Ideal Magneto-Hydro-Dynamics", lecture note
https://kr.fotolia.com/tag/%ED%92%8D%EC%84%A0%EB%B6%88%EA%B8%B0

Ideal MHD Instabilities

- **Edge Localised Modes (ELMs)**
  - current driven (peeling mode) and pressure driven (ballooning mode) combined instability

MAST
MEGA-AMPERE SPHERICAL TOKAMAK



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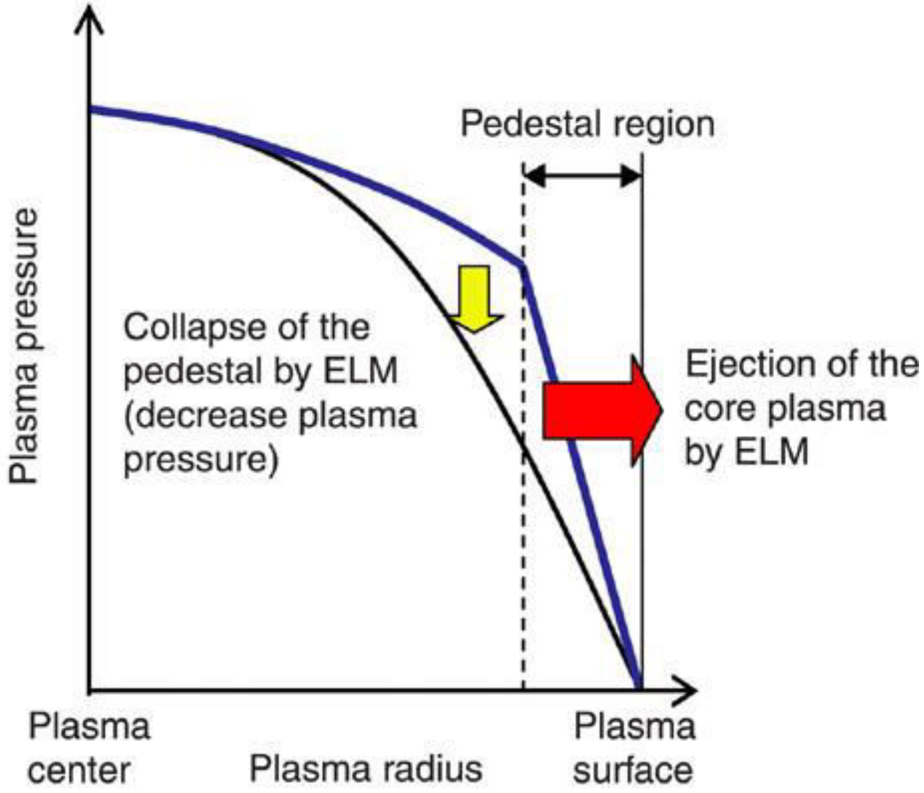


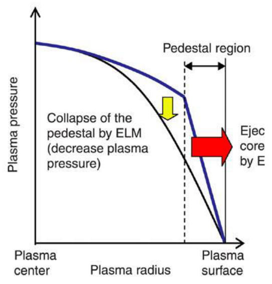
Ideal MHD Instabilities

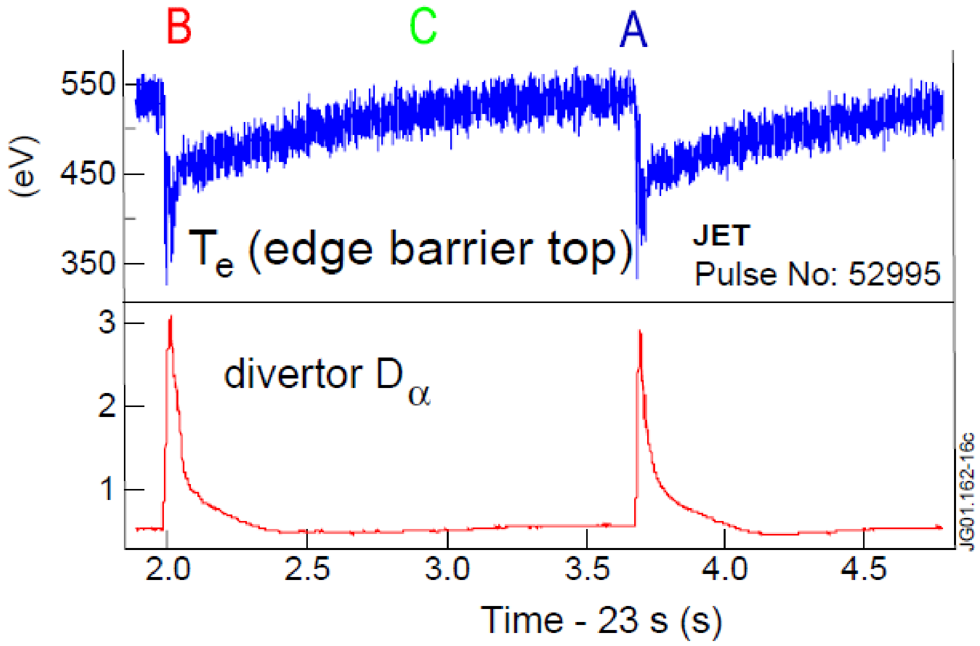
- **Edge Localised Modes (ELMs)**

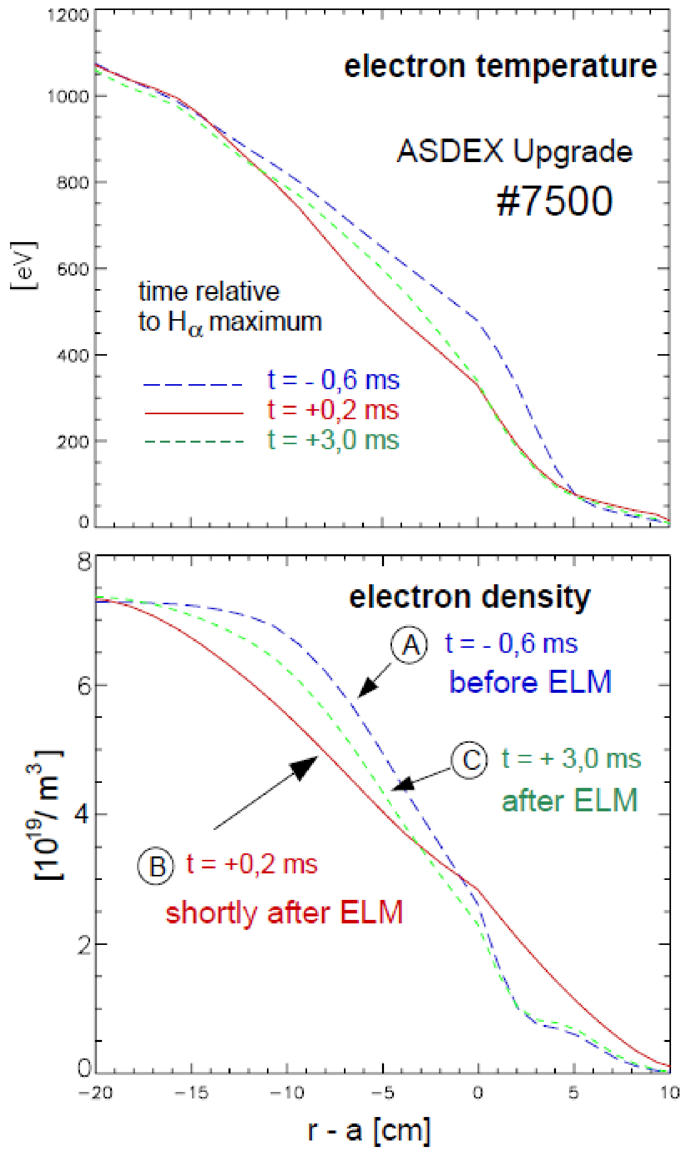
A. Critical ∇p in H-mode barrier region reached
→ short unstable phase (ELM event)

B. Energy and particle loss reduces gradients.

C. Gradients build up during reheat/refuelling phase.







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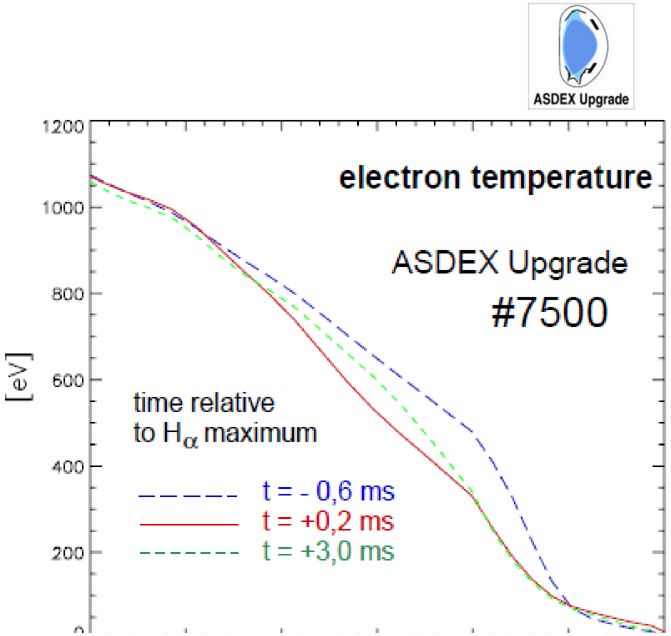
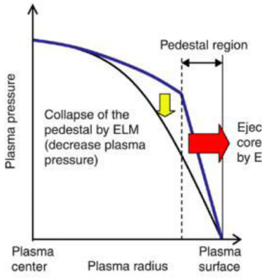
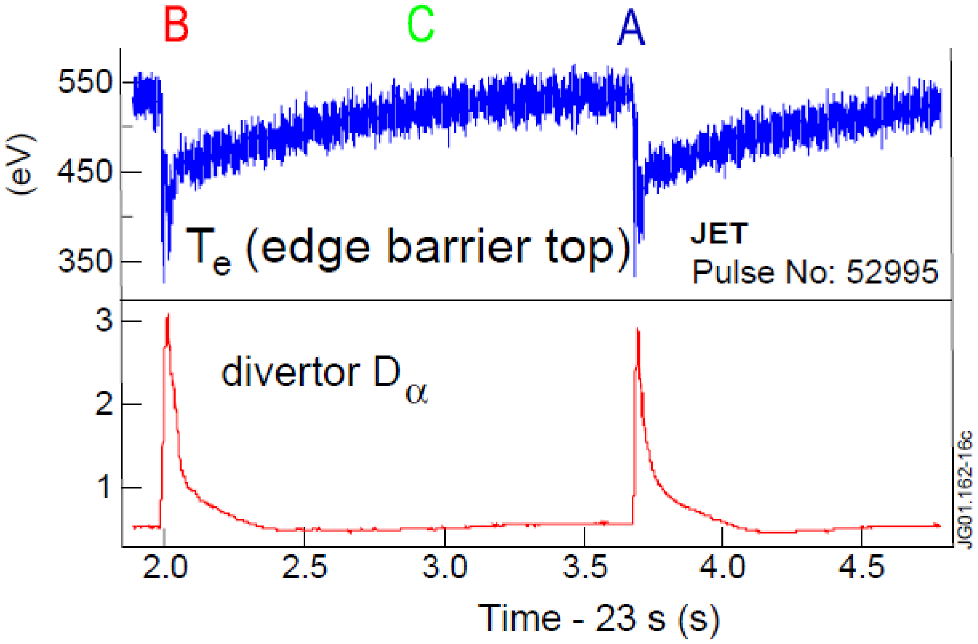
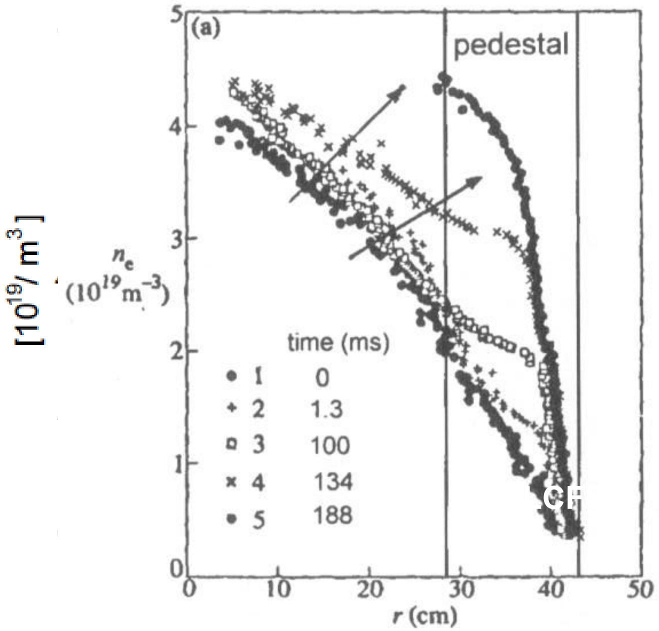


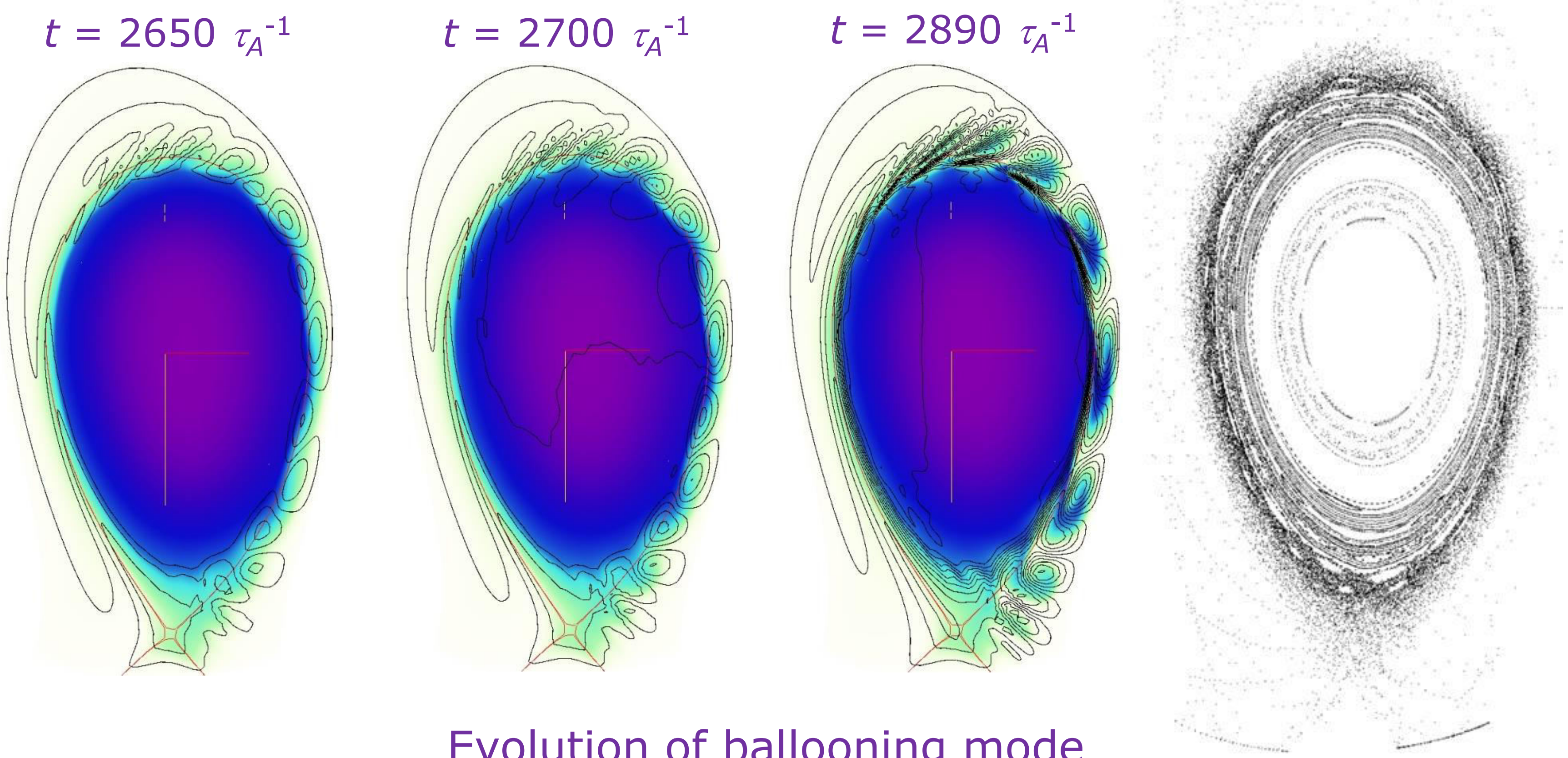






Ideal MHD Instabilities

- **Edge Localised Modes (ELMs)**
  - Non-linear MHD simulations with JOREK

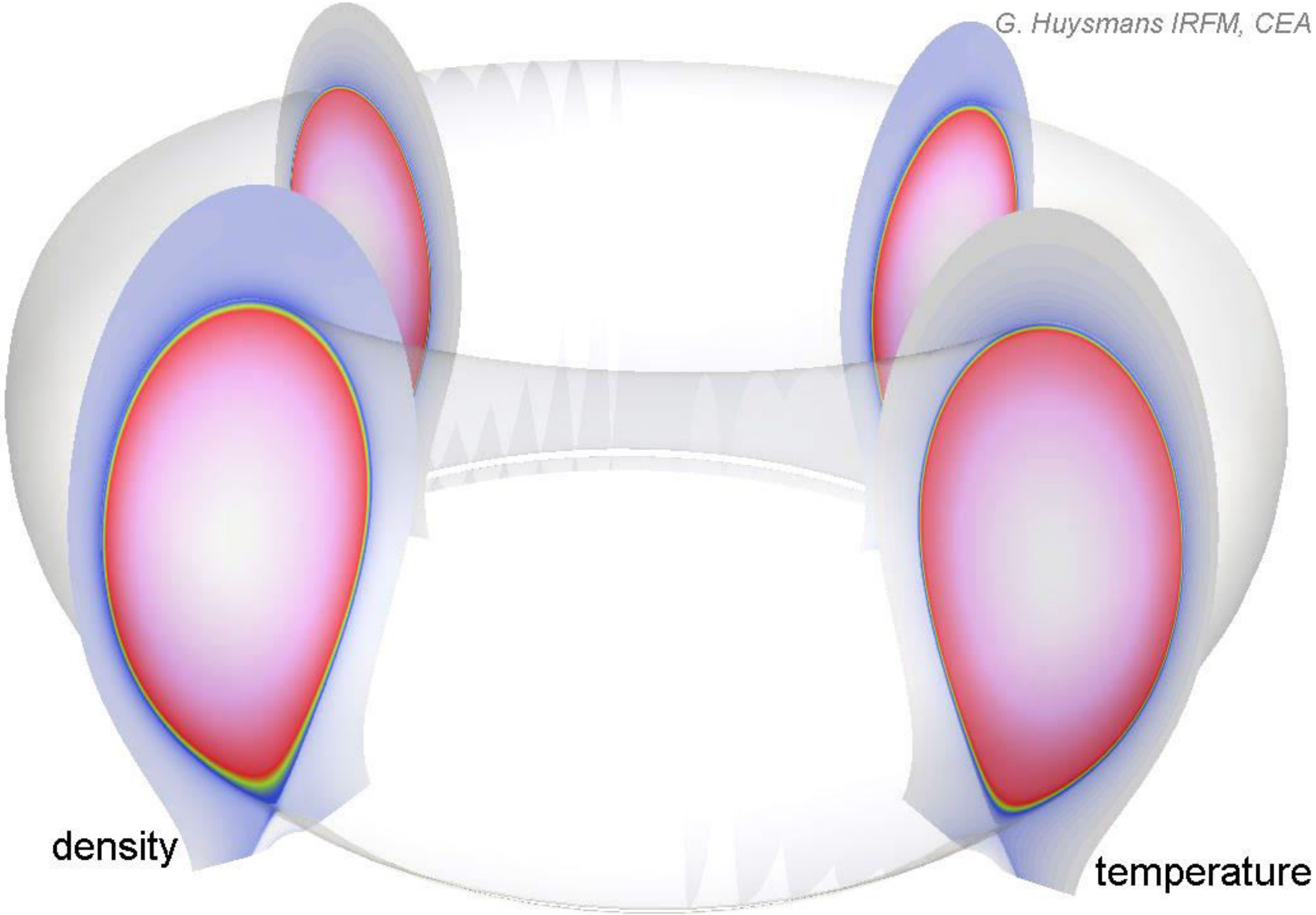


Evolution of ballooning mode

Huysmans, Czarny, NF ***47*** *659 (2007)*

Ideal MHD Instabilities

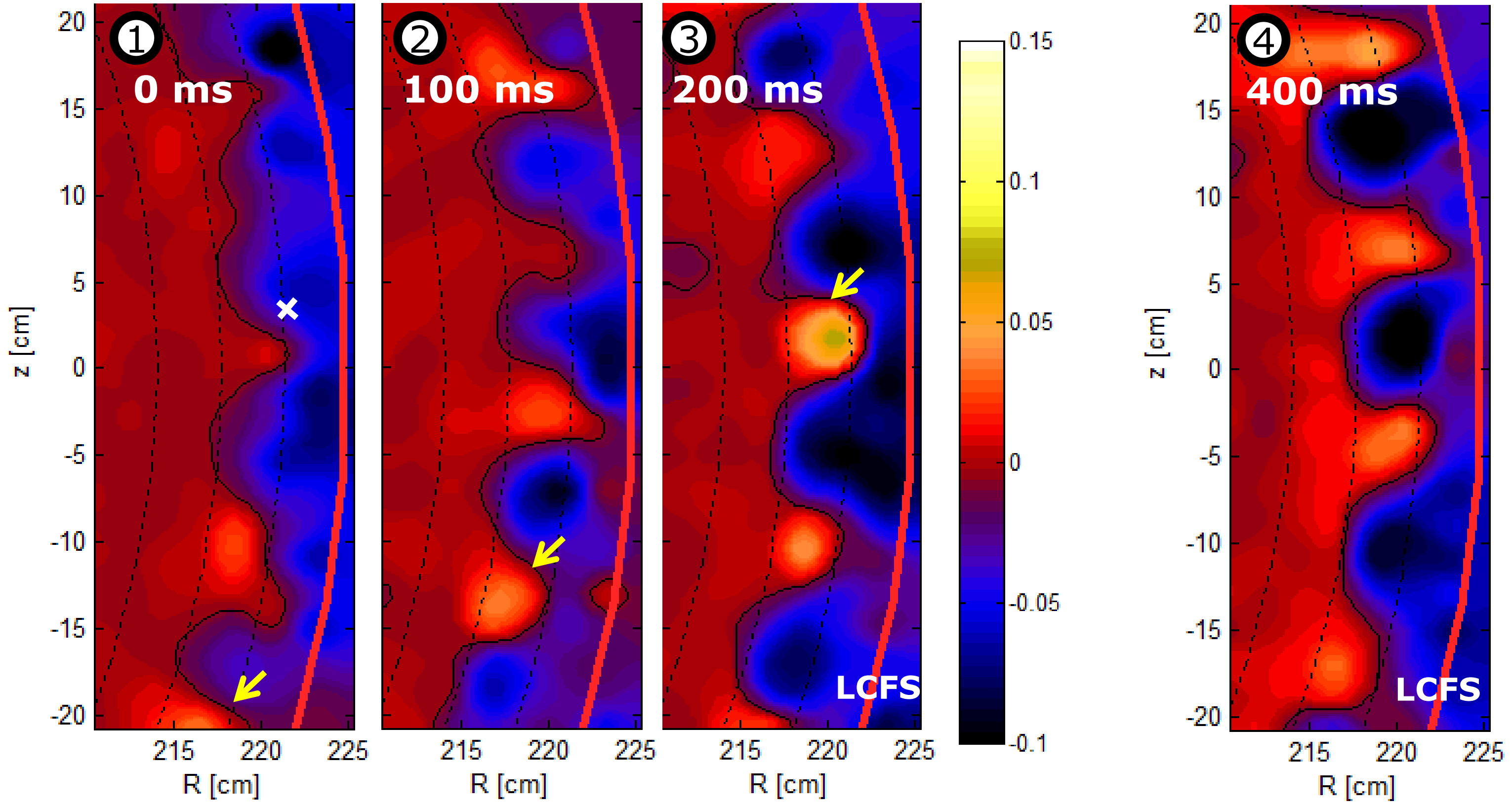
- **Edge Localised Modes (ELMs)**
  - Non-linear MHD simulations with JOREK



Ideal MHD Instabilities

- **Edge Localised Modes (ELMs)**
  - Standard ELM dynamics in the KSTAR visualized by ECEI

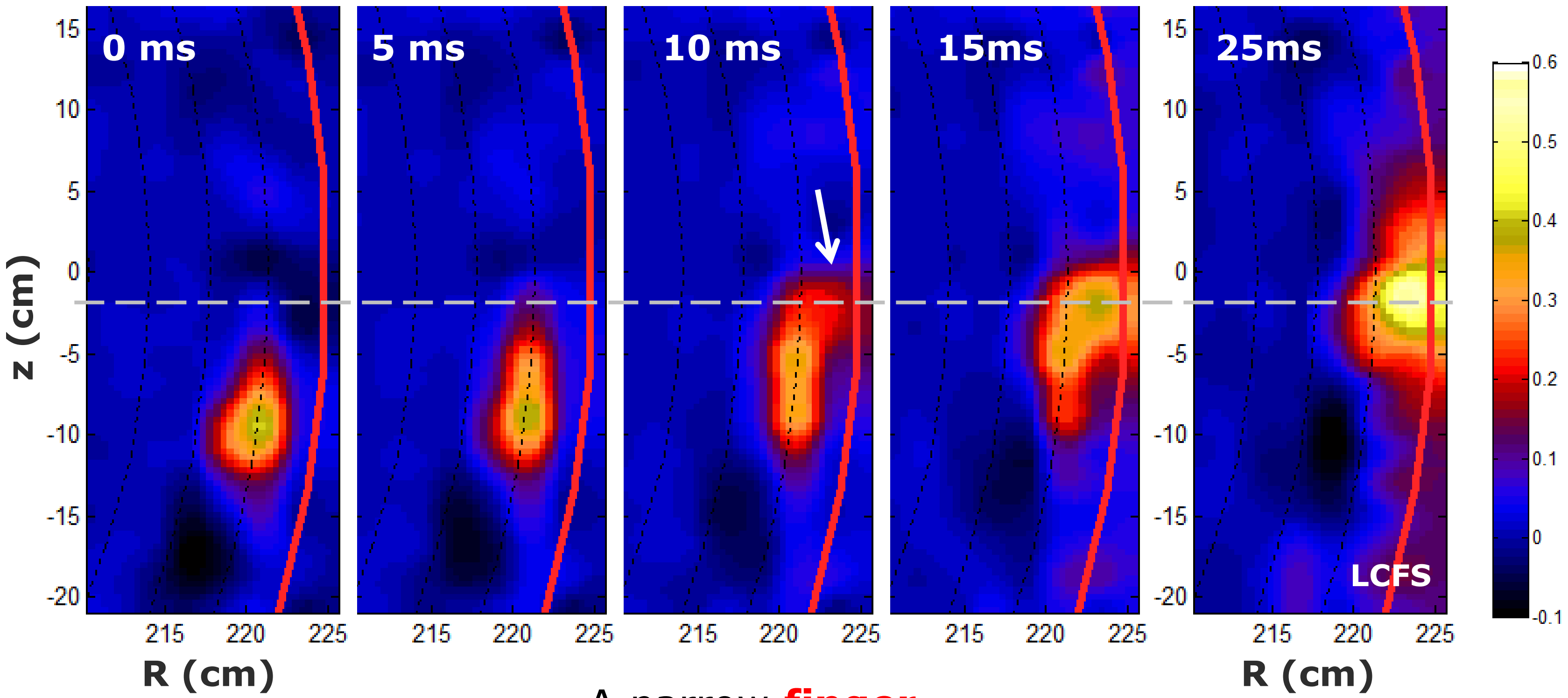
(1) Initial Growth (2) Saturation



G.S. Yun et al., PRL (2011)

Ideal MHD Instabilities

- **Edge Localised Modes (ELMs)**
  - Standard ELM dynamics in the KSTAR visualized by ECEI
    - (3) ELM crash



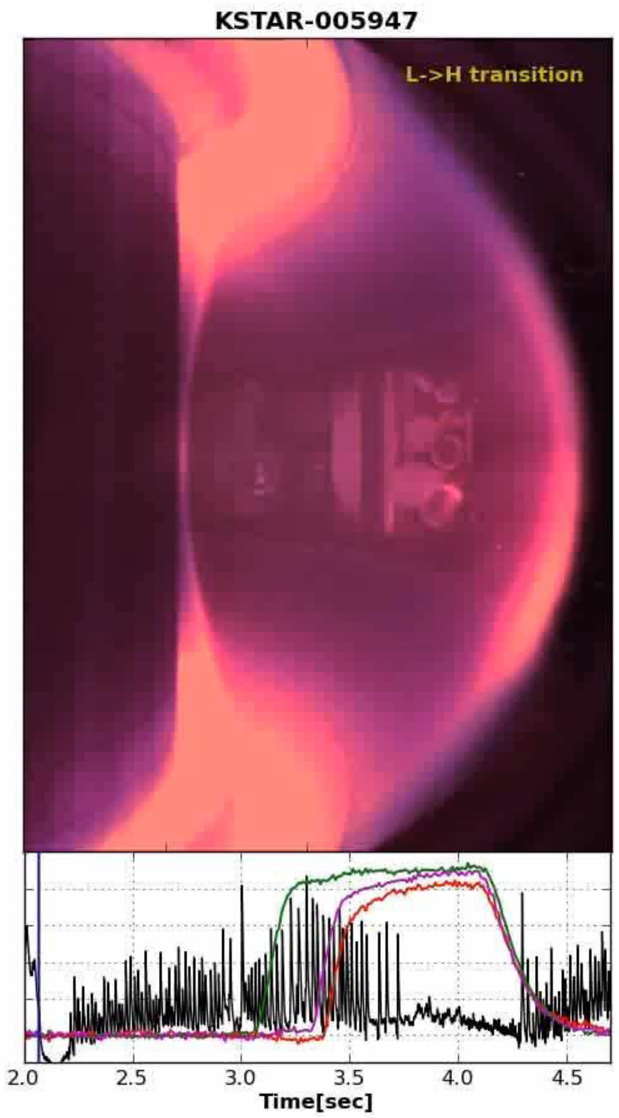
Filaments elongate poloidally ⇨ A narrow **finger**-like structure develops ⇨ Particles/heat transport through the finger

G.S. Yun et al., PRL (2011)

Ideal MHD Instabilities

- **Edge Localised Modes (ELMs)**

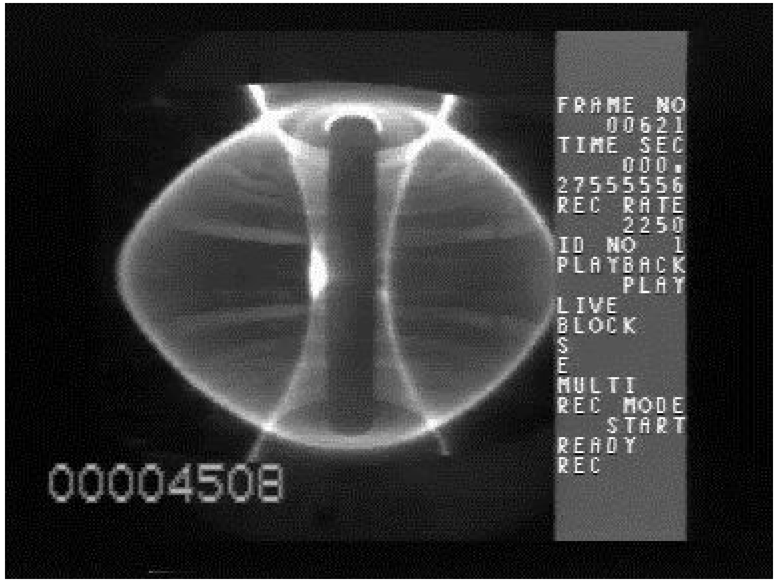
  - Full suppression by 3D magnetic perturbation

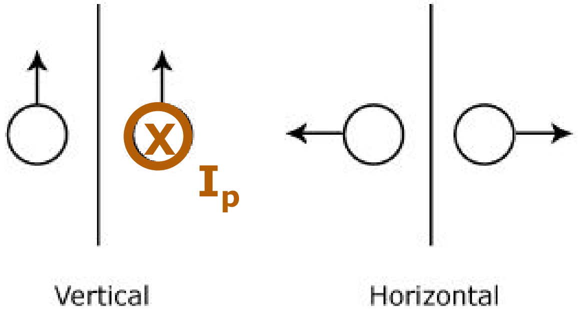


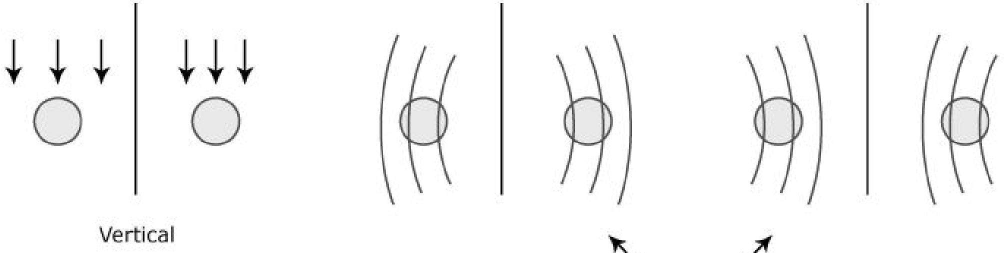
Courtesy of Y. M. Jeon (NFRI)

Ideal MHD Instabilities

- **Vertical Instability**
  - Macroscopic vertical motion of the plasma towards the wall



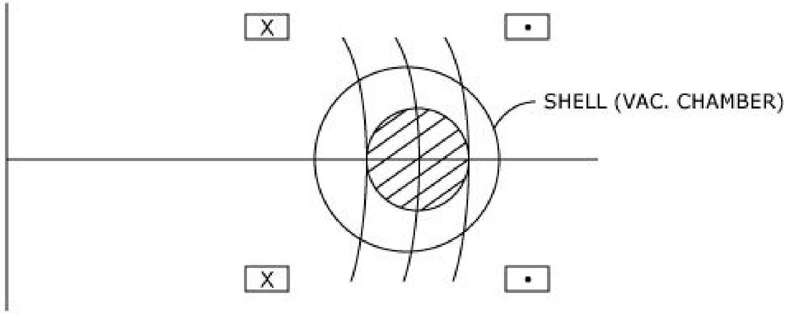


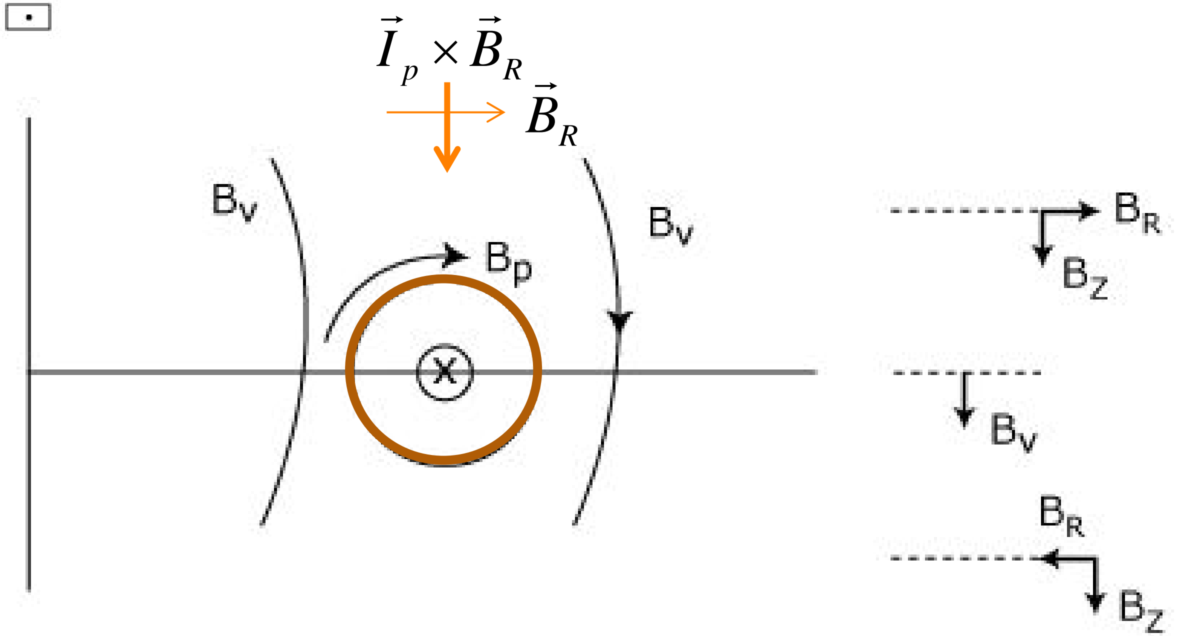


J.P. Freidberg, "Ideal Magneto-Hydro-Dynamics", lecture note

Ideal MHD Instabilities

- **Vertical Instability**





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Ideal MHD Instabilities

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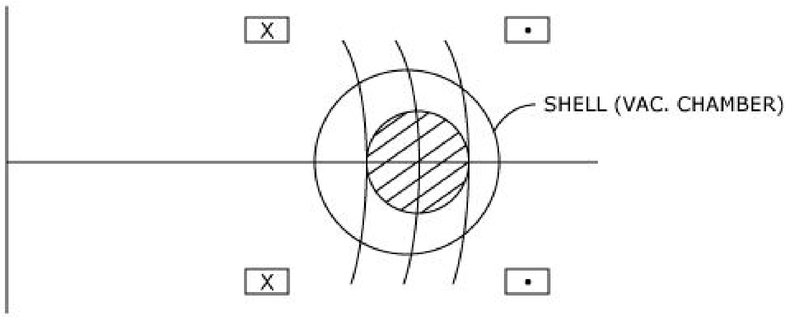

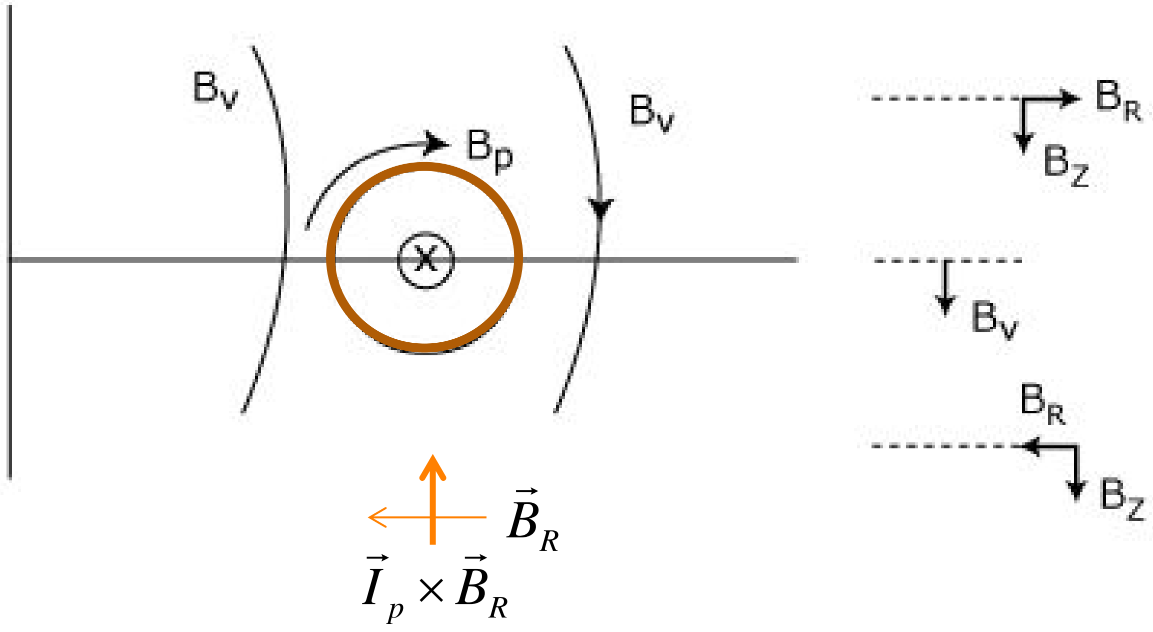

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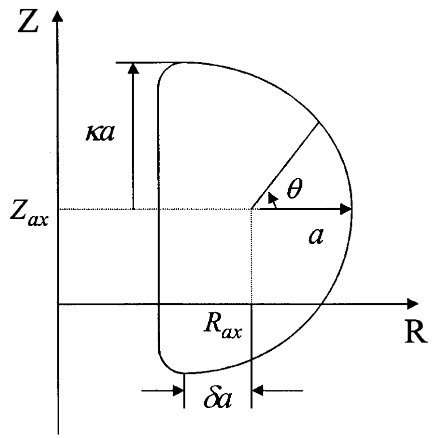
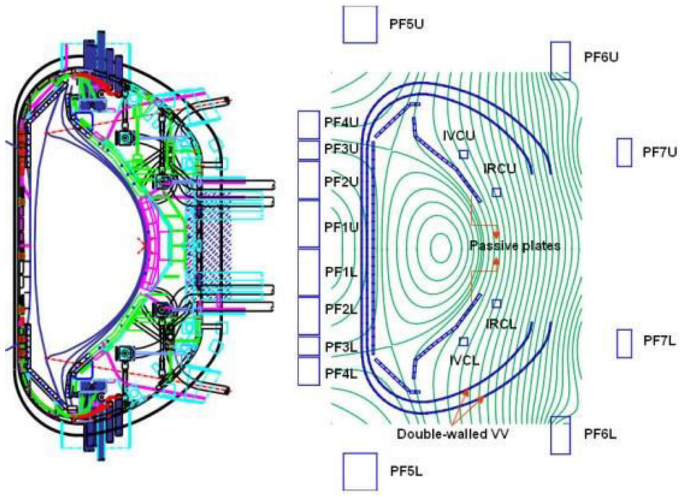
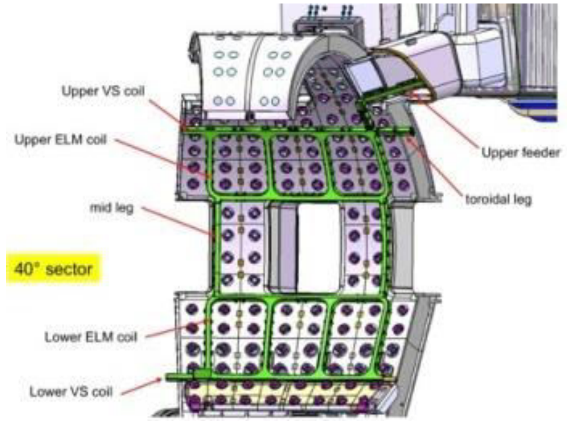
Ideal MHD Instabilities

- **Vertical Instability**
  - For a circular cross sections a moderate shaping of the vertical field should provide stability.
  - For noncircular tokamaks, vertical instabilities produce important limitations on the maximum achievable elongations.
  - Even moderate elongations require a conducting wall or a feedback system for vertical stability.







KSTAR & ITER

J.P. Freidberg, "Ideal Magneto-Hydro-Dynamics", lecture note