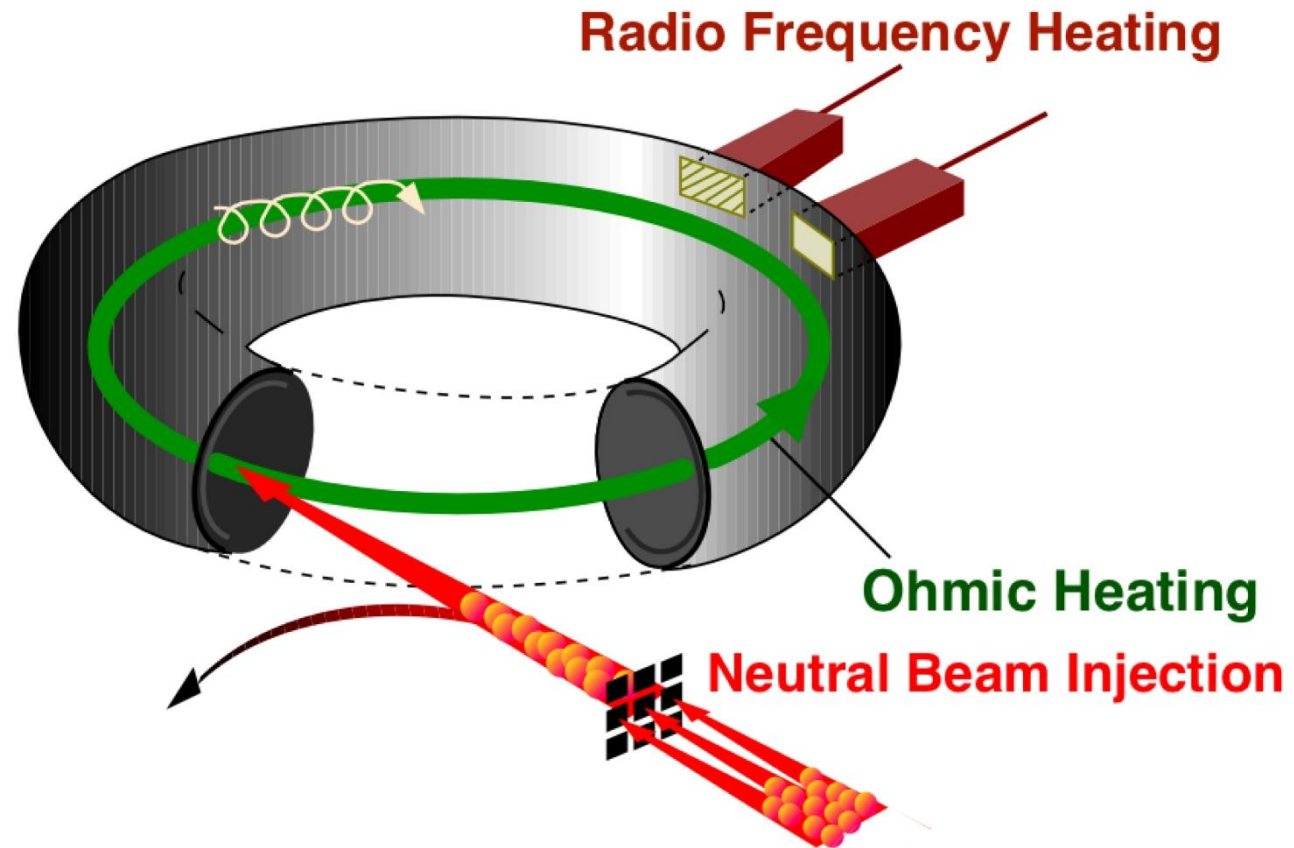


# Introduction to Nuclear Fusion

Prof. Dr. Yong-Su Na

**How to heat up a plasma?**

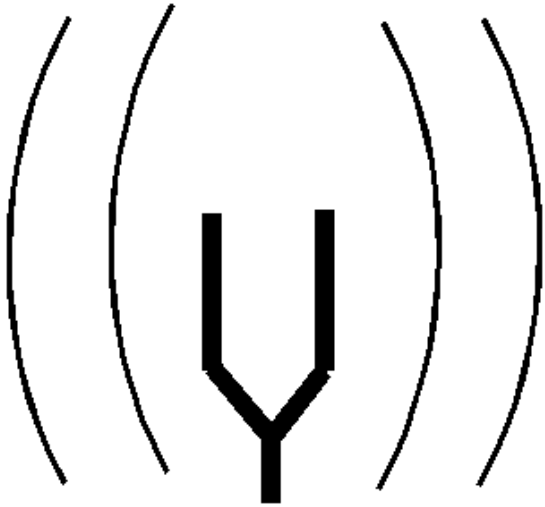
# Plasma Heating



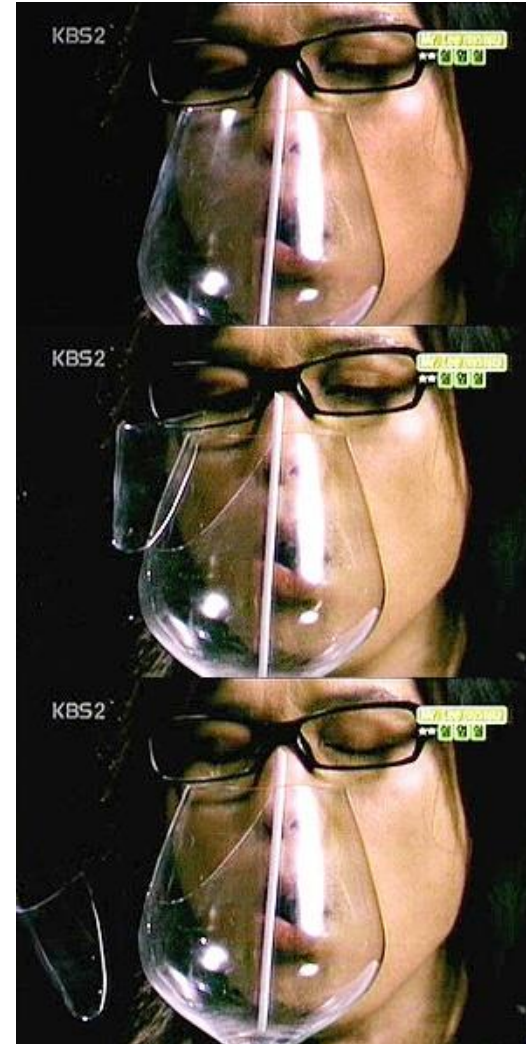
# Wave heating by resonance

# Electromagnetic Waves

Tuning fork



Resonance



*KBS. 스펀지: 목소리로 와인 잔 깨기.  
2006.3.11  
[http://www.kbs.co.kr/end\\_program/2tv/enter/sponge/view/vod/1386311\\_1027.html](http://www.kbs.co.kr/end_program/2tv/enter/sponge/view/vod/1386311_1027.html)*

# Electromagnetic Waves



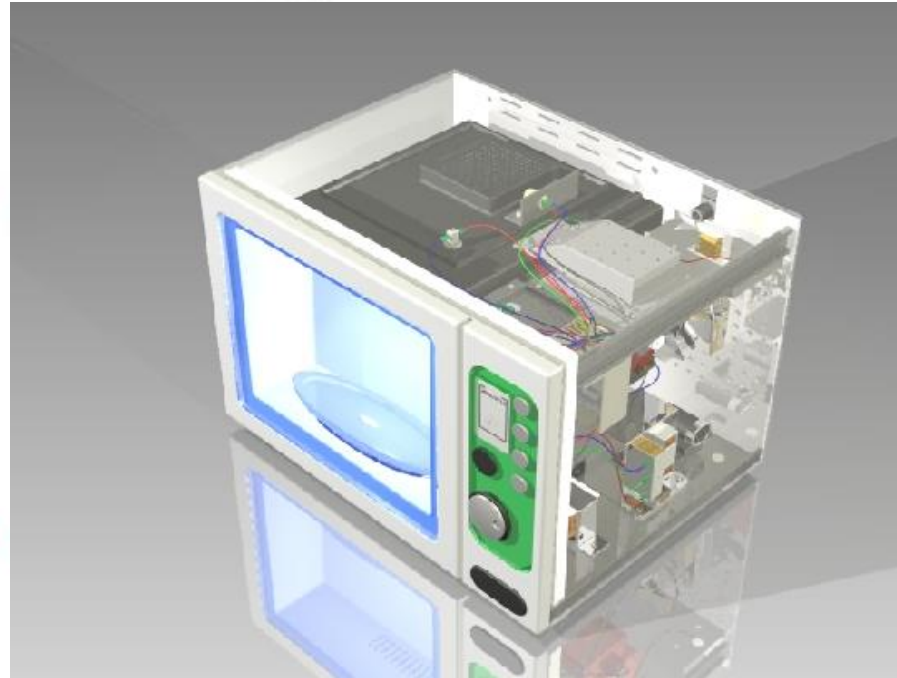
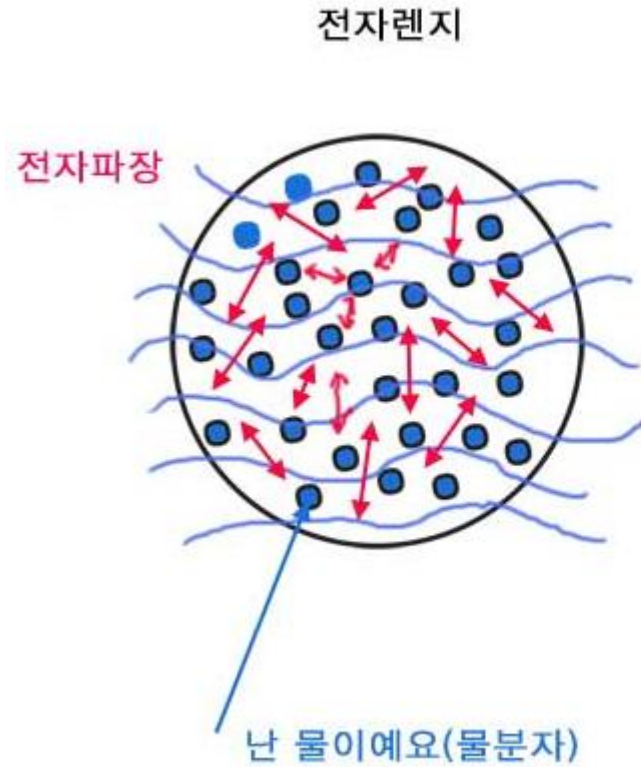
Tacoma Narrows Bridge  
(1940. 11. 4)

# Electromagnetic Waves



Broughton Suspension Bridge

# Electromagnetic Waves

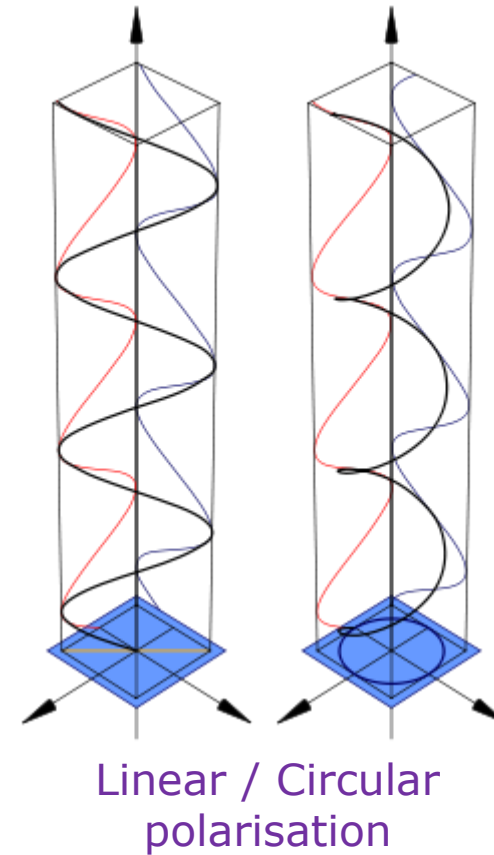
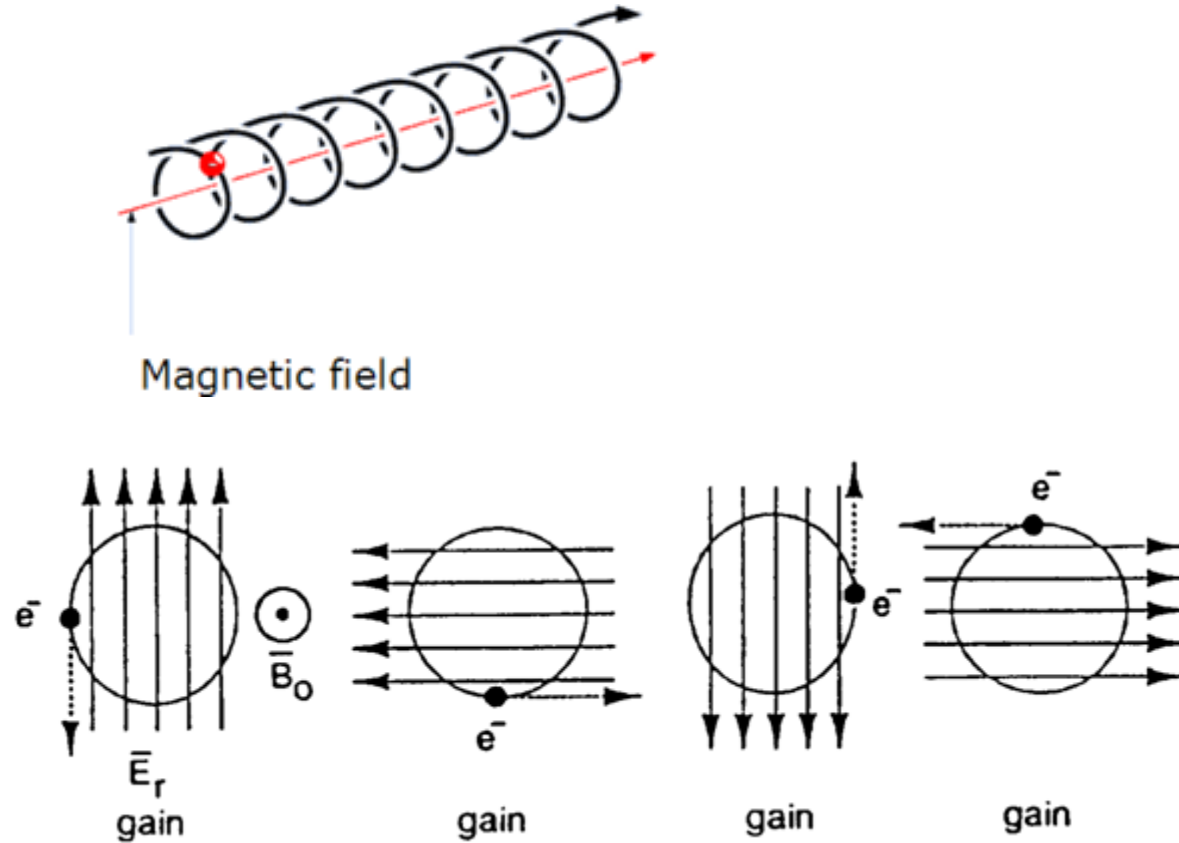


Microwave oven (2.45 GHz)



# Electromagnetic Waves

- Electron Cyclotron Resonance Heating (ECRH)



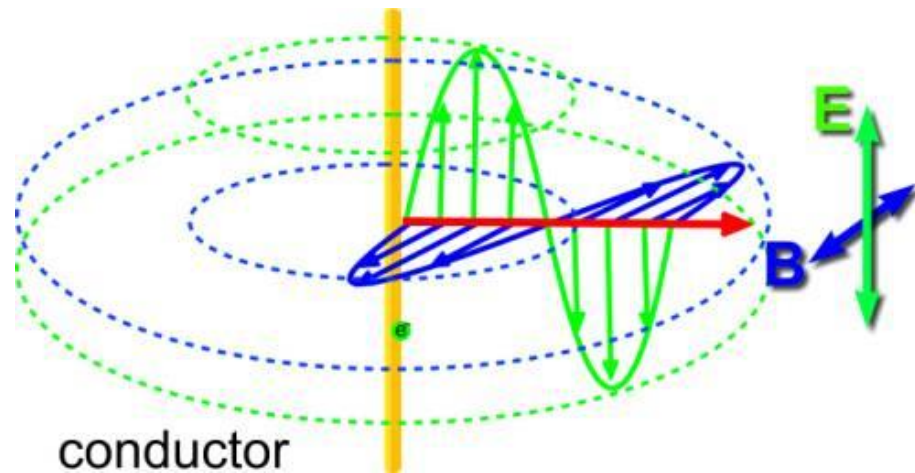
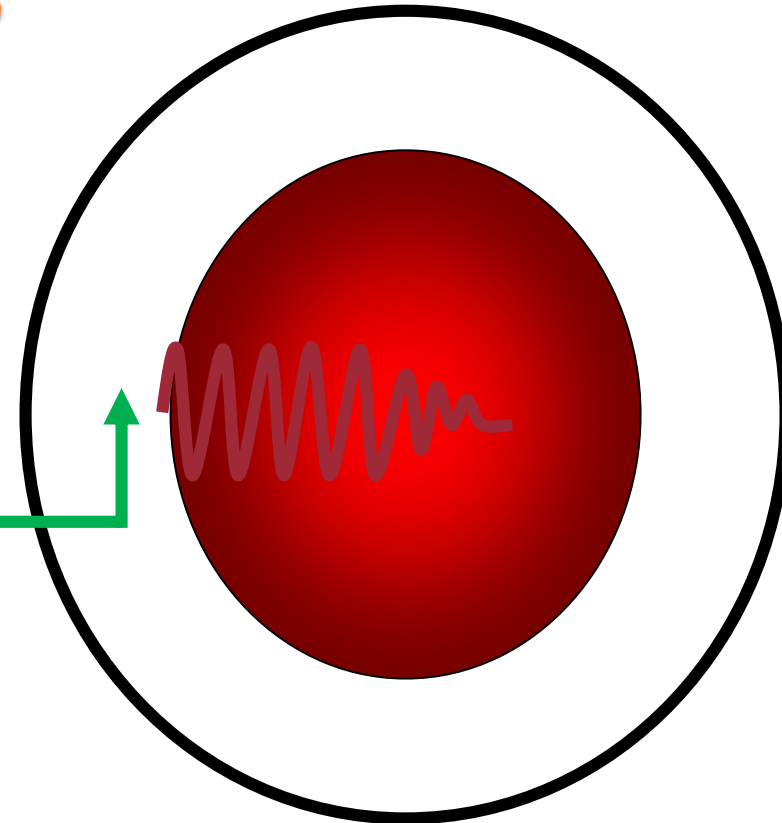
# Electromagnetic Waves

Excitation of plasma wave  
(frequency  $\omega$ ) near plasma edge



wave transports power  
into the plasma center

Antenna  
 $\omega$



# Electromagnetic Waves

Excitation of plasma wave  
(frequency  $\omega$ ) near plasma edge



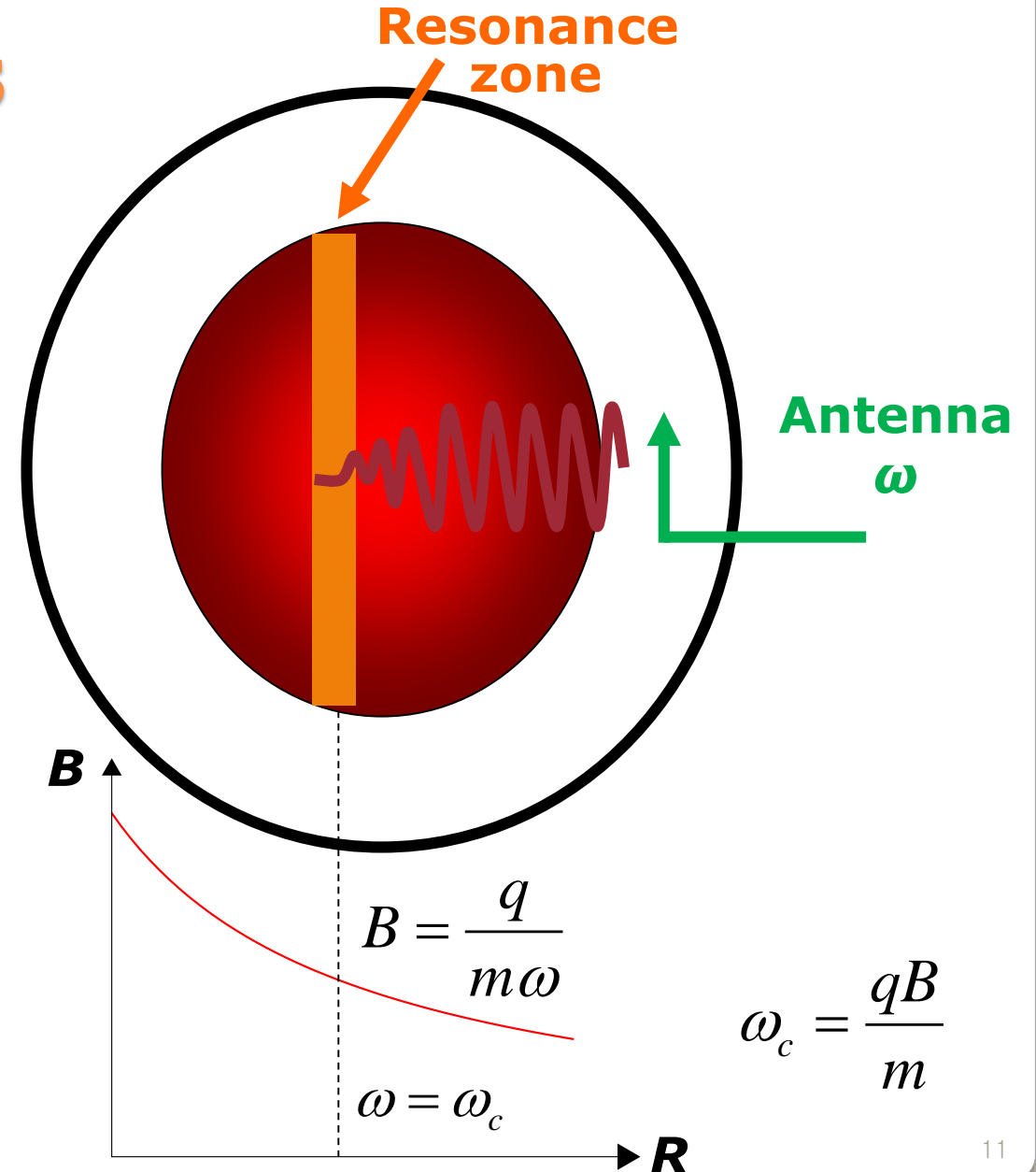
wave transports power  
into the plasma center



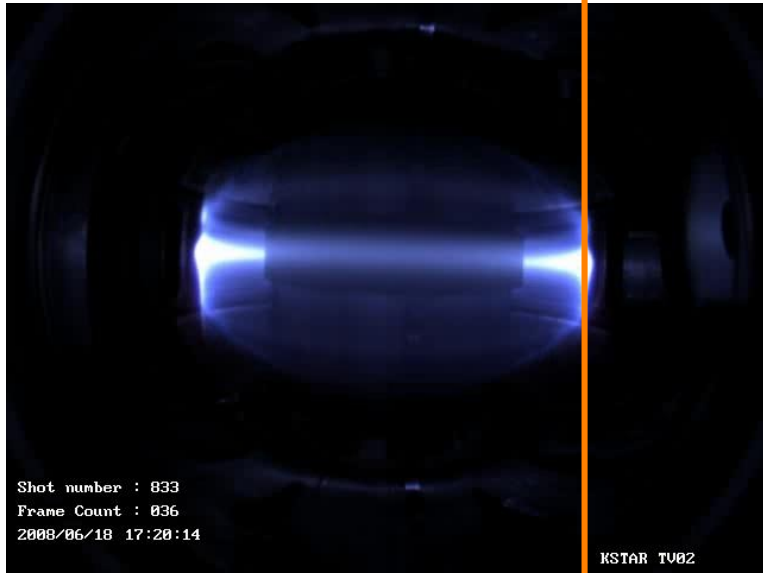
absorption near resonance,  
e.g.  $\omega \approx \omega_c$ ,  
i.e. conversion of wave energy  
into kinetic energy of  
resonant particles



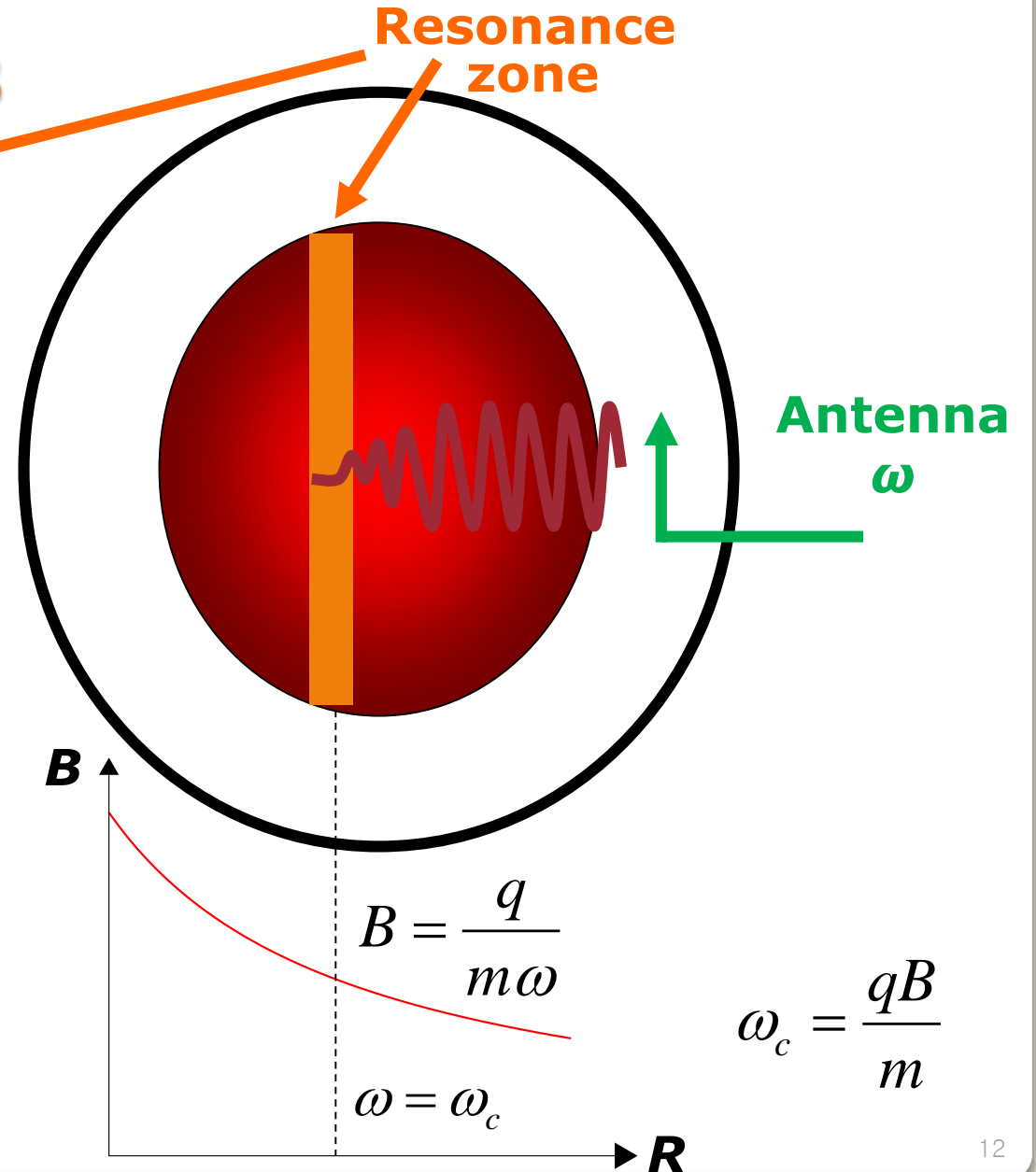
Resonant particles thermalise



# Electromagnetic Waves



KSTAR first plasma



# Waves in a plasma

# Plasma Waves

- Considering externally driven perturbations in the magnetic and electric fields and in the current, relative to an equilibrium condition for a cold plasma w/o external magnetic fields

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$$

Maxwell equation

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\begin{aligned} \nabla \times (\nabla \times \vec{E}) &= \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} \\ &= -\nabla \times \left( \frac{\partial \vec{B}}{\partial t} \right) \\ &= -\frac{\partial}{\partial t} (\nabla \times \vec{B}) \\ &= -\mu_0 \frac{\partial \vec{j}}{\partial t} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} \end{aligned}$$

$$\vec{j} \equiv \sum_j n_j q_j \vec{u}_j$$

Equations of motion:  
cold plasma w/o  $\mathbf{B}$

$$m_j n_j \left( \frac{\partial \vec{u}_j}{\partial t} + (\vec{u}_j \cdot \nabla) \vec{u}_j \right) = n_j q_j (\vec{E} + \vec{u}_j \times \vec{B}) - \nabla p_j$$

$$\Rightarrow \frac{\partial \vec{u}_j}{\partial t} = \frac{q_j}{m_j} \vec{E} \quad \Rightarrow \frac{\partial \vec{j}}{\partial t} = \sum_j n_j q_j \frac{\partial \vec{u}_j}{\partial t} = \sum_j \frac{n_j q_j^2}{m_j} \vec{E}$$

# Plasma Waves

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \left( \sum_j \frac{n_j q_j^2}{m_j} \right) \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \frac{\partial \vec{j}}{\partial t} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

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# Plasma Waves

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \left( \sum_j \frac{n_j q_j^2}{m_j} \right) \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$
$$-\vec{k}(\vec{k} \cdot \vec{E}) + k^2 \vec{E} = -\mu_0 \left( \sum_j \frac{n_j q_j^2}{m_j} \right) \vec{E} + \frac{\omega^2}{c^2} \vec{E}$$

Plane waves with space  
and time dependences

$$\exp[i(\vec{k} \cdot \vec{r} - \omega t)]$$

$\vec{k}$  : wave vector

$\omega$  : frequency

$$\nabla = i\vec{k}, \quad \frac{\partial}{\partial t} = -i\omega$$



# Plasma Waves

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \left( \sum_j \frac{n_j q_j^2}{m_j} \right) \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}$$

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$$-\vec{k}(\vec{k} \cdot \vec{E}) + k^2 \vec{E} + \mu_0 \left( \sum_j \frac{n_j q_j^2}{m_j} \right) \vec{E} - \frac{\omega^2}{c^2} \vec{E} = 0$$

$$\rightarrow (\dots) \vec{E} = 0$$

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Determinant (...) = 0

→ Dispersison relation  $D(\omega, k) = 0$

# Plasma Waves

$$-\vec{k}(\vec{k} \cdot \vec{E}) + k^2 \vec{E} + \mu_0 \left( \sum_j \frac{n_j q_j^2}{m_j} \right) \vec{E} - \frac{\omega^2}{c^2} \vec{E} = 0$$

$$\rightarrow (\dots) \vec{E} = 0$$

- $\vec{k} \parallel \vec{E}$

$$\omega^2 = \frac{1}{\epsilon_0} \sum_j \frac{n_j q_j^2}{m_j} \equiv \omega_p^2$$

Plasma frequency

- $\vec{k} \perp \vec{E}$

$$\omega^2 = c^2 k^2 + \omega_p^2$$

Plasma wave

Plane waves with space and time dependences

$$\exp[i(\vec{k} \cdot \vec{r} - \omega t)]$$

$\vec{k}$  : wave vector

$\omega$  : frequency

# Dispersion Relation

- Plasma waves are solutions of dispersion relation  $D(\omega, k)=0$ .

Generally:

$\omega$  given by generator  
 $k_{||}$  given by antenna  
where  $k_{||} > k_{||, \text{Vacuum}}$

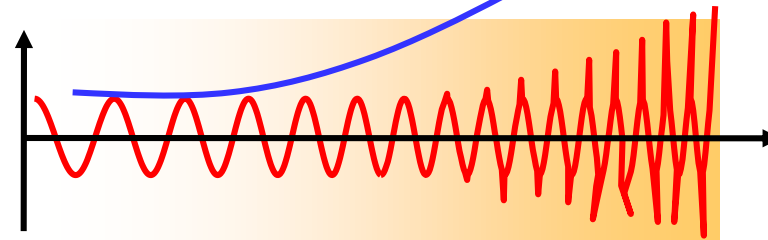
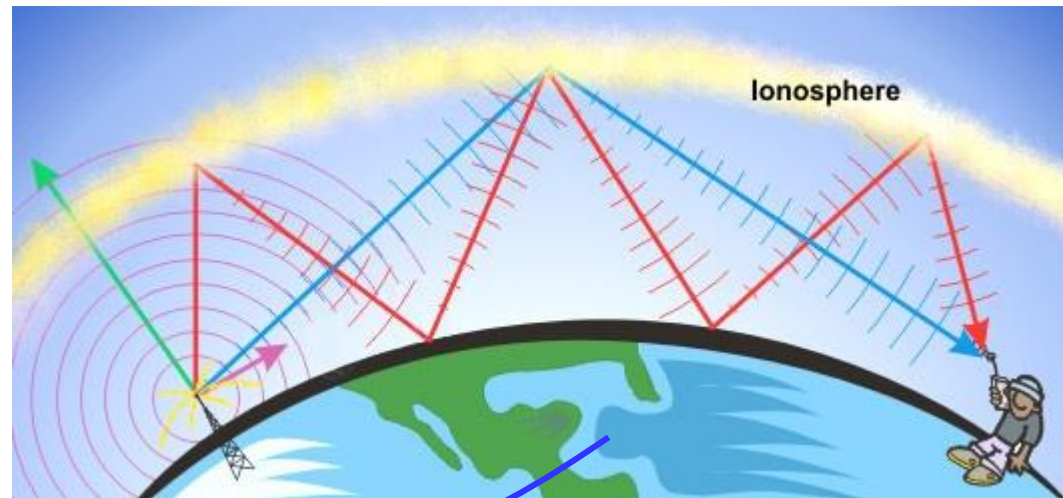


solution:  $k_{\perp} = k_{\perp}(\omega, k_{||})$

Special cases:

1.  $k_{\perp} \rightarrow 0$  „cutoff“

2.  $k_{\perp} \rightarrow \infty$  „resonance“



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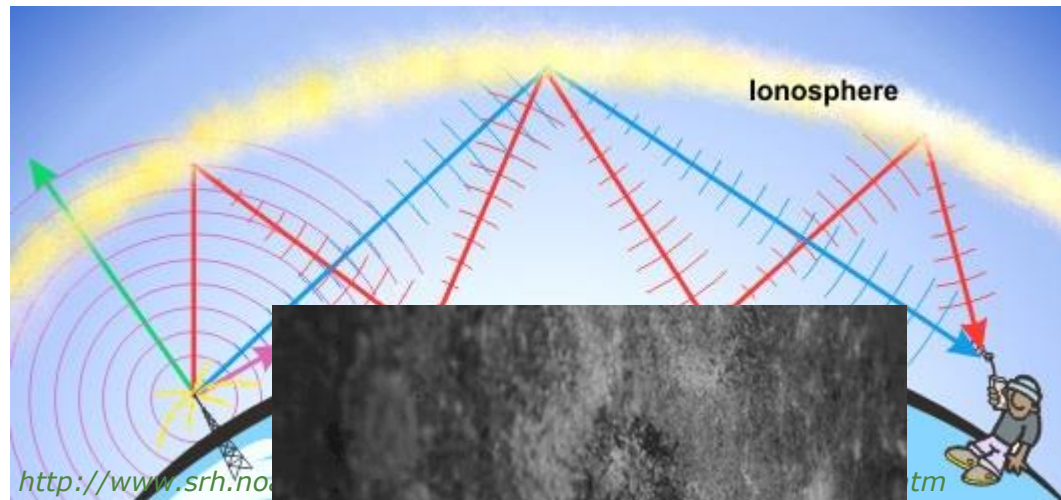


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Special cases:

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2.  $k_{\perp} \rightarrow \infty$  „resonance“



# Wave heating in a tokamak

# Electromagnetic Wave

- **Ion Cyclotron Resonance Heating (ICRH):**

occurring only when two or more ion species are present

$\omega \sim \omega_{ci}$ , 30 MHz – 120 MHz ( $\lambda \sim 10$  m)

$$\omega_{ii}^2 = \frac{\omega_{c1}\omega_{c2}(1+n_2m_2/n_1m_1)}{(m_2Z_1/m_1Z_2+n_2Z_2/n_1Z_1)}, \quad \omega_{ci} = \frac{Z_i e B}{m_i} \quad \text{: Ion-ion resonance frequency}$$

- **Lower Hybrid (LH) Resonance Heating:**

$\omega_{ci} < \omega < \omega_{ce}$ , 1 GHz – 8 GHz ( $\lambda \sim 10$  cm)

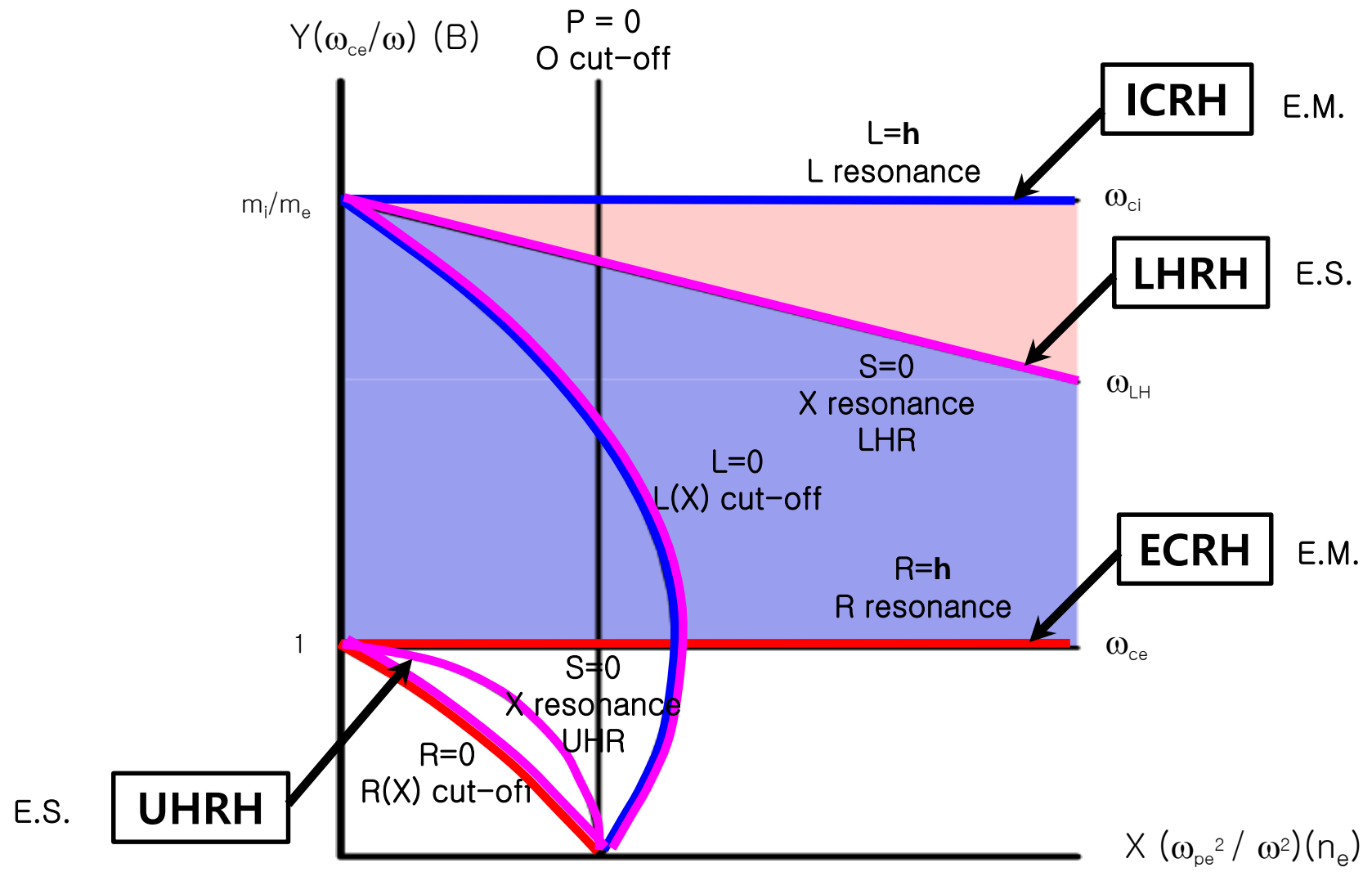
$$\omega_{LH}^2 \approx \omega_{pi}^2 / (1 + \omega_{pi}^2 / \omega_{ce}^2), \quad \omega_{pi}^2 \gg \omega_{ci}^2$$

- **Electron Cyclotron Resonance Heating (ECRH):**

$\omega \sim \omega_{ce}$ , 100 GHz – 200 GHz ( $\lambda \sim$  mm)

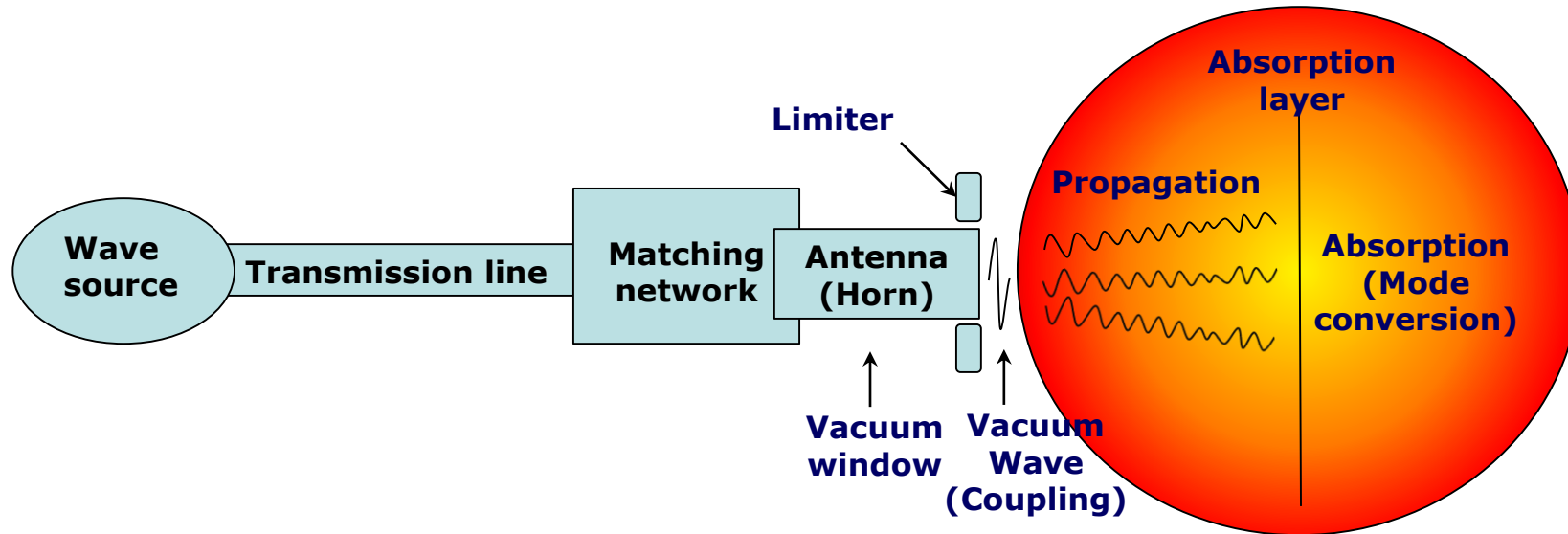
$$\omega_{UH}^2 \approx \omega_{pe}^2 + \omega_{ce}^2$$

# Electromagnetic Wave



# Wave System

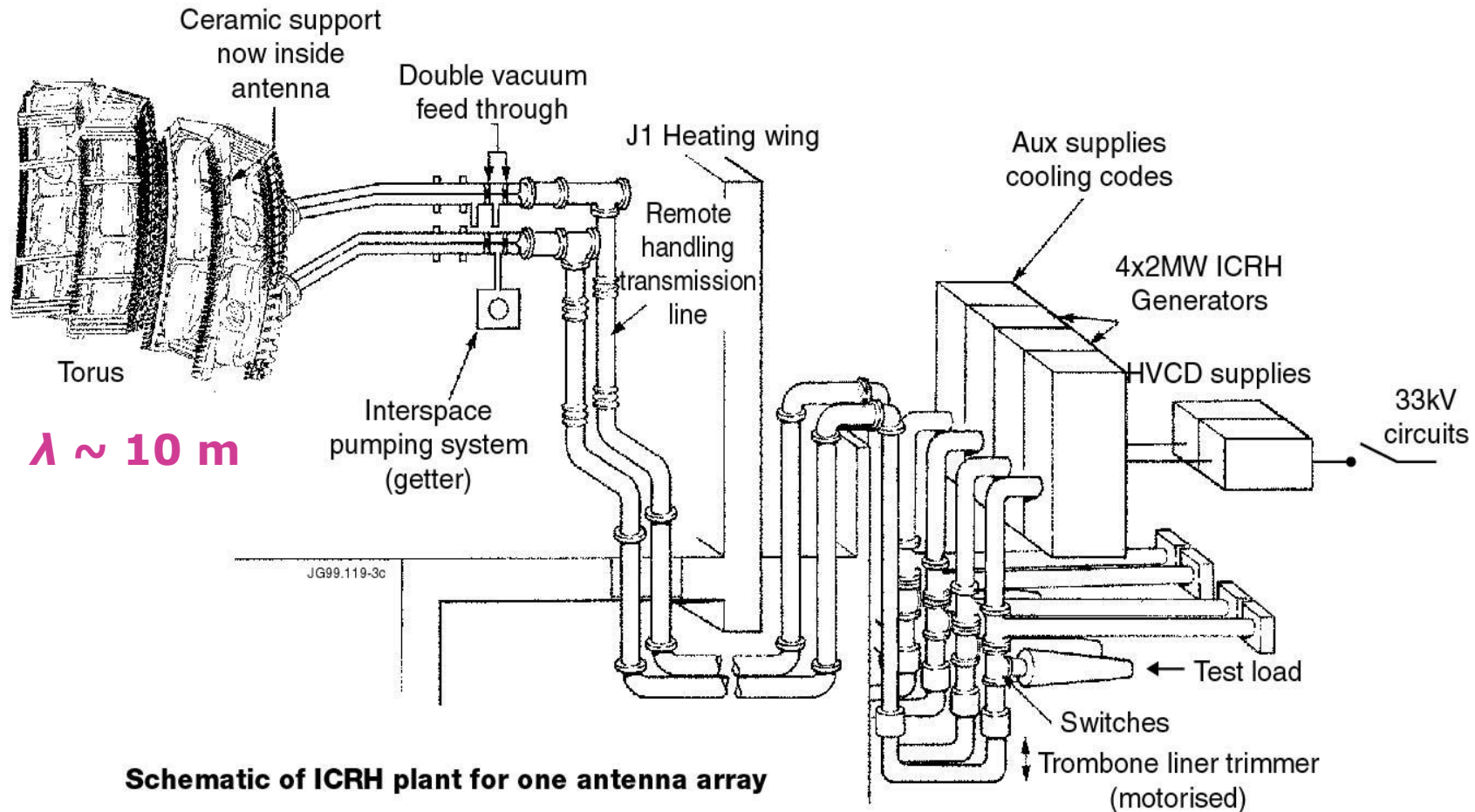
- Wave launching, propagation, absorption in fusion plasmas





# Ion Cyclotron Resonance Heating

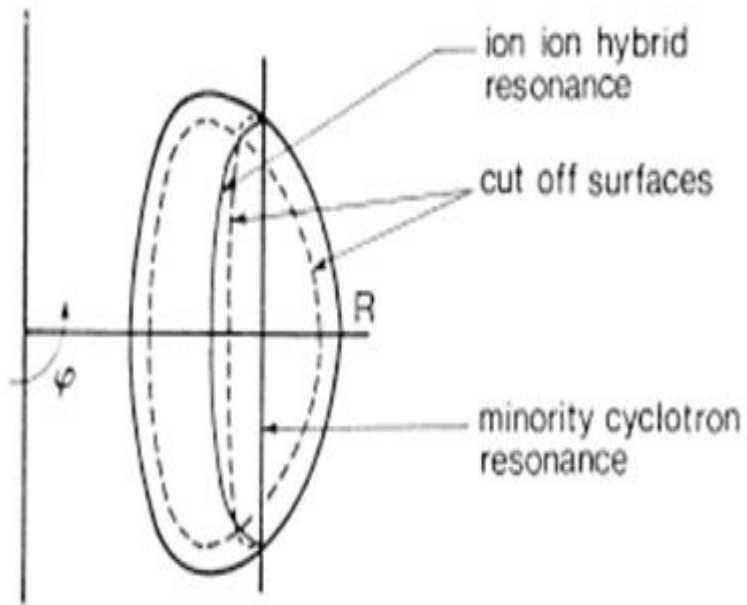
## • JET ICRH System



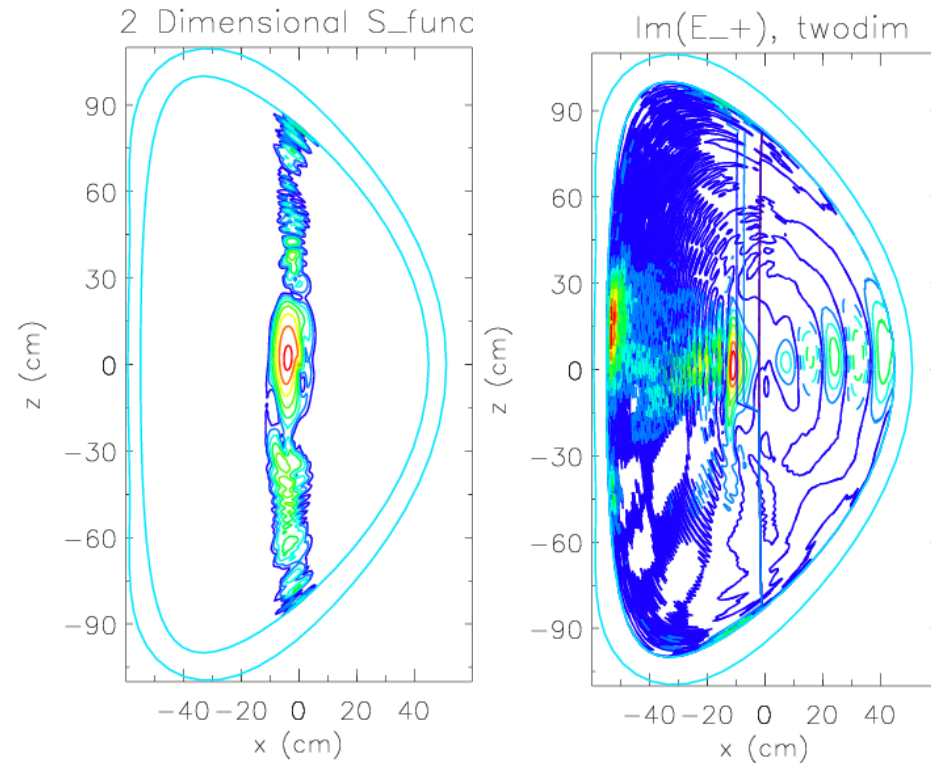
KSTAR ICRF wave generator: Transmitter (Tetrode tube)

# Ion Cyclotron Resonance Heating

- Propagation and absorption



Cut-off/Resonances in minority heating scheme

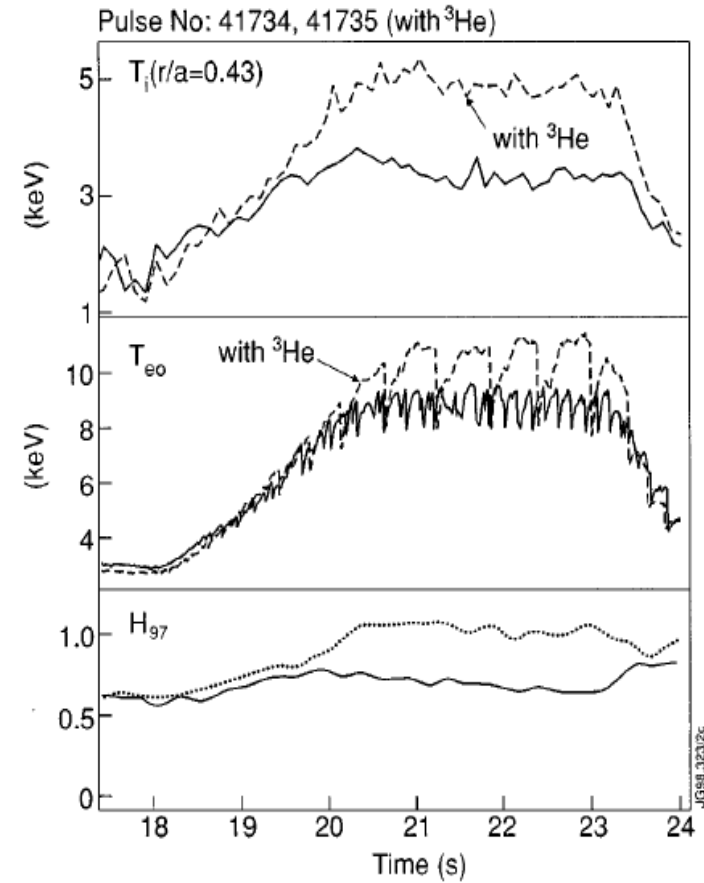
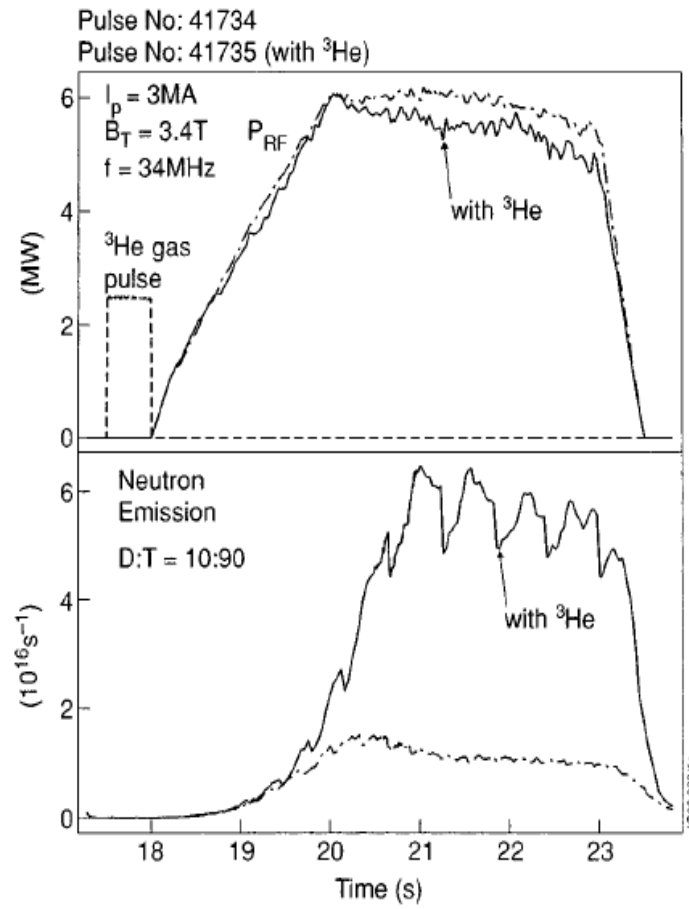


D(H) Minority Heating Scheme  
in KSTAR  
Wang, 2009

# Ion Cyclotron Resonance Heating

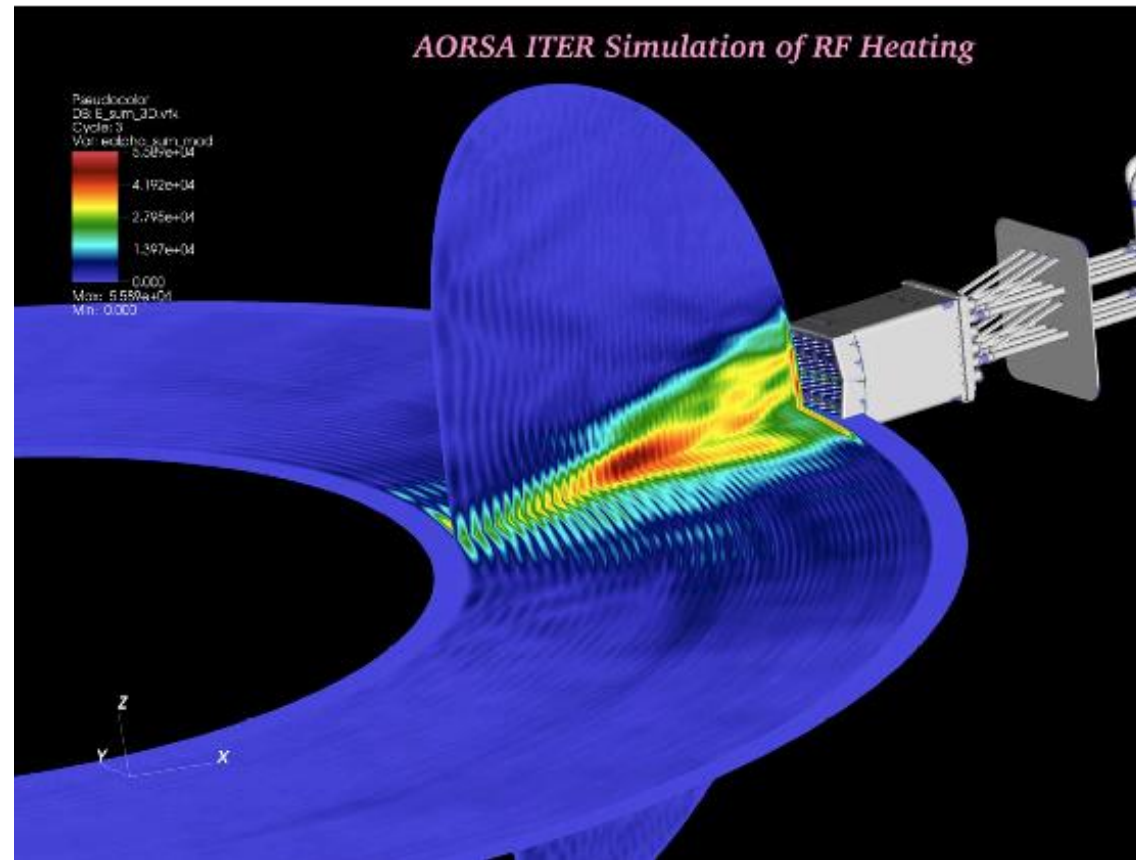
- Experimental results

T 2<sup>nd</sup> Harmonic + He3 minority Heating



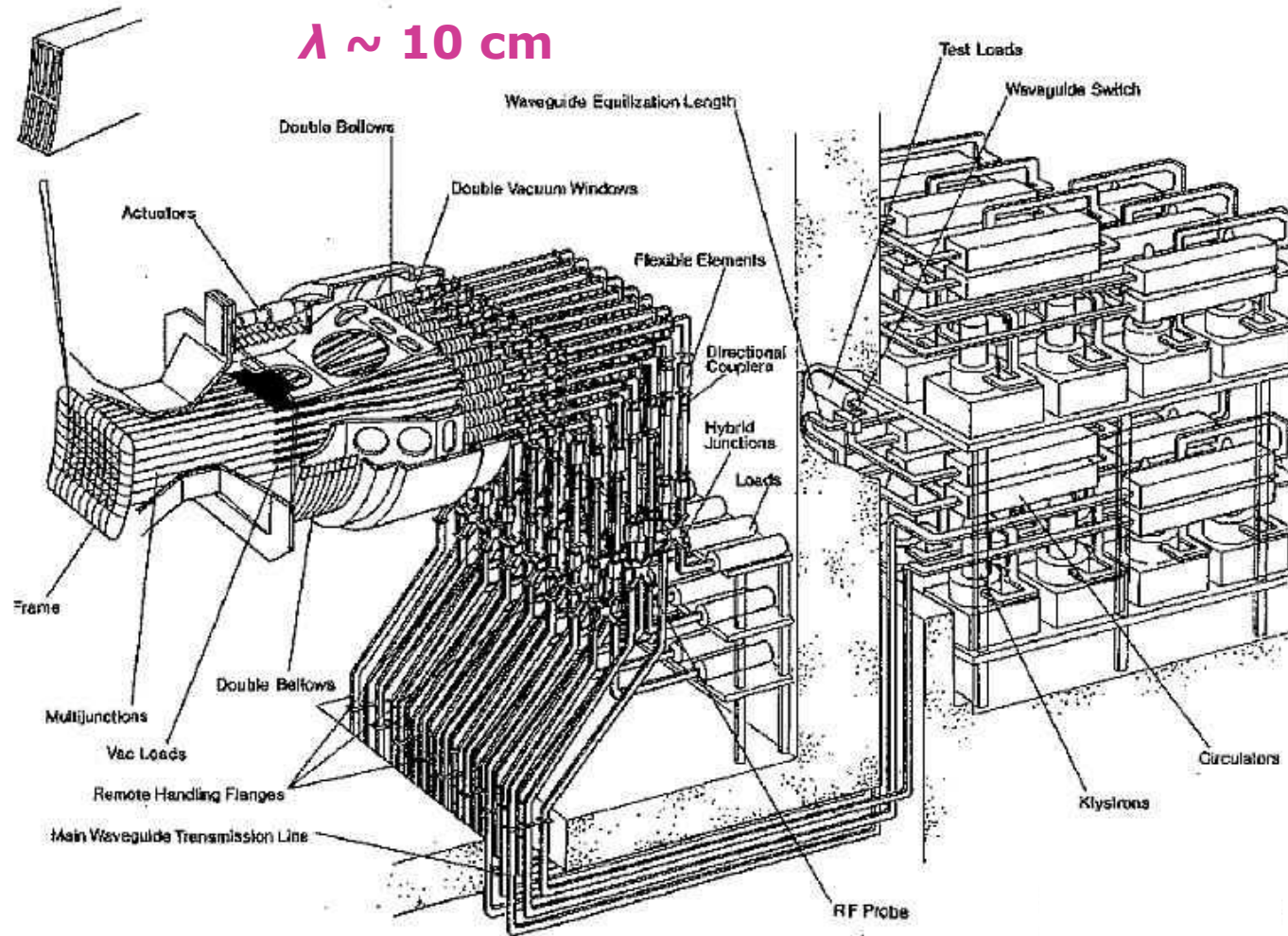
JET (Start et al. 1999)

# Ion Cyclotron Resonance Heating



# Lower Hybrid Heating

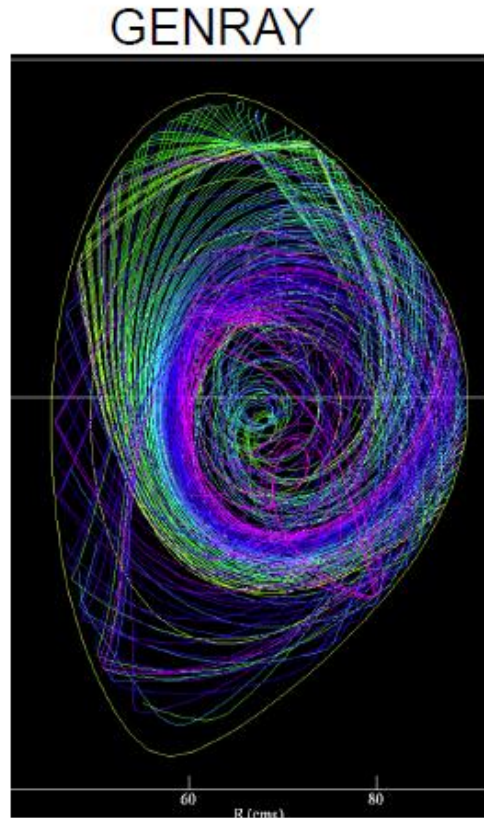
## • JET LH System



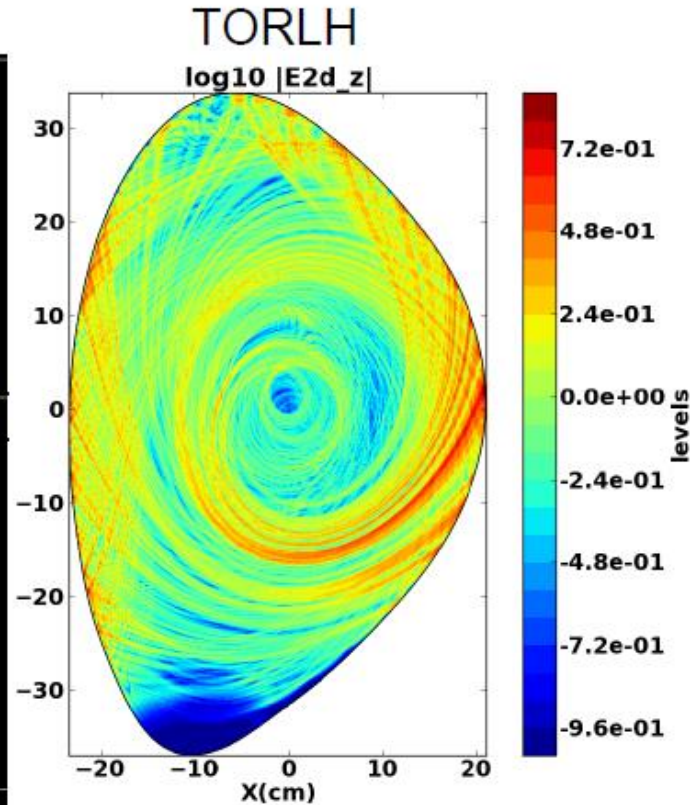
500 kW klystron  
for ITER

# Lower Hybrid Heating

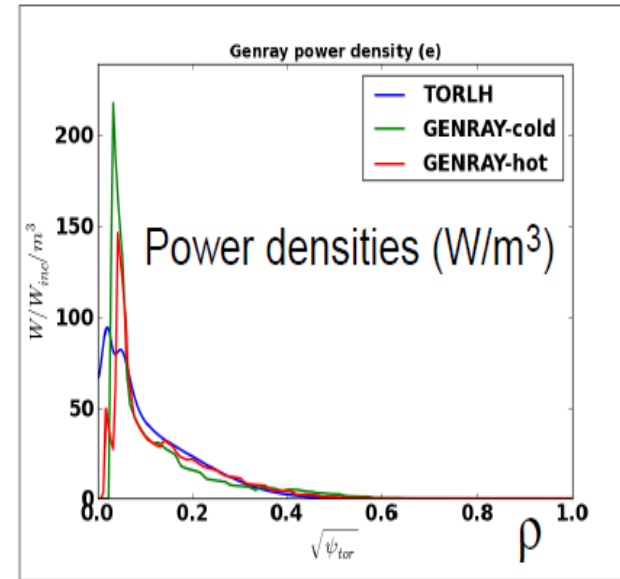
- Propagation and absorption



Petrov, CompX



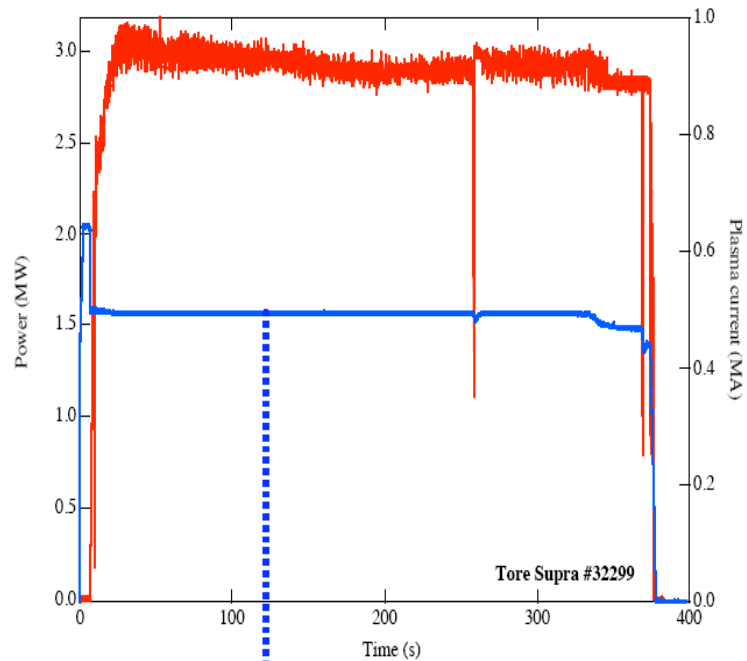
J. C. Wright, POP (2009)



Petrov, CompX

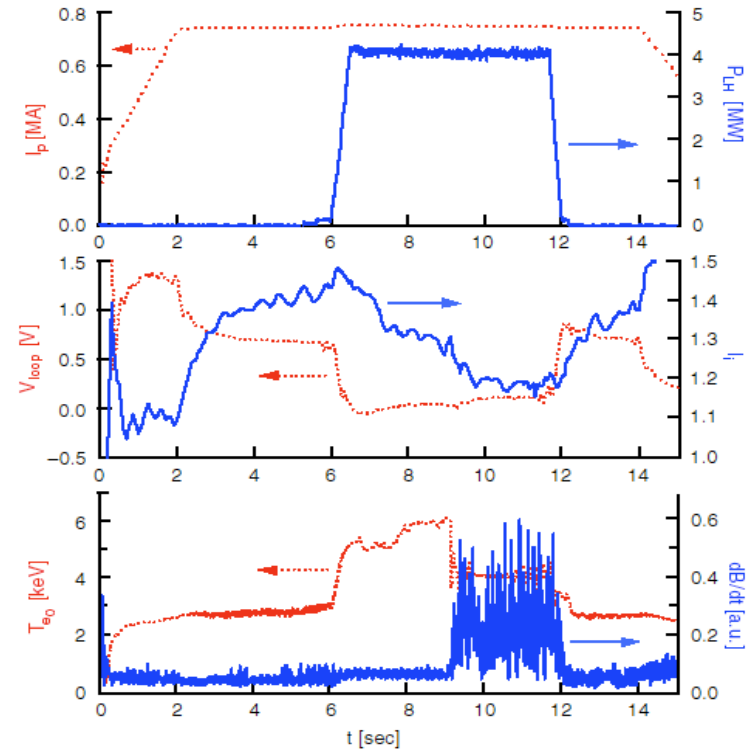
# Lower Hybrid Heating

- Experimental results



- Full LHCD (6 min.)
- 2 antennas
- $n_{||0} = 1.7 \pm 0.2$
- $P_{lh} = 3$  MW
- directivity: 0.6 & 0.7

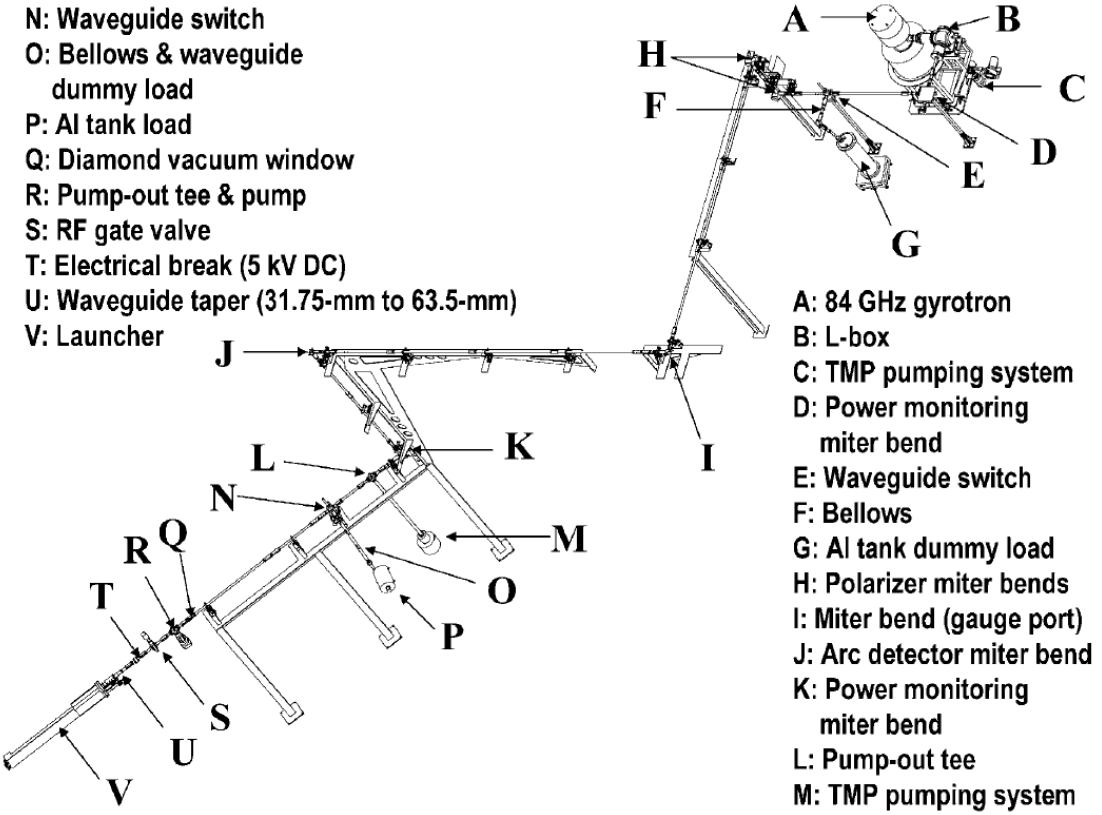
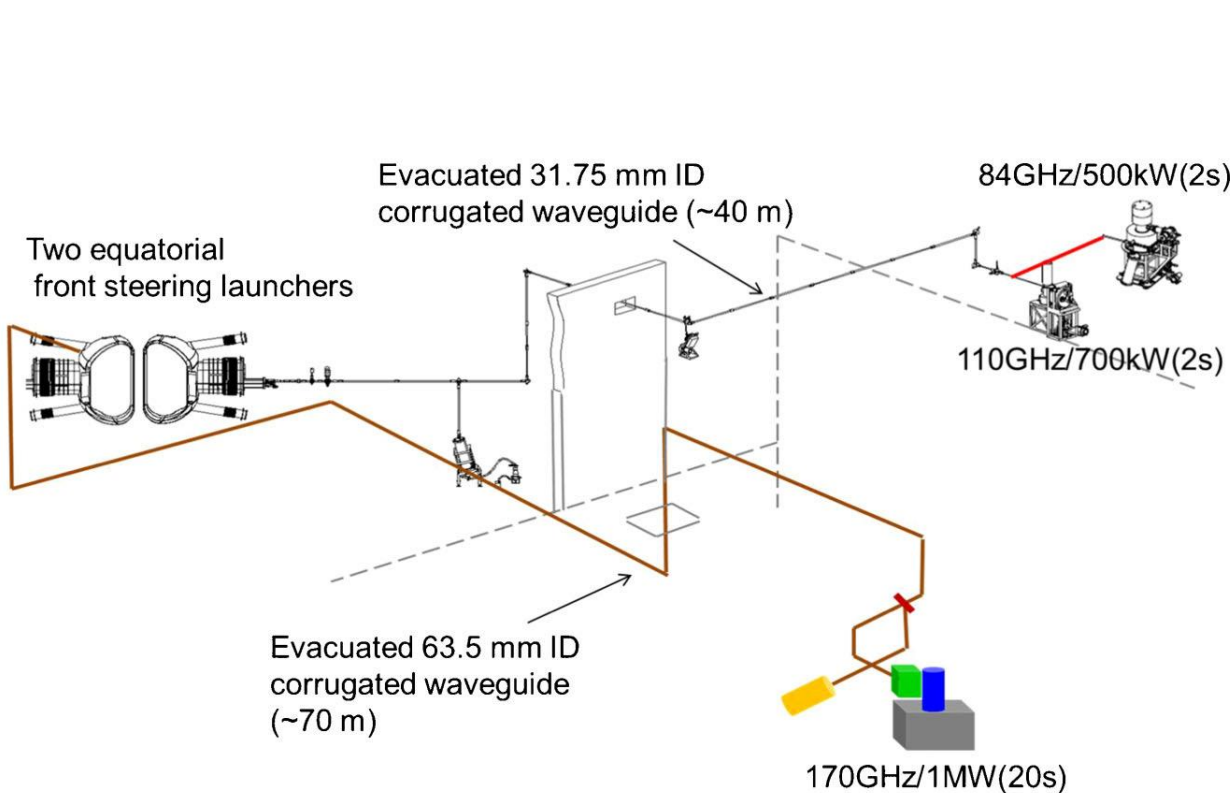
$I_p \approx 500$  kA



# Electron Cyclotron Heating

## • KSTAR ECH System

$\lambda \sim \text{mm}$





# Electron Cyclotron Heating

## High-Power Gyrotrons for Fusion Plasma Applications



**ITER: TOSHIBA/JAEA (JA)**  
 170 GHz, 1 (0.8) MW  
 800 (3600) s, 55 (57) %



**ITER: GYCOM/IAP (RF)**  
 170 GHz, 1.05 (0.83) MW  
 116 (203) s, 52 (48) %



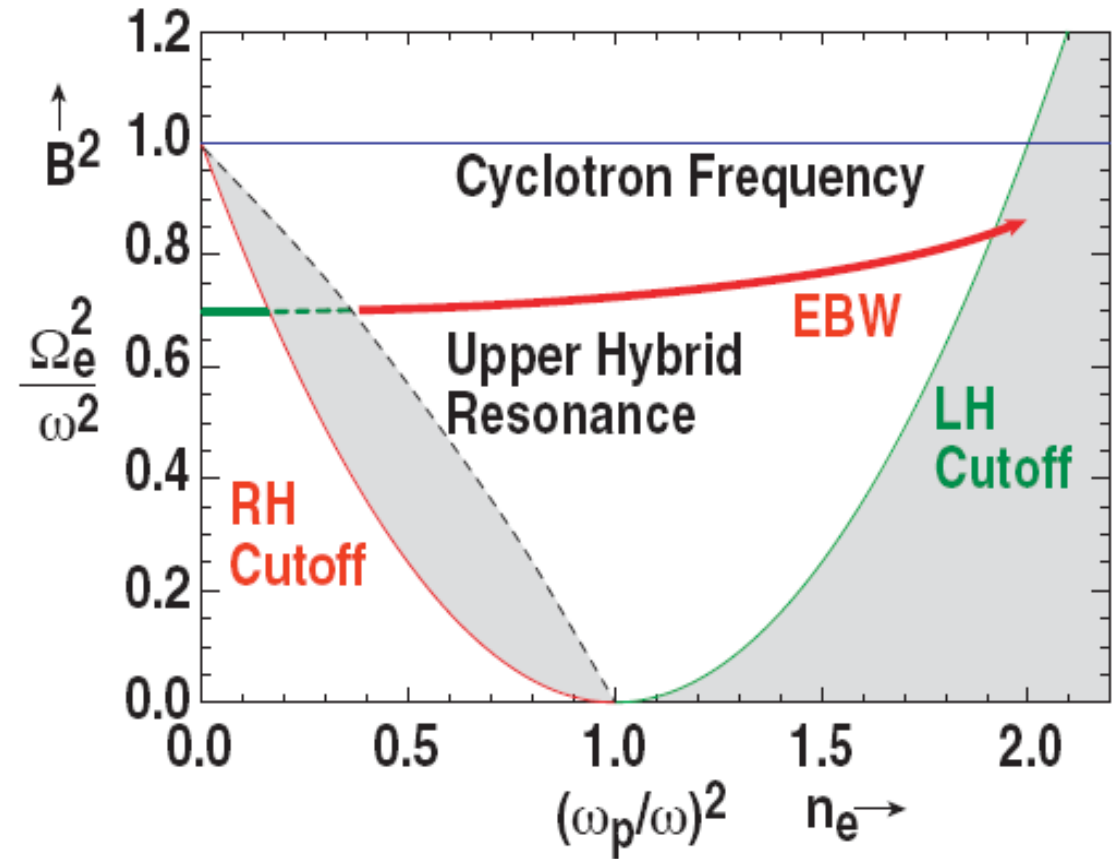
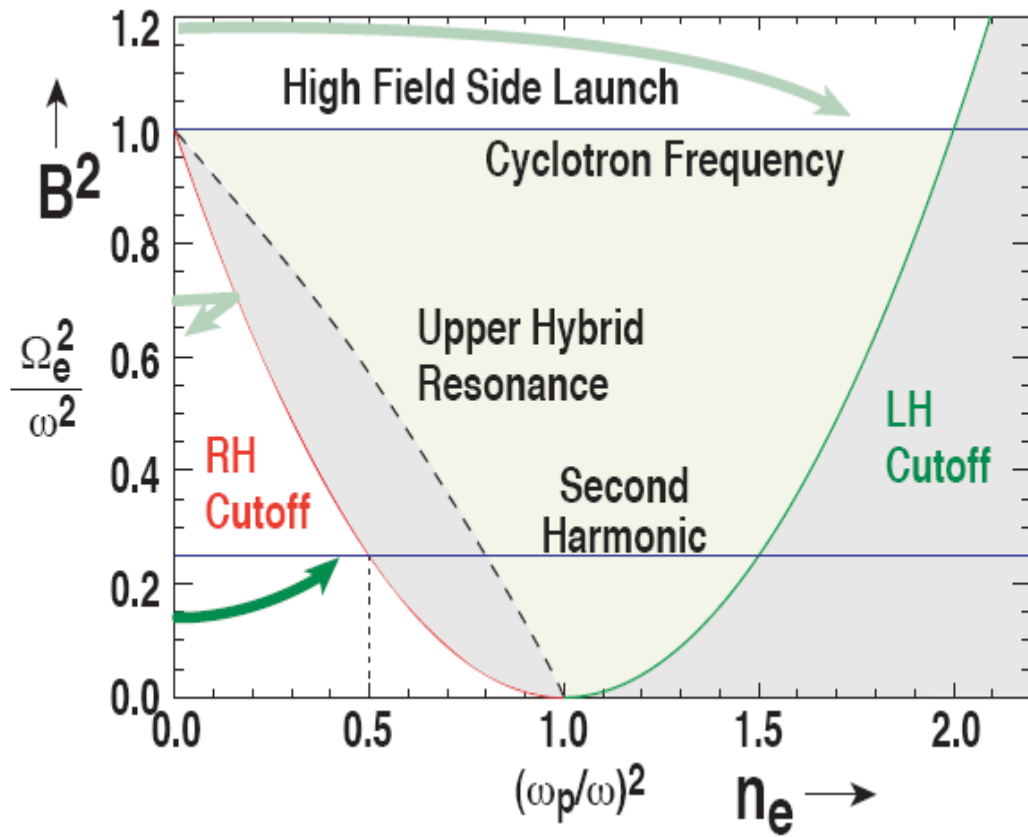
**W7-X: CPI (USA)**  
 140 GHz, 0.9 MW  
 1800 s, 35 %



**W7-X: TED/FZK/CRPP (EU)**  
 140 GHz, 0.92 MW  
 1800 s, 45 %

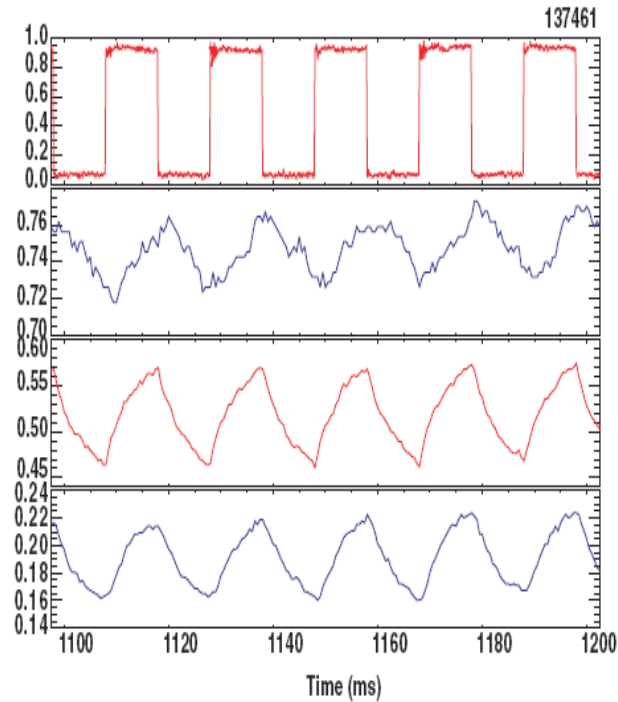
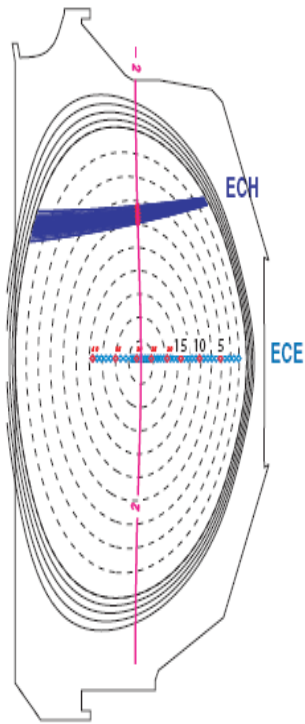
# Electron Cyclotron Heating

- Propagation and absorption

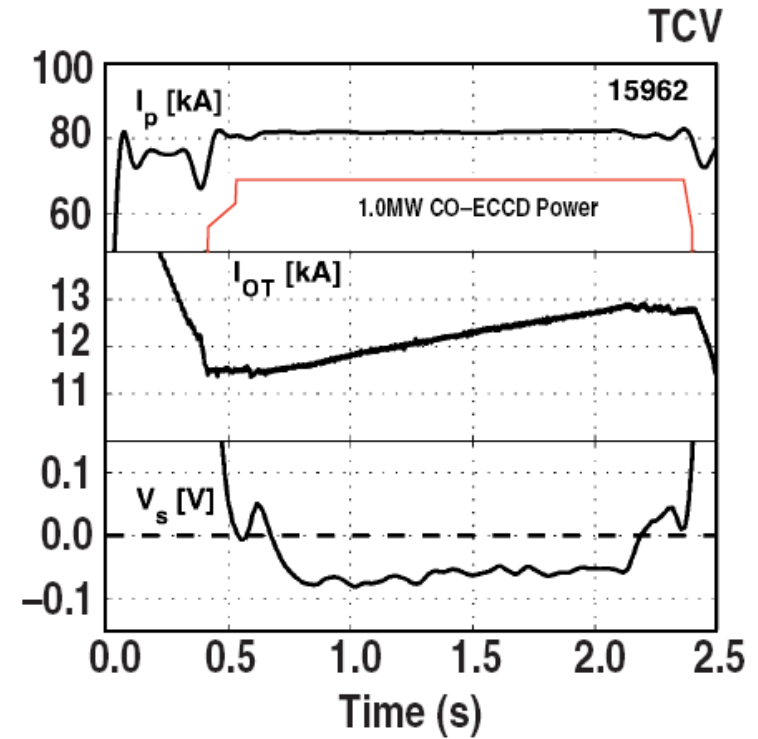


# Electron Cyclotron Heating

- Experimental results



X2 heating in DIII-D



Full non-inductive CD in TCV

# Alpha particle heating

# $\alpha$ -Particle Heating

- Intrinsic self-heating by Coulomb collision of fusion  $\alpha$  particles with plasma particles in D-T reactions



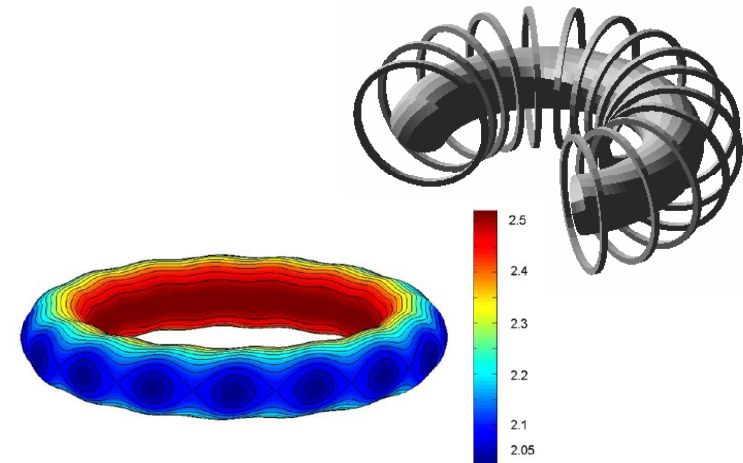
↓ leaves plasma      ↓ heats plasma if sufficiently long confined

- Heating power density:  $N_D N_T \langle \sigma v \rangle \frac{Q_{DT}}{5}$

where  $\langle \sigma v \rangle \propto T_i$

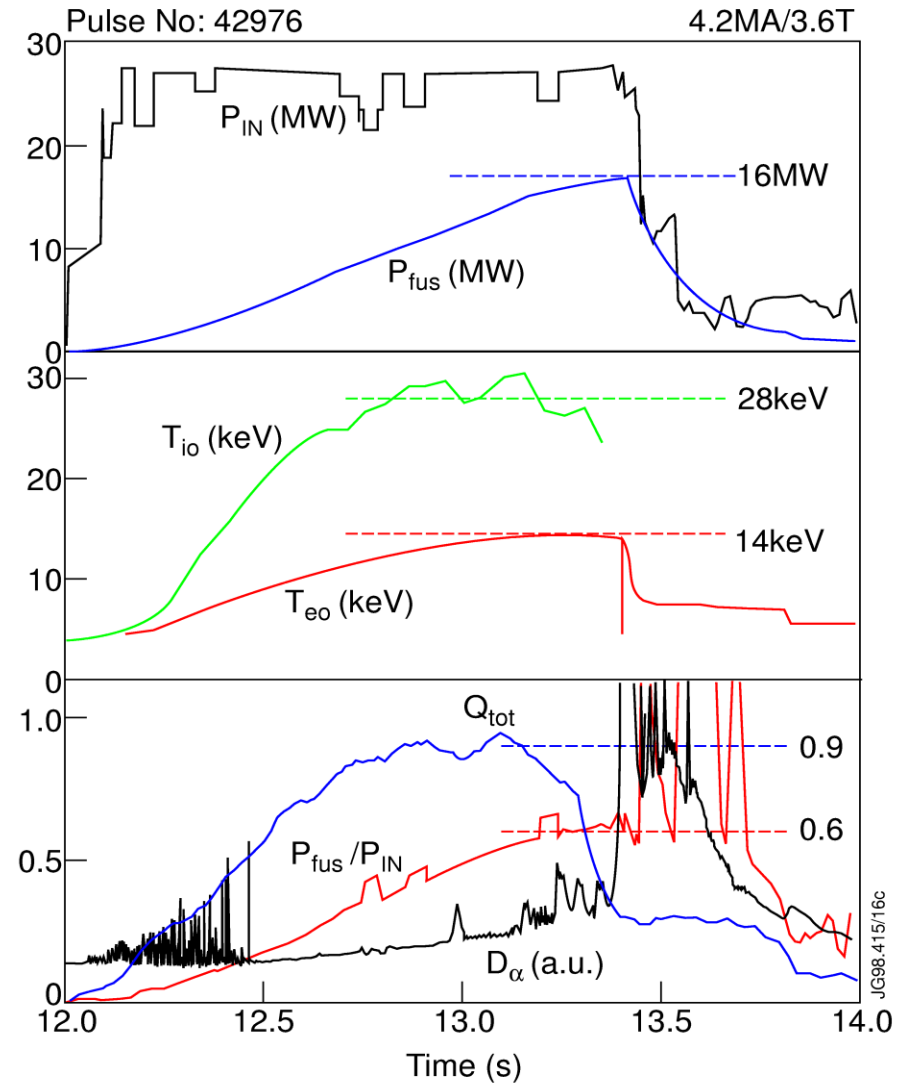
⇒ peaked heating profile

- $\alpha$ -particle loss mechanisms: field ripples  
MHD events e.g. Alfvén Eigenmode (AE)



# $\alpha$ -Particle Heating

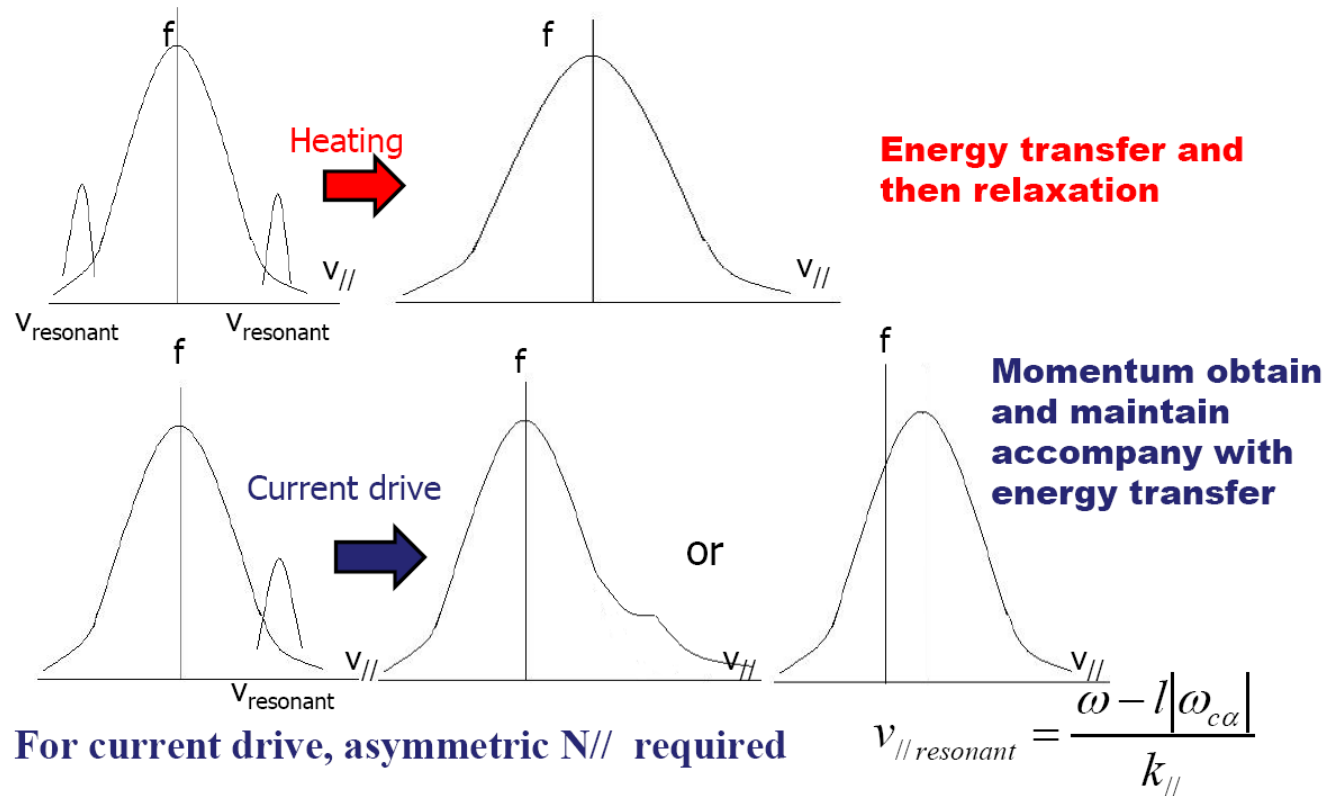
- DT-Experiments only in
  - JET
  - TFTR
- with world records in JET:
  - $P_{fusion} = 16$  MW
  - $Q = 0.64$



# Current drive

# Non-inductive Current Drive

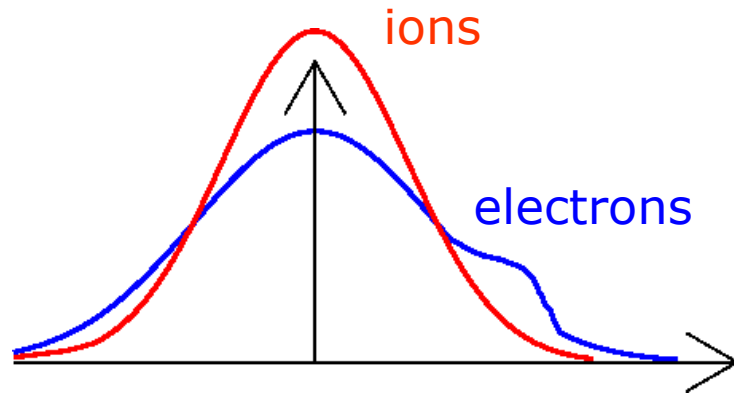
## Heating and current drive





# Non-inductive Current Drive

- Asymmetric velocity distribution can be a side effect of plasma heating.

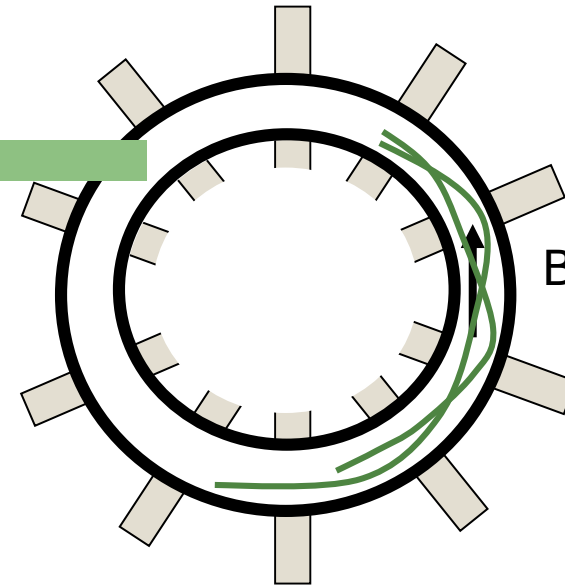
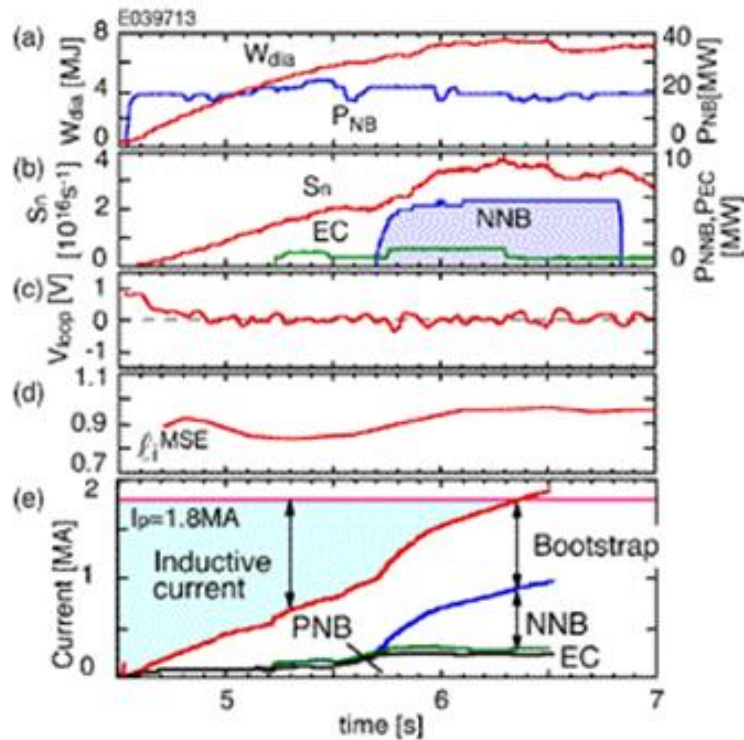


$$j = \sum_s q_s n_s \int v_{\parallel} f(v_{\parallel}) dv$$

- **Needed for:** Steady-state tokamak  
current profile control in tokamaks  
bootstrap current compensation in stellarators

# Neutral Beam Current Drive

Tangential injection



JT-60U high  $\beta_p$  ELMy H-mode

# Heating and Current Drive

Heating Scheme	Advantages	Disadvantages
OH	Efficient	Cannot reach ignition
NBI	Reliable	Close to torus, Negative ion source necessary
LH	Efficient current drive (CD)	Antenna close to plasma, off-axis CD
ECRH	Reliable, Flexible Localised CD	Electron heating
ICRH	Ion heating Central heating	Antenna close to plasma, Antenna coupling