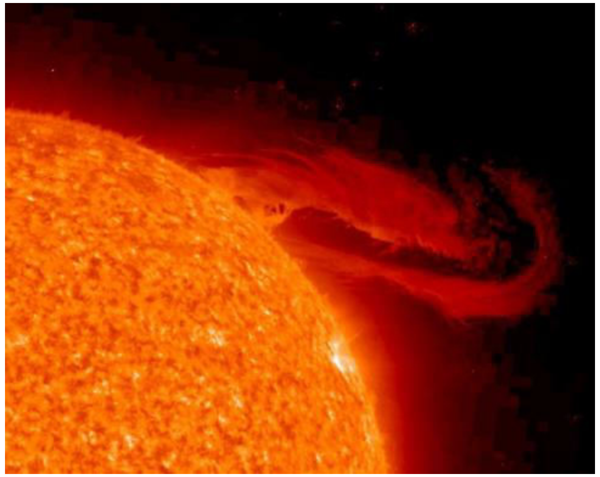
Introduction to Nuclear Fusion

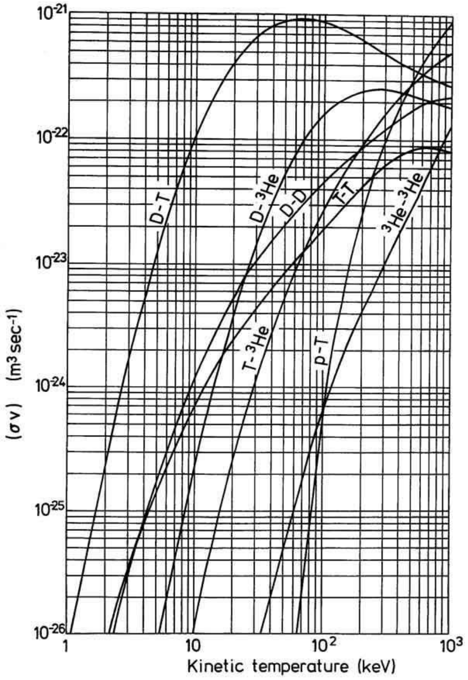
Prof. Dr. Yong-Su Na

How to describe a plasma?

Description of a Plasma

- **Thermonuclear fusion with high energy particles of 10-20 keV energy → plasma**





- **Three approaches to describe a plasma**

- Single particle approach
- Kinetic theory
- Fluid theory

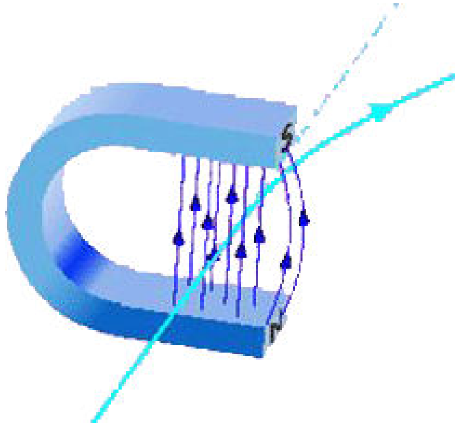
http://http://dz-dev.net/blog/tag/eruption-solaire

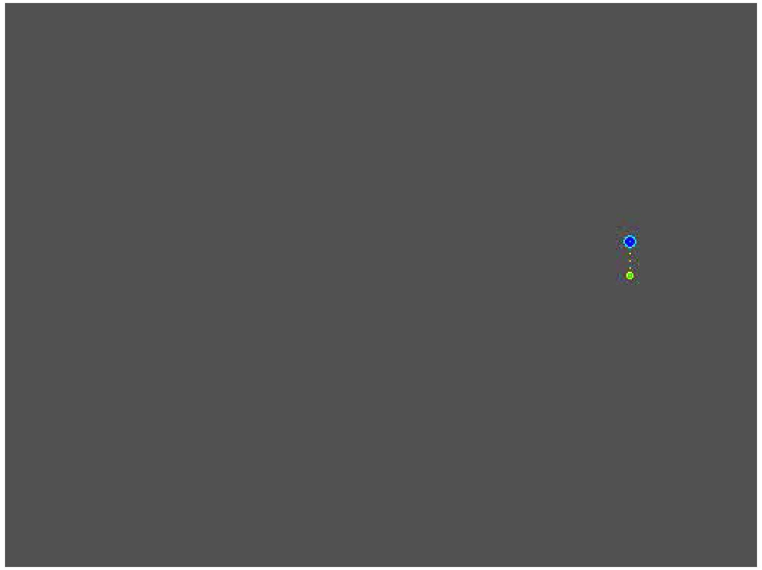
Individual Charge Trajectories

- **Equation of motion**

- Basic relation determining the motion of an individual charged particle of mass *m* and charge *q* in a combined electric (**E**) and magnetic (**B**) field
- Neglecting electromagnetic fields generated by movement of the charge itself

$$m\frac{d\mathbf{v}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$





Individual Charge Trajectories

- **Homogeneous electric field**

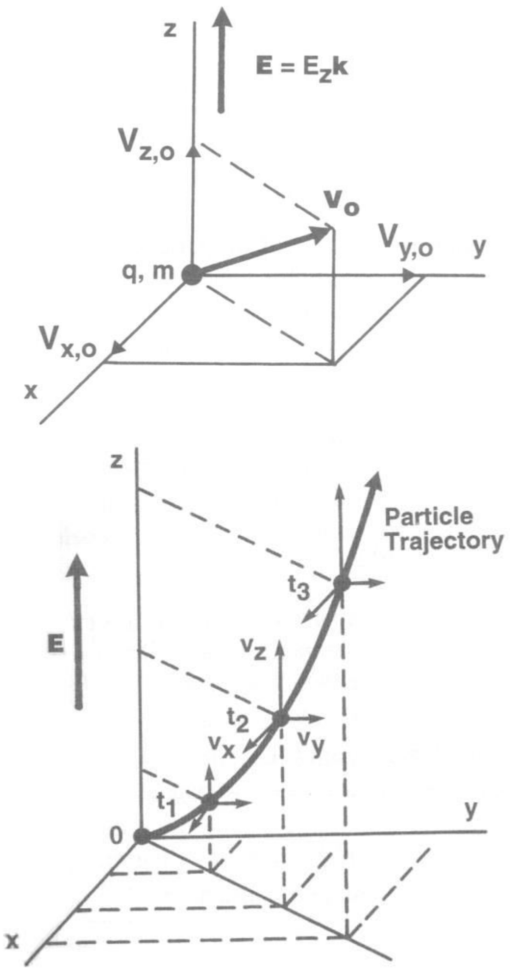
$$m\frac{d\mathbf{v}}{dt} = q\mathbf{E} = qE_z\mathbf{k}$$

$$v_x(0) = v_{x,0}, \quad v_y(0) = v_{y,0}, \quad v_z(0) = v_{z,0}$$

$$v_x(t) = v_{x,0}, \quad v_y(t) = v_{y,0}, \quad v_z(t) = v_{z,0} + \frac{q}{m}E_z t$$

$$x(t) = v_{x,0}t, \quad y(t) = v_{y,0}t,$$

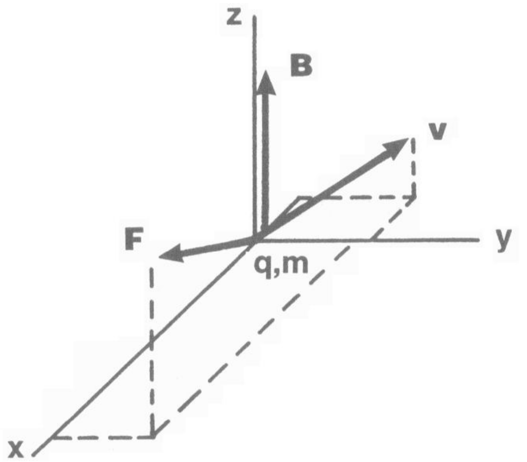
$$z(t) = v_{z,0}t + \frac{1}{2}\left(\frac{q}{m}E_z\right)t^2$$



A. A. Harms et al, "Principles of Fusion Energy", World Scientific (2000)

Individual Charge Trajectories

- **Homogeneous magnetic field**



$$m\frac{d\mathbf{v}}{dt} = q(\mathbf{v}\times\mathbf{B}) = q(\mathbf{v}\times B_z\mathbf{k})$$

$$v_x(0) = v_{x,0},\quad v_y(0) = v_{y,0},\quad v_z(0) = v_{z,0} = v_\parallel$$

$$v_x(t) = v_\perp\cos(\omega_c t+\phi),\quad v_y(t) = \mp v_\perp\sin(\omega_c t+\phi),\quad v_z(t) = v_{z,0} = v_\parallel$$

$$v_\perp = \sqrt{v_x^2+v_y^2},\quad \tan(\phi) = \mp\frac{v_{y,0}}{v_{x,0}},\quad \omega_c = \frac{|q|B_z}{m}$$ Cyclotron frequency

$$x(t) = x_0 + \frac{v_\perp}{\omega_c}\sin(\omega_c t+\phi),\quad y(t) = y_0 \pm \frac{v_\perp}{\omega_c}\cos(\omega_c t+\phi),\quad z(t) = z_0 + v_\parallel t$$

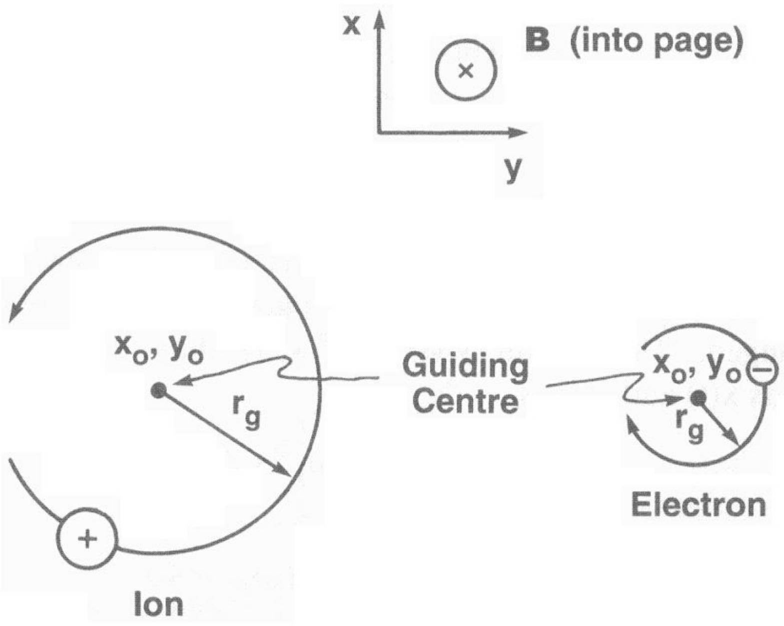
$$(x-x_0)^2+(y-y_0)^2 = \left(\frac{v_\perp}{\omega_c}\right)^2 = \left(\frac{v_\perp m}{|q|B_z}\right)^2 = r_L^2$$ Larmor radius

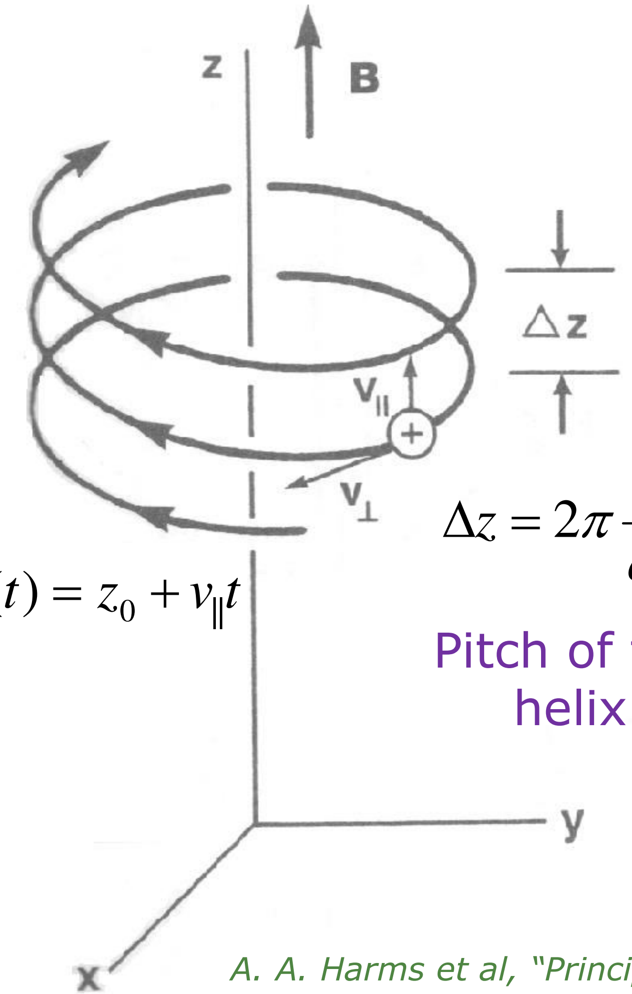
5 T, 10 keV
r_L = 0.05 mm for e
r_L = 2.9 mm for d
r_L = 3.5 mm for t

A. A. Harms et al, "Principles of Fusion Energy", World Scientific (2000)

Individual Charge Trajectories

- **Homogeneous magnetic field**



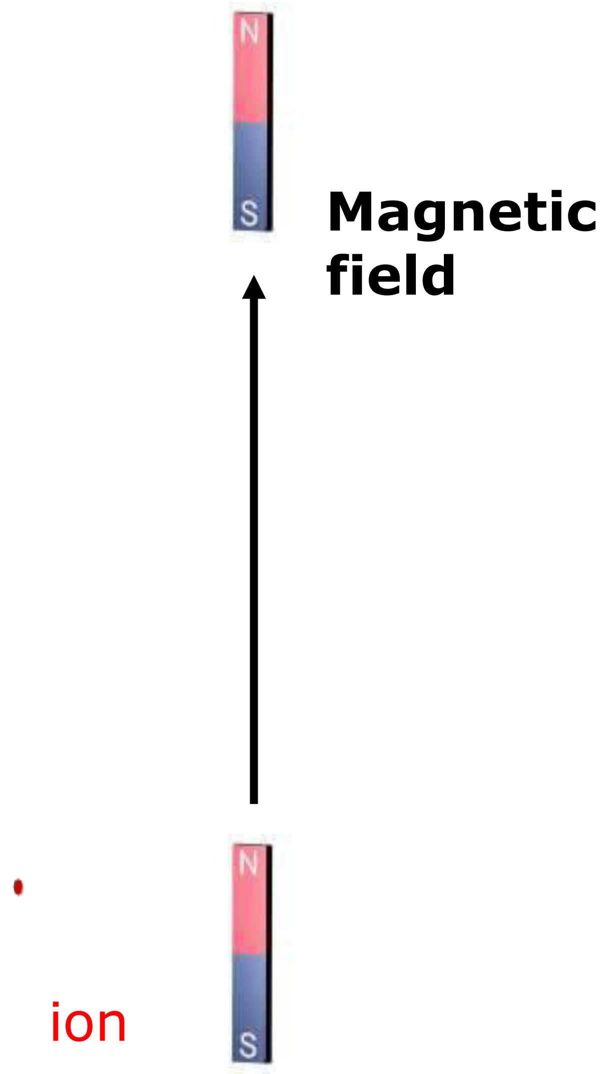


$$z(t) = z_0 + v_{\parallel} t$$

$$\Delta z = 2\pi \frac{v_{\parallel}}{\omega_c}$$

Pitch of the helix

$$(x - x_0)^2 + (y - y_0)^2 = \left(\frac{v_{\perp} m}{|q| B_z} \right)^2 = r_L^2$$



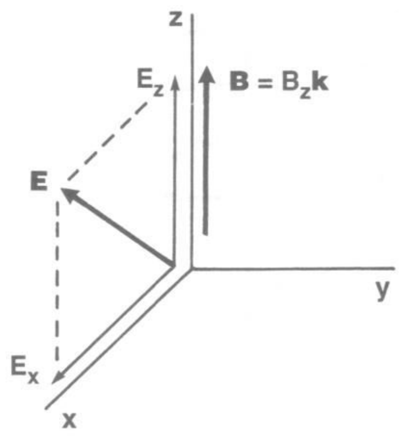
A. A. Harms et al, "Principles of Fusion Energy", World Scientific (2000)

Individual Charge Trajectories

- **Combined homogeneous electric and magnetic field**

$$m\frac{d\mathbf{v}}{dt} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) = q[(E_x\mathbf{i} + E_z\mathbf{k}) + \mathbf{v}\times B_z\mathbf{k}]$$

$$v_x(0) = v_{x,0},\quad v_y(0) = v_{y,0},\quad v_z(0) = v_{z,0} = v_\parallel$$



$$v_x(t) = v_\perp^* \cos(\omega_c t + \phi^*),\quad v_y(t) = \mp v_\perp^* \sin(\omega_c t + \phi^*) - \frac{E_x}{B_z},$$

$$v_z(t) = v_\parallel + \left(\frac{qE_z}{m}\right)t$$

$$v_\perp^* = \sqrt{v_x^2 + \left(v_y + \frac{E_x}{B_z}\right)^2} = \frac{v_\perp \cos\phi}{\cos\phi^*},\quad \tan(\phi^*) = \mp\frac{v_{y,0} + \dfrac{E_x}{B_z}}{v_{x.0}}$$

A. A. Harms et al, "Principles of Fusion Energy", World Scientific (2000)

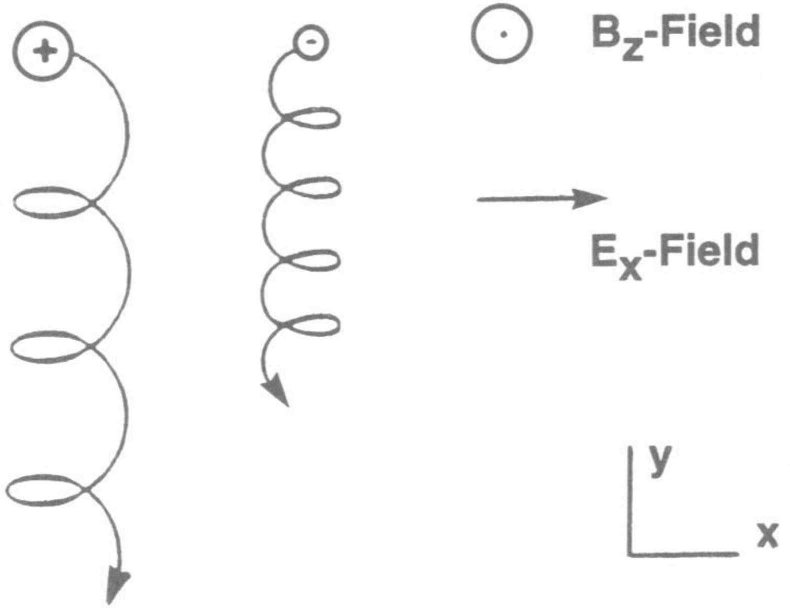
Individual Charge Trajectories

- **Combined homogeneous electric and magnetic field**

$$x(t) = x_0 + \frac{v_\perp^*}{\omega_c}\sin(\omega_c t + \phi^*), \quad y(t) = y_0 \pm \frac{v_\perp^*}{\omega_c}\cos(\omega_c t + \phi^*) - \frac{E_x}{B_z}t,$$

$$z(t) = z_0 + v_\parallel t + \frac{1}{2}\left(\frac{qE_z}{m}\right)t^2$$

$$(x - x_0)^2 + \left(y - y_0 + \frac{E_x}{B_z}t\right)^2 = \left(\frac{v_0^*}{\omega_c}\right)^2$$



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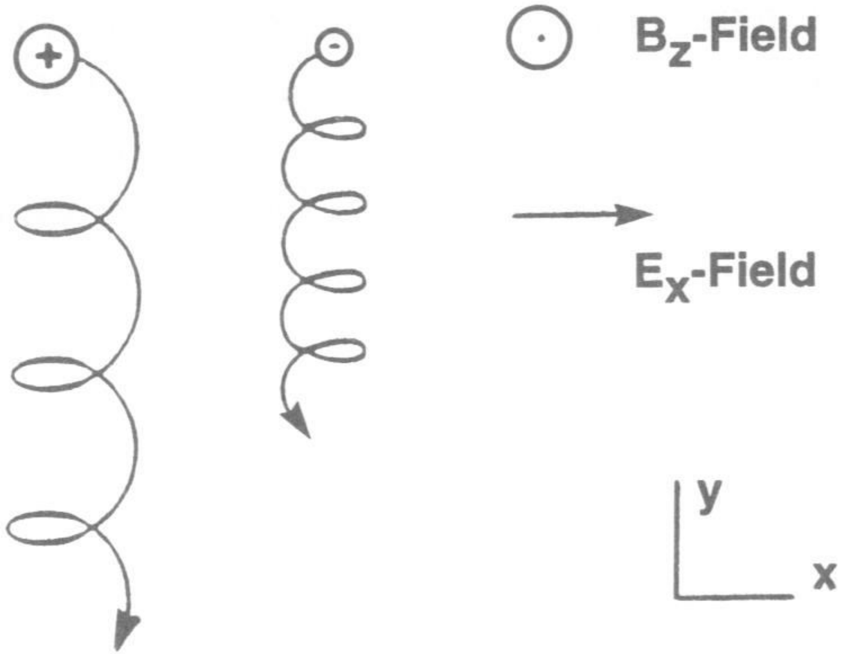
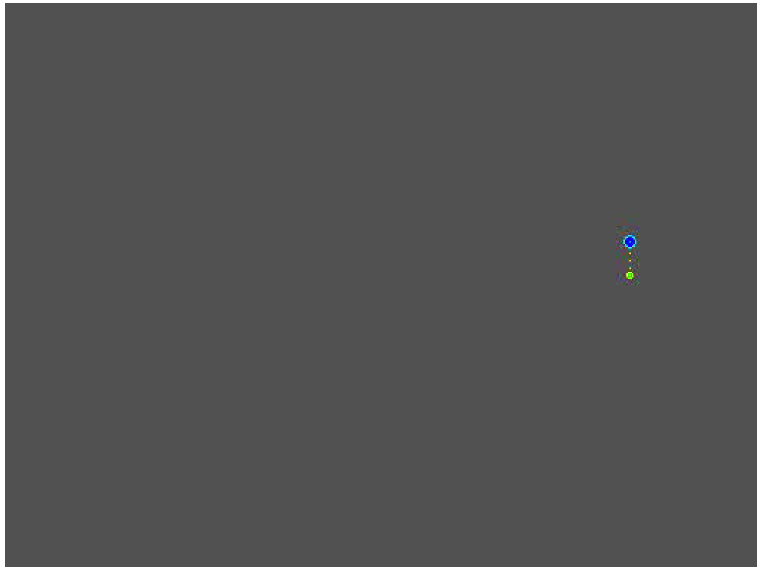
Individual Charge Trajectories

- **Combined homogeneous electric and magnetic field**

$$\mathbf{v} = \mathbf{v}_{gc} + \mathbf{v}_g$$: **Guiding centre + Gyro motion**





Individual Charge Trajectories

- **Combined homogeneous electric and magnetic field**

$$\mathbf{v} = \mathbf{v}_{gc} + \mathbf{v}_g$$ **: Guiding centre + Gyro motion**

2.2 IN MAGNETIC FIELDS 17

Because of this change of momentum, the *centre of curvature* of the projection on the *xy*-plane of the path is displaced a distance

$$\mathbf{D} = \boldsymbol{\rho}' - \boldsymbol{\rho} \tag{9}$$

or, because of (6)–(8),

$$\mathbf{D} = -\frac{c}{eH^2}[\mathbf{H}\Delta\mathbf{p}_\perp], \tag{10}$$

$$\mathbf{D} = -\frac{c}{eH^2}\left[\mathbf{H}\int \mathbf{f}_\perp\, dt\right]. \tag{11}$$

This formula holds for a single collision, but of course also for a series of collisions. If **f** is a continuous force, **D** gives the displacement of that point where the centre of curvature would be if **f** vanished for a moment. This point shall be called the *guiding centre*. If **f** is continuous, *the guiding centre drifts with the velocity*

$$\mathbf{U}_\perp = \frac{d\mathbf{D}}{dt} = -\frac{c}{eH^2}[\mathbf{H}\mathbf{f}]. \tag{12}$$

H. Alfvén, Cosmical Electrodynamics, Oxford (1950)

Hannes Alfvén (1908-1995)

"Nobel prize in Physics (1970)"

WIKIPEDIA

Individual Charge Trajectories

- **Combined homogeneous electric and magnetic field**

$$\mathbf{v} = \mathbf{v}_{gc} + \mathbf{v}_{g}$$ **: Guiding centre + Gyro motion**

$$m\frac{d\mathbf{v}}{dt} = \mathbf{F} + q\left(\mathbf{v}\times\mathbf{B}\right)$$

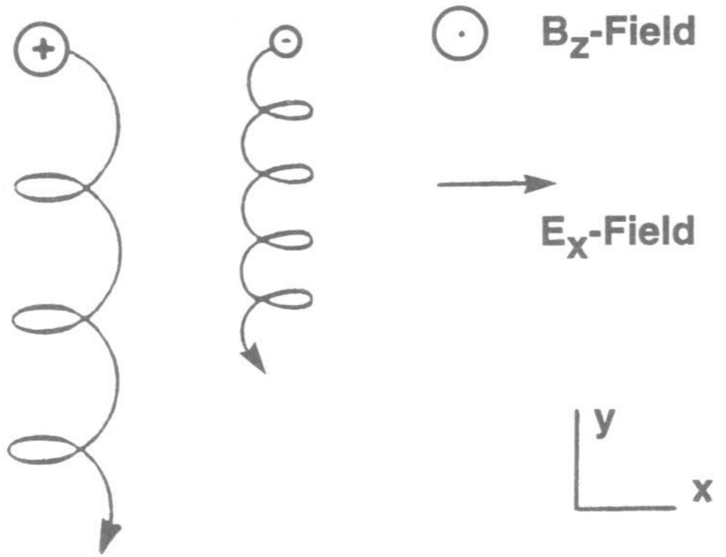
$$m\frac{d\mathbf{v}_{gc}}{dt} + m\frac{d\mathbf{v}_{g}}{dt} = \mathbf{F} + q\left(\mathbf{v}_{gc}\times\mathbf{B}\right) + q\left(\mathbf{v}_{g}\times\mathbf{B}\right)$$

$$v_{gc,\parallel}(t) = v_{gc,\parallel}(0) + \frac{1}{m}\int F_{\parallel}dt$$

$$\overline{\mathbf{v}}_{gc,\perp} = \frac{\overline{\mathbf{F}}\times\mathbf{B}}{qB^2} = \mathbf{v}_{DF}$$

$$\mathbf{F} = \mathbf{E}q$$

$$\mathbf{v}_{D,E} = \frac{\mathbf{E}\times\mathbf{B}}{B^2}$$



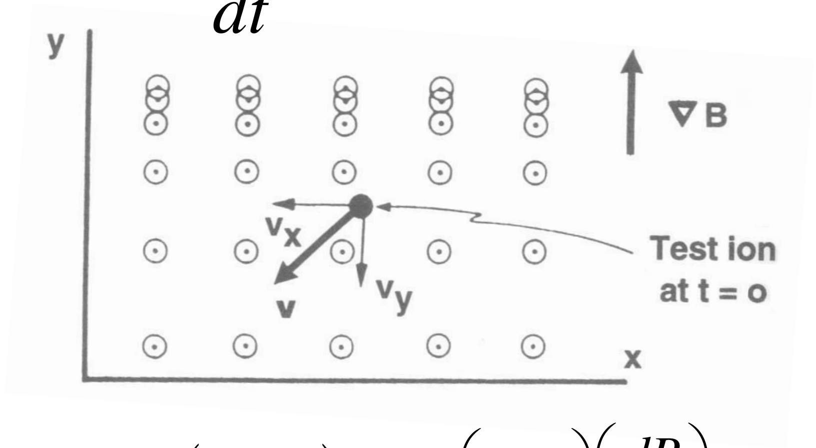
A. A. Harms et al, "Principles of Fusion Energy", World Scientific (2000)

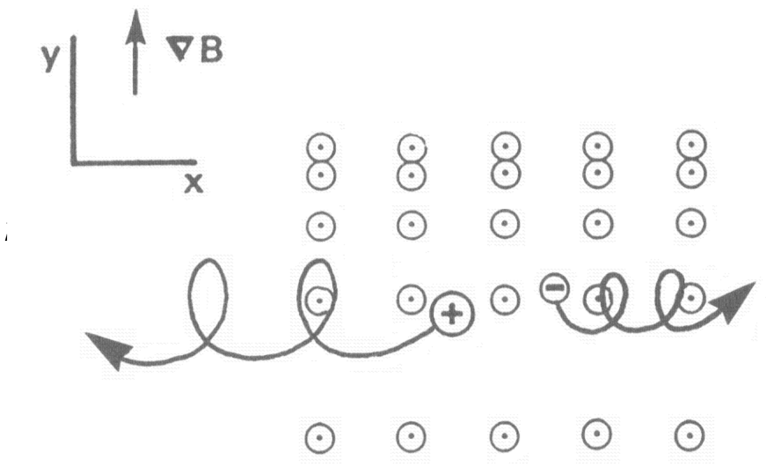
Individual Charge Trajectories

- **Spatially varying magnetic field**

$$m\frac{d\mathbf{v}}{dt} = q[\mathbf{v}\times\mathbf{B}(r)]$$

$$\nabla B = \frac{\partial B_z}{\partial y}\mathbf{j}, \quad v_\parallel = 0$$





$$\overline{\mathbf{F}} = q\left(\vec{v}_\perp\times\vec{B}\right) \approx qv_\perp\left(-\frac{r_L}{2}\right)\left(\frac{dB}{dy}\right)\mathbf{j}$$

$$= -|q|\left(\frac{v_\perp^2}{2\omega_c}\right)\left(\frac{dB}{dy}\right)\mathbf{j} = -|q|\frac{v_\perp^2}{2\omega_c}\nabla B$$

$$= -\frac{|q|}{2}v_\perp r_L \nabla B = -\frac{mv_\perp^2/2}{B}\nabla B$$

$$\mathbf{v}_{D,\nabla B} = \frac{\overline{\mathbf{F}}\times\mathbf{B}}{qB^2}$$

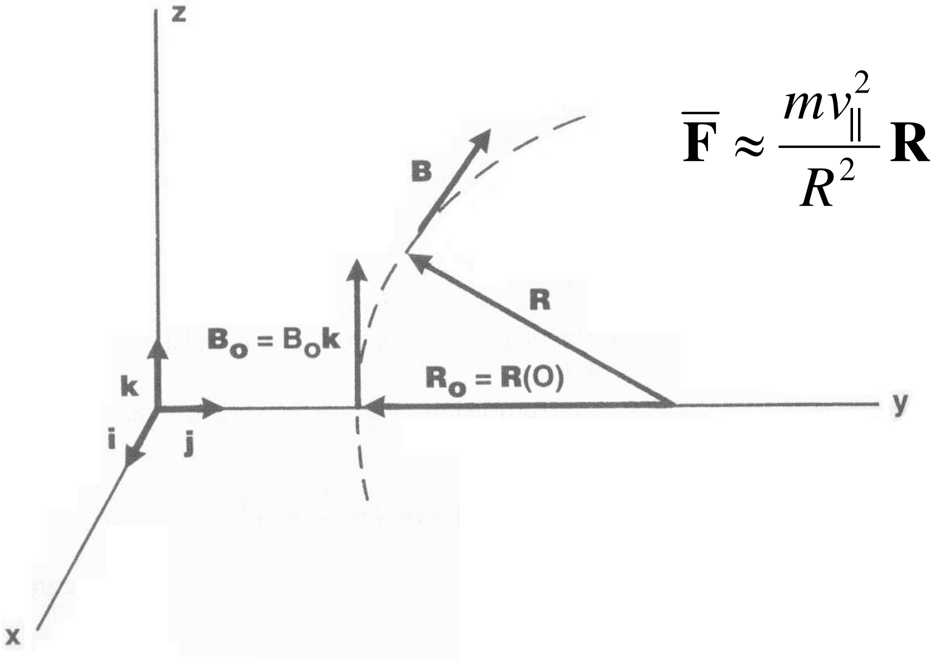
$$= \pm\frac{v_\perp^2}{2\omega_c}\frac{\mathbf{B}\times\nabla B}{B^2} = \pm\frac{1}{2}v_\perp r_L\frac{\mathbf{B}\times\nabla B}{B^2}$$

A. A. Harms et al, "Principles of Fusion Energy", World Scientific (2000)

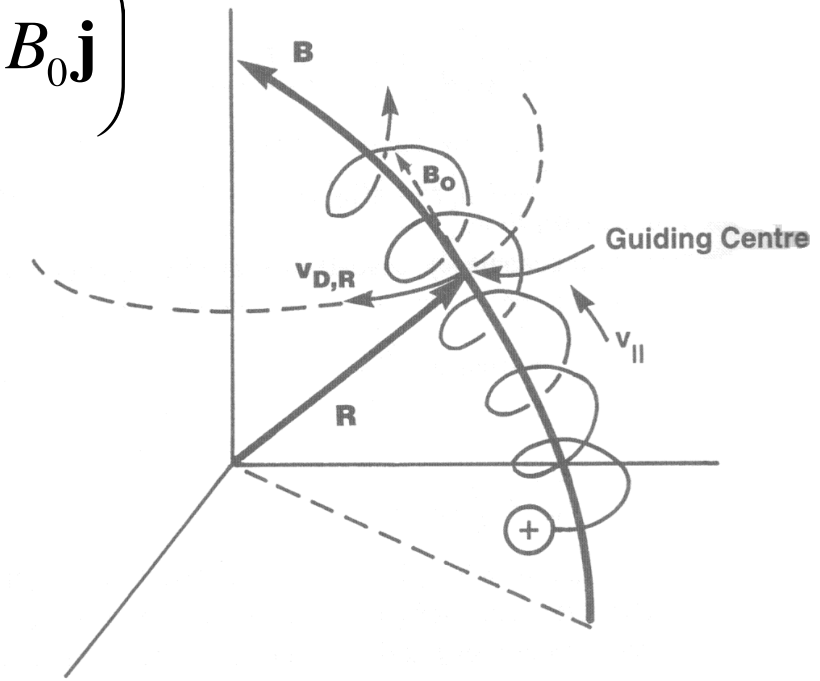
Individual Charge Trajectories

- **Curvature drift**

$$m\frac{d\mathbf{v}}{dt} = q(\mathbf{v}\times\mathbf{B}) = q(\mathbf{v}\times B_0\mathbf{k}) + q\left(\mathbf{v}\times\frac{z}{R}B_0\mathbf{j}\right)$$



$$\overline{\mathbf{F}} \approx \frac{mv_{\parallel}^2}{R^2}\mathbf{R}_0$$



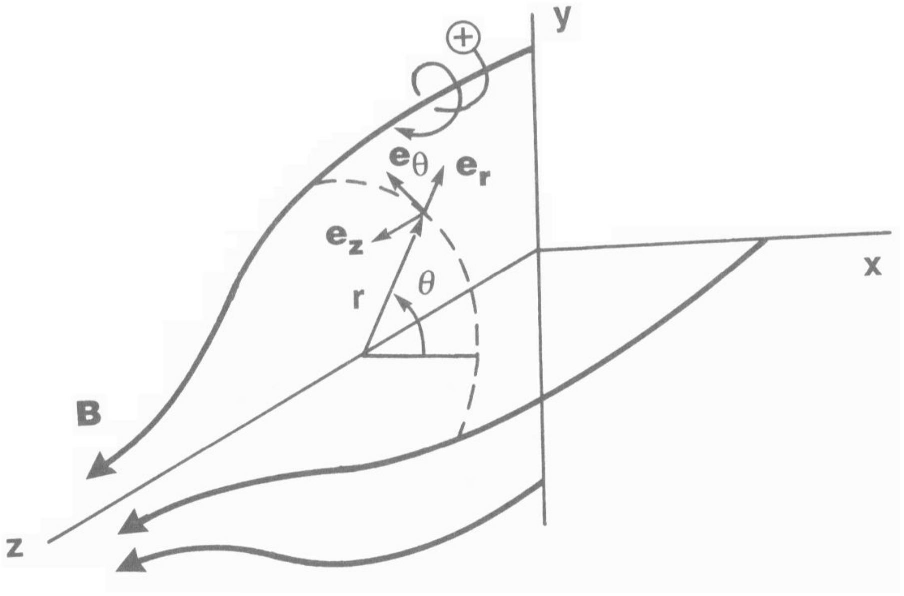
$$\mathbf{v}_{D,R} = \overline{\mathbf{v}}_{gc,x} = \frac{mv_{\parallel}^2}{qB_0^2}\frac{\mathbf{R}_0\times\mathbf{B}_0}{R^2}$$

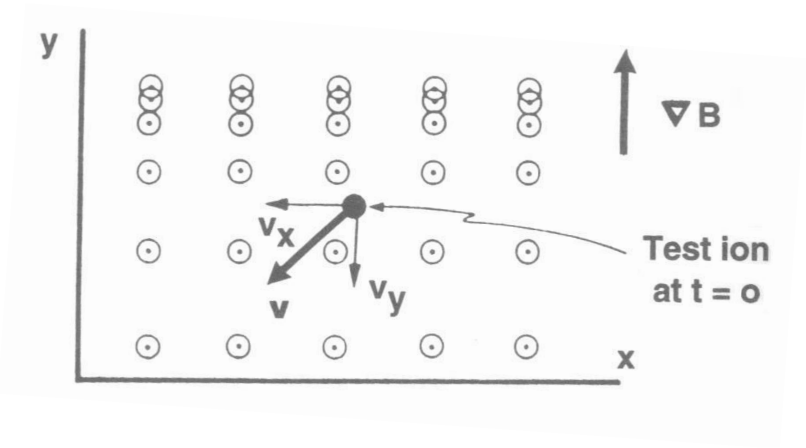
A. A. Harms et al, "Principles of Fusion Energy", World Scientific (2000)

Individual Charge Trajectories

- **Axial field variation**

$$m\frac{d\mathbf{v}}{dt} = q\left(\mathbf{v}\times B_z\mathbf{e}_z\right) + q\left(\mathbf{v}\times B_r\mathbf{e}_r\right)$$





$$\overline{\mathbf{F}} = q\left(\vec{v}_\perp\times\vec{B}\right) \approx qv_\perp\left(-\frac{r_L}{2}\right)\left(\frac{dB}{dy}\right)\mathbf{j}$$

$$= -|q|\left(\frac{v_\perp^2}{2\omega_c}\right)\left(\frac{dB}{dy}\right)\mathbf{j} = -|q|\frac{v_\perp^2}{2\omega_c}\nabla B$$

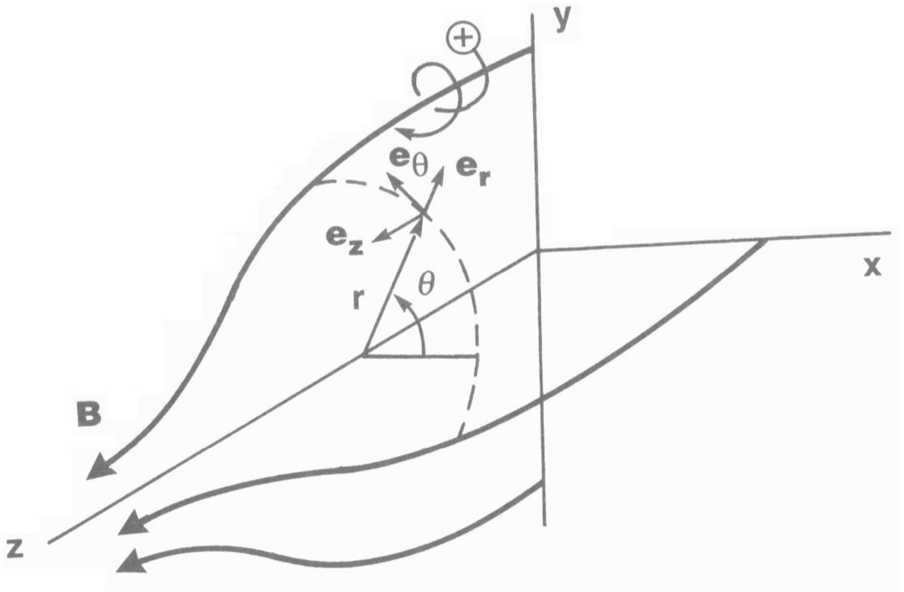
$$= -\frac{|q|}{2}v_\perp r_L\nabla B = -\frac{mv_\perp^2/2}{B}\nabla B$$

A. A. Harms et al, "Principles of Fusion Energy", World Scientific (2000)

Individual Charge Trajectories

- **Axial field variation**

$$m\frac{d\mathbf{v}}{dt} = q\left(\mathbf{v}\times B_z\mathbf{e}_z\right) + q\left(\mathbf{v}\times B_r\mathbf{e}_r\right)$$



$$\nabla\cdot\mathbf{B} = 0 \rightarrow B_r \approx -\frac{r}{2}\frac{\partial B_z}{\partial z} \qquad B_\theta = 0,\quad B_r(r=0) = 0$$

$$\mathbf{F}_\parallel = -\frac{1}{2}\frac{mv_\perp^2}{B}\nabla_\parallel B = -\mu\nabla_\parallel B$$

μ : magnetic moment of the gyrating particle

A. A. Harms et al, "Principles of Fusion Energy", World Scientific (2000)

Individual Charge Trajectories

- **Invariant of motion**

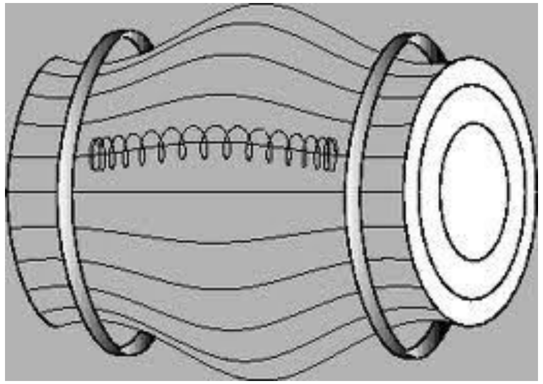
$$\frac{d}{dt}E_0 = \frac{d}{dt}\left(\frac{1}{2}mv_\perp^2 + \frac{1}{2}mv_\parallel^2\right) = 0 \qquad \mu = \frac{mv_\perp^2/2}{B} \qquad m\frac{dv_\parallel}{dt} = -\frac{\mu}{v_\parallel}\frac{dB}{dt} \qquad mv_\parallel\frac{dv_\parallel}{dt} = -\mu\frac{dB}{dt}$$

$$\mathbf{F}_\parallel = m\frac{d\mathbf{v}_\parallel}{dt} = -\mu\nabla_\parallel\mathbf{B} = -\mu\frac{\partial\mathbf{B}}{\partial s} = -\mu\frac{\partial\mathbf{B}}{\partial s}\cdot\frac{ds}{dt}\cdot\frac{dt}{ds} = -\mu\frac{\partial\mathbf{B}}{\partial s}\cdot\frac{ds}{dt}\cdot\frac{1}{v_\parallel} = -\frac{\mu}{v_\parallel}\frac{d\mathbf{B}}{dt} \longrightarrow \frac{d}{dt}\left(\frac{1}{2}mv_\parallel^2\right) = -\mu\frac{dB}{dt}$$

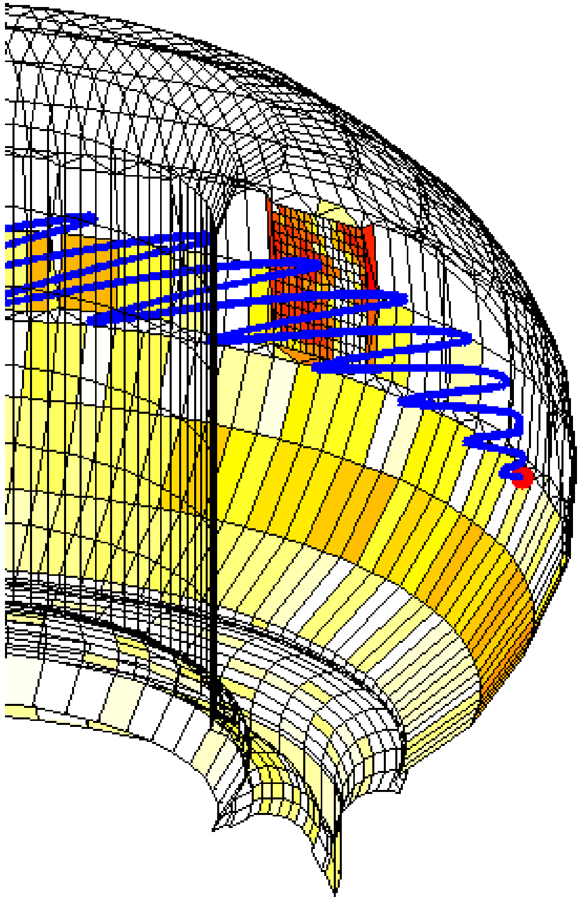
$$\longrightarrow \frac{d}{dt}\left(\frac{1}{2}mv_\perp^2 + \frac{1}{2}mv_\parallel^2\right) = \frac{d}{dt}(\mu B) + \left(-\mu\frac{dB}{dt}\right) = 0$$

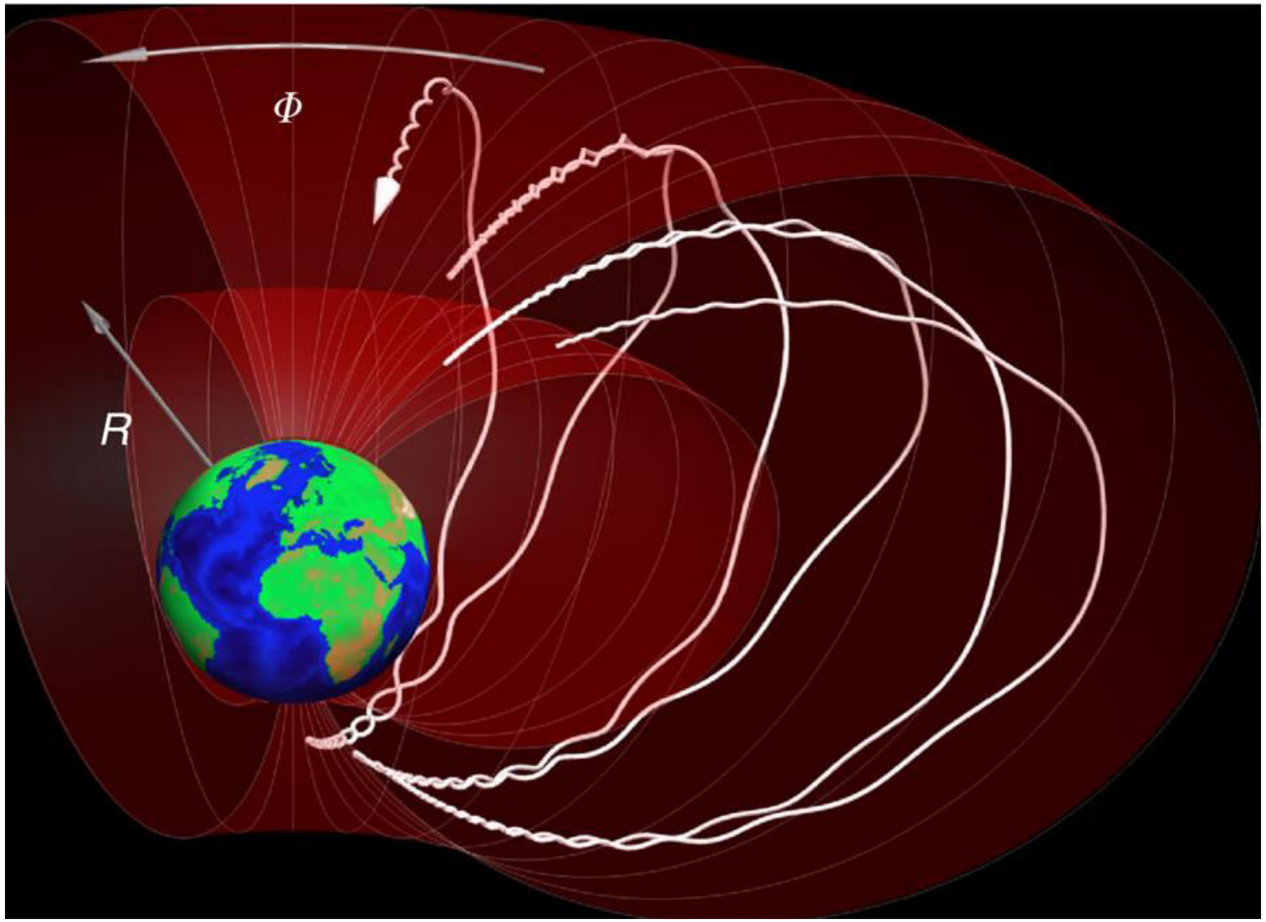
$$\frac{d}{dt}(\mu B) = \mu\frac{dB}{dt} + B\frac{d\mu}{dt}$$

$$\longrightarrow \frac{d\mu}{dt} = 0$$: adiabatic invariant



Individual Charge Trajectories

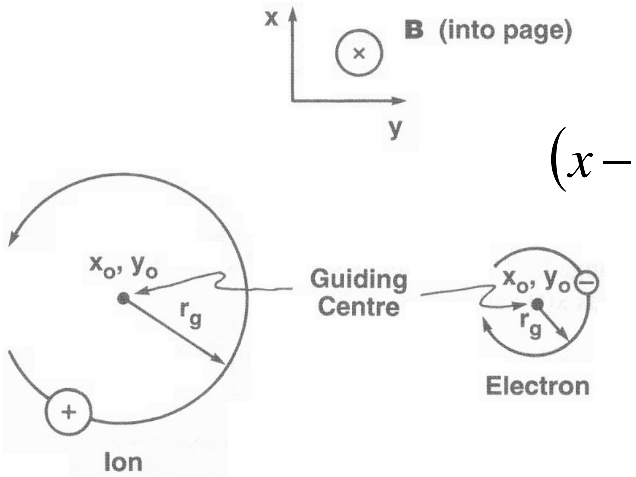




http://www.physics.ucla.edu/icnsp/Html/spong/spong.htm

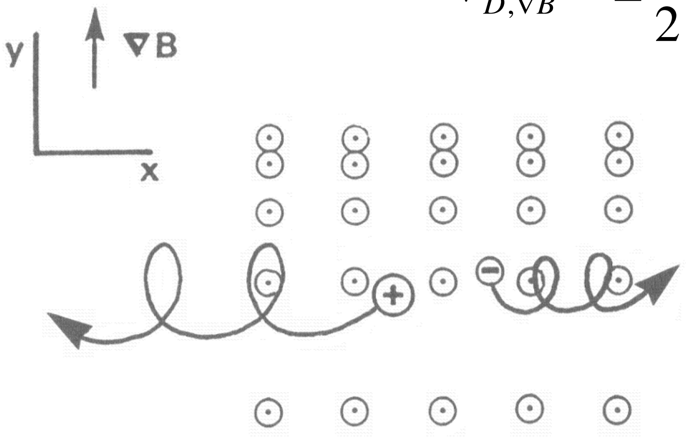
J. P. Graves et al, Nature Communications ***3*** *624 (2012)*

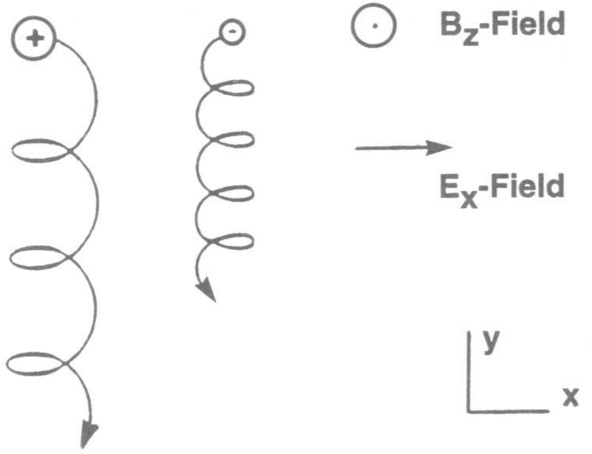
Individual Charge Trajectories



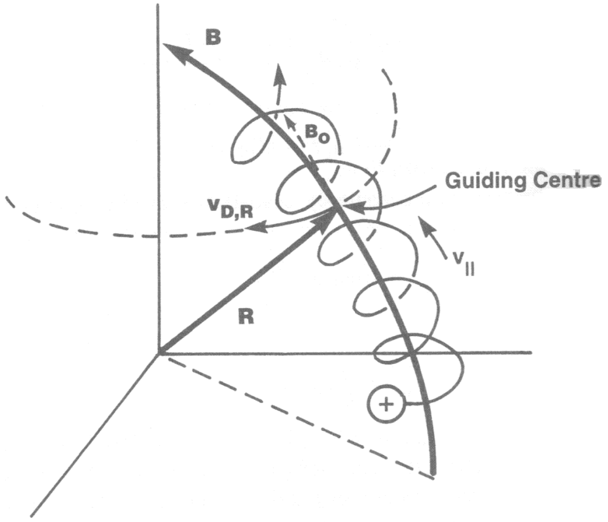
$$\left(x-x_0\right)^2+\left(y-y_0\right)^2=r_L^2$$

$$\mathbf{v}_{D,\nabla B}=\pm\frac{1}{2}v_\perp r_L\frac{\mathbf{B}\times\nabla B}{B^2}$$





$$\mathbf{v}_{D,E}=\frac{\mathbf{E}\times\mathbf{B}}{B^2}$$



$$\mathbf{v}_{D,R}=\frac{mv_\parallel^2}{qB_0^2}\frac{\mathbf{R}_0\times\mathbf{B}_0}{R^2}$$