Introduction to Nuclear Fusion

Prof. Dr. Yong-Su Na

How to describe a plasma?

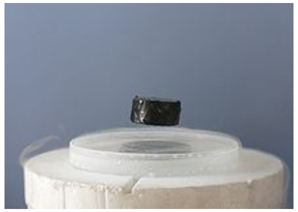
Plasmas as Fluids

- **Ideal MHD**

- Single-fluid model

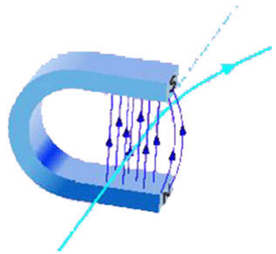
- Ideal:

  Perfect conductor with zero resistivity



- MHD:

  Magnetohydrodynamic (magnetic fluid dynamic)

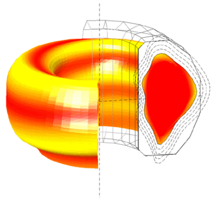


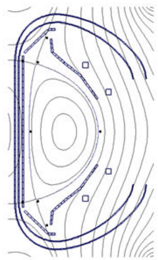
- Assumptions:

  Low-frequency, long-wavelength
  collision-dominated plasma

- Applications:

  Equilibrium and stability in fusion plasmas





$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} = 0$$

$$\rho \frac{d\vec{v}}{dt} = \vec{J} \times \vec{B} - \nabla p$$

$$\frac{d}{dt}\left(\frac{p}{\rho^{\gamma}}\right) = 0$$

$$\vec{E} + \vec{v} \times \vec{B} = 0$$

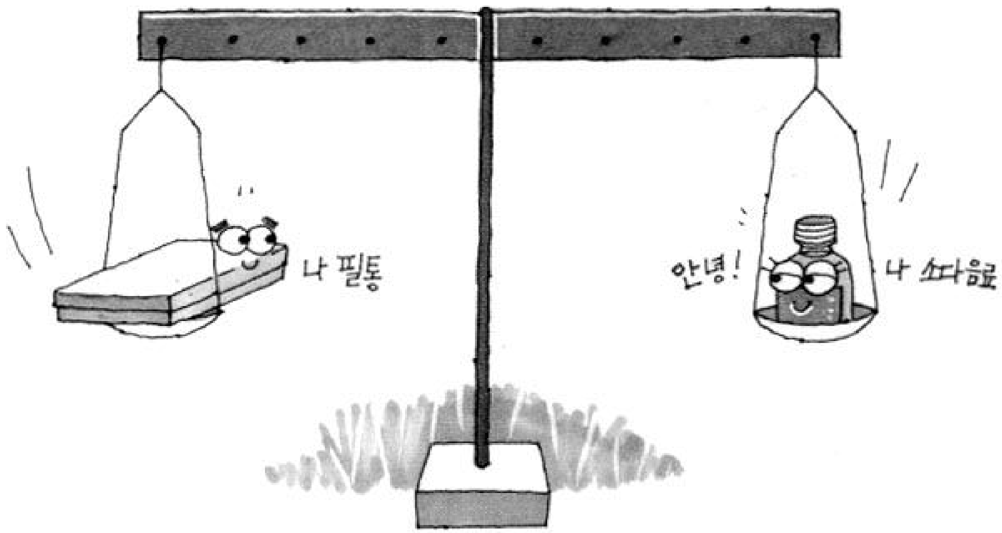
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\nabla \cdot \vec{B} = 0$$

What is plasma equilibrium?

Equilibrium and Stability



Equilibrium?   Yes! Forces are balanced

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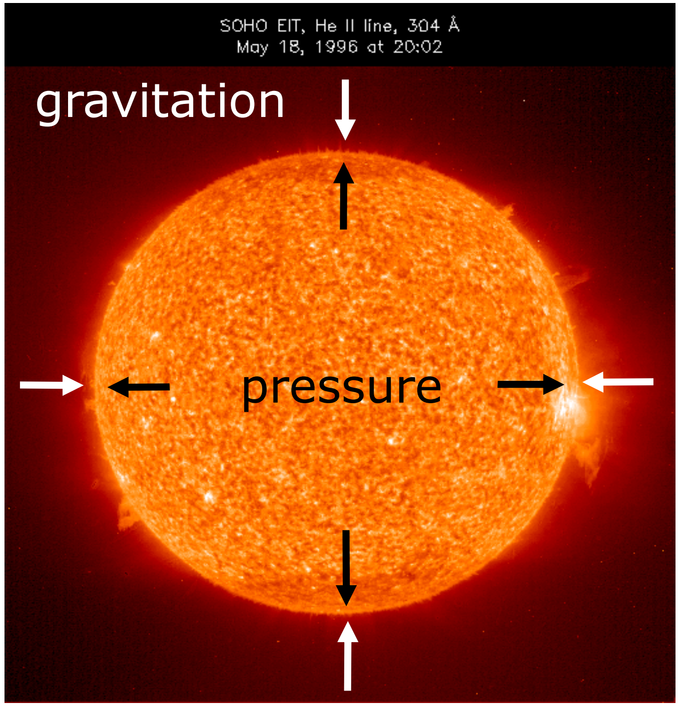
Equilibrium and Stability



Equilibrium in the sun

We need a fusion device which confines the plasma particles to some region for a sufficient time period by making equilibrium.

Equilibrium

- **Basic Equations**

- MHD equilibrium equations:
time-independent with **v** = 0 (static)

$$\boxed{\begin{array}{c}\nabla p = \vec{J}\times\vec{B} \\ \nabla\times\vec{B} = \mu_0\vec{J} \\ \nabla\cdot\vec{B} = 0\end{array}}$$

→ Force balance

→ Ampere's law

→ Closed magnetic field lines

$$\cancel{\frac{\partial\rho}{\partial t}} + \nabla\cancel{\cdot\rho\vec{v}} = 0$$

$$\rho\cancel{\frac{d\vec{v}}{dt}} = \vec{J}\times\vec{B} - \nabla p$$

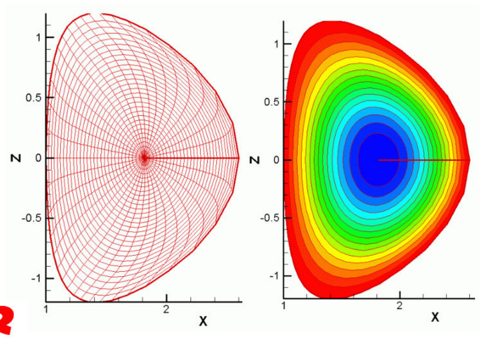
$$\cancel{\frac{d}{dt}\left(\frac{p}{\rho^\gamma}\right)} = 0$$

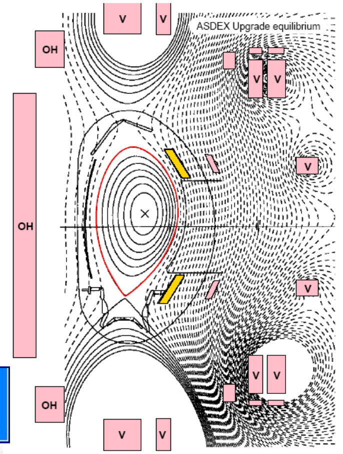
$$\vec{E} + \cancel{\vec{v}\times\vec{B}} = 0$$

$$\nabla\times\cancel{\vec{E}} = -\cancel{\frac{\partial\vec{B}}{\partial t}}$$

$$\nabla\times\vec{B} = \mu_0\vec{J}$$

$$\nabla\cdot\vec{B} = 0$$





ASDEX Upgrade

IPP

KSTAR

Magnetic and Kinetic Pressure

- **Plasma Equilibrium**

$$\boxed{\begin{aligned}\nabla p &= \vec{J}\times\vec{B} \\ \nabla\times\vec{B} &= \mu_0\vec{J} \\ \nabla\cdot\vec{B} &= 0\end{aligned}}$$

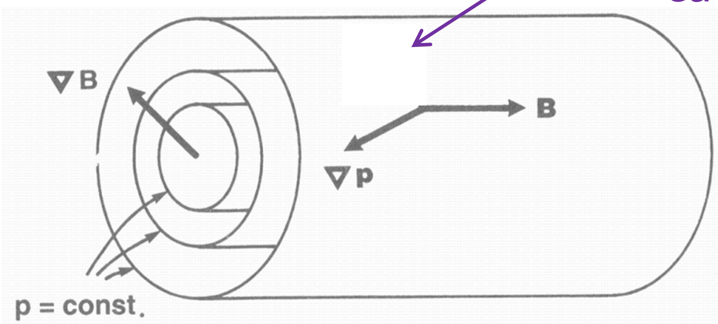
→ Force balance — kinetic pressure balanced by **J**x**B** (Lorentz) force

→ Ampere's law

→ Closed magnetic field lines

$\vec{B}\cdot\nabla p = 0 \quad \vec{J}\cdot\nabla p = 0$

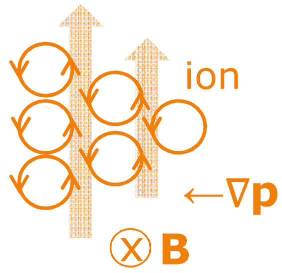
induced by the pressure gradient:
causing a decrease in **B** → diamagnetism



Diamagnetic current

$$\vec{v}_{D,\nabla p} = -\frac{\nabla p\times\vec{B}}{nqB^2}$$

$$\vec{J} = n_i q_i \vec{v}_{D,i} + n_e q_e \vec{v}_{D,e} = \frac{\vec{B}\times\nabla p}{B^2}$$



- If B is applied, plasma equilibrium can be built by itself due to induction of diamagnetic current. $\nabla p = \vec{J}\times\vec{B}$

Magnetic and Kinetic Pressure

- **Plasma Equilibrium**

$$\boxed{\begin{aligned} \nabla p &= \vec{J}\times\vec{B} \\ \nabla\times\vec{B} &= \mu_0\vec{J} \\ \nabla\cdot\vec{B} &= 0 \end{aligned}}$$

→ Force balance — kinetic pressure balanced by **J**x**B** (Lorentz) force

→ Ampere's law

→ Closed magnetic field lines

$$\nabla p = (\nabla\times B)\times B/\mu_0$$

$$= [(B\cdot\nabla)B - \nabla(B^2/2)]/\mu_0$$

$$\nabla(p + B^2/2\mu_0) = \cancel{(B\cdot\nabla)B/\mu_0}$$

Assuming the field lines are straight and parallel

$$\frac{E^*_{mag}}{V} = \frac{BH}{2} = \frac{B^2}{2\mu_0}$$

$$p + \frac{B^2}{2\mu_0} = \text{constant}$$

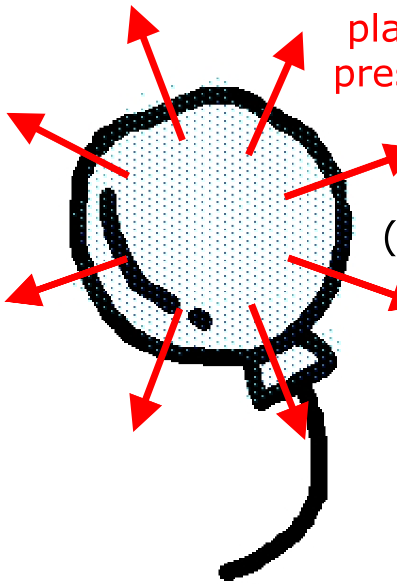
Total sum of kinetic pressure and magnetic field energy density (magnetic pressure) will be a constant

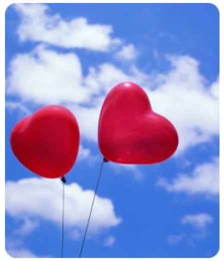
Magnetic and Kinetic Pressure

- **Concept of Beta**

$$\beta = \frac{p}{B^2/2\mu_0} = \frac{(n_i + n_e)kT}{B^2/2\mu_0}$$

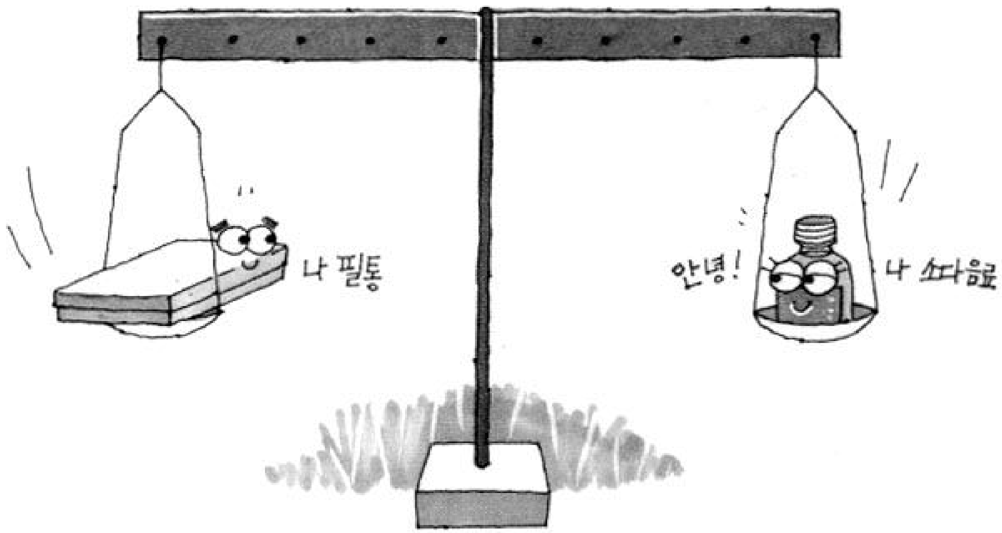
- The ratio of the plasma pressure to the magnetic field pressure
- A measure of the degree to which the magnetic field is holding a non-uniform plasma in equilibrium.
- In most magnetic configurations, fusion plasma confinement requires an imposed magnetic pressure significantly exceeding the particle kinetic pressure.





What is plasma stability?

Equilibrium and Stability



Equilibrium? Yes! Forces are balanced

Stable? No!

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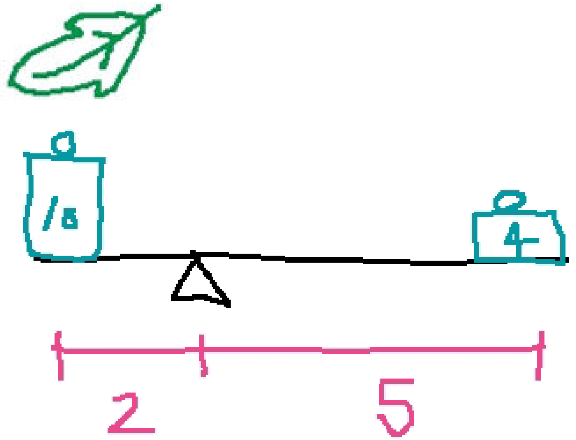
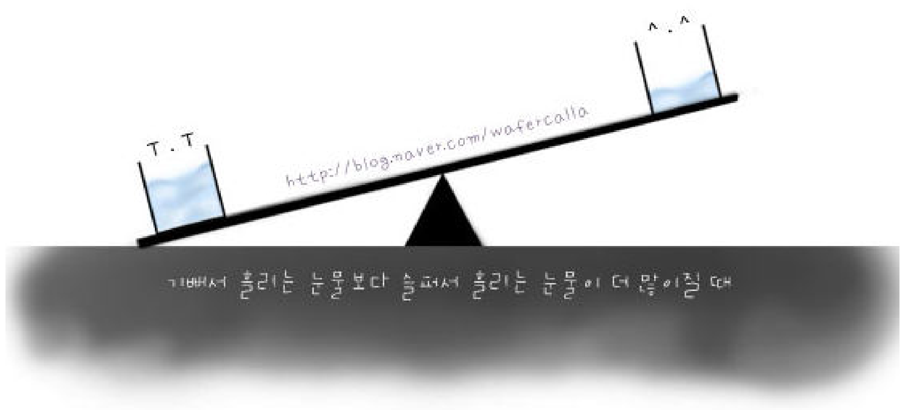
Equilibrium and Stability



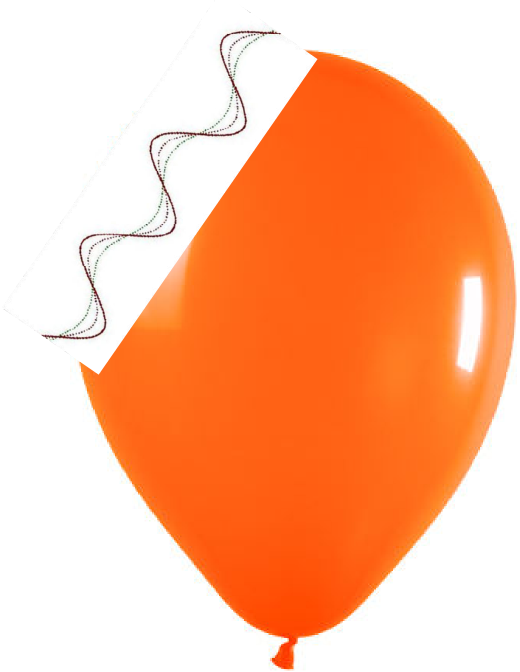


Equilibrium? Yes! Forces are balanced

Stable? No! The system cannot recover.

We need a fusion device which confines the plasma particles to some region for a sufficient time period in a stable way.

Stability

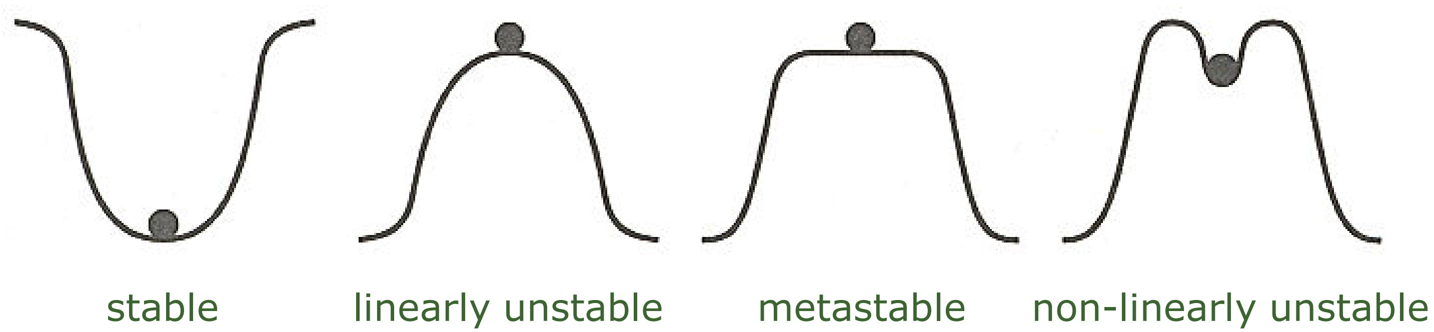


http://www.amazon.co.uk/11Inch-Latex-Orange-Wedding-Balloons/dp/B004JUQG4Q
http://www.psdgraphics.com/backgrounds/blue-water-drop-background/

Stability

- **Definition of Stability**

  - A small change (disturbance) of a physical system at some instant changes the behavior of the system only slightly at all future times *t*.
  - The fact that one can find an equilibrium does not guarantee that it is stable. Ball on hill analogies:

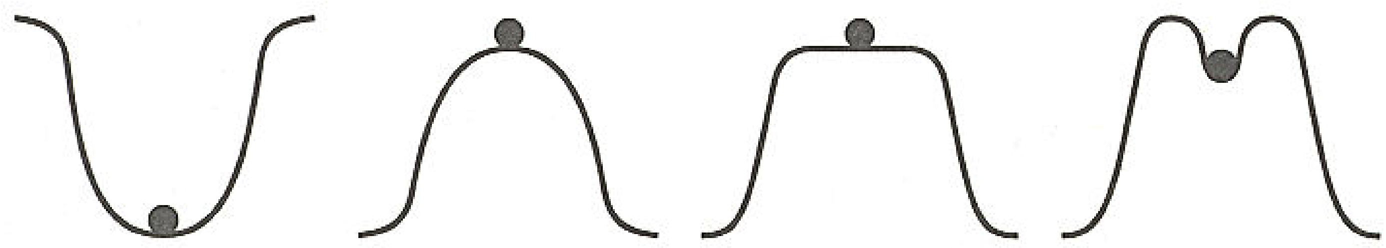


linear: with small perturbation
non-linear: with large perturbation

  - Generation of instability is the general way of redistributing energy which was accumulated in a non-equilibrium state.

Stability

- **Definition of Stability**



- Assuming all quantities of interest linearised about their equilibrium values.

$$Q(\vec{r},t) = Q_0(\vec{r}) + \tilde{Q}_1(\vec{r},t)$$ small 1st order perturbation $$\tilde{Q}_1 / |Q_0| << 1$$

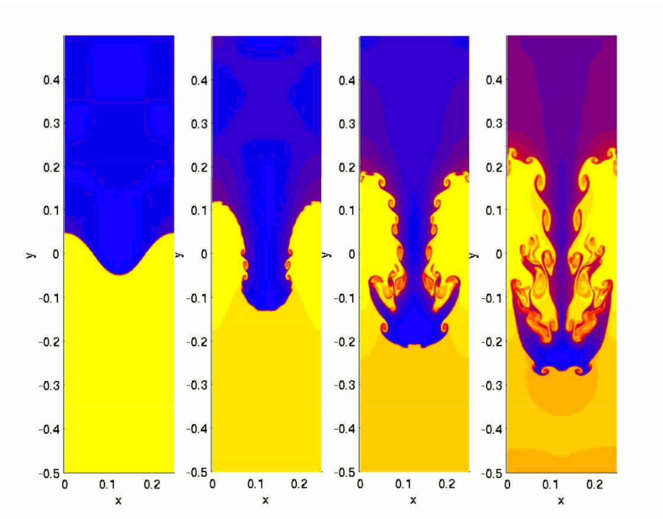
$$\tilde{Q}_1(\vec{r},t) = Q_1(\vec{r})e^{-i\omega t} = Q_1(\vec{r})e^{-i(\omega_r + i\omega_i)t} = Q_1(\vec{r})e^{-i\omega_r t}e^{\omega_i t} \qquad \omega = \omega_r + i\omega_i$$

Im $\omega > 0$ ($\omega_i > 0$): exponential instability

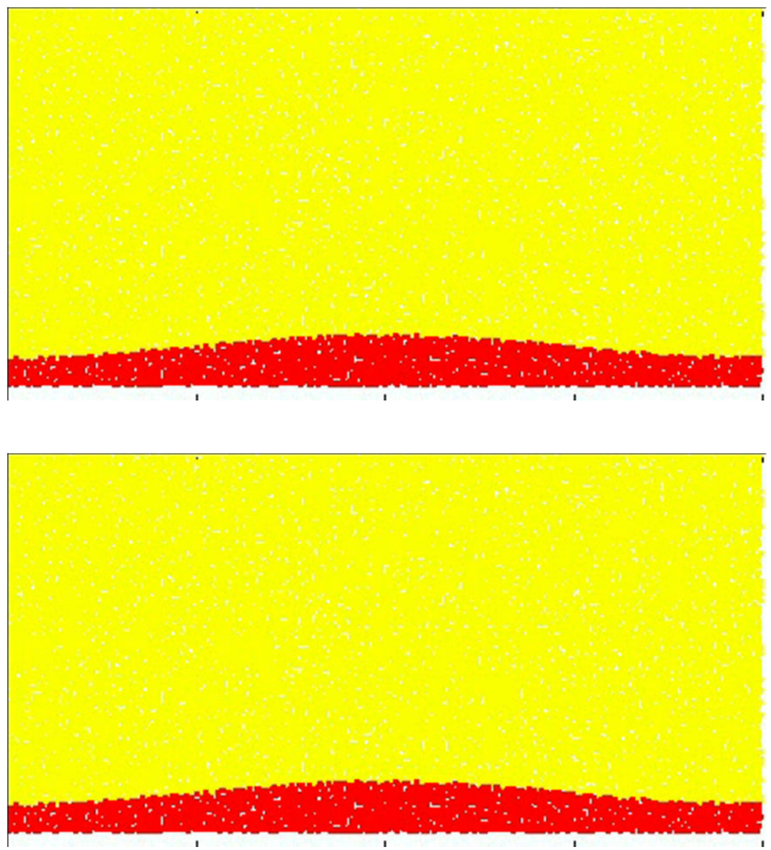
Im $\omega \leq 0$ ($\omega_i \leq 0$): exponential stability

Stability

- **Gravitational Instability**



Rayleigh-Taylor instability



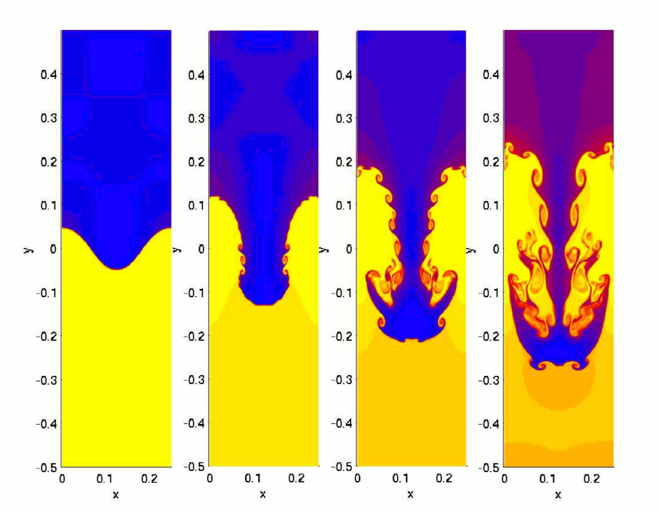
http://user.uni-frankfurt.de/~schmelin/presentations/Rayleigh-Taylornew.html

Stability

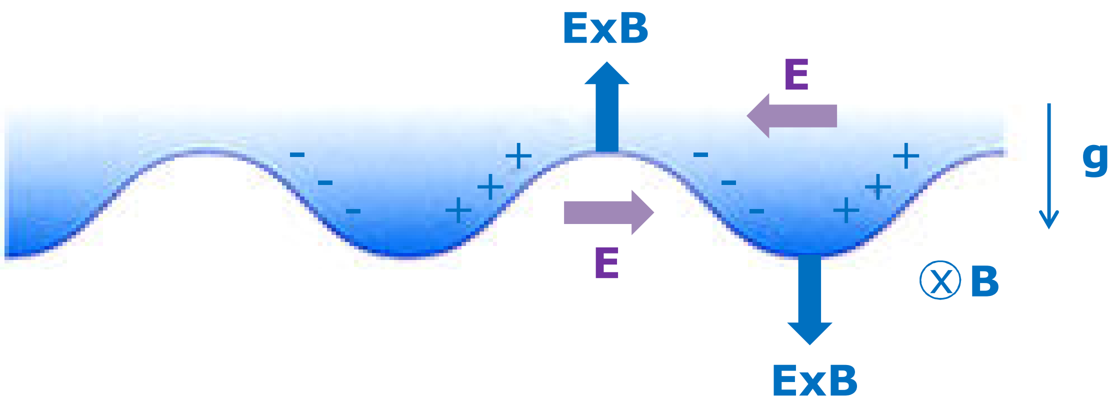
- **Gravitational Instability**

$$\mathbf{v}_{DF} = \frac{\overline{\mathbf{F}} \times \mathbf{B}}{qB^2}$$

$$\mathbf{v}_{D,g} = \frac{m\mathbf{g} \times \mathbf{B}}{qB^2} \qquad \mathbf{v}_{D,E} = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$



Rayleigh-Taylor instability



What is plasma transport?

Plasma Transport

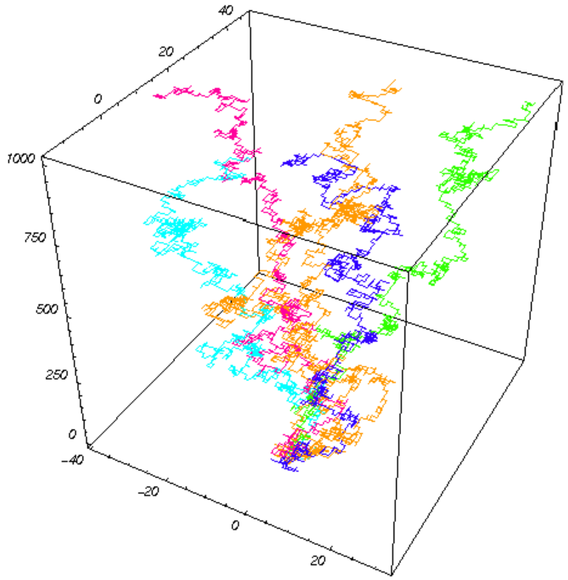
http://www.amazon.co.uk/11Inch-Latex-Orange-Wedding-Balloons/dp/B004JUQG4Q

Plasma Transport

- **Classical Transport**
  - Particle transport

    random walk: no net flux (zero average)

    with gradient: net flux down the gradient (diffusion)



http://functions.wolfram.com/Constants/Pi/visualizations/2/ShowAll.html

Plasma Transport

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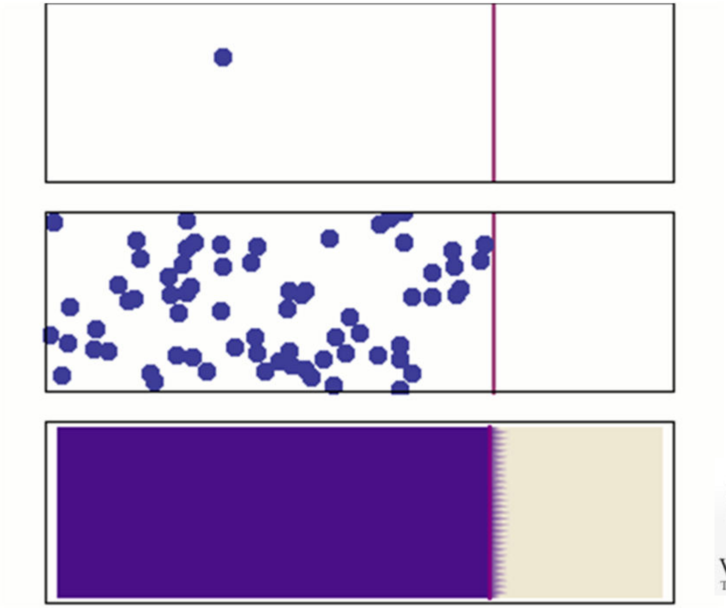
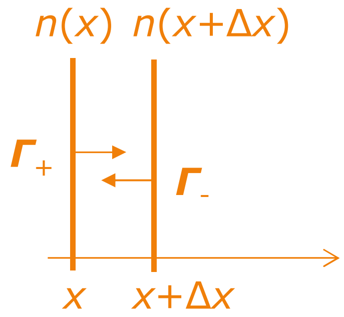


Plasma Transport

- **Classical Transport**
  - Particle transport

Particle flux: $\vec{\Gamma} = n\vec{v}$ $[\#/\mathrm{m}^2\mathrm{s}]$



$$\Gamma_+ = \frac{n(x)}{2}\frac{\Delta x}{\tau}, \quad \Gamma_- = \frac{n(x+\Delta x)}{2}\frac{\Delta x}{\tau}$$

$$\Gamma = \Gamma_+ - \Gamma_- = \frac{\Delta x}{2\tau}\left[n(x) - n(x+\Delta x)\right] \qquad n(x+\Delta x) \approx n(x) + \Delta x\frac{\partial n}{\partial x}$$

$$= -\frac{(\Delta x)^2}{2\tau}\frac{\partial n}{\partial x} = -D\frac{\partial n}{\partial x}$$: Fick's law

$$D = \frac{(\Delta x)^2}{2\tau}$$: diffusion coefficient (m^2/s)

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Adolf Eugen Fick
(1829-1901)

The heat and momentum fluxes can be estimated in the similar fashion.

Plasma Transport

- **Classical Transport**

  - Heat transport

Heat flux

$$q = -\kappa \frac{\partial T}{\partial x}$$: Fourier's law

$$\kappa \sim \frac{n(\Delta x)^2}{\tau} \sim nD$$: thermal conductivity

Jean-Baptiste
Joseph Fourier
(1768-1830)

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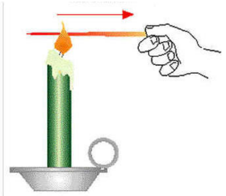
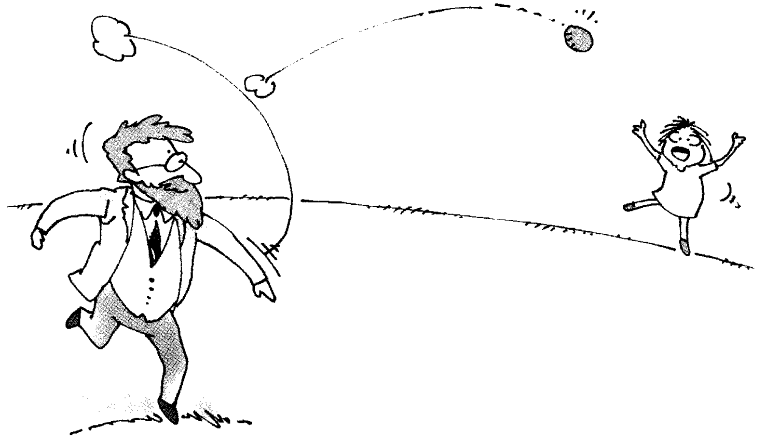
Plasma Transport

- **Classical Transport**

  - Heat transport





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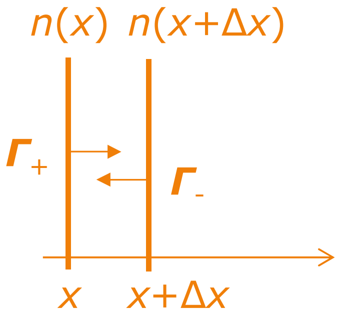
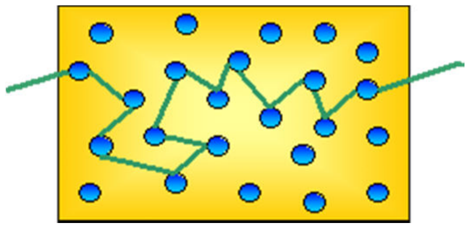
Plasma Transport

- **Classical Transport**
  - Particle transport in weakly ionised plasmas



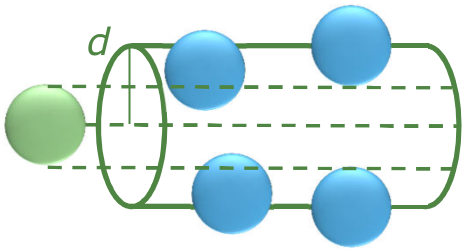
$$D = \frac{(\Delta x)^2}{2\tau}$$

Estimate transport coefficients: *Δx* from mean free path



$$\Delta x = \lambda_m = \frac{1}{n_n \sigma}$$

$$\frac{vt}{n_n \pi d^2 vt} = \frac{1}{n_n \pi d^2} = \frac{1}{n_n \sigma}$$: particle approach



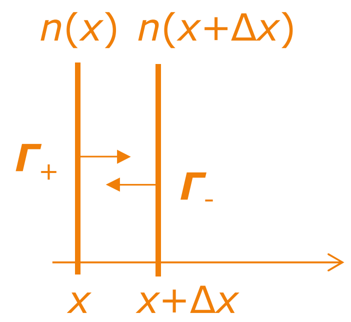
$$\Gamma = \Gamma_0 e^{-n_n \sigma x} \equiv \Gamma_0 e^{-x/\lambda_m}$$: fluid approach

$$d\Gamma = -\sigma n_n \Gamma dx$$

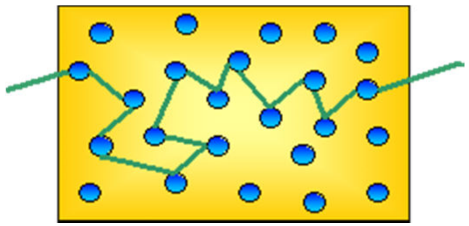
Plasma Transport

- **Classical Transport**
  - Particle transport in weakly ionised plasmas

$$D = \frac{(\Delta x)^2}{2\tau}$$



Estimate transport coefficients: τ from collision frequency with neutrals



Neutral particles

Plasma Transport

- **Classical Transport**
  - Particle transport in weakly ionised plasmas

$$n_\alpha m_\alpha\left(\frac{\partial \vec{v}_\alpha}{\partial t}+\vec{v}_\alpha\cdot\nabla\vec{v}_\alpha\right)=nq_\alpha\left(\vec{E}+\vec{v}_\alpha\times\vec{B}\right)-\nabla p_\alpha-n_\alpha m_\alpha \nu_{\alpha n}\left(\vec{v}_\alpha-\vec{v}_n\right)$$

$$0=nq_\alpha\vec{E}-kT_\alpha\nabla n_\alpha-n_\alpha m_\alpha\nu_{\alpha n}\vec{v}_\alpha$$

$$n_\alpha\vec{v}_\alpha=\frac{nq_\alpha\vec{E}}{m_\alpha\nu_{\alpha n}}-\frac{kT_\alpha}{m_\alpha\nu_{\alpha n}}\nabla n_\alpha$$

$$\vec{\Gamma}_\alpha=n_\alpha\vec{v}_\alpha=\pm\mu_\alpha n_\alpha\vec{E}-D_\alpha\nabla n_\alpha$$

$$\mu\equiv\frac{|q_\alpha|}{m_j\nu_{\alpha n}}$$: Mobility

$$D=\frac{kT_\alpha}{m_\alpha\nu_{\alpha n}}\sim v_{th}^2\tau\sim\frac{\lambda_m^2}{\tau}$$: Diffusion coefficient

Plasma Transport

- **Classical Transport**

  - Particle transport in weakly ionised plasmas

Ambipolar Diffusion

Faster electrons
slower ions → Charge separation → E-field induction

→ Electrons decelerated, ions accelerated → Electrons and ions diffuse together

$$\vec{\Gamma}_i = \vec{\Gamma}_e$$

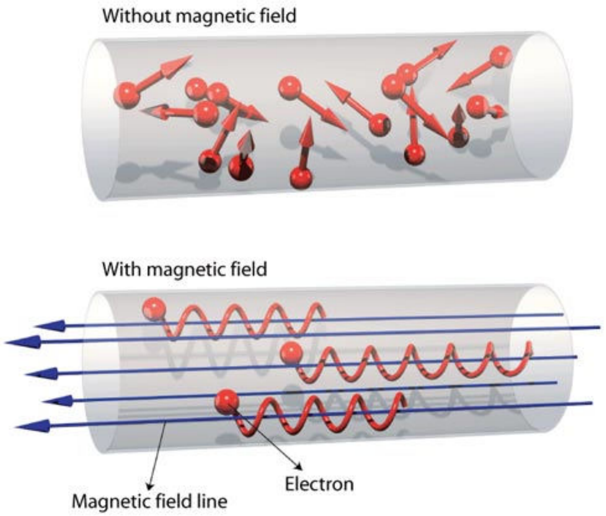
$$\vec{\Gamma} = -D_a \nabla n$$

$$D_a \equiv \frac{\mu_i D_e + \mu_e D_i}{\mu_i + \mu_e} \sim D_i + \frac{T_e}{T_i} D_i$$

Plasma Transport

- **Classical Transport**
  - Particle transport in weakly ionised plasmas with magnetic field

$$\vec{\Gamma}_{\perp\alpha} = n\vec{v}_{\perp\alpha} = \pm\mu_{\perp\alpha}n_\alpha\vec{E} - D_{\perp\alpha}\nabla n_\alpha + \frac{n(\vec{v}_E + \vec{v}_D)}{1+(\nu^2/\omega_c^2)}$$



$$\mu_\perp \equiv \frac{\mu}{1+\omega_c^2\tau^2}$$

$$\omega_c = \frac{|q|B}{m}$$

$$D_\perp = \frac{D}{1+\omega_c^2\tau^2} \sim \frac{kT\nu}{m_j\omega_c^2} \sim v_{th}^2\frac{r_L^2\nu}{v_{th}^2} \sim \frac{r_L^2}{\tau}$$

$$r_L = \frac{mv_\perp}{|q|B}$$

$$\mu \equiv \frac{|q_\alpha|}{m_\alpha\nu_{\alpha n}}$$

$$\vec{\Gamma}_\alpha = n_\alpha\vec{v}_\alpha = \pm\mu_\alpha n_\alpha\vec{E} - D_\alpha\nabla n_\alpha$$

$$D = \frac{kT_\alpha}{m_j\nu_{\alpha n}} \sim v_{th}^2\tau \sim \frac{\lambda_m^2}{\tau}$$

Plasma Transport

- **Classical Transport**
  - Particle transport in fully ionised plasmas with magnetic field

$$\vec{\Gamma}_\perp = n\vec{v}_\perp = -D_\perp \nabla n$$

$$D_\perp = \frac{\eta_\perp n \sum kT}{B^2}$$

τ from collision frequency

$$\nu_{ee} \approx \nu_{ei} \propto \frac{ne^4}{\sqrt{m_e} T_e^{3/2}}$$

$$\nu_{ie} = \left(\frac{m_e}{m_i}\right)\nu_{ee}$$

$$\nu_{ii} = \left(\frac{m_e}{m_i}\right)^{1/2}\left(\frac{T_e}{Ti}\right)^{3/2}\nu_{ee}$$