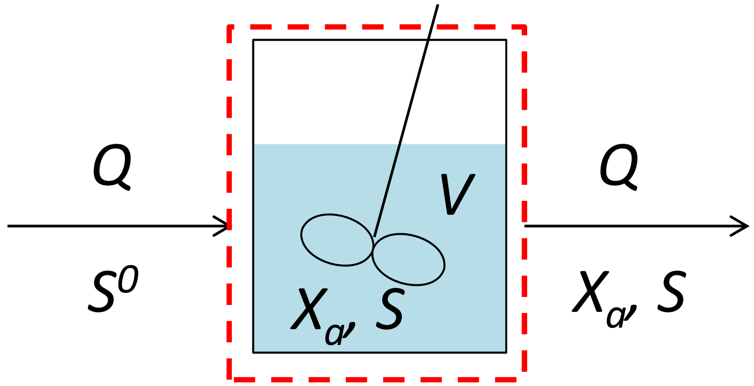
Microbial kinetics in reactors I

Today's lecture

- Master equations and key trends
- Including inert biomass
- Observed yield
- Soluble microbial products

CSTR, Monod: Master equations



Assumptions:

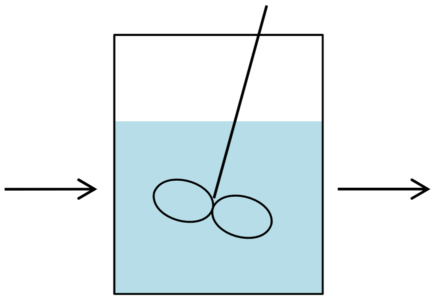
- Steady state
- $X_a = 0$ in the influent
  (negligible influent active biomass)

$$S = K \frac{1 + b\theta}{Y\hat{q}\theta - (1 + b\theta)}$$

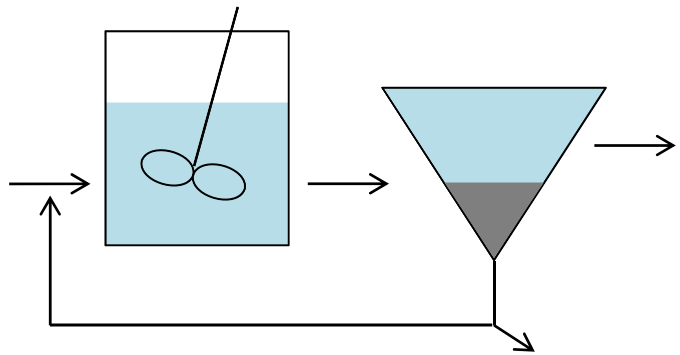
$$X_a = Y \frac{S^0 - S}{1 + b\theta}$$

HRT vs. SRT

- HRT (θ): **H**ydraulic **R**etention **T**ime; the average time the water stays in the system
- SRT (θ_x): **S**olids **R**etention **T**ime (or mean cell residence time, MCRT); the average time the biomass stays in the system



A CSTR without sludge return (chemostat): HRT = SRT



A CSTR with sludge return: HRT < SRT

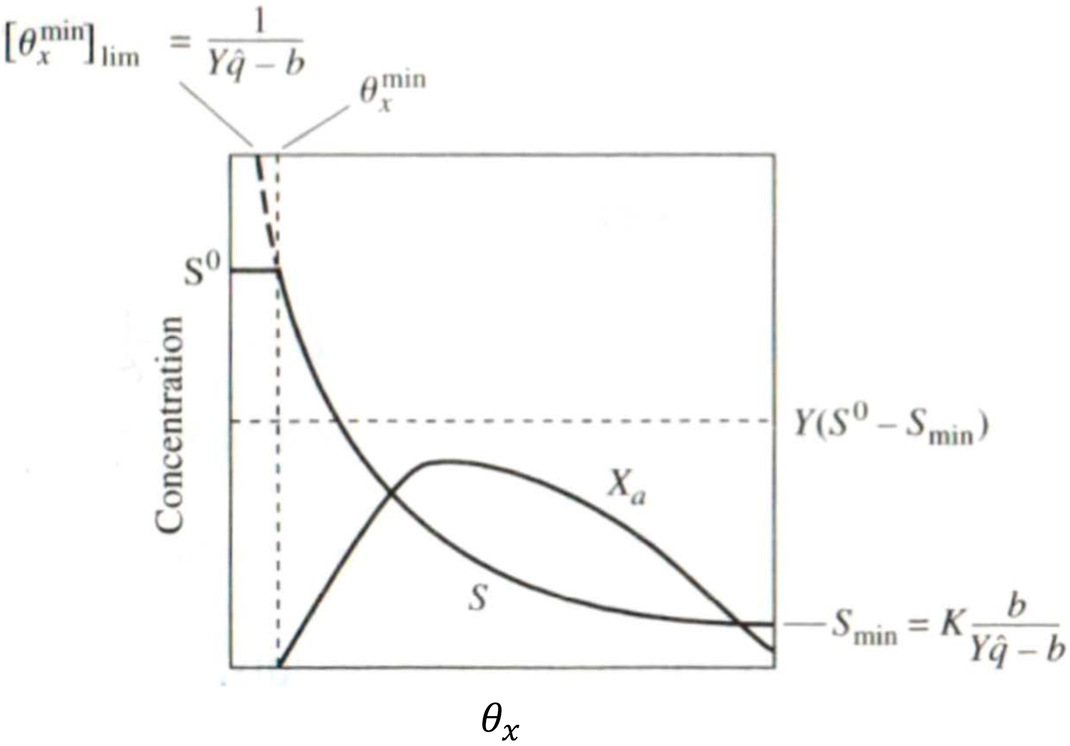
CSTR, Monod: Master equations

It is more useful (we'll see!) to substitute θ_x with θ:

$$S = K \frac{1 + b\theta_x}{Y\hat{q}\theta_x - (1 + b\theta_x)}$$

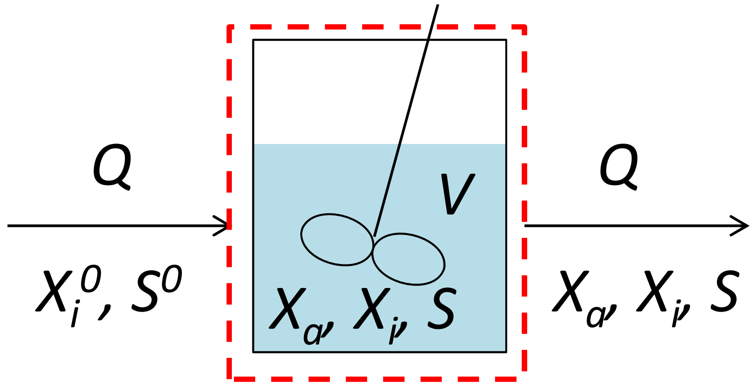
$$X_a = Y \frac{S^0 - S}{1 + b\theta_x}$$

θ_x vs. S: key trends



1) $\Theta_x \leq \Theta_x^{min}$: washout
2) $\Theta_x \rightarrow \infty$: $S = S_{min}$
3) For $\Theta_x^{min} < \Theta_x$, S decreases with increase in Θ_x, but X_a peaks at some point

Including inert biomass



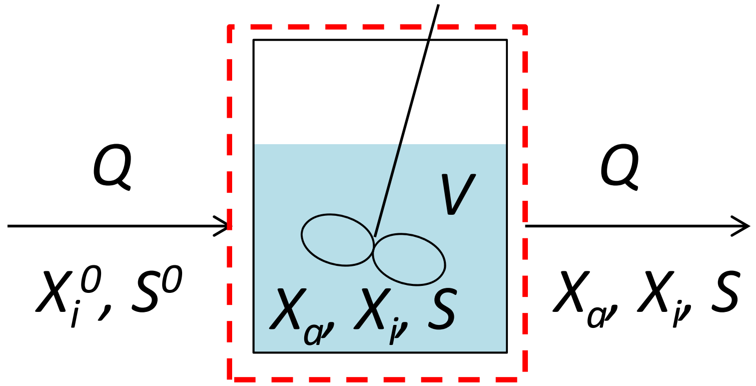
Influent contains some non-biodegradable, particulate organics: this is included when you measure VSS

Recall:

$$\left(\frac{1}{X_a}\frac{dX_a}{dt}\right)_{inert} = -\frac{1}{X_a}\frac{dX_i}{dt} = -(1-f_d)b$$

$$\frac{dX_i}{dt} = (1-f_d)bX_a$$

Including inert biomass: mass balance



Steady-state mass balance for X_i:

$$0 = QX_i^{\,0} - QX_i + (1 - f_d)bX_aV$$

$$\Rightarrow \quad X_i = X_i^{\,0} + (1 - f_d)bX_a\theta$$

Including inert biomass: solutions

Substituting θ_x with θ:

Solution for inert biomass: $X_i = {X_i}^0 + X_a(1 - f_d)b\theta_x$

Solution for total VSS:

$$X_v = X_i + X_a = {X_i}^0 + Y(S^0 - S)\frac{1 + (1 - f_d)b\theta_x}{1 + b\theta_x}$$

Observed yield

The VSS in a CSTR is calculated as:

VSS gained in the reactor

$$X_v = X_i + X_a = X_i^{\,0} + Y(S^0 - S)\frac{1 + (1 - f_d)b\theta_x}{1 + b\theta_x}$$

Influent inert biomass

substrate utilized in the reactor

The net yield in the CSTR is: $Y\dfrac{1 + (1 - f_d)b\theta_x}{1 + b\theta_x}$

This value is more often called as ***Observed Yield (Y_{obs})***

Soluble microbial products (SMP)

- Cell components that released during cell lysis, diffuse through the cell membrane, are lost during synthesis, or are excreted for some purpose
- Does not include intermediates of degradation pathway
- MW = 100s – 1000s
- Biodegradable

Significance of SMP

- Appear in all cases
- Constitute the majority of the effluent COD & BOD in many cases
- Can complex metals, foul membranes, & cause color or foaming

Two types of SMP

- UAP (substrate-**U**tilization-**A**ssociated **P**roducts):
  produced directly during substrate metabolism

$$r_{UAP} = -k_1 r_{ut}$$

r_{UAP} = rate of UAP formation $[M_p L^{-3} T^{-1}]$
k_1 = UAP-formation coefficient $[M_p M_s^{-1}]$

- BAP (**B**iomass-**A**ssociated **P**roducts):
  formed directly from biomass as part of maintenance and decay

$$r_{BAP} = k_2 X_a$$

r_{BAP} = rate of BAP formation $[M_p L^{-3} T^{-1}]$
k_2 = BAP-formation coefficient $[M_p M_x^{-1} T^{-1}]$

SMP biodegradation

- Assume degradation of both UAP and BAP follow Monod kinetics (with own parameter values for each):

$$r_{deg-UAP} = -\frac{\hat{q}_{UAP} UAP}{K_{UAP} + UAP} X_a$$

$$r_{deg-BAP} = -\frac{\hat{q}_{BAP} BAP}{K_{BAP} + BAP} X_a$$

UAP = concentration of UAP [$M_p L^{-3}$]

BAP = concentration of BAP [$M_p L^{-3}$]

SMP in a CSTR: mass balance

- Steady-state mass balance for UAP & BAP:

$$0 = 0 - Q \cdot UAP - k_1 r_{ut} V - \frac{\hat{q}_{UAP} UAP}{K_{UAP} + UAP} X_a V$$

$$0 = 0 - Q \cdot BAP + k_2 X_a V - \frac{\hat{q}_{BAP} BAP}{K_{BAP} + BAP} X_a V$$

SMP in a CSTR: solutions

$$UAP = -\frac{\hat{q}_{UAP}X_a\theta + K_{UAP} + k_1 r_{ut}\theta}{2} + \frac{\sqrt{(\hat{q}_{UAP}X_a\theta + K_{UAP} + k_1 r_{ut}\theta)^2 - 4K_{UAP}k_1 r_{ut}\theta}}{2}$$

$$BAP = -\frac{K_{BAP} + (\hat{q}_{BAP} - k_2)X_a\theta}{2} + \frac{\sqrt{(K_{BAP} + (\hat{q}_{BAP} - k_2)X_a\theta)^2 + 4K_{BAP}k_2 X_a\theta}}{2}$$

SMP in a CSTR: note for the solutions

[Note 1]

Keep the form θ in the solutions, not θ_x

[Note 2]

The substrate utilization rate, r_{ut}, can be computed from

$$r_{ut} = -\frac{S^0 - S}{\theta} = -\frac{\hat{q}S}{K + S}X_a$$