

## Microbial kinetics in reactors I: additional note

Master equations:

$$S = K \frac{1 + b\theta_x}{Y\hat{q}\theta_x - (1 + b\theta_x)}$$

$$X_a = Y \frac{S^0 - S}{1 + b\theta_x}$$

### Special cases

1. When  $\Theta = \theta_x$  is very small,  $S=S_0$  and  $X_a=0$ : washout; not substrate removal and accumulation of active biomass

The active biomass is washed out before substantial growth occurs to show observable substrate utilization

Denote the  $\theta_x$  at washout as  $\theta_x^{\min}$ , then, using the equation for S:

$$S = K \frac{1 + b\theta_x}{Y\hat{q}\theta_x - (1 + b\theta_x)}$$

$$S^0 = K \frac{1 + b\theta_x^{\min}}{Y\hat{q}\theta_x^{\min} - (1 + b\theta_x^{\min})}$$

This gives

$$\theta_x^{\min} = \frac{K + S^0}{S^0(Y\hat{q} - b) - bK}$$

2. For  $\Theta > \theta_x$ , S declines with the increase in  $\theta_x$ .  $X_a$  initially increases, but reaches a maximum and then decreases as decay becomes dominant.

3. When  $\Theta = \theta_x$  is very large, the decay predominates and substrate concentration is not further reduced (S reaches  $S_{\min}$ )

$$S = K \frac{1 + b\theta_x}{Y\hat{q}\theta_x - (1 + b\theta_x)}$$

$$\theta_x^{min} \rightarrow \infty$$

$$S_{min} = K \frac{b}{Y\hat{q} - b}$$

### **Bacterial growth following Monod kinetics including inert biomass**

Mass balance for inert biomass (assume steady state):

$$0 = QX_i^0 - QX_i + (1 - f_d)bX_aV$$

$$X_i = X_i^0 + X_a(1 - f_d)b\theta$$

(This shows that operating at large  $\Theta$  results in large accumulation of inert biomass)

The total volatile suspended solids (VSS) concentration,  $X_v$  is calculated as (used  $\Theta$  instead of  $\Theta_x$ )

$$X_v = X_i + X_a = X_i^0 + X_a(1 - f_d)b\theta_x + X_a = X_i^0 + Y(S^0 - S) \frac{1 + (1 - f_d)b\theta_x}{1 + b\theta_x}$$