Reactor analysis II

Some definitions

- **Hydraulic retention time**

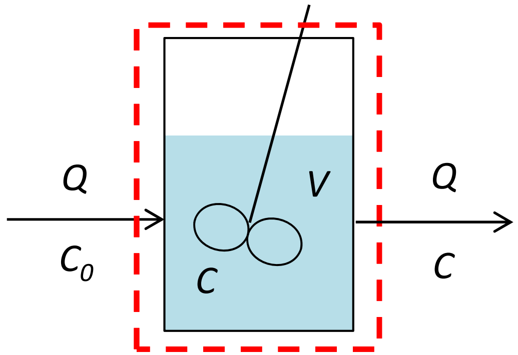
$$\tau = V/Q$$

τ = hydraulic retention time [T]; V = volume of the reactor [L^3]
Q = flowrate [L^3/T]

- **Conservative tracers**: substances that do neither chemically transform nor partition from water; used to analyze the flow characteristics either in natural/engineered systems

Ideal CSTR – tracer response

i) Draw schematic, identify CV



Assumptions:

- $C = 0$ at $t = 0$
- Step input of tracer: at $t \geq 0$ in the influent with a concentration of C_0
- Complete mixing in the reactor
- No reaction (conservative tracer)

ii) Write mass balance eq.

(rate of accumulation)

= (rate of inflow) – (rate of outflow) + (rate of generation)

iii) Solve the eq.

Ideal CSTR – tracer response

Solution (step input):

$$\frac{C}{C_0} = 1 - e^{-t/\tau}$$

cf) solution (slug input):

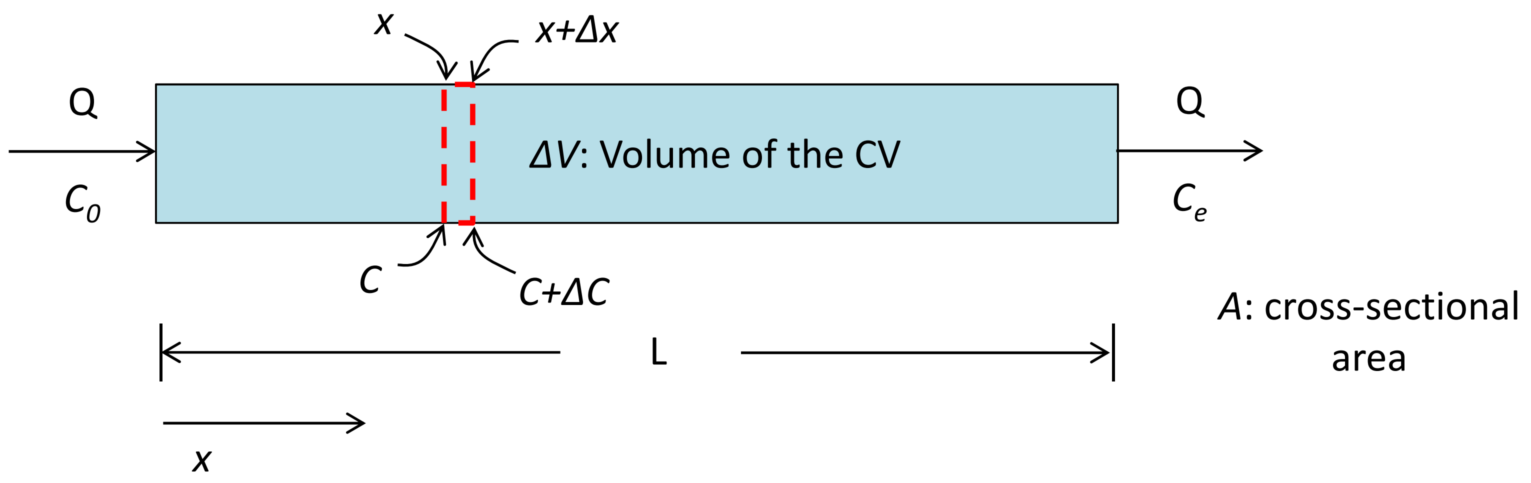
$$\frac{C}{C_0} = e^{-t/\tau}$$

C_0 = concentration at t=0 due to slug input of tracer

Ideal PFR – tracer response

Assume any type of tracer input in the influent, described as $C_0=F(t)$

i) Draw schematic, identify CV



ii) Write mass balance eq.

iii) Solve the eq.

Ideal PFR - tracer response

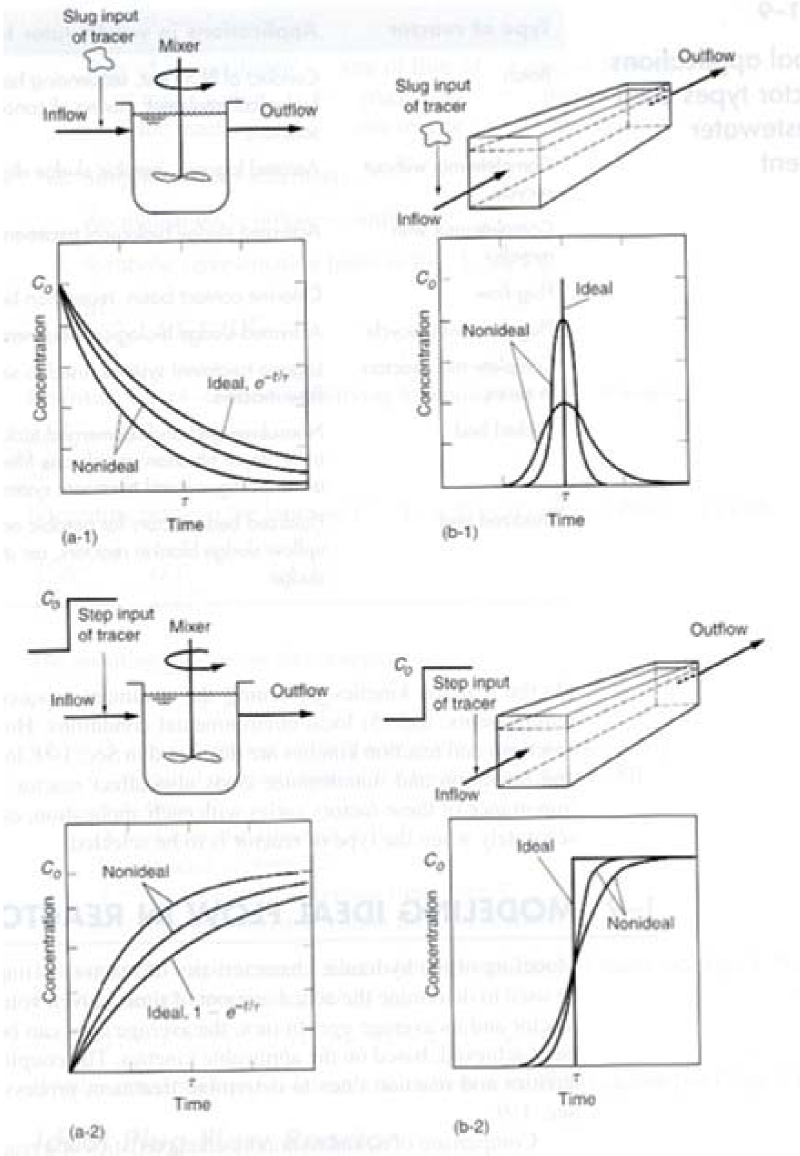
For any $C_0 = C(x = 0, t) = F(t)$:

$$C_e = C(x = L, t) = F(t - \tau)$$

For a PFR, the inflow concentration profile of a tracer is observed exactly the same in the outflow with a time shift of τ

Non-ideal flow in CSTR & PFR

- In practice, the flow in CSTR and PFR is seldom ideal – there are some extent of deviations from the ideal cases



Non-ideal flow in CSTR & PFR

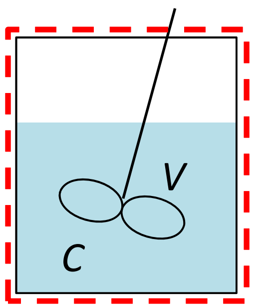
- Factors leading to non-ideal flow (short-circuiting)
  - **Temperature differences**: temperature difference developed within a reactor → density currents occur → water does not flow at a full depth
  - **Wind-driven circulation patterns**: wind creates a circulation cell which acts as a dead space
  - **Inadequate mixing**: insufficient mixing of some portions of the reactor
  - **Poor design**: dead zones developed at the inlet and the outlet of the reactor
  - **Axial dispersion in PFRs**: mechanical dispersion and molecular diffusion in the direction of the flow

Reactor analysis - treatment processes

- Incorporate the reaction rate expression into the mass balance equation!

- Batch reactor with first-order reaction

i) Draw schematic, identify CV



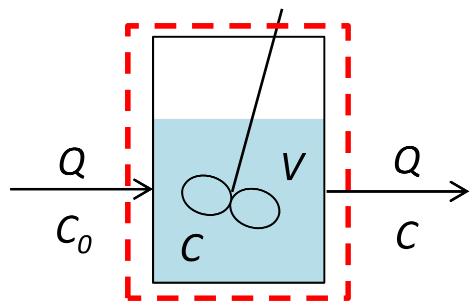
ii) Write mass balance eq.

iii) Solve the eq.

Solution: $C/C_0 = e^{-kt}$

CSTR, 1st order reaction

i) Draw schematic, identify CV



ii) Write mass balance eq.

iii) Solve the eq.

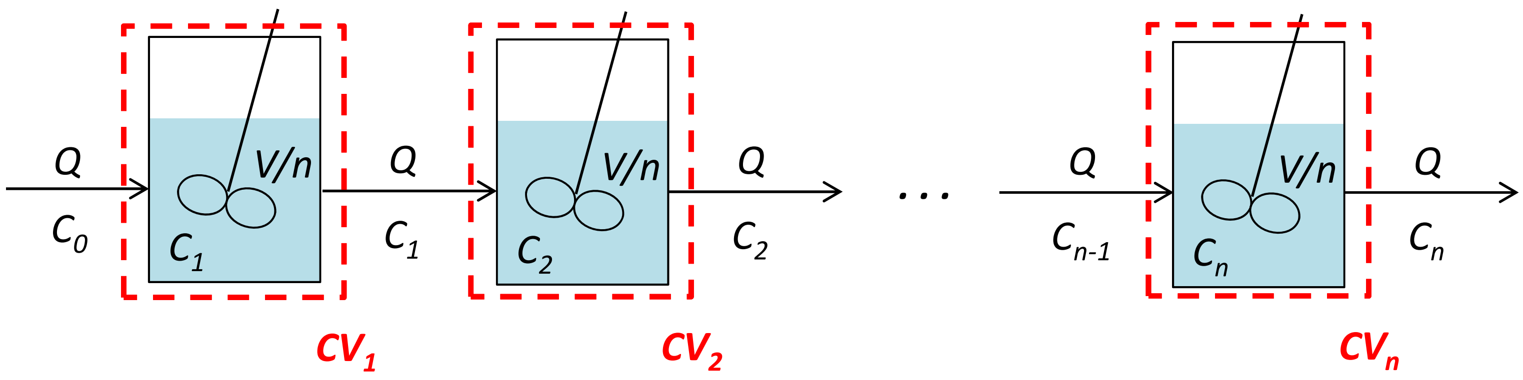
General solution:

$$C/C_0 = \frac{1}{1+k\tau}\left(1 - e^{-(k+1/\tau)t}\right) + e^{-(k+1/\tau)t}$$

Steady-state solution:

$$C/C_0 = \frac{1}{1+k\tau}$$

CSTR in series, 1st order reaction



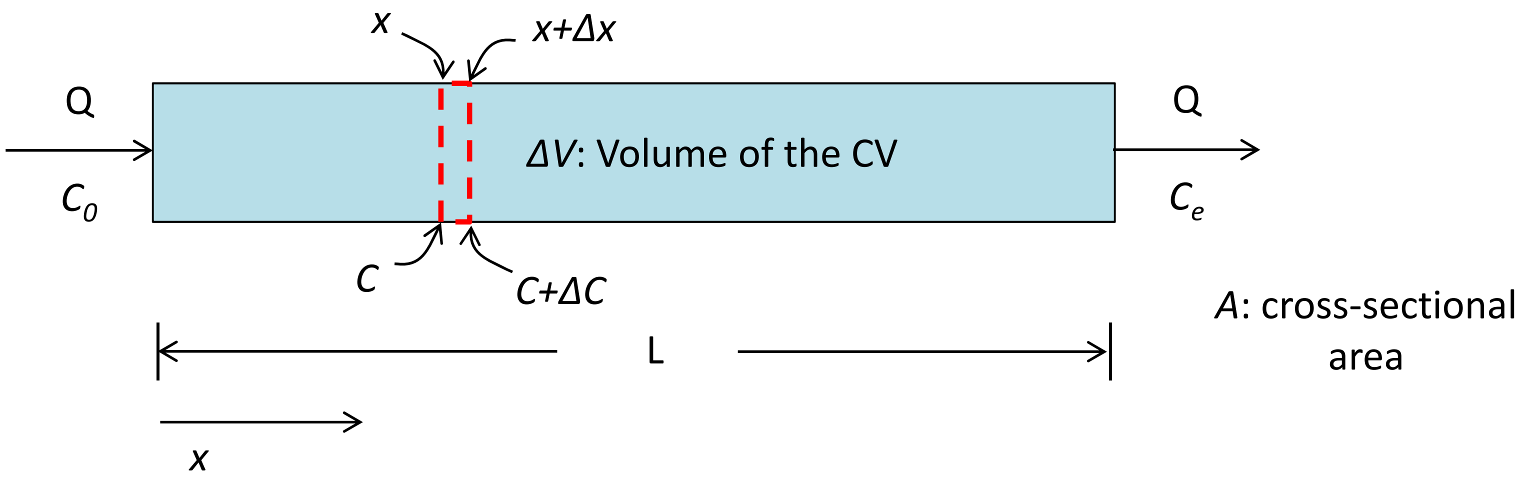
Steady-state solution:

$$C_n/C_0 = \frac{1}{(1 + kV/nQ)^n}$$
$$= \frac{1}{(1 + k\tau/n)^n}$$

V = sum of all reactor volumes

τ = hydraulic retention time in the entire system

PFR, 1^{st} order reaction



Steady-state solution:

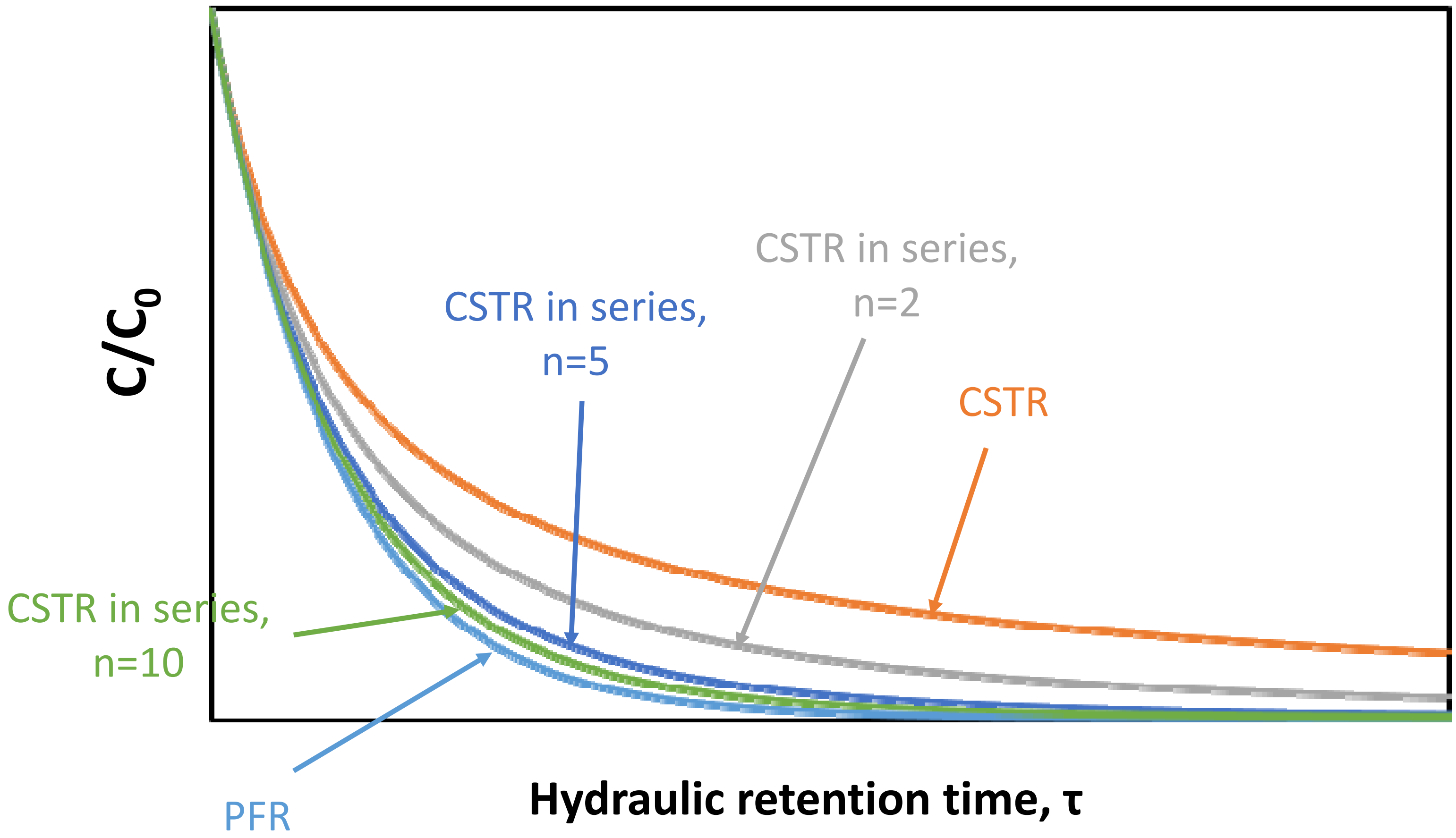
$$\frac{C_e}{C_0} = e^{-k\tau}$$

– Equivalent to the batch reactor solution (why??)

Treatment process modeling

Q: Compare the performance of i) a CSTR, ii) three CSTRs in series, and iii) a PFR having the same hydraulic retention time of 0.2 days when the first-order reaction rate coefficient, k, is 10 day^{-1}. Assume steady state.

CSTR vs. PFR



C/C_0
CSTR in series,
n=2
CSTR in series,
n=5
CSTR
CSTR in series,
n=10
PFR
Hydraulic retention time, τ