

Microbial kinetics in reactors II

Today's lecture

- Nutrient & e^- acceptor consumption
- Hydrolysis
- Alternate rate expressions
- Example question

Nutrient consumption

- For the consumption of nutrients for biomass production:

$$r_n = \gamma_n \cdot Y_{obs} \cdot r_{ut} = \gamma_n \cdot Y \cdot r_{ut} \frac{1 + (1 - f_d)b\theta_x}{1 + b\theta_x}$$

r_n = rate of nutrient consumption [$M_n L^{-3} T^{-1}$]

γ_n = the stoichiometric ratio of nutrient mass to VSS for biomass [$M_n M_x^{-1}$]

Using $C_5H_7O_2N$ as cell formula:

$$\gamma_N = 14 \text{ g N} / 113 \text{ g VSS} = 0.124 \text{ g N/g VSS}$$

$$\gamma_P = 0.2 \times 0.124 = 0.025 \text{ g P/g VSS (assume } g P = 0.2 \times g N)$$

Nutrient consumption in a CSTR

- Steady-state mass balance:

$$0 = QC_n^0 - QC_n + r_n V$$

C_n^0 = nutrient conc. in the influent [M_oL^{-3}]

C_n = nutrient conc. in the reactor [M_oL^{-3}]

$$C_n = C_n^0 + r_n \theta$$

if $C_n < 0$, nutrient-limiting

e^- acceptor consumption

(e^- acceptor used in a reactor)

*= [(total O.D. in the influent) - (total O.D. in the effluent)]
x (conversion factor)*

In terms of the use rate for a reactor ($\Delta S_a/\Delta t$):

$$\begin{aligned}\frac{\Delta S_a}{\Delta t} &= \gamma_a [Q(S^0 + 1.42X_v^0) - Q(S + SMP + 1.42X_v)] \\ &= \gamma_a Q [S^0 - S - SMP + 1.42(X_v^0 - X_v)]\end{aligned}$$

γ_a = the stoichiometric ratio of acceptor mass to oxygen demand

for oxygen: 1 g O_2 /g COD

for nitrate: 0.35 g NO_3^- -N/g COD

e⁻-acceptor consumption

To estimate the required mass rate of acceptor supply (ex: aeration [O₂] requirement), the calculated e⁻ acceptor use rate, $\Delta S_a/\Delta t$ can be written as:

$$\frac{\Delta S_a}{\Delta t} = \gamma_a [Q(S^0 + 1.42X_v^0) - Q(S + SMP + 1.42X_v)]$$

$$= Q(S_a^0 - S_a) + R_a$$

(mass flow rate in) -
(mass flow rate out)

Requirement of e⁻
acceptor addition

[M_aL⁻³]

*S_a⁰ = e⁻ acceptor conc. in the
influent [M_aL⁻³]*

*S_a = e⁻ acceptor conc. in the
reactor [M_aL⁻³]*

Hydrolysis of particulates & polymers

- Particulates and polymeric substances account for a significant portion of BOD in wastewater
- >50% of BOD in typical sewage is particulates (SS)
- Particulates and large-MW compounds cannot penetrate the cell membrane
 - needs to be hydrolyzed to smaller molecules
- Catalyzed by extracellular enzymes
- The mechanism and kinetics of hydrolysis is not fully understood

Hydrolysis of particulates & polymers

One simple way of describing hydrolysis is to assume first-order kinetics for particulates (or polymers):

$$r_{hyd} = -k_{hyd}S_p$$

r_{hyd} = rate of accumulation of particulates ($=dS_p/dt$) [$M_sL^{-3}T^{-1}$]

k_{hyd} = first-order hydrolysis rate coefficient [T^{-1}]

S_p = concentration of particulates in the reactor [M_sL^{-3}]

In a steady-state CSTR,

$$0 = Q(S_p^0 - S_p) - k_{hyd}S_pV$$

S_p^0 = concentration of particulates in the influent [M_sL^{-3}]



$$S_p = \frac{S_p^0}{1 + k_{hyd}\theta}$$

Hydrolysis of particulates & polymers

- Effect of hydrolysis on dissolved substrates mass balance in a steady-state CSTR

$$0 = Q(S^0 - S) - \frac{\hat{q}S}{K + S}X_aV + k_{hyd}S_pV$$

$$0 = (S^0 - S) - \frac{\hat{q}S}{K + S}X_a\theta + k_{hyd}S_p\theta$$

→ Increase in S^0 by $k_{hyd}S_p\theta$

→ Increased biomass, but no change in dissolved substrates in the reactor

So: to include the effect of hydrolysis of particulates,

substitute S^0 with $(S^0 + k_{hyd}S_p\theta)$

Alternate rate expressions

- Contois equation

$$r_{ut} = -\frac{\hat{q}S}{BX_a + S}X_a \quad B = \text{constant } [M_s/M_x]$$

$$\text{When } X_a \rightarrow \infty, \quad r_{ut} = -\frac{\hat{q}}{B}S$$

(at high biomass concentrations substrate utilization depends on S , not X_a)

Alternate rate expressions

- Moser equation

$$r_{ut} = -\frac{\hat{q}S}{K + S^{-\gamma}}X_a \quad \gamma = \text{constant [unitless]}$$

- Tessier equation

$$r_{ut} = -\hat{q}(1 - e^{S/K})X_a$$

*Just **REMEMBER** that Monod Eq. is **NOT** the only option!!!*

Dual Monod equation

$$r_{ut} = -\hat{q} \frac{S}{K + S} \frac{A}{K_A + A} X_a$$

A = e⁻ acceptor concentration [M_A/L³]

K_A = half-saturation coefficient for e⁻ acceptor [M_A/L³]

- e⁻ acceptor can also be limiting!
- Can be reduced to single Monod eq. if $A \gg K_A$
- Terms for other limiting substances can be added as well

Analyzing CSTR (Chemostat) Performance

Q: A chemostat having $V=2,000 \text{ m}^3$ receives a flow rate of $Q = 1,000 \text{ m}^3/\text{d}$ of wastewater containing $S^0 = 500 \text{ mg BOD}_L/\text{L}$. Also included in the wastewater is the inert biomass $X_i^0 = 50 \text{ mg VSS}/\text{L}$. The following parameters are found for aerobic biodegradation:

$$\hat{q} = 20 \text{ g BOD}_L / \text{g VSS}_a - d$$

$$Y = 0.42 \text{ g VSS}_a / \text{g BOD}_L$$

$$K = 20 \text{ mg BOD}_L / \text{L}$$

$$b = 0.15 / d$$

$$f_d = 0.8$$

$$k_1 = 0.12 \text{ g COD}_p / \text{g BOD}_L$$

$$k_2 = 0.09 \text{ g COD}_p / \text{g VSS}_a - d$$

$$\hat{q}_{UAP} = 1.8 \text{ g COD}_p / \text{g VSS}_a - d$$

$$K_{UAP} = 100 \text{ mg COD}_p / \text{L}$$

$$\hat{q}_{BAP} = 0.1 \text{ g COD}_p / \text{g VSS}_a - d$$

$$K_{BAP} = 85 \text{ mg COD}_p / \text{L}$$

Analyzing CSTR (Chemostat) Performance

Questions:

1. Calculate S_{min} , θ_x^{min} , and θ_x of the chemostat
2. Calculate effluent VSS, COD, and BOD_L .
3. Calculate the effluent N and P concentrations when influent concentrations are 50 mg NH_4^+ -N/L and 10 mg PO_4^{3-} -P/L, respectively.
4. Calculate the amount of O_2 that should be supplied to the reactor when influent and effluent DO are 6 and 2 mg/L, respectively.
5. Assuming that the influent also contains biodegradable particulate organic matter with a concentration of 100 mg COD/L and the hydrolysis rate coefficient is $k_{hyd} = 0.2/d$, recalculate the effluent VSS, COD, and BOD_L .

Hints

- Effluent VSS should include both active and inert biomass
($X_v = X_a + X_i$)
- Effluent COD should include COD of the substrate, SMP, and VSS
(eff. COD = substrate COD + SMP COD + VSS COD)
 - Conversion needed for VSS: recall 1.42 g COD/g VSS for biomass ($C_5H_7O_2N$)
- BOD_L stands for “ultimate BOD”, the oxygen demand for all biodegradable organic matter
 - S^0 is given as “ BOD_L/L ”, so substrate is assumed to be fully biodegradable
 - SMP is fully biodegradable
 - active biomass is partially biodegradable (biodegradable fraction = f_d)
 - inert biomass is non-biodegradableSo: eff. BOD_L
= substrate BOD_L (=COD) + SMP BOD_L (=COD) + $f_d \times$ active biomass COD