

Microbial kinetics in reactors III: additional note

From active biomass mass balance,

$$0 = 0 - (Q^e X_a^e + Q^w X_a^w) + r_{net} V$$

$$r_{net} = \text{net rate of active biomass growth} = Y(-r_{ut}) - bX_a$$

$$(Q^e X_a^e + Q^w X_a^w) = \{Y(-r_{ut}) - bX_a\}V$$

$$\frac{Q^e X_a^e + Q^w X_a^w}{X_a V} = \frac{Y(-r_{ut})}{X_a} - b$$

$$\frac{1}{\theta_x} = \frac{Y(-r_{ut})}{X_a} - b = Y \frac{\hat{q}S}{K + S} - b$$

(1)

$$S = K \frac{1 + b\theta_x}{\theta_x(Y\hat{q} - b) - 1}$$

(solution for S)

From (1),

$$X_a = \theta_x \frac{Y(-r_{ut})}{1 + b\theta_x}$$

$$\text{Use } r_{ut} = -\frac{S^0 - S}{\theta} :$$

$$X_a = \frac{\theta_x Y(S^0 - S)}{\theta (1 + b\theta_x)}$$