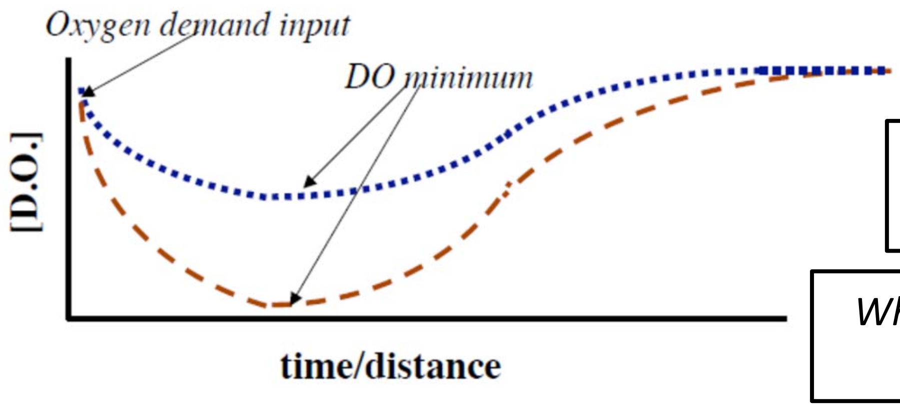
# Oxygen dynamics in a river

# Stream dissolved oxygen

- **DO is a critical indicator for stream "health"**
- **Classic well studied problem**
  - Wastewater/WWTP effluent discharge to streams very common
  - Fish kills, odors are easily noticed
  - Big issue in developed countries in mid-20th century; ~90s in Korea
    - Wastewater treatment investments; regulations have largely solved the problem in U.S. & Europe
    - Still a big issue in many regions of the world
- **Amenable to analytical models employing multiple 1st order rate processes**



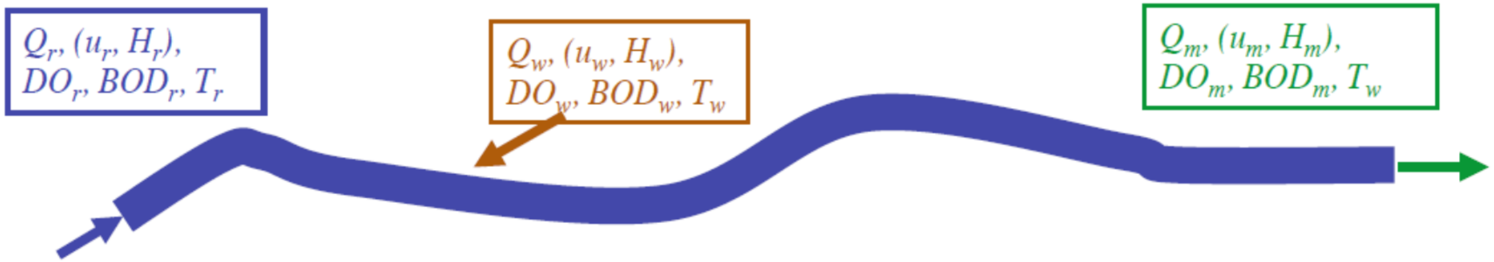
*How much $O_2$ demand can the stream handle?*

*What factors influence the stream's capacity?*

*What are the relevant processes?*

# $O_2$ dynamics in a river

- **Consider effects of:**
  - BOD
  - Re-aeration
  - Stream geometry
  - Temperature
- **Streeter-Phelps model: from 1925; study of Ohio River**
  - 1st order processes
  - Single stream segment
  - No longitudinal mixing, complete transverse & vertical mixing
  - Spatially & temporally invariant conditions



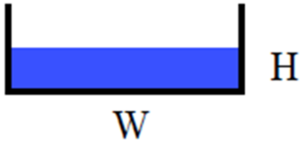
# Modeling

## 1. Assume Manning-like, plug flow in stream

$$\boxed{u = \frac{R^{2/3}S^{1/2}}{n}}$$

$u$ = stream velocity (m/s)
$R$ = hydraulic radius (m)
$S$ = slope (-)
$n$ = roughness factor (-)

$$R = \frac{H \times W}{2H + W}$$



*If W>>H & W constant:* $H \sim Q^{3/5}$, $u \sim Q^{2/5}$

*No longitudinal mixing*

*u constant everywhere, no spatial, temporal variability*

## 2. Employ DO deficit, *D*

$$\boxed{D = C_{sat} - C}$$

$D$ = DO deficit (mg/L)
$C_{sat}$ = saturated DO concentration (mg/L)
$C$ = actual DO concentration (mg/L)

# Modeling

## 3. Assume 1st order processes adequately mimic the system

*For: Biodegradation*
*Air-water mass transfer*

## 4. Employ temperature corrections of the form:

$$k_{i_T} = k_{i_{20}} \theta^{(T-20)}$$

$k_i$ = 1st-order rate constant ($s^{-1}$)
$T$ = temperature (°C)
$\theta$ = empirical correction factor; generally $1.02 < \theta < 1.1$

# Procedure

## 1. Do mass balances where stream & waste mix (*x*=0)

$$Q_m = Q_r + Q_w$$

$$C_m = \frac{C_r Q_r + C_w Q_w}{Q_m}$$

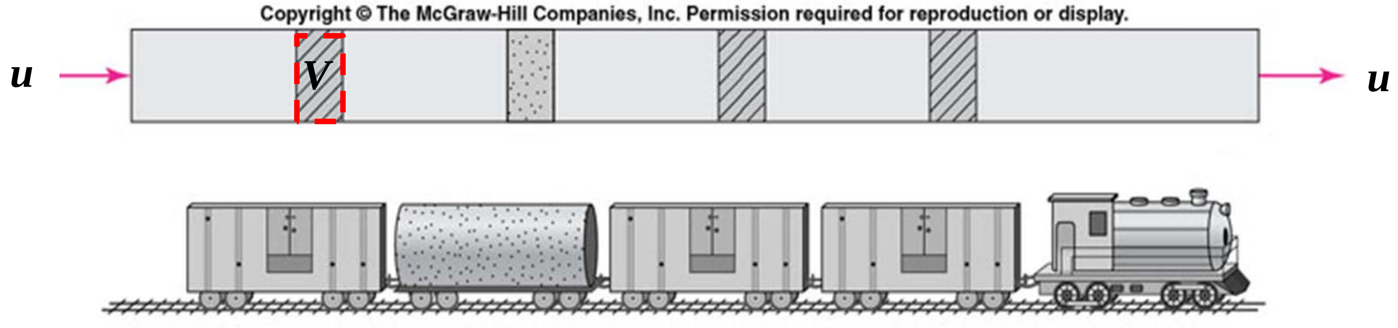
- C: concentration for DO, *D* (oxygen deficit), *L* (ultimate BOD)
- Can also be applied for temperature
- May require stream velocity and/or depth re-calculations

## 2. Determine, adjust model parameters

## 3. Use control volume

- Account for: inputs, outputs, reactions, transfers
- Control volume moves with stream

# Procedure



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**Rate of:**

***Accumulation = Input − Output − Degradation + Reaeration***

$$V\frac{\Delta C}{\Delta t} \quad = \quad 0 \quad - \quad 0 \quad - \quad Vk_1L \quad + \quad Vk_2D$$

$$= V(k_2D - k_1L)$$

$$\boxed{u\frac{dD}{dx} - k_1L + k_2D = 0}$$

$L$ = organic loading (UBOD) (mg/L)

$D$ = DO deficient (mg/L)

$k_1$ = rate constant for organic degradation (d$^{-1}$)

$k_2$ = reaeration rate constant (d$^{-1}$)

$$= \left(D_{w,O_2} \cdot u\right)^{0.5} / H^{1.5}$$

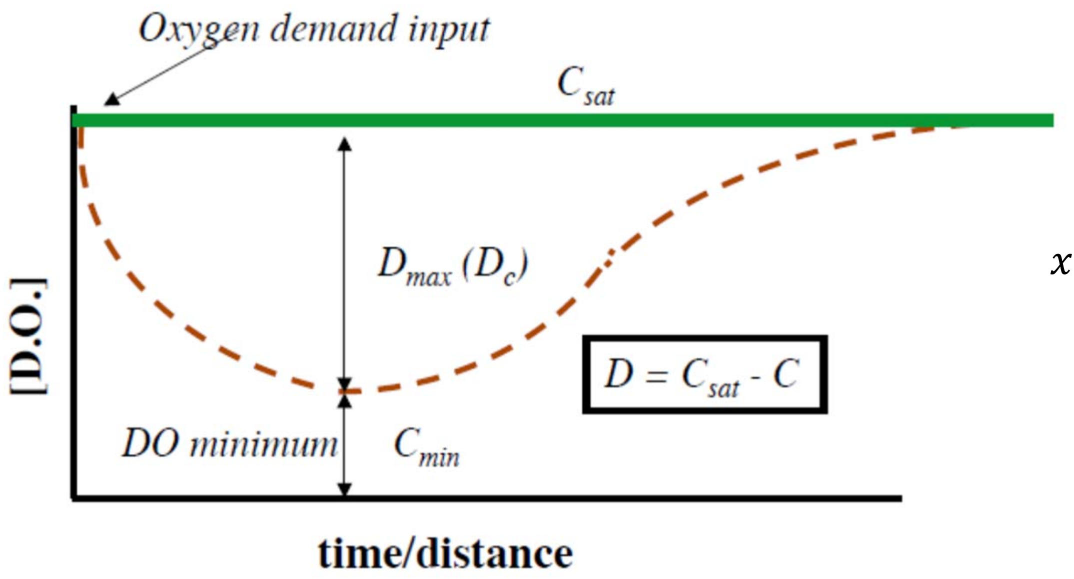
# Procedure

$$\boxed{u\frac{dD}{dx} - k_1L + k_2D = 0}$$

$$L = L_0e^{-k_1t} = L_0e^{-k_1(x/u)}$$

⇨ $$u\frac{dD}{dx} + k_2D = k_1L_0e^{-k_1(x/u)}$$

$$\boxed{\boldsymbol{D(x) = D_0e^{-k_2(x/u)} + \frac{k_1L_0}{k_2 - k_1}\left[e^{-k_1(x/u)} - e^{-k_2(x/u)}\right]}}$$



**Critical distance:**

$$x_c = \frac{u}{k_2 - k_1}ln\left[\left(\frac{k_2}{k_1}\right)\left\{1 - \frac{D_0(k_2 - k_1)}{k_1L_0}\right\}\right]$$

**Critical deficit:**

$$D_c = D(x_c)$$

# Streeter-Phelps: limitations

- **No variability in stream properties & temp.**
- **No longitudinal mixing**
  - Plug flow
- **No microbial growth**
- **No interactions with sediments**
  - No settling of organic matter
- **No photosynthesis**
- **1st order rates, no interdependence**
- **If DO reaches 0, degradation rate ($k_1L$) can be no more than reaeration rate ($k_2D$)**
- **If DO gets sufficiently low (~<1 mg/L) degradation is inhibited**

# Notes

$$\begin{array}{ccccc} & \gg & & \Longrightarrow & D_c \approx C_{sat} \\ k_1 L_0 & \approx & k_2 C_{sat} & \Longrightarrow & D_c \approx 0.5 C_{sat} \\ & \ll & & \Longrightarrow & D_c \approx 0 \end{array}$$

$\frac{k_1 x}{u} \gg 1$ *reaction proceeds far*

$\frac{k_1 x}{u} \ll 1$ *reaction negligible within time frame*

***Damköhler# (I)***

$\frac{k_1}{k_2} \gg 1$ *reaction faster than mass transfer*

$\frac{k_1}{k_2} \ll 1$ *mass transfer faster than reaction*

***Damköhler# (II)***

$\frac{k_2 x}{u} \gg 1$ *mass transfer proceeds far*

$\frac{k_2 x}{u} \ll 1$ *mass transfer negligible within time frame*

***Stanton#***

# Example question

**Q:** A city disposes of 1.0 $m^3/s$ of treated sewage having UBOD of 28.0 mg/L and DO of 1.8 mg/L into a river. At the upstream from the outfall, the river flowrate is 7.0 $m^3/s$. The UBOD and DO of the river are 3.6 and 7.6 mg/L, respectively. The width of the river is 40 m for the entire distance of consideration. At the river temperature, the saturation value of DO is 8.5 mg/L, the biodegradation rate constant, $k_1$, 0.61 $day^{-1}$, and the reaeration rate constant, $k_2$, 0.76 $day^{-1}$.

1) Calculate the ultimate BOD and DO just downstream from the outfall. Assume complete mixing.
2) Calculate the velocity of the river flow after the mixing. Assume n=0.03 and S=0.02.
3) Calculate the DO 16 km downstream from the outfall.
4) Calculate the critical time, distance, and the minimum DO.