

# Air pollution II

# Adiabatic lapse rate

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→ When air moves vertically in the atmosphere, how does its temperature change?

→ This is close to the adiabatic expansion; the air adjusts to the decreasing atmospheric pressure by expanding in volume, with a negligible exchange of heat between it and the surrounding air



# Adiabatic lapse rate

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- First law of thermodynamics

$$dU = dQ - dW$$

change in internal  
energy of the  
system

Heat transfer to  
the system

Change in energy  
due to work done  
by the system

For a adiabatic system,  $dQ = 0$

$$dW = pdV \quad p = \text{pressure}; V = \text{volume}$$

$$dU = C_v dT \quad C_v = \text{heat capacity}; T = \text{temperature}$$

So:  $C_v dT = -pdV$

As air moves up,  $V \uparrow \Rightarrow T \downarrow$

# Adiabatic lapse rate and air stability

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- Dry adiabatic lapse rate

$$\Gamma = -\frac{dT}{dz} = 1^{\circ}C / 100 m$$

- Air stability based on dry adiabatic lapse rate

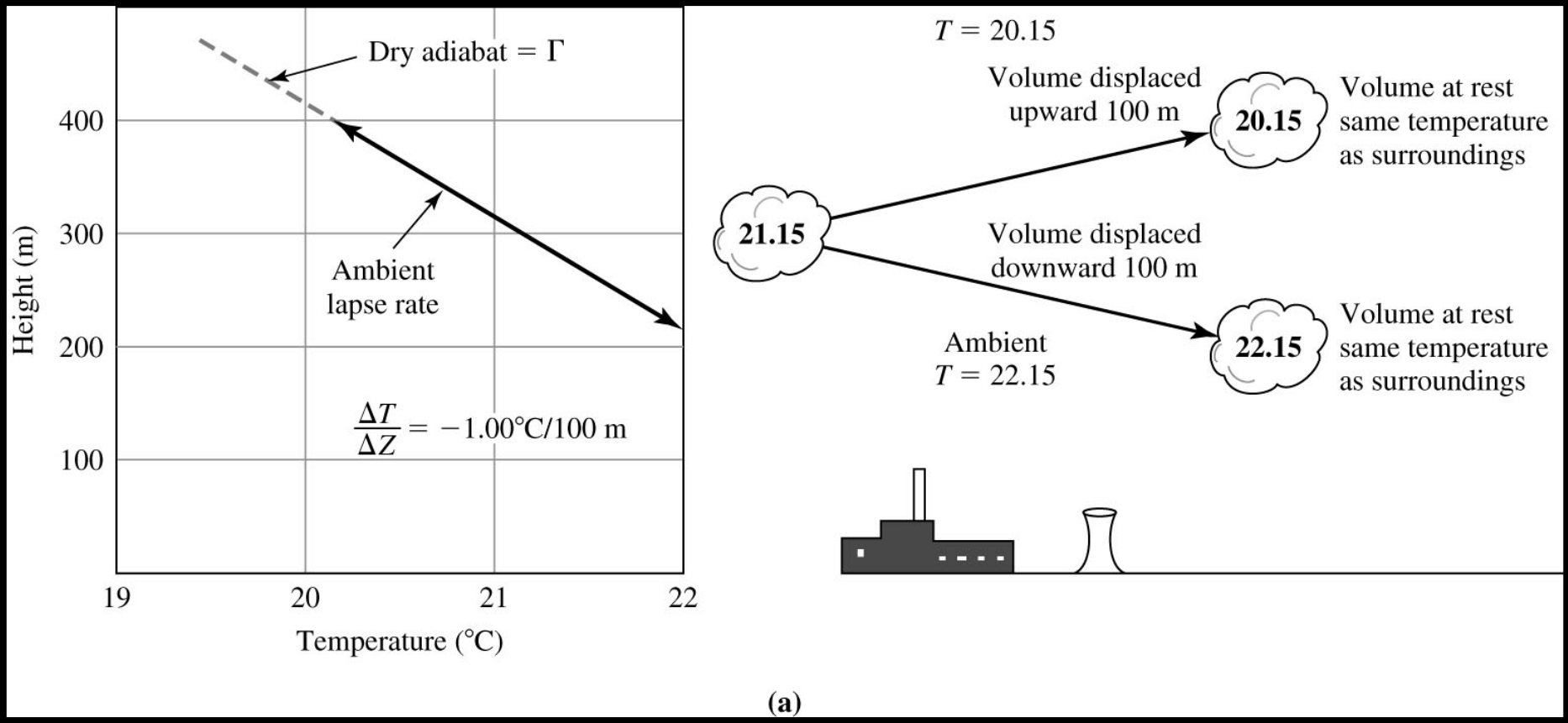
$\Lambda = \Gamma$       Neutral stability

$\Lambda > \Gamma$       Unstable atmosphere

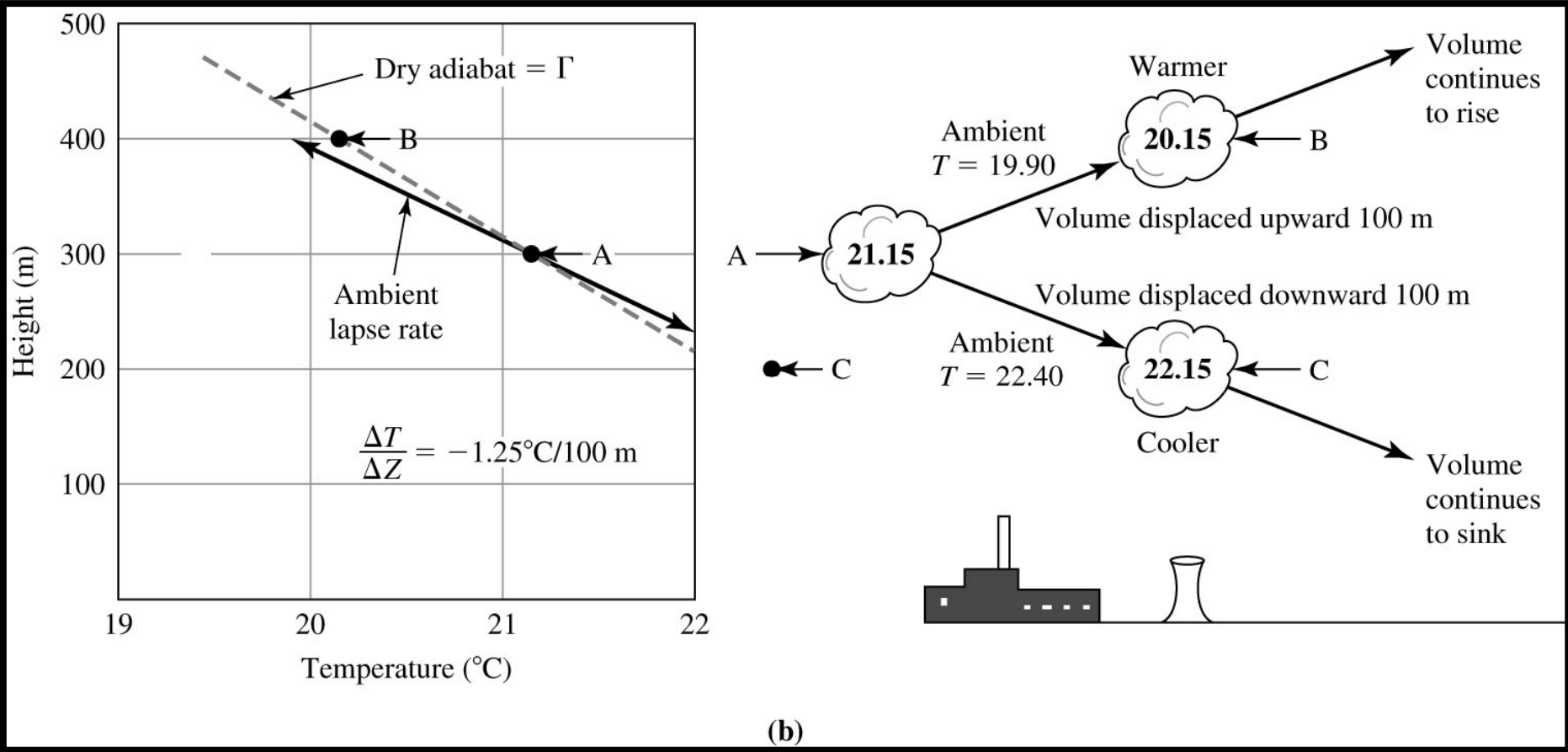
$\Lambda < \Gamma$       Stable atmosphere

$\Lambda = \text{actual lapse rate of the atmosphere}$

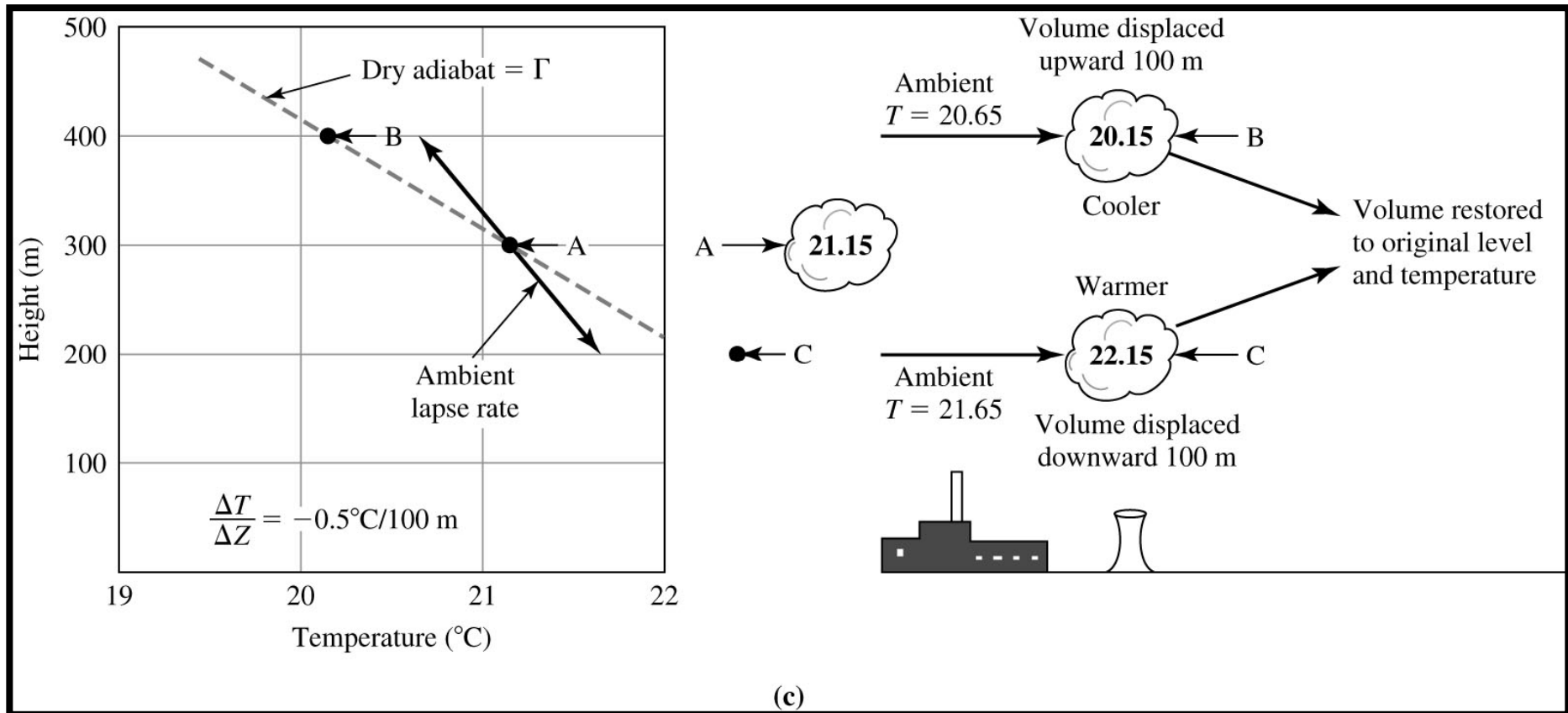
# Neutral stability



# Unstable

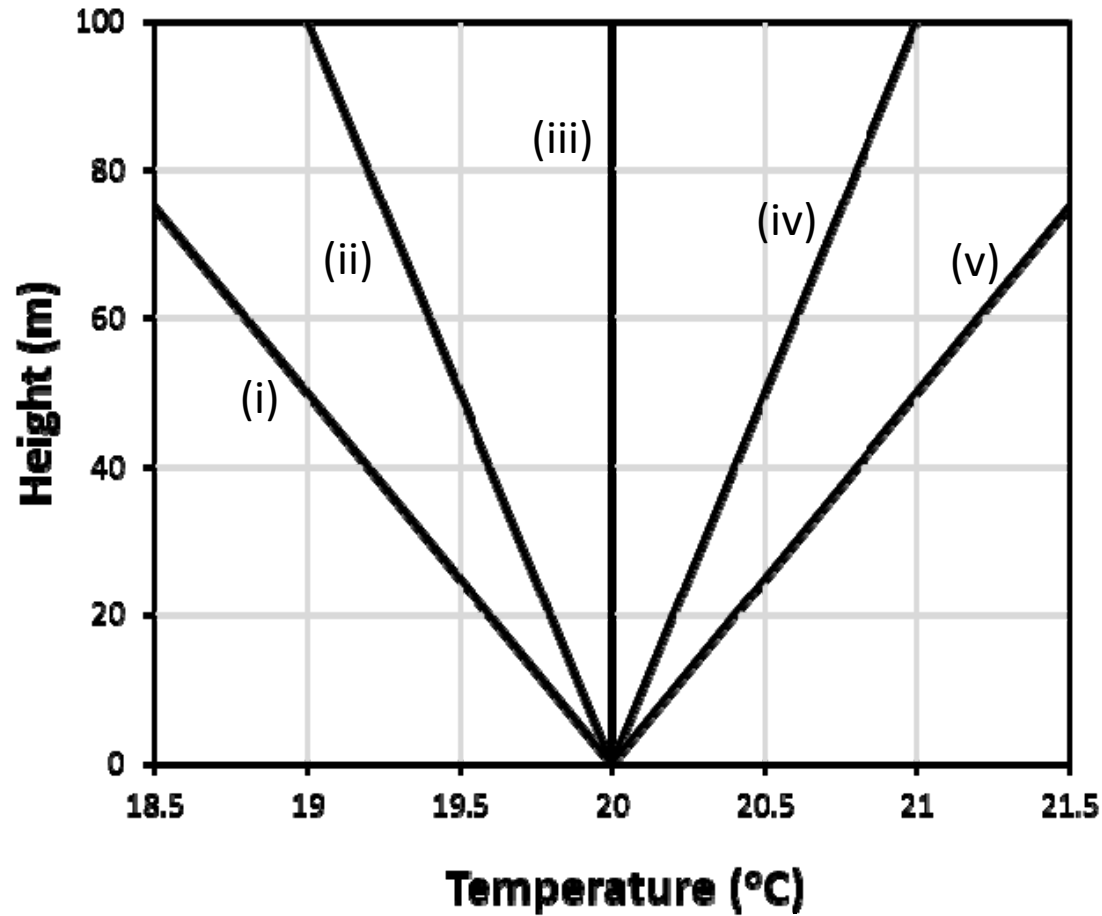


# Stable



# Air stability

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Neutral:

Unstable:

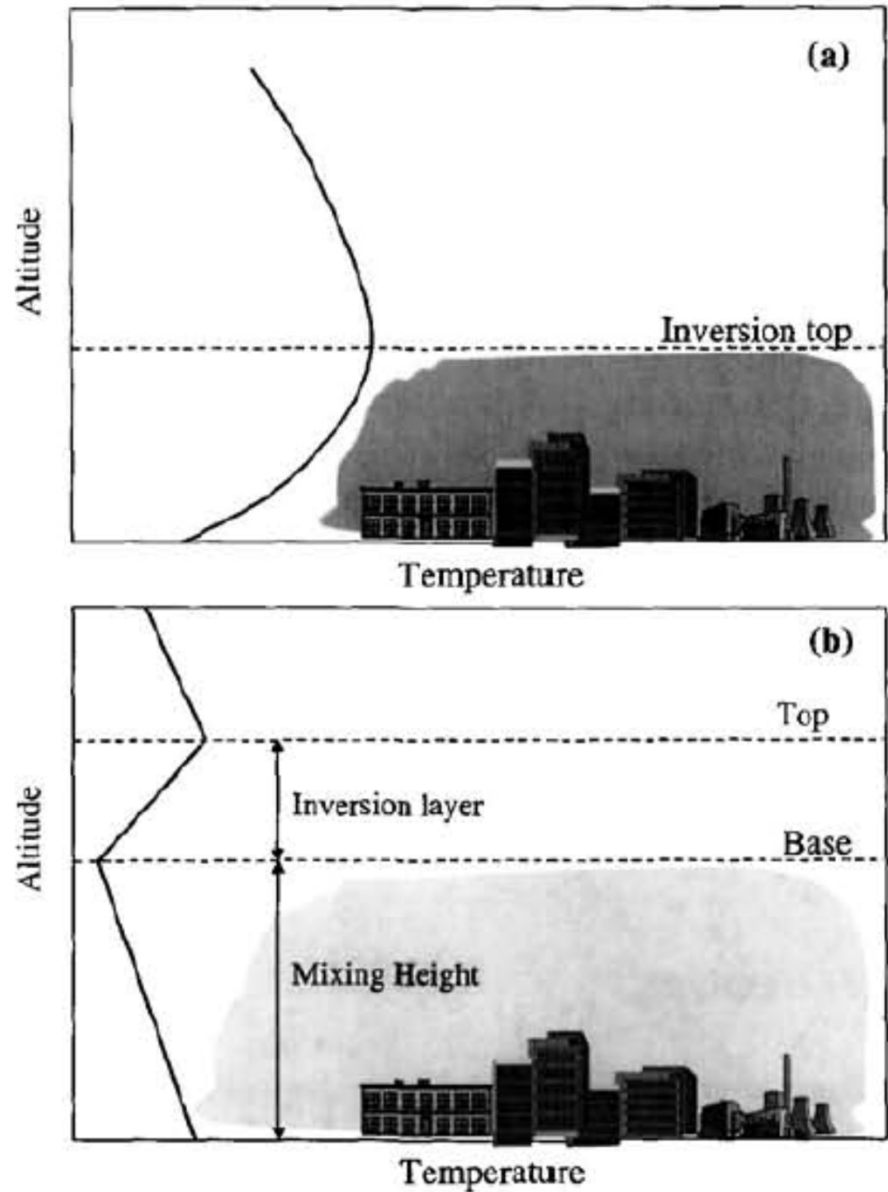
Stable:



# Inversion layer

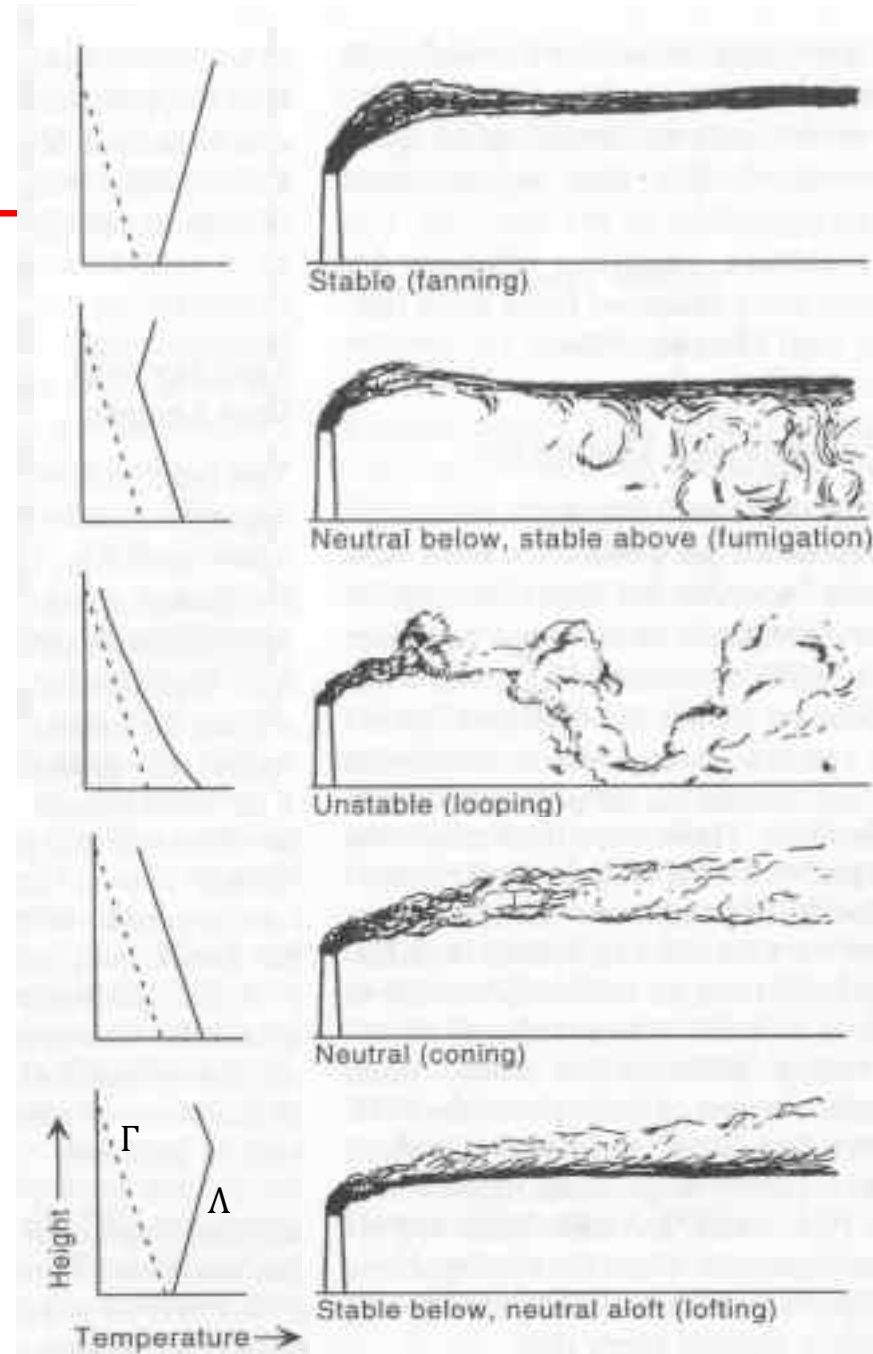
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- Inversion: an increase in atmospheric temperature with height
- Radiation inversion: form by nighttime cooling of the ground
- Subsidence inversion: form by sinking (→ warming) of air



# Plume behavior

- Effect of atmospheric lapse rates on plume behavior

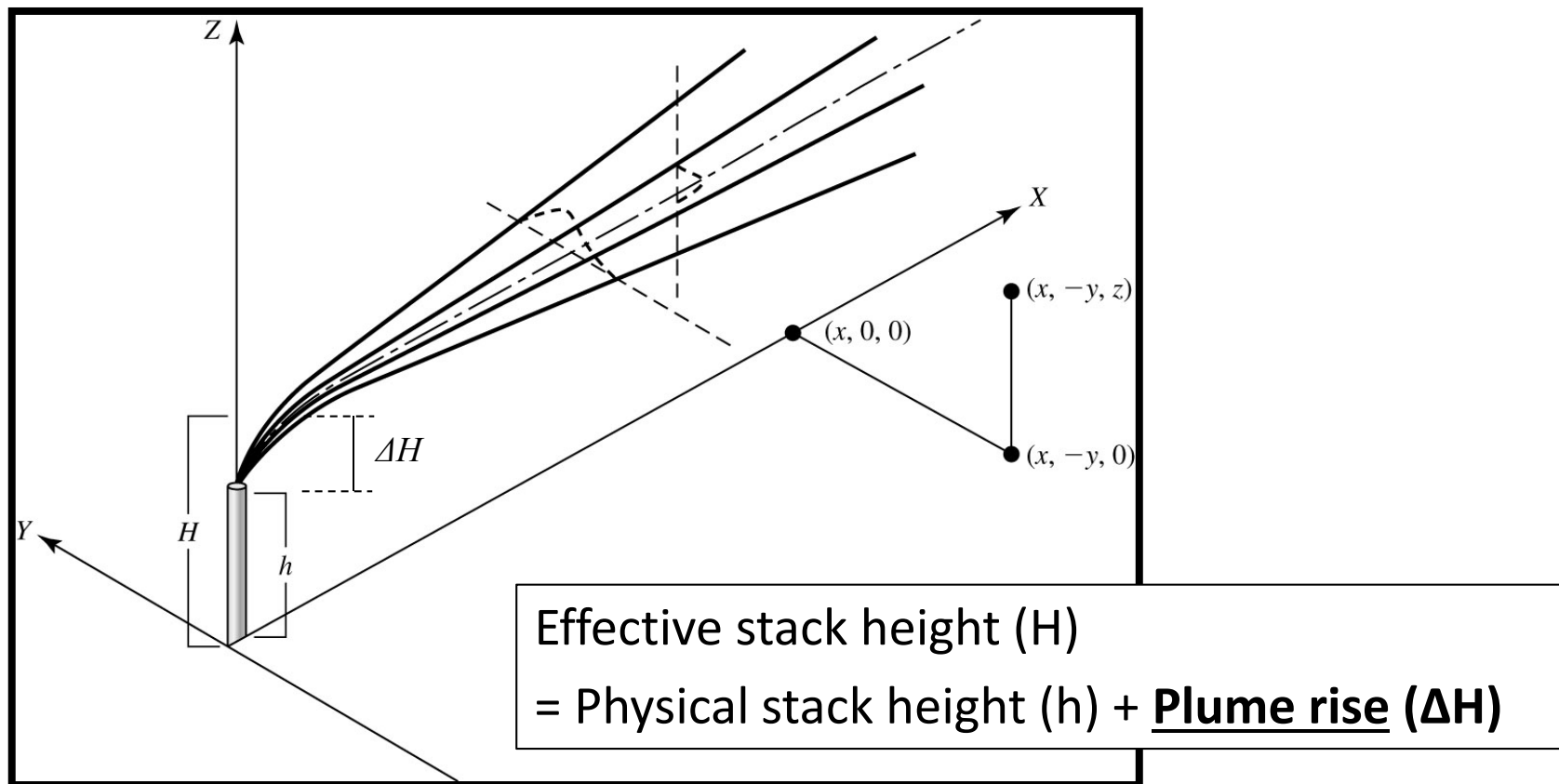


Slade, 1967

# Atmospheric dispersion of air pollutants

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- Coordination system



# Atmospheric dispersion of air pollutants

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- Plume rise,  $\Delta H$ 
  - Due to buoyancy of the hot gas and inertia of the gas leaving the stack

$$\Delta H = \frac{v_s d}{u} \left[ 1.5 + \left( 2.68 \times 10^{-2} \cdot P \cdot \left( \frac{T_s - T_a}{T_s} \right) \cdot d \right) \right]$$

$v_s$  = stack velocity (m/s)

$d$  = stack diameter (m)

$u$  = wind speed (m/s)

$P$  = pressure (kPa)

$T_s$  = stack temperature (K)

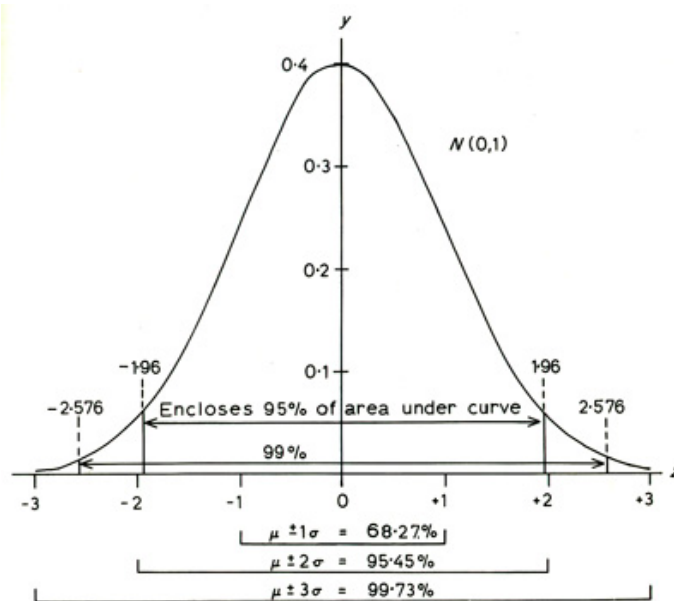
$T_a$  = air temperature (K)

# Atmospheric dispersion of air pollutants

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- We assume Gaussian (normal) distribution of pollutants

## *Gaussian distribution*



$$y = \frac{1}{\sigma(2\pi)^{1/2}} \exp \left[ \frac{-(x - \mu)^2}{2\sigma^2} \right]$$

When the area of the curve is 1

$\mu = \text{mean}$

$\sigma = \text{standard deviation}$

# Atmospheric dispersion of air pollutants

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... But in two directions (y and z) at a certain distance downwind

So, for a continuous point source of an air pollutant located at (x, y, z) = (0, 0, H):

$$C(x, y, z) = \left(\frac{E}{u}\right) \times \left(\frac{1}{\sigma_y(2\pi)^{1/2}} \exp\left[\frac{-(y-0)^2}{2\sigma_y^2}\right]\right) \times \left(\frac{1}{\sigma_z(2\pi)^{1/2}} \exp\left[\frac{-(z-H)^2}{2\sigma_z^2}\right]\right)$$

↑  
Total mass of pollutant  
between  $x$  m and  $(x+1)$  m

$C$  = concentration (g/m<sup>3</sup>)  
 $E$  = emission rate (g/s)  
 $u$  = wind speed (m/s)

# Atmospheric dispersion of air pollutants

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Therefore,

$$C(x, y, z) = \left( \frac{E}{2\pi u \sigma_y \sigma_z} \right) \left[ \exp \left( \frac{-y^2}{2\sigma_y^2} \right) \right] \left[ \exp \left( \frac{-(z - H)^2}{2\sigma_z^2} \right) \right]$$

where  $\sigma_y$  and  $\sigma_z$  are functions of  $x$  and atmospheric conditions (wind speed, stability, etc.)

# Atmospheric dispersion of air pollutants

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How do we account for the ground?

1) No reflection: apply the equation we derived!

$$C(x, y, z) = \left( \frac{E}{2\pi u \sigma_y \sigma_z} \right) \left[ \exp \left( \frac{-y^2}{2\sigma_y^2} \right) \right] \left[ \exp \left( \frac{-(z - H)^2}{2\sigma_z^2} \right) \right]$$

2) Total reflection: assume a virtual source at  $(x, y, z) = (0, 0, -H)$

$$C(x, y, z) = \left( \frac{E}{2\pi u \sigma_y \sigma_z} \right) \left[ \exp \left( \frac{-y^2}{2\sigma_y^2} \right) \right] \\ \times \left( \left[ \exp \left( \frac{-(z - H)^2}{2\sigma_z^2} \right) \right] + \left[ \exp \left( \frac{-(z + H)^2}{2\sigma_z^2} \right) \right] \right)$$



# Atmospheric dispersion of air pollutants

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Concentration of a pollutant at the ground level with total reflection:

$$C(x, y, 0) = \left( \frac{E}{\pi u \sigma_y \sigma_z} \right) \left[ \exp \left( \frac{-y^2}{2\sigma_y^2} \right) \right] \left[ \exp \left( \frac{-H^2}{2\sigma_z^2} \right) \right]$$

Eq. (12-18) of the textbook

Now, how do we obtain  $\sigma_y$  and  $\sigma_z$  (functions of  $x$  and atmospheric stability)?

## Key to stability categories

Surface wind speed (at 10 m) (m/s)	Day <sup>a</sup>			Night <sup>a</sup>	
	Incoming solar radiation			Thinly overcast or	
	Strong	Moderate	Slight	$\geq 1/2$ Low cloud	$\leq 3/8$ Cloud
<2	A	A-B	B	—	—
2-3	A-B	B	C	E	F
3-5	B	B-C	C	D	E
5-6	C	C-D	D	D	D
>6	C	D	D	D	D

<sup>a</sup>The neutral class, D, should be assumed for overcast conditions during day or night. Note that “thinly overcast” is not equivalent to “overcast.”

*Notes:* Class A is the most unstable and class F is the most stable class considered here. Night refers to the period from one hour before sunset to one hour after sunrise. Note that the neutral class, D, can be assumed for overcast conditions during day or night, regardless of wind speed.

“Strong” incoming solar radiation corresponds to a solar altitude greater than 60° with clear skies; “slight” insolation corresponds to a solar altitude from 15° to 35° with clear skies. Table 170, Solar Altitude and Azimuth, in the Smithsonian Meteorological Tables, can be used in determining solar radiation. Incoming radiation that would be strong with clear skies can be expected to be reduced to moderate with broken (5/8 to 7/8 cloud cover) middle clouds and to slight with broken low clouds.

(Source: Turner, 1967.)

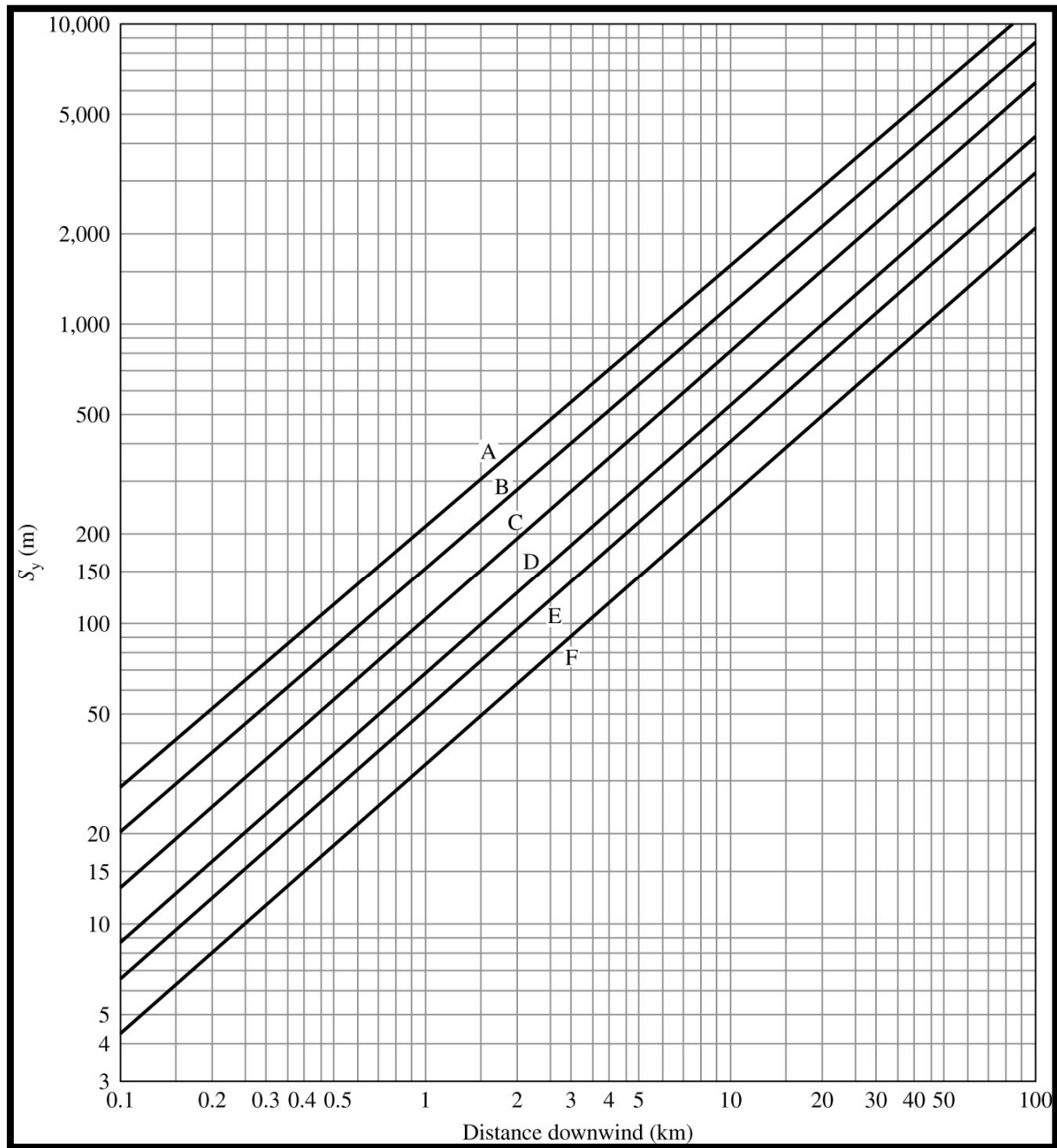
$$\sigma_y = ax^{0.894}$$

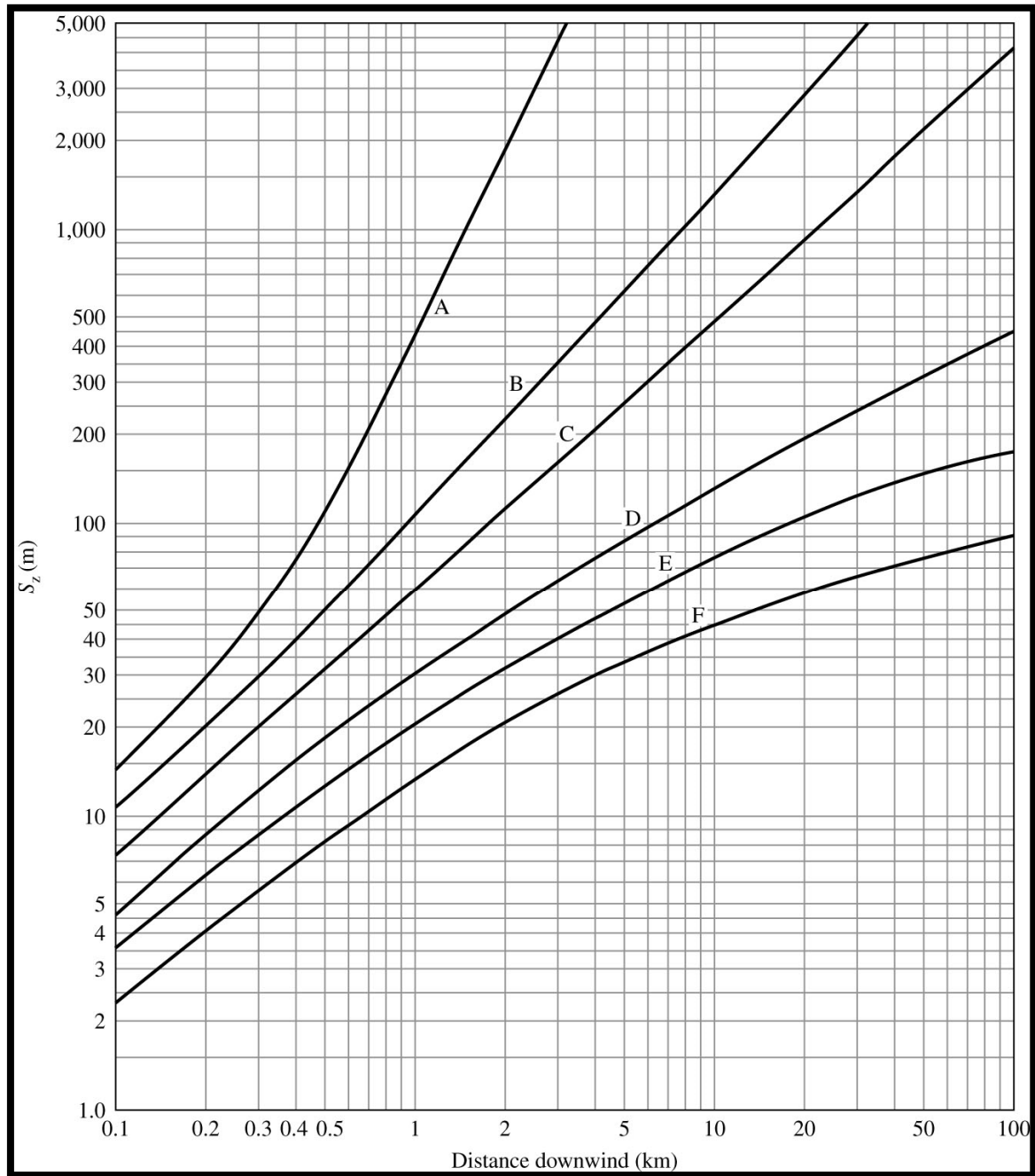
$$\sigma_z = cx^d + f \quad x = \text{downwind distance (km)}$$

**Values of  $a$ ,  $c$ ,  $d$ , and  $f$  for calculating  $s_y$  and  $s_z$**

Stability class	$x \leq 1 \text{ km}$				$x > 1 \text{ km}$		
	$a$	$c$	$d$	$f$	$c$	$d$	$f$
A	213	440.8	1.941	9.27	459.7	2.094	-9.6
B	156	100.6	1.149	3.3	108.2	1.098	2
C	104	61	0.911	0	61	0.911	0
D	68	33.2	0.725	-1.7	44.5	0.516	-13.0
E	50.5	22.8	0.678	-1.3	55.4	0.305	-34.0
F	34	14.35	0.74.0	-0.35	62.6	0.18	-48.6

(Source: Martin, 1976.)





# Atmospheric dispersion of air pollutants

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**Q:** A coal-fired power plant emits  $\text{SO}_2$  at a rate of 1656 g/s. At 3 km downwind on an overcast summer afternoon, what is the concentration of  $\text{SO}_2$  on the ground? The following parameters apply.

**Stack parameters:**

Height = 120 m

Diameter = 1.2 m

Exit velocity = 10 m/s

Temperature = 315°C

**Atmospheric conditions:**

Pressure = 95.0 kPa

Temperature = 25°C

Wind speed = 4.5 m/s

# Reading assignment

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Textbook Ch 12 p. 621-631