

**2016 Fall**

# **“Phase Transformation *in* Materials”**

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## Summary for previous class

- **Thermally Activated Migration of Grain Boundaries:**

Metastable equilibrium of grain boundary (Balances of 1) boundary E + 2) surface tension)

→ **real curvature** ( $\Delta P \rightarrow \Delta G$ : Gibbs Thomson Eq.) →  $F = 2\gamma/r = \Delta G/V_m$  (by curvature)

(Pulling force per unit area of boundary)

→ **Grain coarsening at high T annealing**

- **Kinetics of Grain Growth**

- **Grain boundary migration ( $v$ ) by thermally activated atomic jump**

Boundary velocity  $v = \frac{A_2 n_1 v_1 V_m^2}{N_a RT} \exp\left(-\frac{\Delta G^a}{RT}\right) \frac{\Delta G}{V_m}$

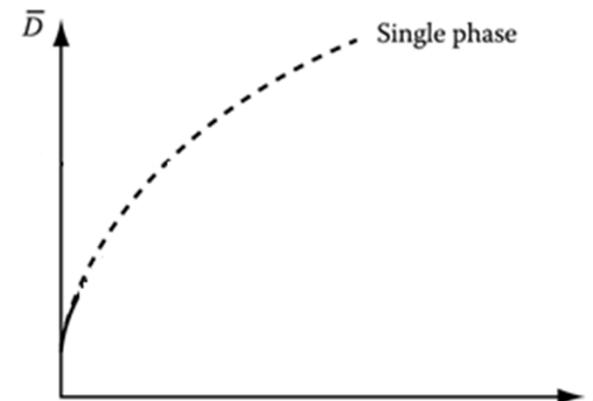
$v \sim \Delta G/V_m$  driving force  
 $\rightarrow F = \Delta G/V_m$

$M$  : mobility = velocity under unit driving force  $\sim \exp(-1/T)$

rate of grain growth  $d\bar{D}/dt \sim 1/\bar{D}$ , exponentially increase with  $T$

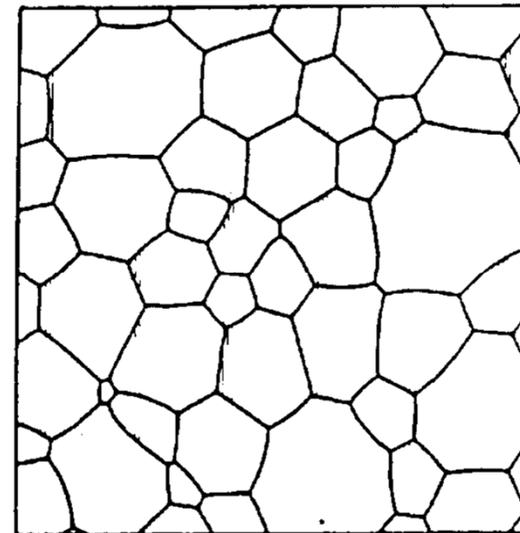
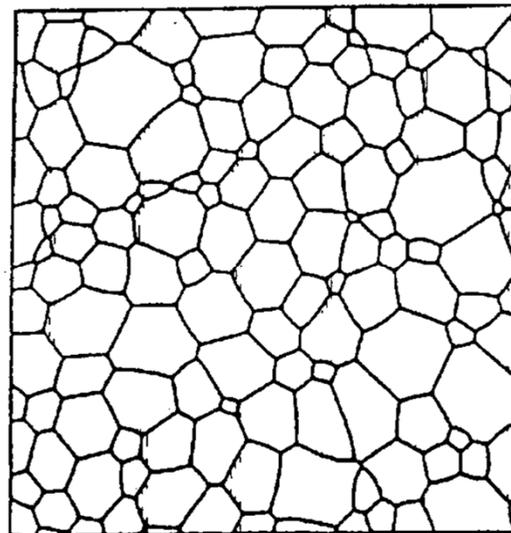
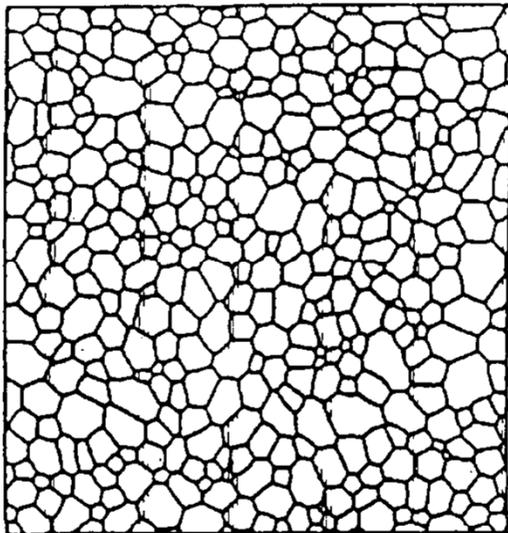
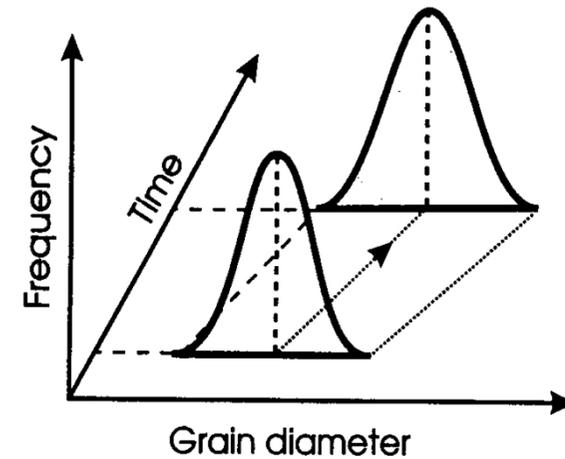
→  $\bar{D} = k't^n$

( Experimental:  $n < 1/2$ ,  $1/2$  at pure metals or high Temp.)



# Normal Grain Growth

- Grain boundary moves to reduce area and total energy
- Large grain grow, small grains shrink
- Average grain size increases
- Little change of size distribution



## Considering factors of G.B. growth

(a) Impurity (solute) drag

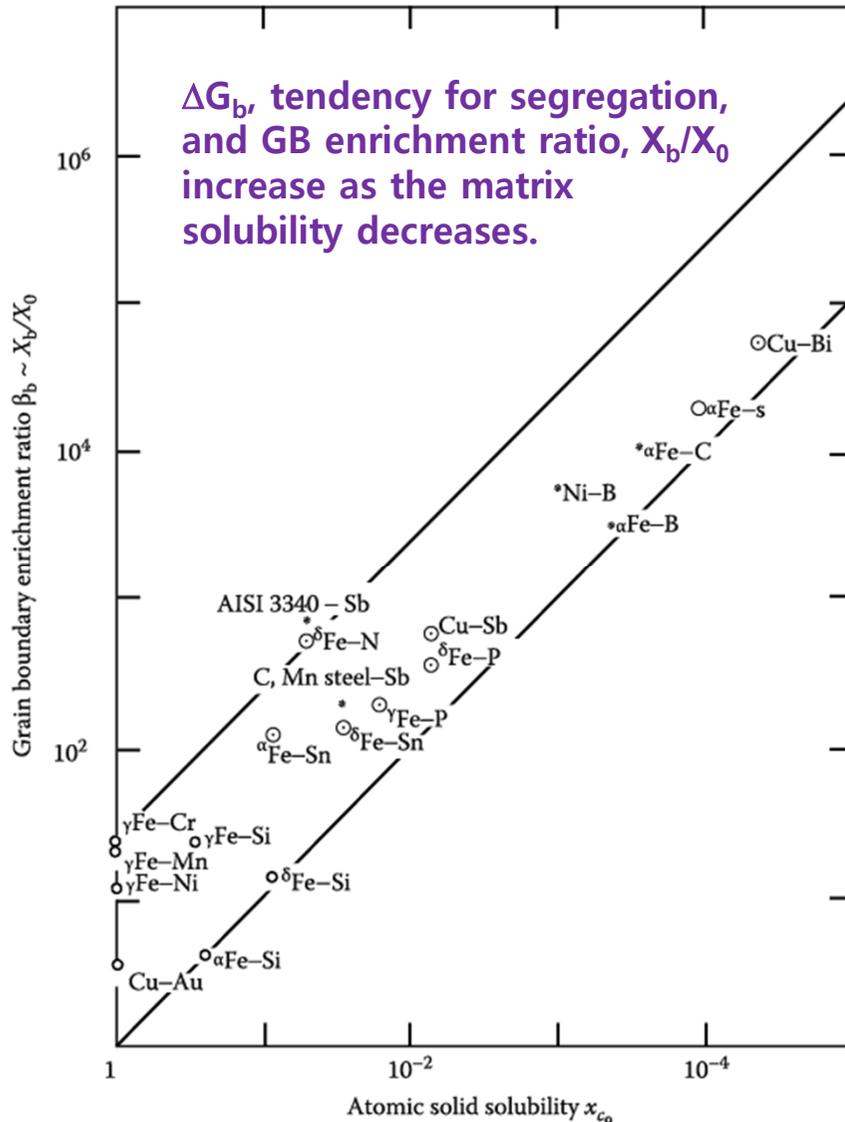
(b) Pinning particle

(c) 2<sup>nd</sup> phases

(d) Anisotropic  $\sigma$ , M

(e) Strain energy

(f) Free surface



<Increasing GB enrichment with decreasing solid solubility in a range of system>

$X_0$  : matrix solute concentration/  $X_b$  : boundary solute concentration

$\Delta G_b$  : free energy reduced when one mole of solute is moved to GB from matrix.

( $\Delta G_b$ ) → The high mobility of special boundaries can possibly be attributed to a low solute drag on account of the relatively more close-packed structure of the special boundaries.

## (a) Impurity (solute) drag

\* Solute drag effect

In general,

$G_b$  (grain boundary E) and mobility of pure metal decreases on alloying.

~Impurities tend to stay at the GB.

Generally,  $\Delta G_b$ , tendency of segregation, increases as the matrix solubility decreases.

$$X_b = X_0 \exp \frac{\Delta G_b}{RT}$$

$X_b/X_0$ : GB enrichment ratio

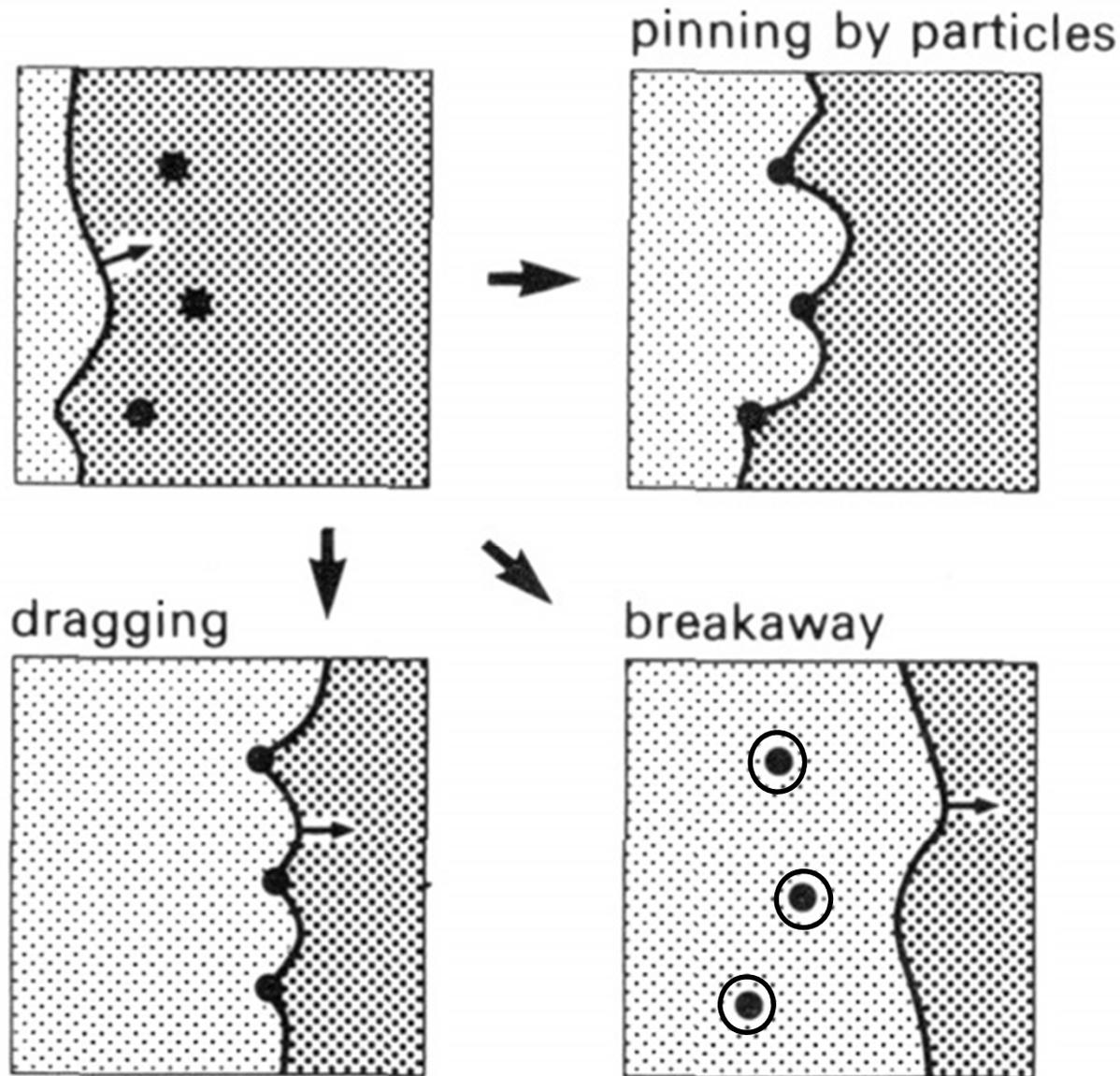
- Decreases as temp. increases, i.e., the solute "evaporates" into the matrix

Low T or  $\Delta G_b$  ↑  $X_b$  ↑ Mobility of G.B. ↓

→ Alloying elements affects mobility of G.B.

**(b) Pinning particle or (c) 2<sup>nd</sup> phases**

**Schematic diagram illustrating the possible interactions of second phase particles and migrating grain boundaries.**

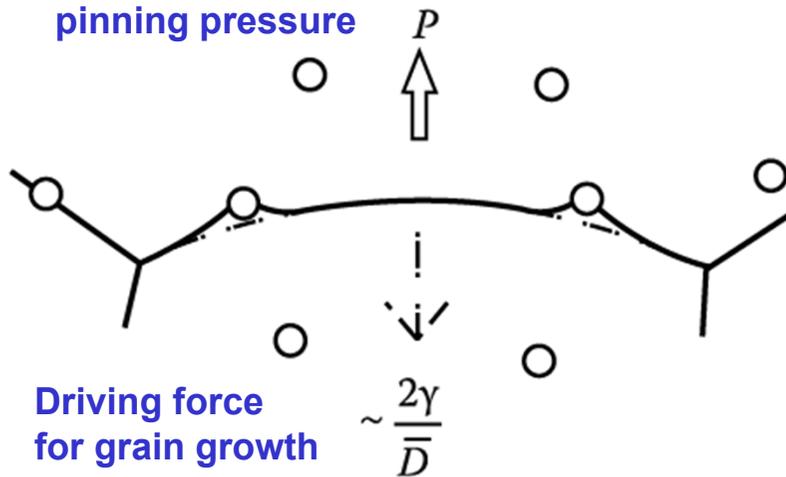


## Interaction with particles

## Zener Pinning

$$P = \frac{3f_v}{2\pi r^2} \cdot \pi r \gamma = \frac{3f_v \gamma}{2r}$$

This force will oppose the driving force for grain growth,  $2\gamma/\bar{D}$ .



$$\frac{2\gamma}{\bar{D}} = \frac{3f_v \gamma}{2r} \rightarrow \bar{D}_{\max} = \frac{4r}{3f_v}$$

Driving force will be insufficient to overcome the drag of the particles and grain growth stagnates.

For fine grain size

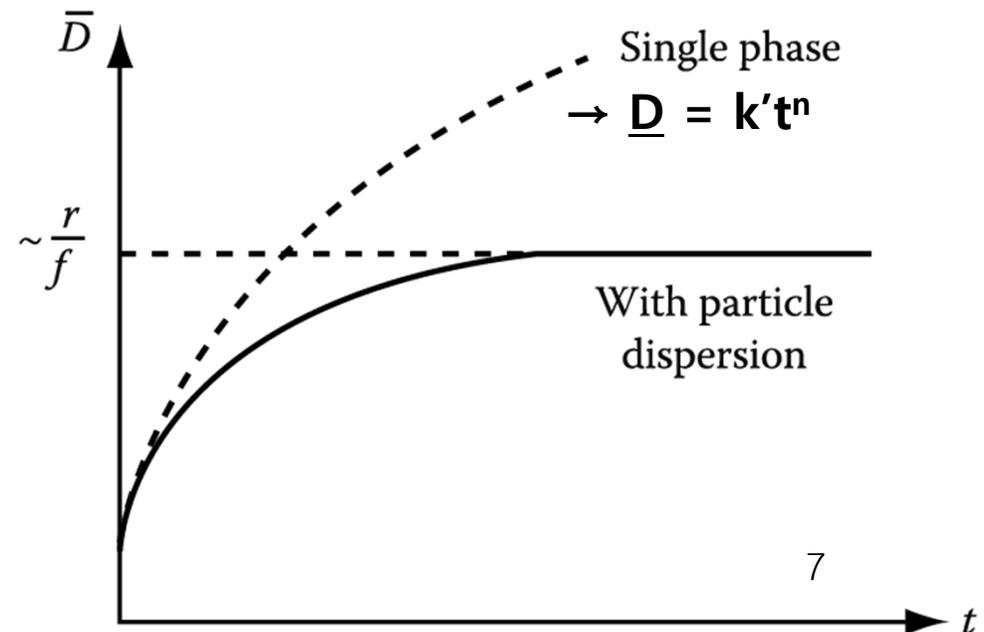
$\rightarrow$  a large volume fraction of very small particles

## \* Effect of second-phase particles on grain growth

$$\bar{D}_{\max} = \frac{4r}{3f_v}$$

: Stabilization of a fine grain size during heating at high temp.  $\rightarrow$  large volume fraction ( $f \uparrow$ ) of very small particles ( $r \downarrow$ ).

$$\bar{D}_{\max} = \frac{4r}{3f_v} \downarrow$$

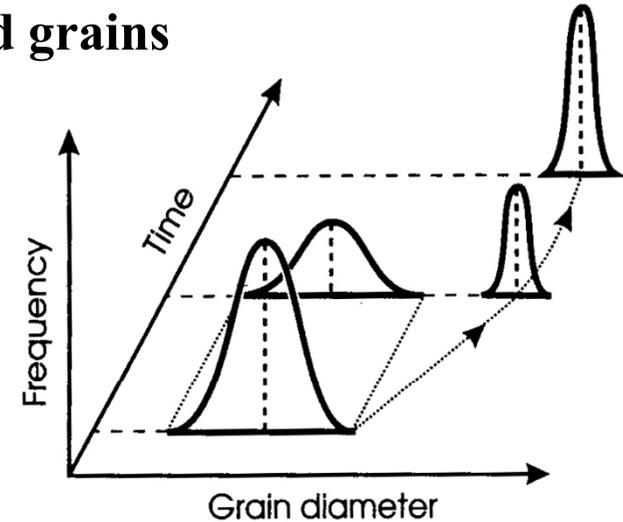


# Abnormal Grain Growth

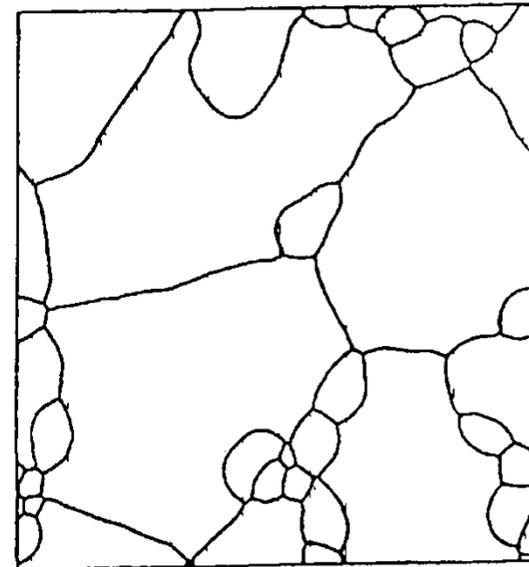
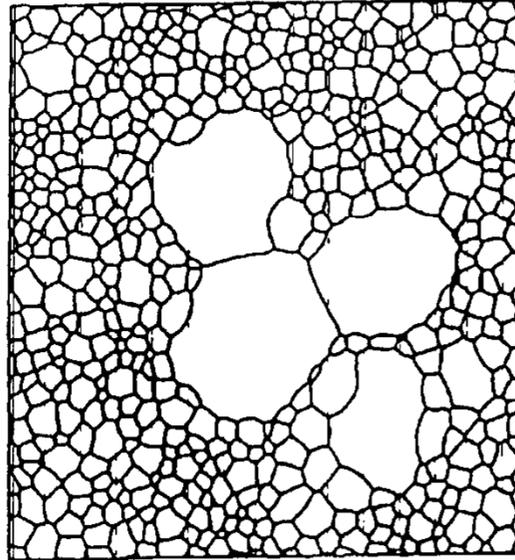
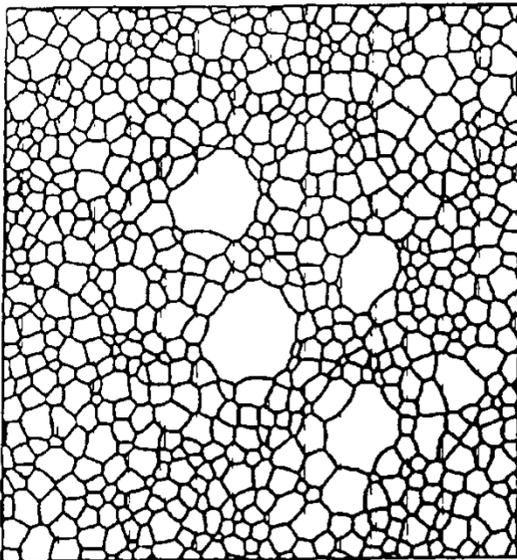
(high mobility of special GBs → development of recrystallization textures)

## ❑ Discontinuous grain growth of a few selected grains

- Local breaking of pinning by precipitates
- Anisotropy of grain boundary mobility
- Anisotropy of surface & grain boundary energy
- Selective segregation of impurity atoms
- Inhomogeneity of strain energy

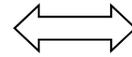


## ❑ Bimodal Size distribution



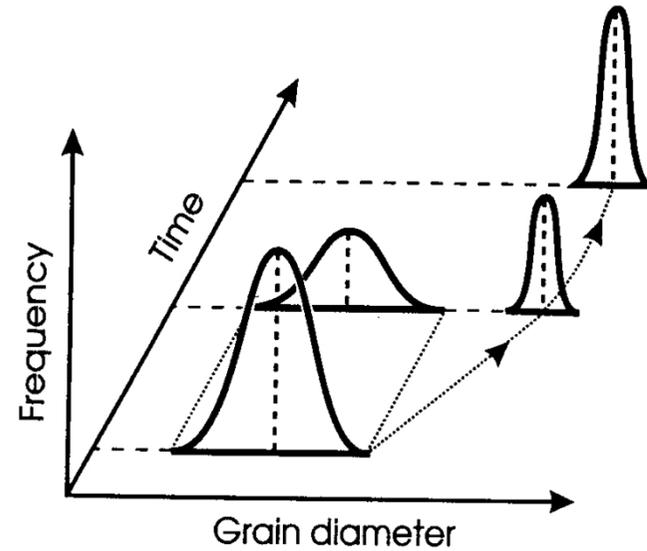
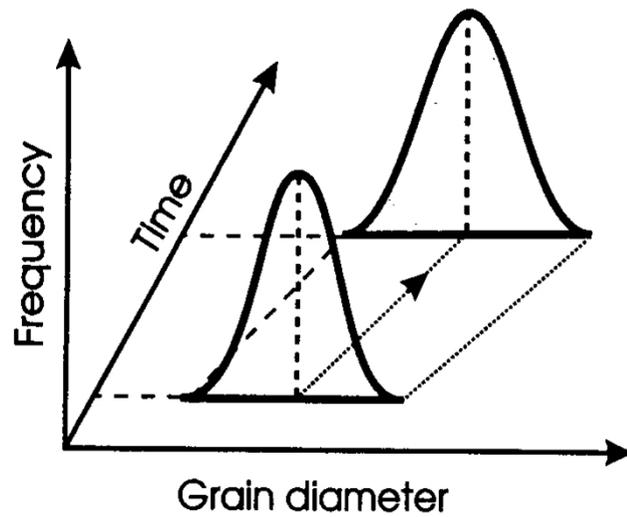
# • Grain Growth

- Normal grain growth

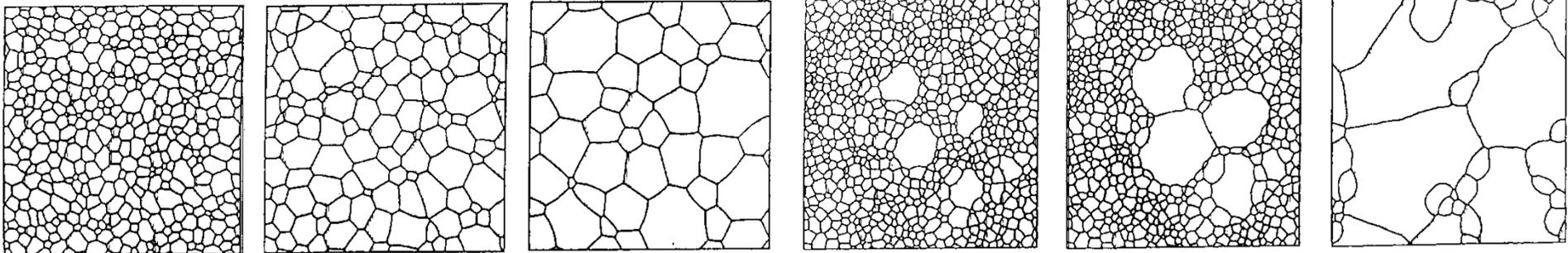


**Abnormal grain growth**

(high mobility of special GBs  
→ development of recrystallization textures)



< Bimodal Size distribution >



# Contents for today's class

- **Interphase Interfaces in Solid ( $\alpha/\beta$ )**

- **Types of interphase interfaces in solid ( $\alpha/\beta$ )**

- **Second-Phase Shape** { **Interface Energy Effects**  
Coherent / Semi-coherent / incoherent  
**Misfit Strain Effects**

$$\sum A_i \gamma_i + \Delta G_S = \text{minimum}$$

- **Coherency Loss**

- **Glissil Interfaces**  $\longleftrightarrow$  **Solid/Liquid Interfaces**

- **Interface migration**

- **Interface controlled growth**  $\longleftrightarrow$  **Diffusion controlled growth**

**Q: What kind of interphase interfaces  
in solid ( $\alpha/\beta$ ) exist?**

**= coherent/ semi-coherent / incoherent/ complex semi-coherent**

**→ different interfacial free energy,  $\gamma$**

## 3.4 Interphase Interfaces in Solids

Interphase boundary

- different two phases : **different crystal structure**  
**different composition**

coherent,  
semicoherent  
incoherent

### 3.4.1 Coherent interfaces

Disregarding chemical species, if the interfacial plane has the same atomic configuration in both phases,

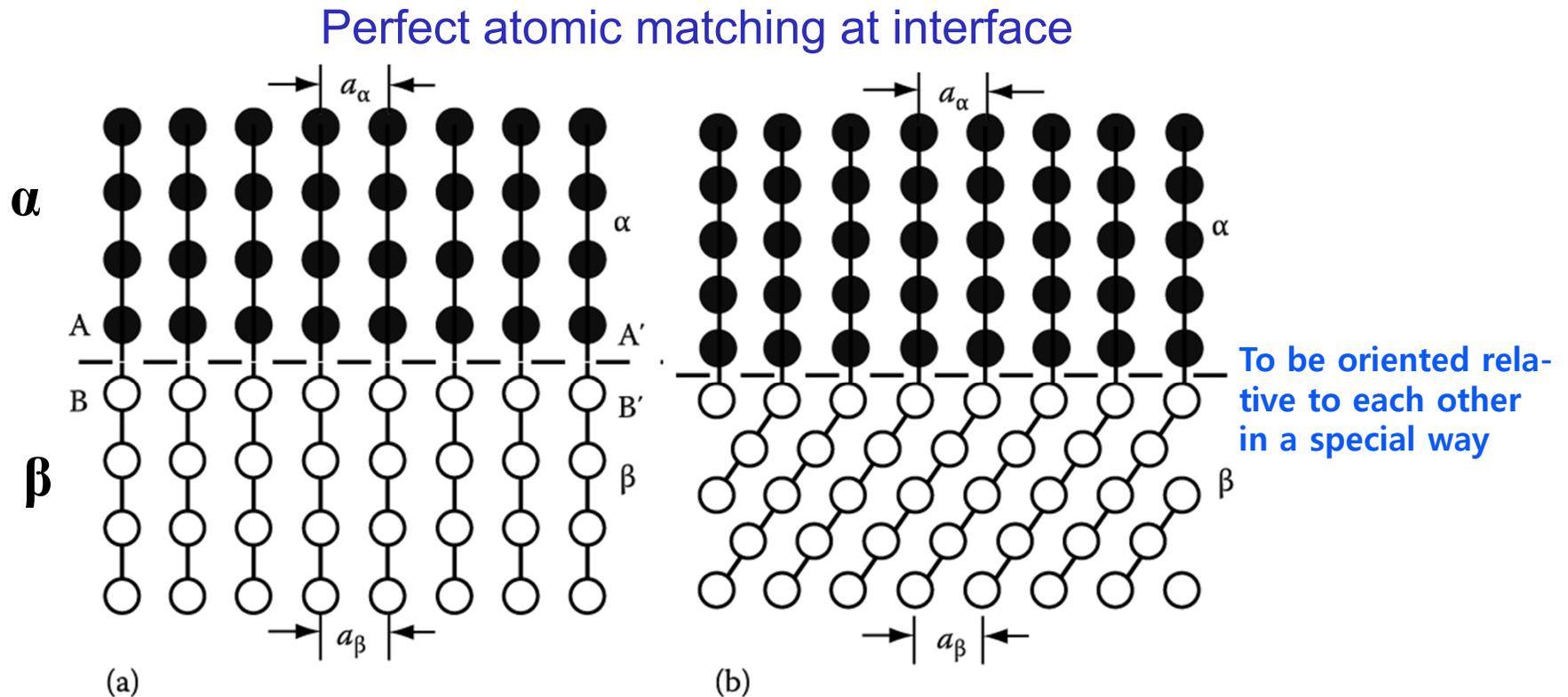


Fig. 3.32 Strain-free coherent interfaces. (a) Each crystal has a different chemical composition but the same crystal structure. (b) The two phases have different lattices

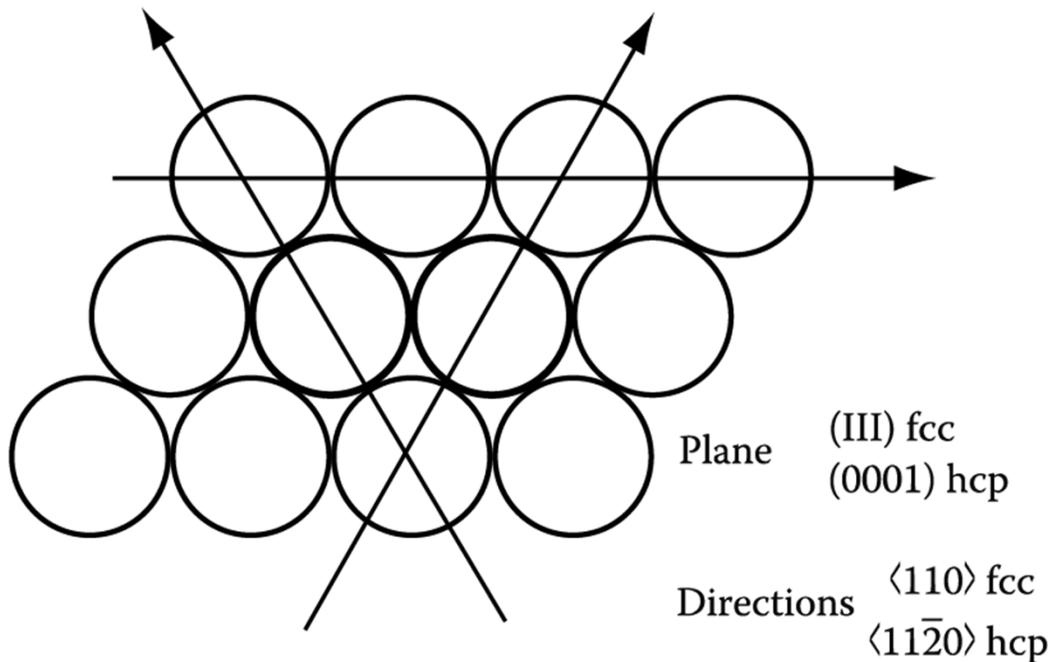
### 3.4.1 Coherent interfaces

Which plane and direction will be coherent between FCC and HCP?

: Interphase interface will make lowest energy and thereby the lowest nucleation barrier

ex) hcp silicon-rich  $\kappa$  phase in fcc copper-rich  $\alpha$  matrix of Cu-Si alloy

→ the same atomic configuration  
& interatomic distance



→ Orientation relation

$$\text{Cu } (111)_{\alpha} // (0001)_{\kappa} \text{ Si}$$

$$[\bar{1}10]_{\alpha} // [11\bar{2}0]_{\kappa}$$

$$\gamma_{\alpha-\kappa} \text{ of Cu-Si} \sim 1 \text{ mJm}^{-2}$$

In general,

$$\gamma (\text{coherent}) \sim 200 \text{ mJm}^{-2}$$

$$\begin{aligned} \gamma_{\text{coherent}} &= \gamma_{\text{structure}} + \gamma_{\text{chemical}} \\ &= \gamma_{\text{chemical}} \end{aligned}$$

Fig. 3.33 The close-packed plane and directions in fcc and hcp structures.

hcp/ fcc interface: only one plane that can form a coherent interface

$$\gamma (\text{coherent}) = \gamma_{\text{ch}} \quad 13$$

When the atomic spacing in the interface is not identical between the adjacent phase, what would happen?

Possible to maintain coherency by straining one or both crystal lattices.

→ lattice distortion

→ Coherency strain

→ strain energy

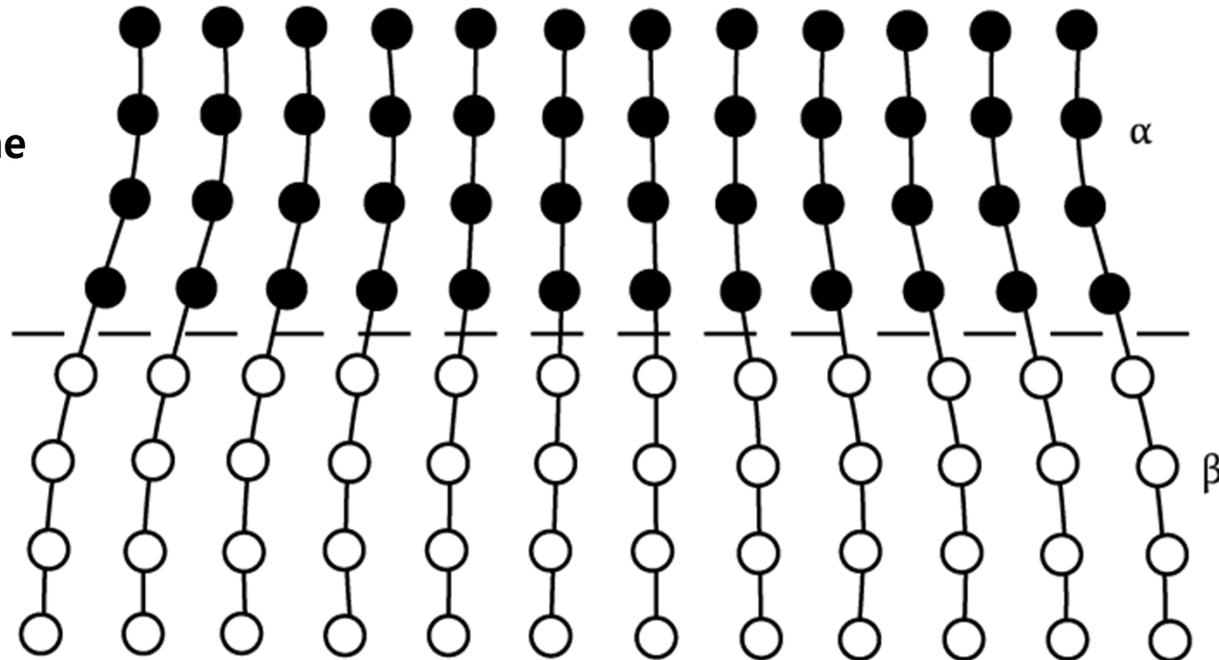
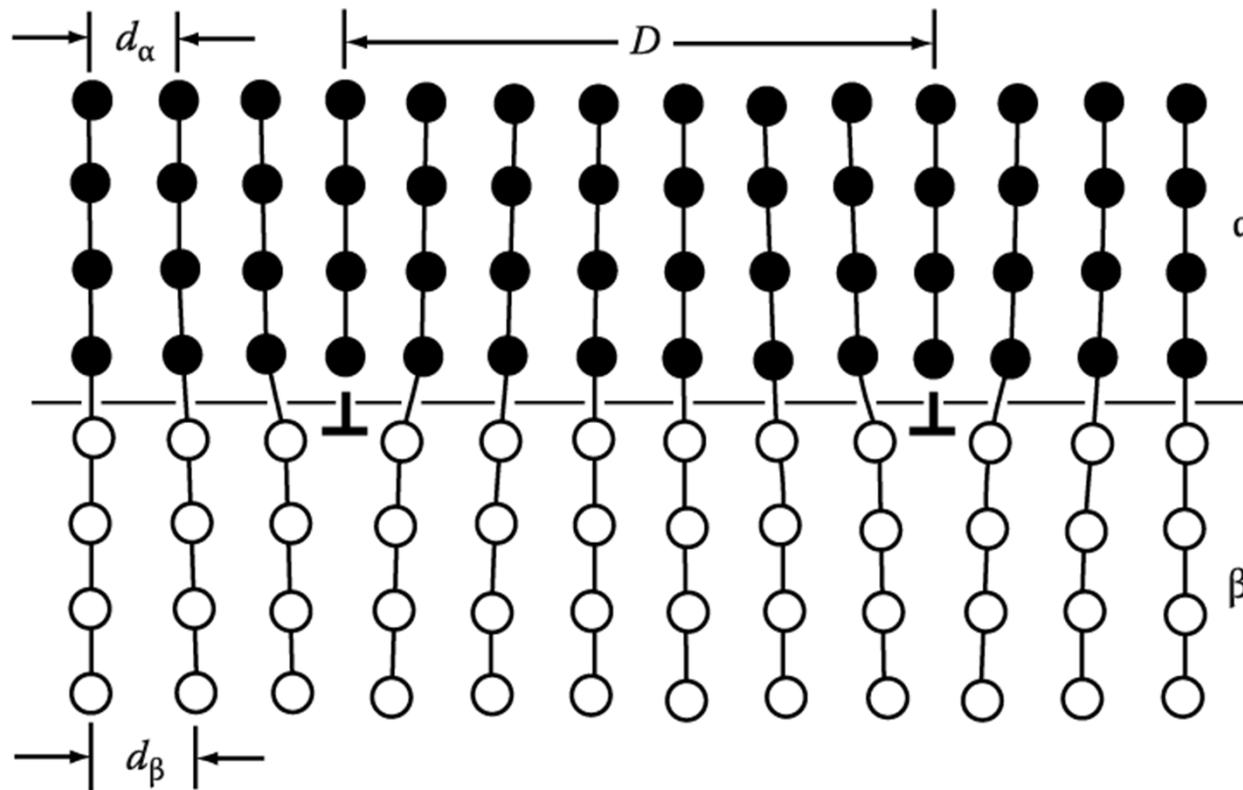


Fig. 3.34 A coherent interface with slight mismatch leads to coherency strains in the adjoining lattices.

**The strains associated with a coherent interface raise the total energy of the system.**

**How can this coherent strain can be reduced?**

If coherency strain energy is sufficiently large, → “misfit dislocations”  
 → semi-coherent interface



Misfit between the two lattices

$$\delta = \frac{d_\beta - d_\alpha}{d_\alpha}$$

$\delta \sim \text{small,}$

$$D = \frac{b}{\delta}$$

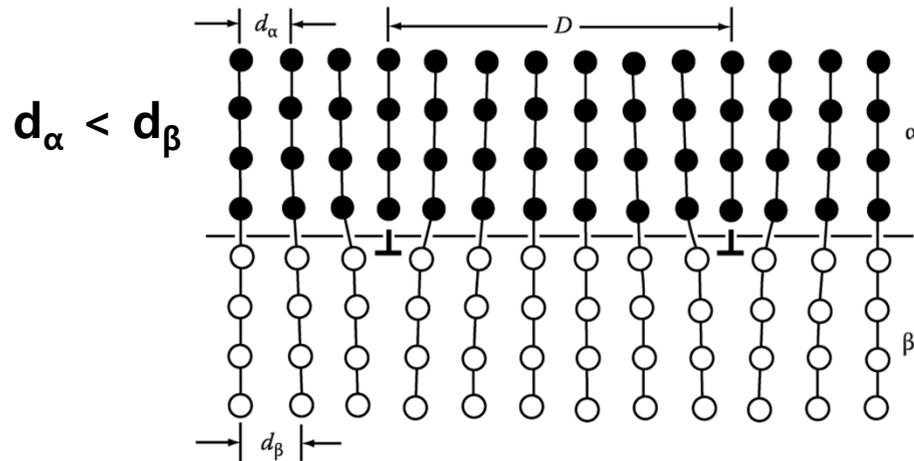
$\delta$ : misfit (disregistry)

**b**: Burgers vector of disl.

$$[\mathbf{b} = (\mathbf{d}_\alpha + \mathbf{d}_\beta) / 2]$$

Fig. 3.35 A semi-coherent interface. The misfit parallel to the interface is accommodated by a series of edge dislocations.

## (2) Semicoherent interfaces



$$\delta = (d_\beta - d_\alpha) / d_\alpha : \text{misfit}$$

→ D vs.  $\delta$  vs. n

$$(n+1) d_\alpha = n d_\beta = D$$

$$\delta = (d_\beta / d_\alpha) - 1, (d_\beta / d_\alpha) = 1 + 1/n = 1 + \delta$$

→  $\delta = 1/n$

$$D = d_\beta / \delta \approx b / \delta \quad [b = (d_\alpha + d_\beta) / 2]$$

$\delta \sim$  small,

Burgers vector of dislocation

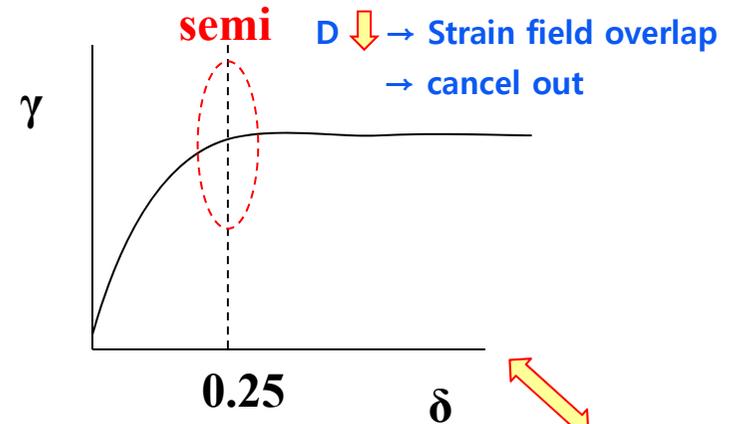
$$\gamma(\text{semicoherent}) = \gamma_{ch} + \gamma_{st}$$

$\gamma_{st}$  → due to **structural distortions** caused by the misfit dislocations

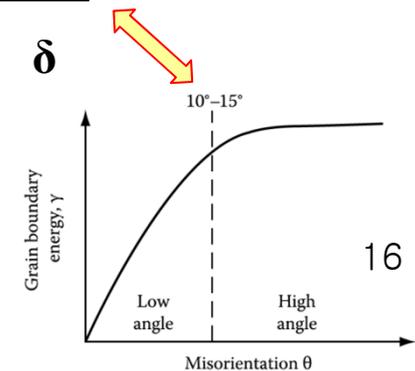
$$\gamma_{st} \propto \delta \text{ for small } \delta$$

In general,

$\gamma(\text{semicoherent}) \sim 200 \sim 500 \text{ mJm}^{-2}$



**1 dislocation per 4 lattices**  
**n=4**



16

### 3) Incoherent Interfaces ~ high angle grain boundary

1)  $\delta > 0.25$  No possibility of good matching across the interface

2) different crystal structure (in general)

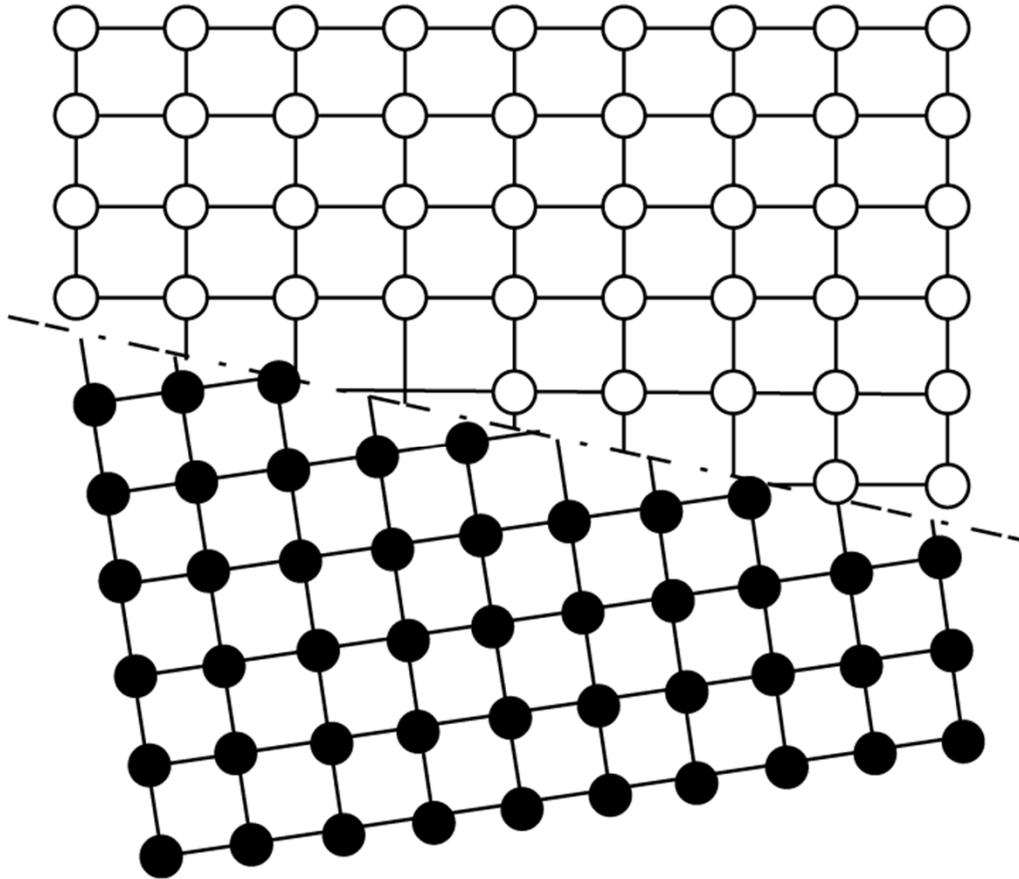
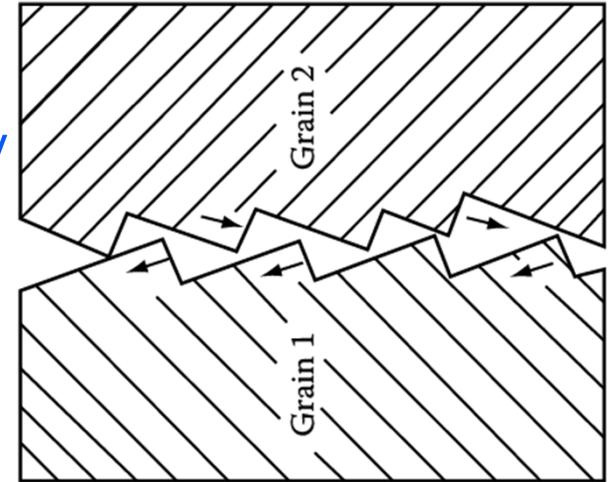


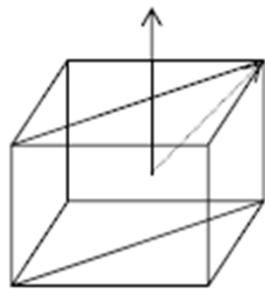
Fig. 3.37 An incoherent interface.



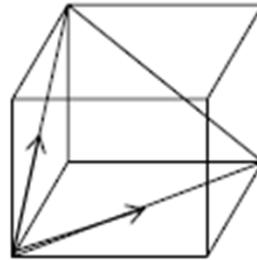
In general,  
 $\gamma$  (incoherent)  $\sim 500\sim 1000 \text{ mJm}^{-2}$

incoherent

## 4) Complex Semicoherent Interfaces



$$a_{\alpha} = 2.87$$



$$a_{\gamma} = 3.57$$

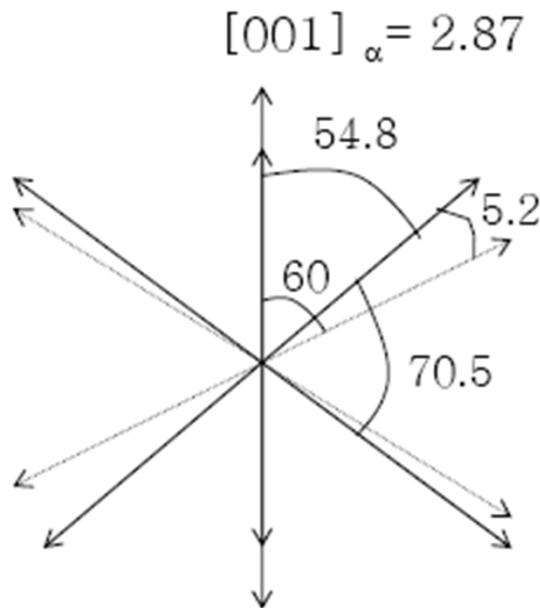
If **bcc**  $\alpha$  is precipitated from **fcc**  $\gamma$ , which interface is expected?

Which orientation would make the lowest interface energy?

For fcc and bcc crystals ~ closest-pack planes in each phase almost parallel to each other

Nishiyama-Wasserman (N-W) Relationship

$$(110)_{bcc} // (111)_{fcc}, [001]_{bcc} // [\bar{1}01]_{fcc}$$



Kurdjumov-Sachs (K-S) Relationships

$$(110)_{bcc} // (111)_{fcc}, [1\bar{1}1]_{bcc} // [0\bar{1}1]_{fcc}$$

(The only difference between these two is a rotation in the closest-packed planes of 5.26°.)

## Complex Semicoherent Interfaces

Semicoherent interface observed at boundaries formed by low-index planes.  
(atom pattern and spacing are almost equal.)

N-W relationship

Good fit is restricted to small diamond-shaped areas that only contain ~8% of the orientation relationship.

A similar situation can be shown to exist for the K-S orientation relationship.

⇒ **But,**  
**impossible to form a large interfaces**  
**→ Incoherent interface**

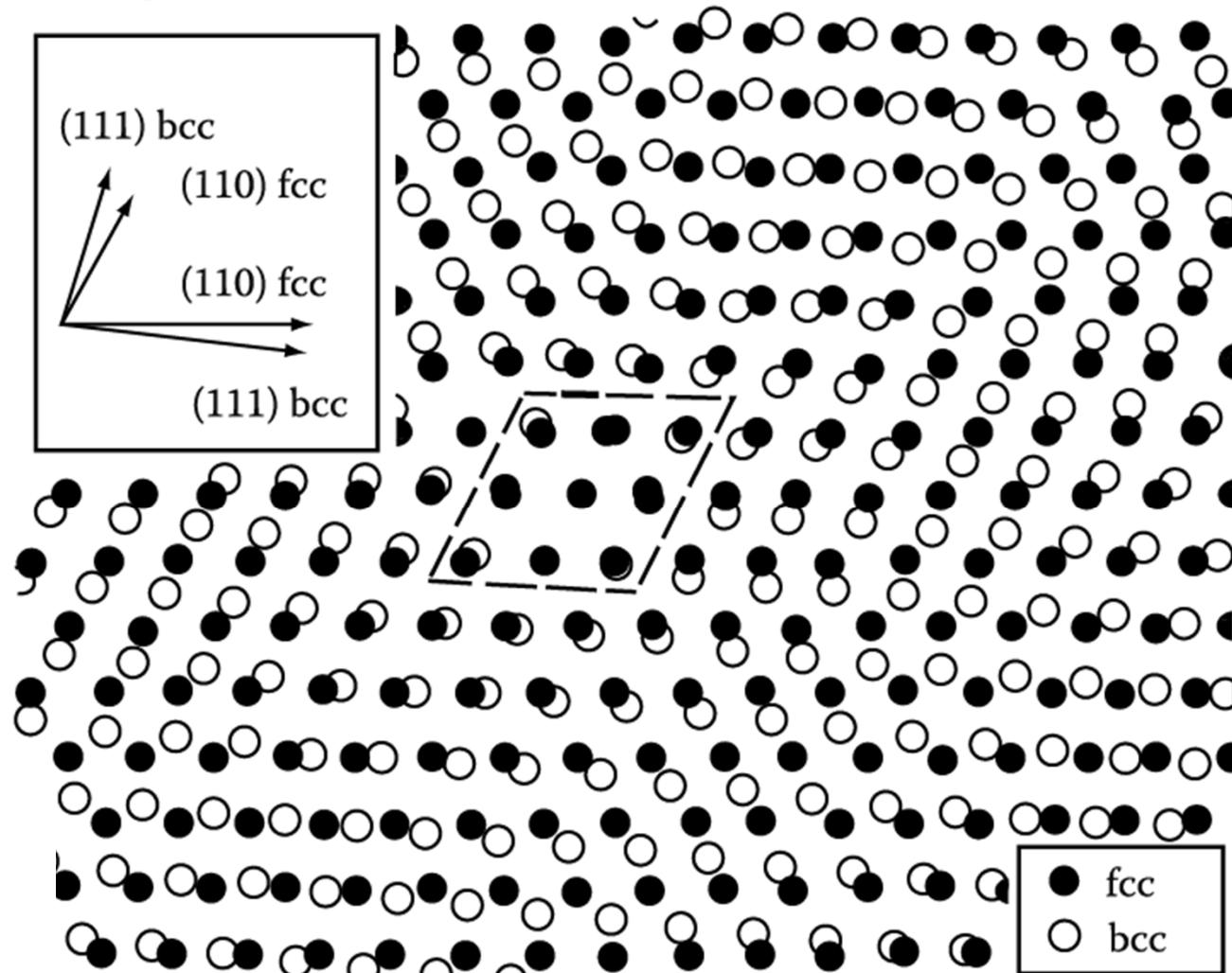
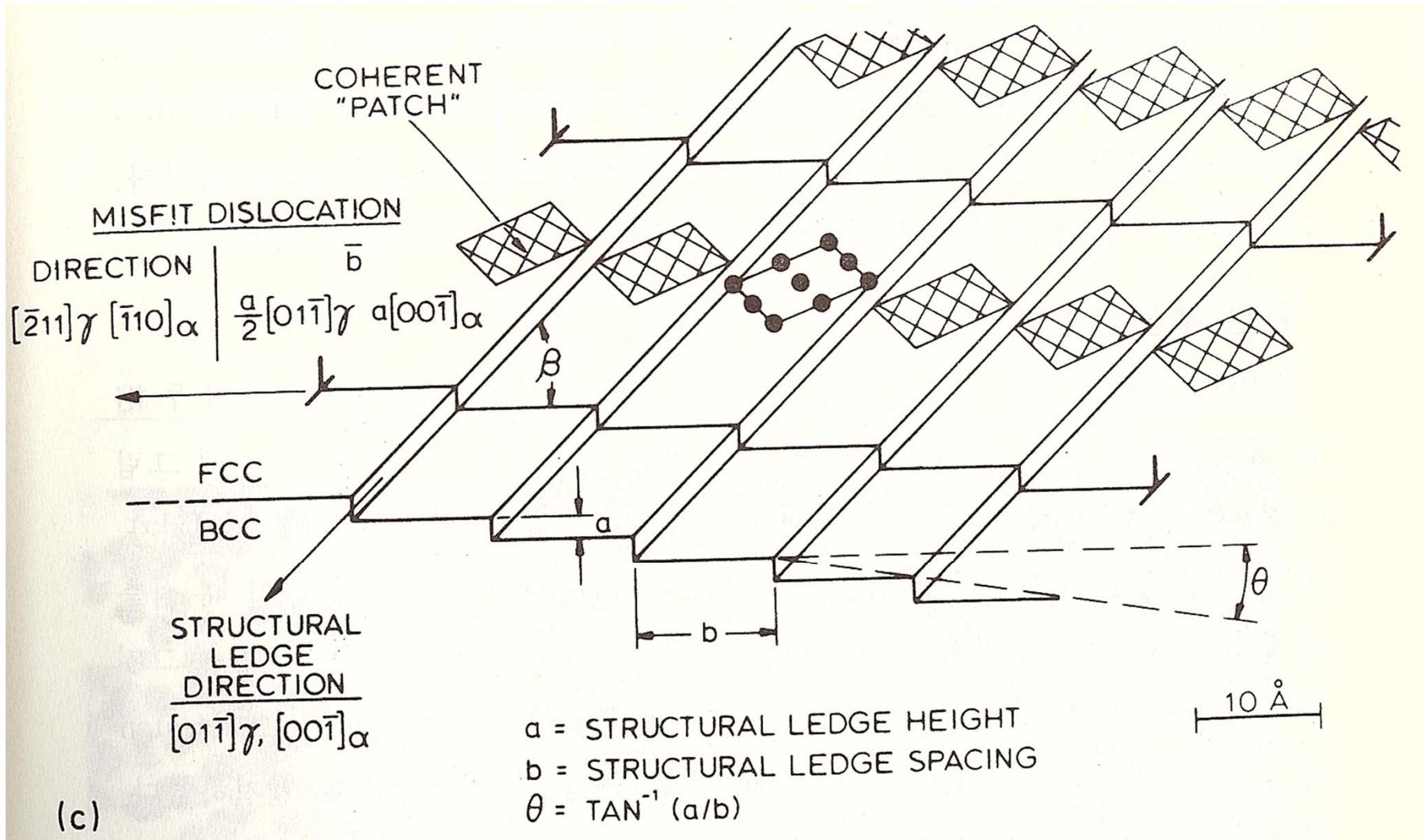


Fig. 3.38 Atomic matching across a (111)fcc/(110)bcc interface bearing the NW orientation relationship for lattice parameters closely corresponding to the case of fcc and bcc iron.

# Complex Semicoherent Interfaces



The degree of coherency can, however, be greatly increased if a macroscopically irrational interface is formed. **The detailed structure of such interfaces is, however, uncertain** due to their complex nature.

# 3.4 Interphase Interfaces in Solids

Interphase boundary - different two phases : **different crystal structure**  
**different composition**

coherent,

Perfect atomic matching at interface

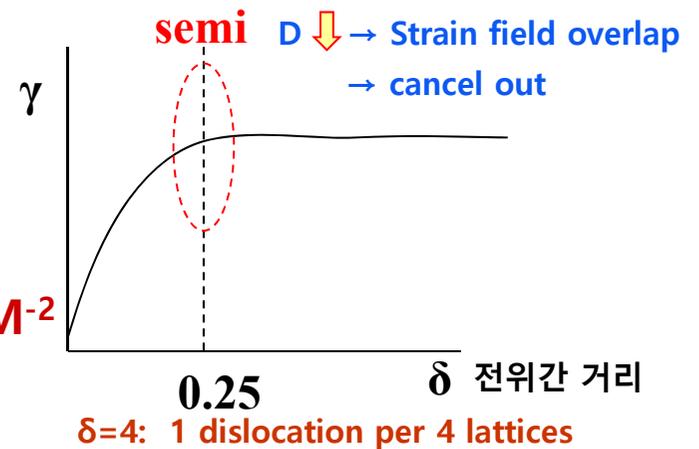
$\gamma$  (coherent) =  $\gamma_{ch}$        $\gamma$  (coherent)  $\sim$  200 mJM<sup>-2</sup>

semicoherent

$\gamma$ (semicoherent) =  $\gamma_{ch} + \gamma_{st}$

$\gamma_{st}$  → due to structural distortions caused by the misfit dislocations

$\gamma$ (semicoherent)  $\sim$  200~500 mJM<sup>-2</sup>



incoherent

1)  $\delta > 0.25$  No possibility of good matching across the interface

2) different crystal structure (in general)

$\gamma$  (incoherent)  $\sim$  500~1000 mJM<sup>-2</sup>

## Complex Semicoherent Interfaces

Nishiyama-Wasserman (N-W) Relationship

Kurdjumov-Sachs (K-S) Relationships

(The only difference between these two is a rotation in the closest-packed planes of 5.26°.)

The degree of coherency can, however, be greatly increased if a macroscopically irrational interface is formed.

# Q: How is the second-phase shape determined?

If misfit is small,  
Equilibrium shape of a coherent  
precipitate or zone **can only**  
**be predicted from the "γ-plot"**

$$\sum A_i \gamma_i$$

⇒  
Misfit

$$\sum A_i \gamma_i + \Delta G_S = \textit{minimum}$$

"γ-plot" + "Elastic strain energy"

Lowest total interfacial free energy  
by optimizing the shape of the precipitate and its orientation relationship

Fully coherent precipitates

$$\gamma_{ch}$$

different composition



$$\gamma_{ch} + \textit{Lattice misfit}$$

Coherency strain energy



Incoherent inclusions

$$\gamma_{ch} + \textit{Volume Misfit } \Delta = \frac{\Delta V}{V}$$

Chemical and structural interfacial E

(a) Precipitate shapes :  $\sum A_i \gamma_i$  ↓

(b) Calculation of misfit strain energy

## 3.4.2 Second-Phase Shape: Interfacial Energy Effects

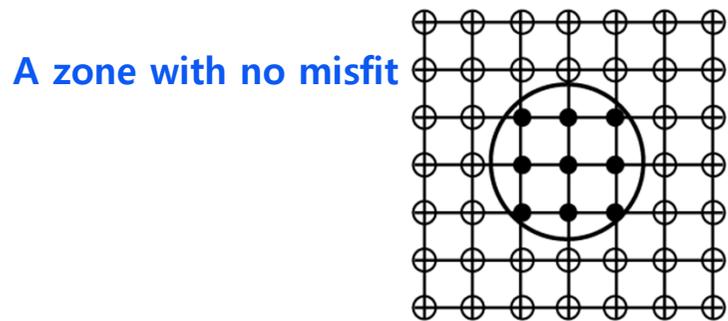
**How is the second-phase shape determined?**  $\sum A_i \gamma_i = \text{minimum}$

Lowest total interfacial free energy

by optimizing the shape of the precipitate and its orientation relationship

### A. Fully Coherent Precipitates (G.P. Zone)

- If  $\alpha$ ,  $\beta$  have the same structure & a similar lattice parameter
- Happens during early stage of many precipitation hardening
- Good match  $\Rightarrow$  can have any shape  $\Rightarrow$  **spherical**

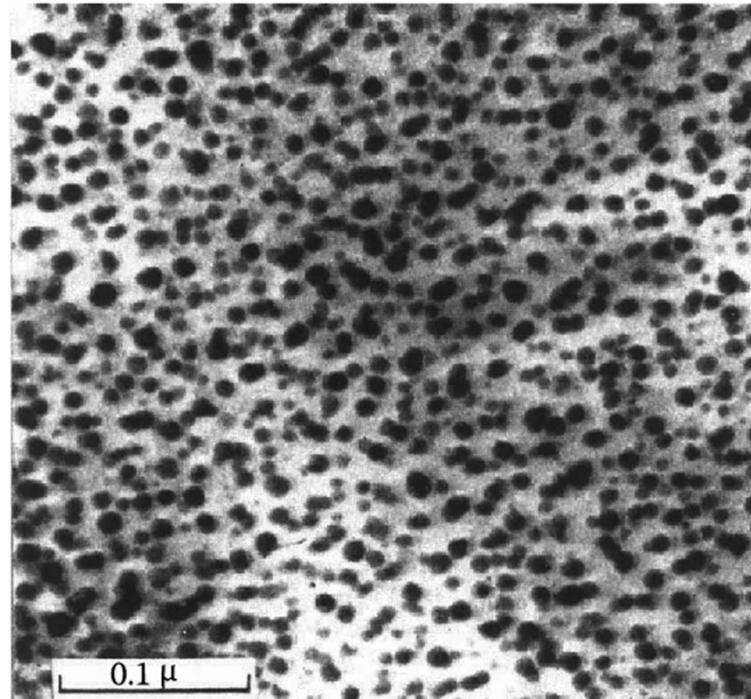


(a)

GP(Guinier- Preston) Zone  
in Al – Ag Alloys

$$\varepsilon_a = \frac{r_A - r_B}{r_A} = 0.7\%$$

$\rightarrow$  negligible contribution  
to the total free energy



(b) Ag-rich GP zones (Dia.  $\sim 10$  nm) in Al-4at% Ag alloy

## B. Partially Coherent Precipitates

- $\alpha$ ,  $\beta$  have different structure and one plane which provide close match
- Coherent or Semi-coherent in one Plane;  
Disc Shape (also plate, lath, needle-like shapes are possible)

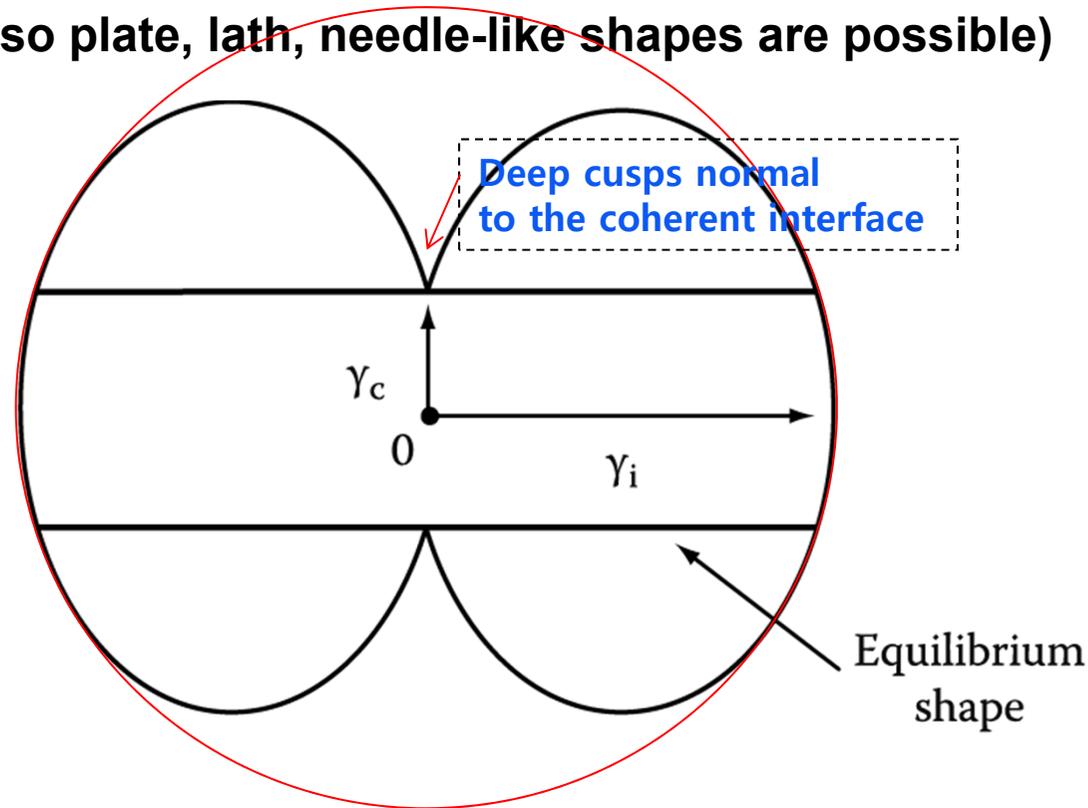


Fig. 3.40 A section through a  $\gamma$ -plot for a precipitate showing one coherent or semi-coherent interface, together with the equilibrium shape (a disc).

### Precipitate shapes observed in practice

- ~ not equilibrium shape through a  $\gamma$ -plot      why? 1) misfit strain  $E$  effects ~ ignored.  
2) different growth rates depending on directions

## *hcp $\gamma'$ Precipitates in Al – 4% Ag Alloys $\rightarrow$ plate*

Semicoherent broad face parallel to the  $\{111\}_\alpha$  matrix planes  
(usual hcp/fcc orientation relationship)

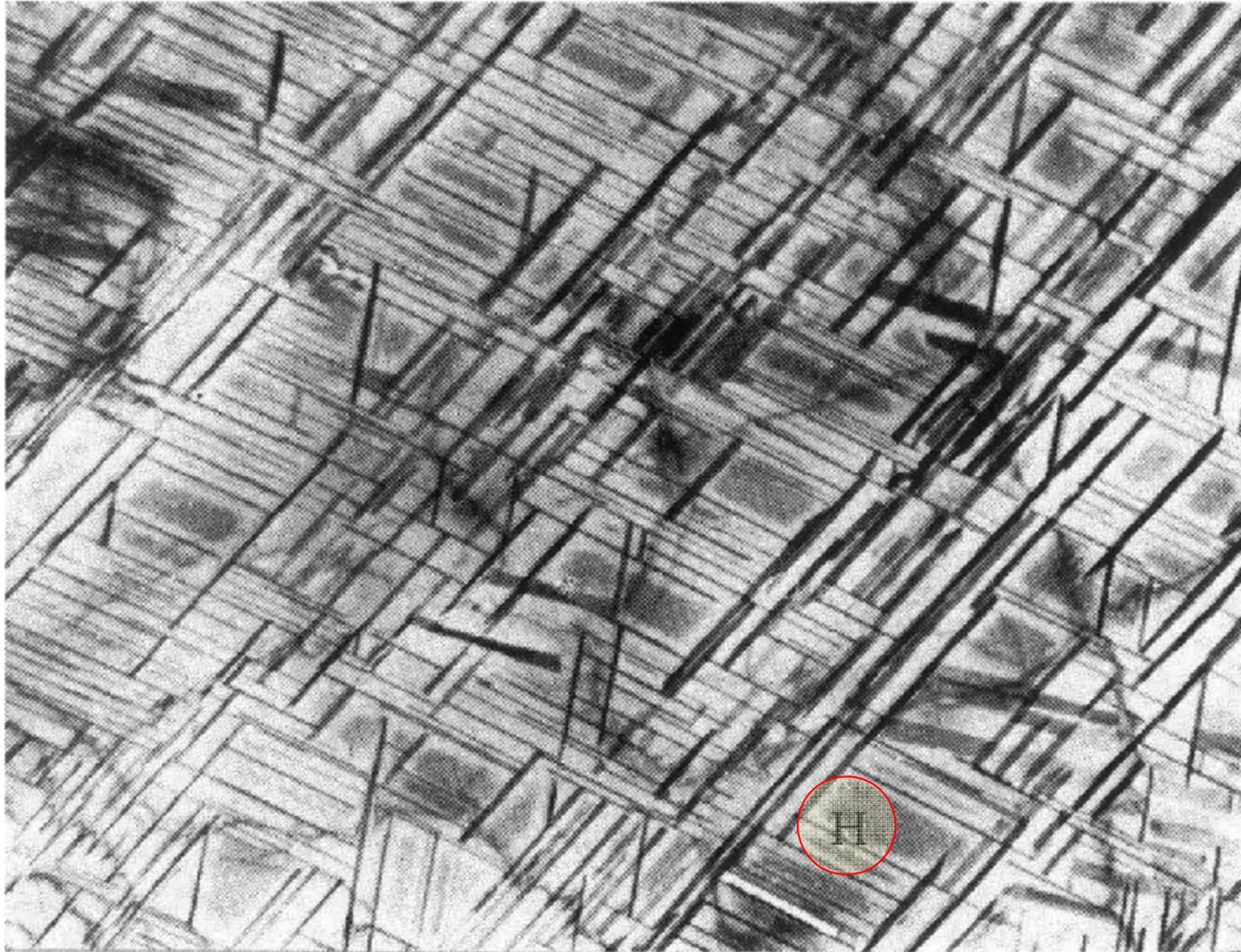
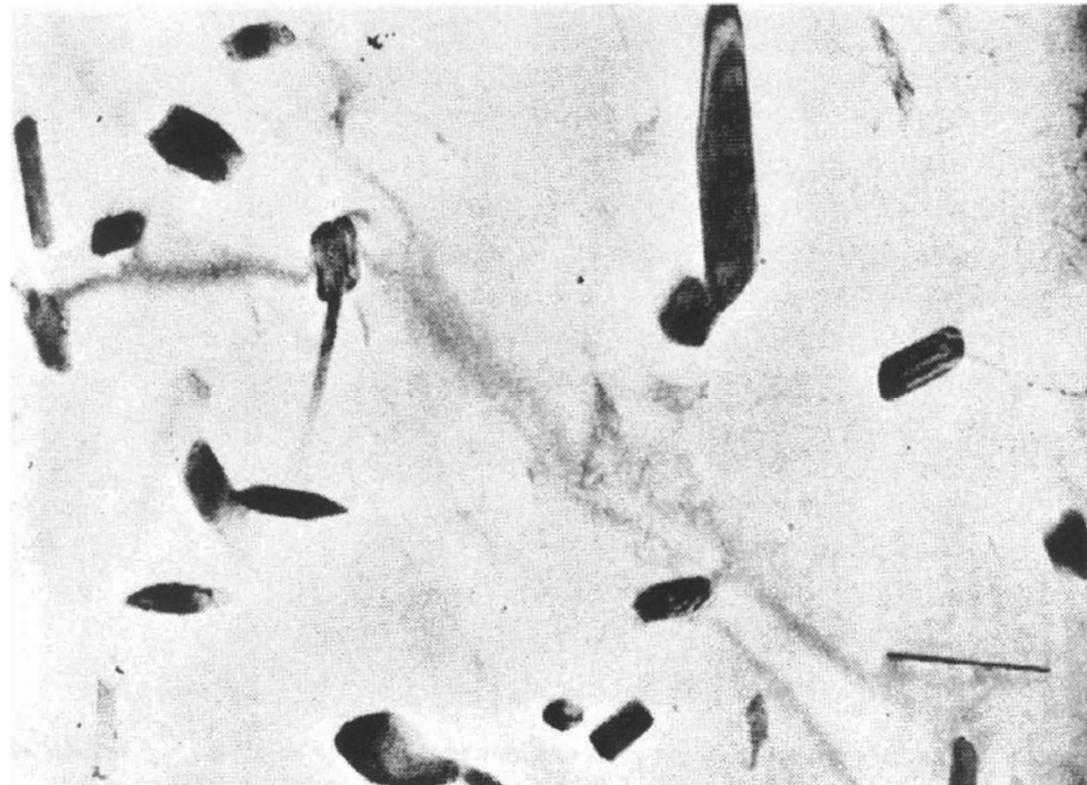
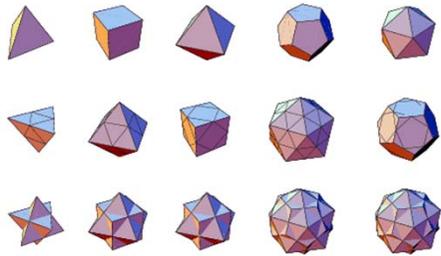


Fig. 3. 42 Electron micrograph showing the Widmanstatten morphology of  $\gamma'$  precipitates in an Al-4 atomic % Ag alloy. GP zones can be seen between the  $\gamma'$  e.g. at H (x 7000).

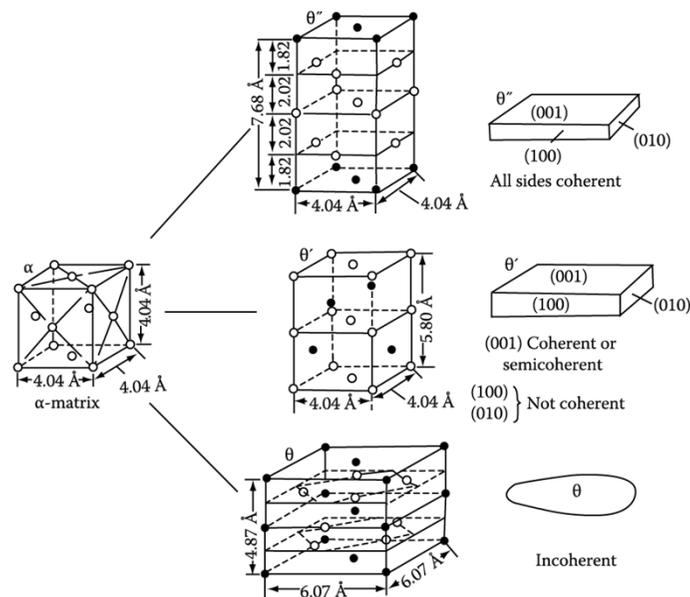
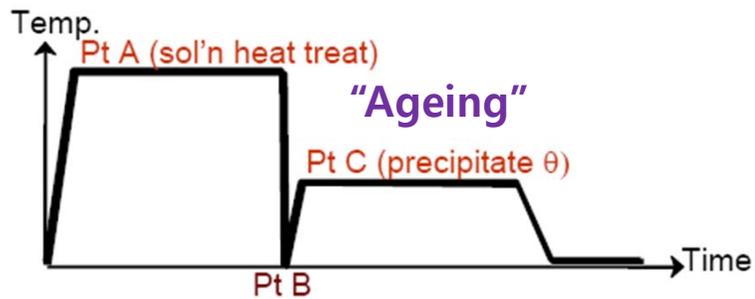
## C. Incoherent precipitates

- when  $\alpha$ ,  $\beta$  have completely different structure  $\Rightarrow$  Incoherent interfaces  
or When the two lattices are in a random orientation
- Interface energy is high for all plane  $\Rightarrow$  spherical shape  
with smoothly curved interface
- Polyhedral shapes: certain crystallographic planes of the inclusion lie at cusps in the  $\gamma$ -plot



$\theta$  phase in Al - Cu alloys ( $\text{Al}_2\text{Cu}$ )

# Q: Example of Second-Phase Shape precipitates from solid solution in Al-Cu alloys



G.P. Zone



$\theta''$ , all coherent



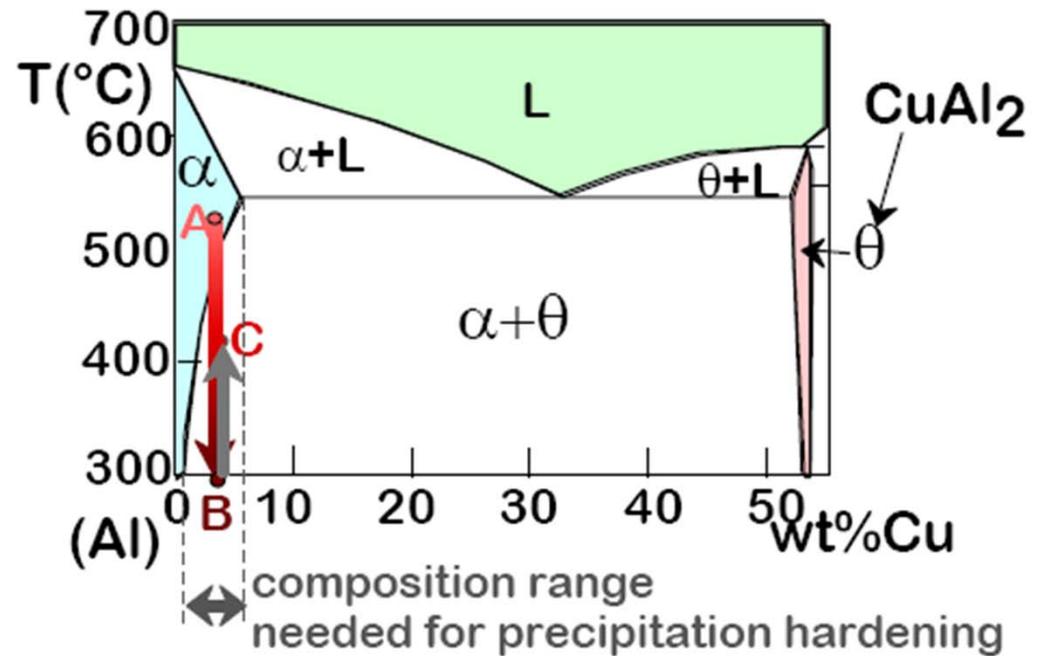
$\theta'$ , partially coherent



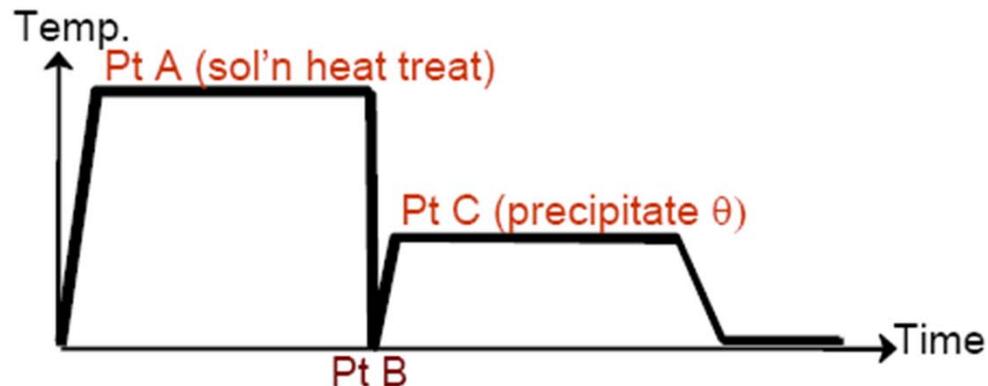
$\theta$ , incoherent

# Precipitation Hardening

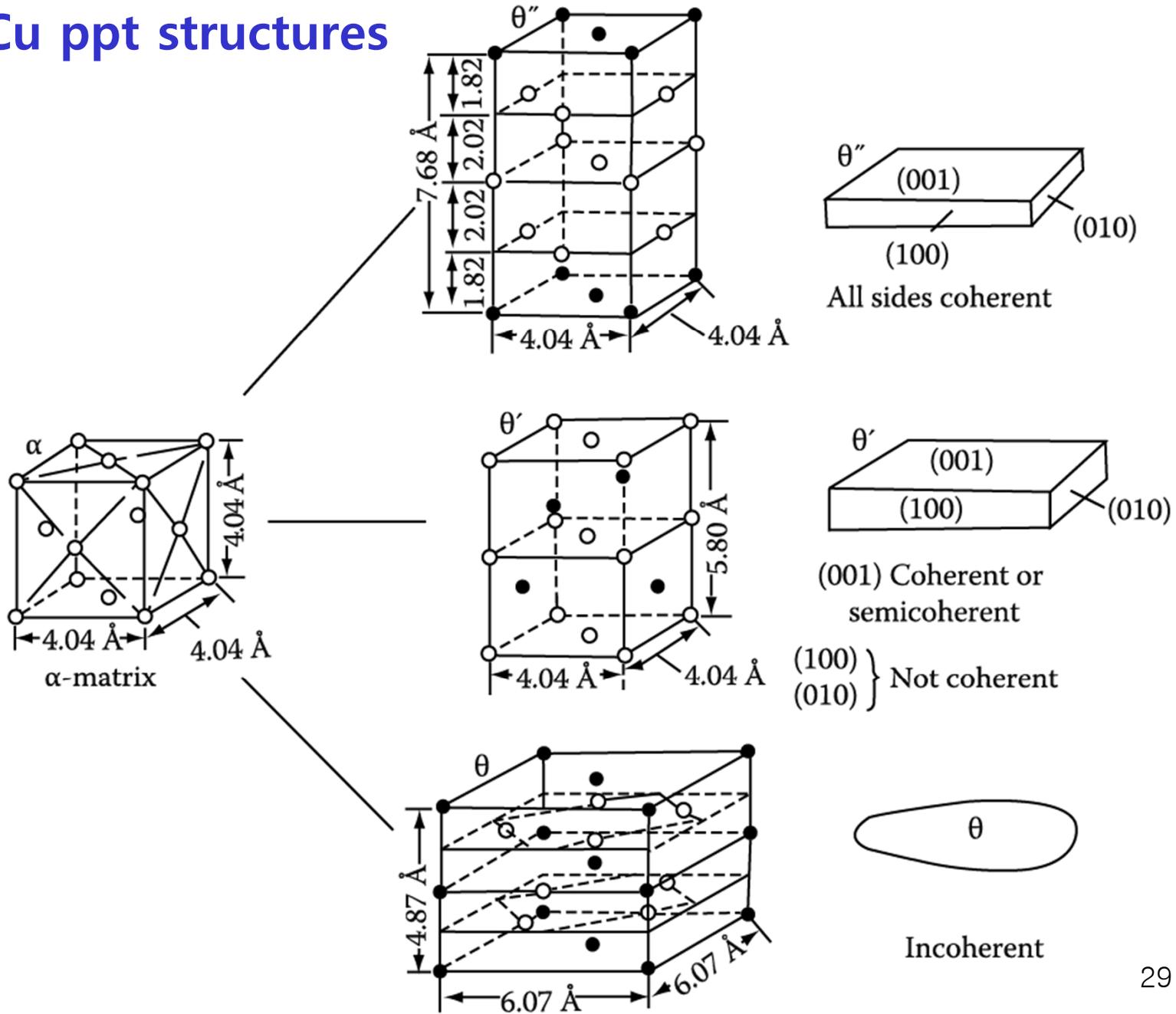
- Ex: Al-Cu system
- Procedure:
  - *Pt A*: solution heat treat (get a solid solution)
  - *Pt B*: quench to room temp.
  - *Pt C*: reheat to nucleate small  $\theta$  crystals within  $\alpha$  crystals.



$\alpha + \theta \rightarrow$  Heat ( $\sim 550^\circ\text{C}$ )  $\rightarrow$  Quench ( $0^\circ\text{C}$ )  $\rightarrow \alpha$  (ssss)  $\rightarrow$  Heat/age ( $\sim 150^\circ\text{C}$ )  $\alpha + \theta_{\text{ppt}}$

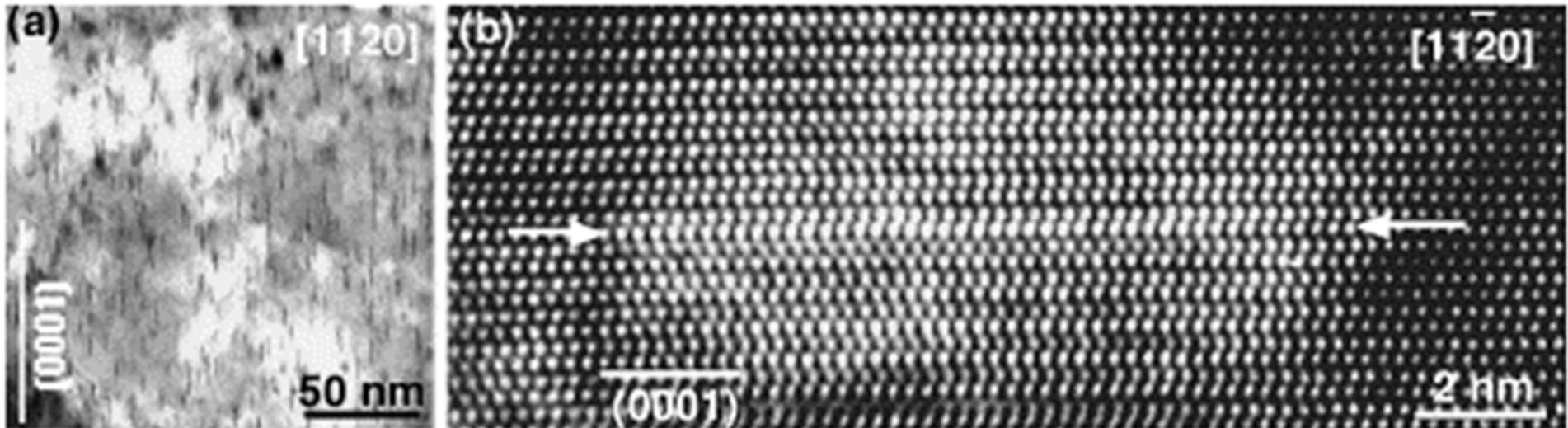
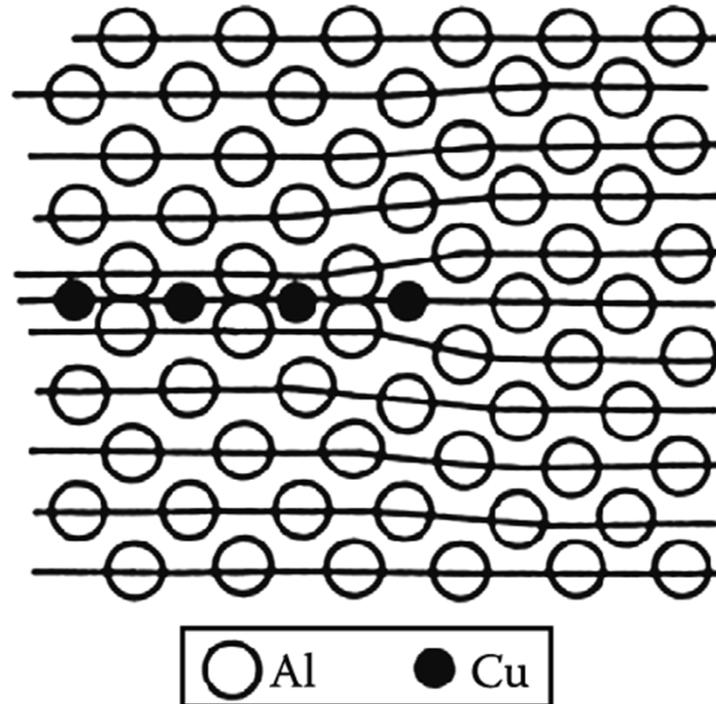


# Al-Cu ppt structures



# Al-Cu ppt structures

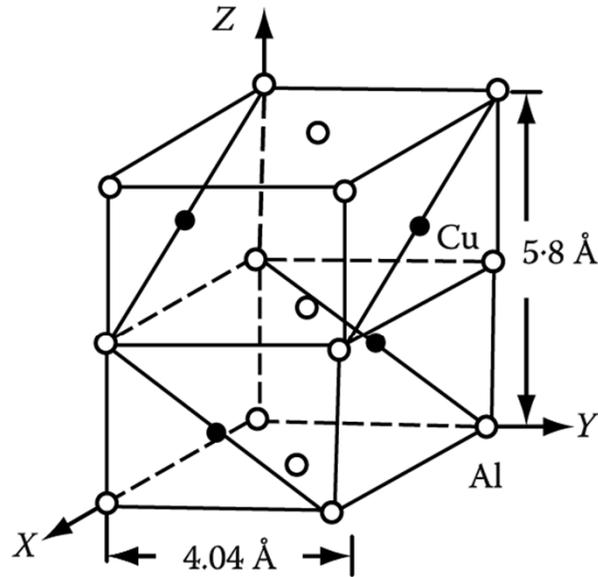
GP zone structure



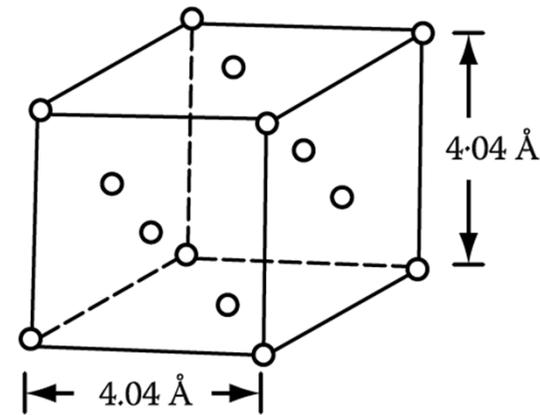
(a) Bright-field TEM image showing G.P. zones, and (b) HRTEM image of a G.P. zone formed on a single (0 0 0 1)<sub>α</sub> plane. Electron beam is parallel to in both (a) and (b).  
30

# $\theta'$ Phase Al-Cu Alloys

Semicoherent broad face parallel to the  $\{100\}_\alpha$  matrix planes (habit plane)



(a) The unit cell of the  $\theta'$  precipitate in Al-Cu alloys



(b) The unit cell of the matrix

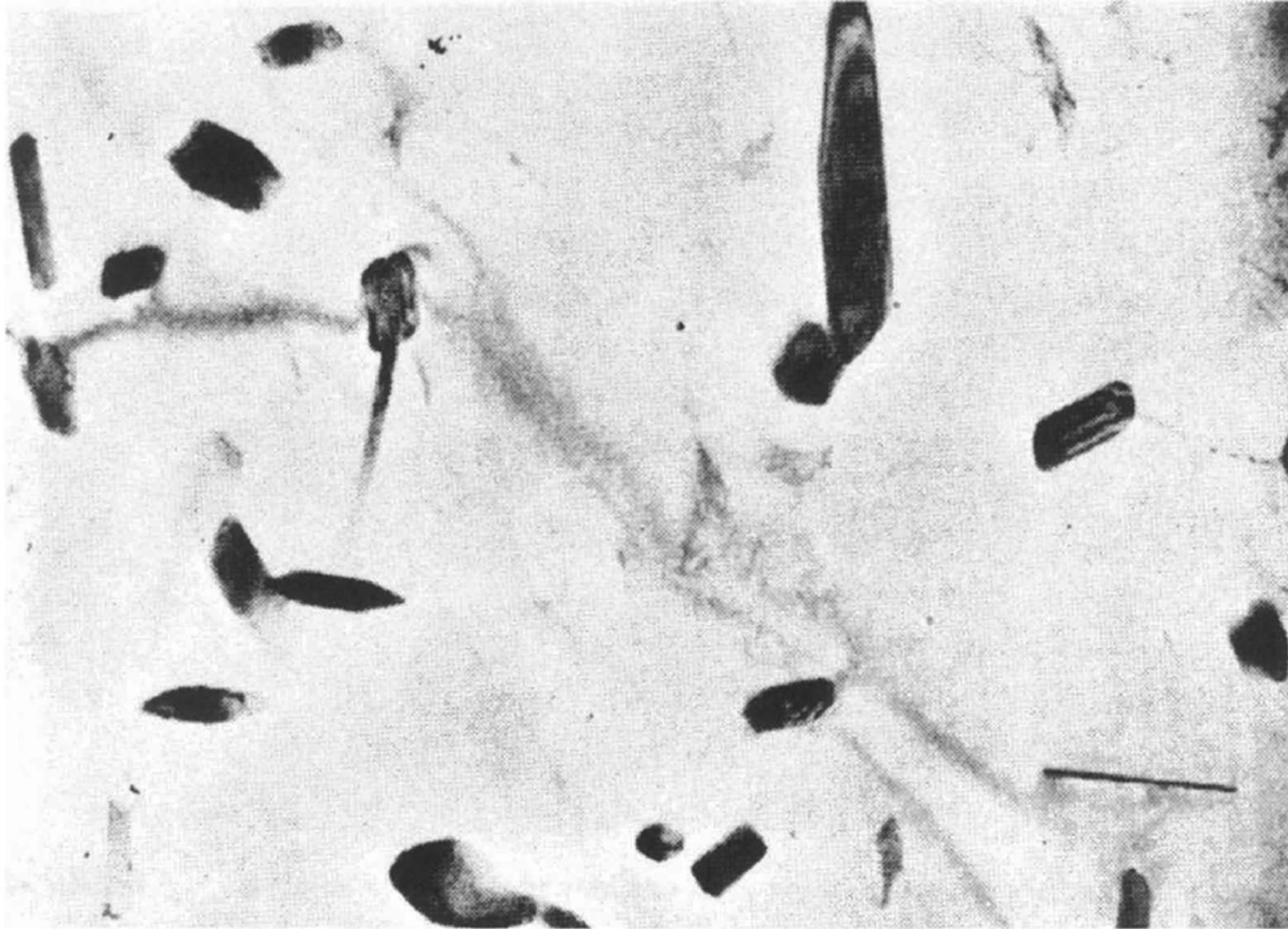
Orientation relationship  
between  $\alpha$  and  $\theta'$

$$(001)_{\theta'} // (001)_{\alpha} \quad [100]_{\theta'} // [100]_{\alpha}$$

→ Cubic symmetry of the Al-rich matrix ( $\alpha$ ) ~ many possible orientations for the precipitate plates within any given grain

S phase in Al-Cu-Mg alloys ; Lath shape  
 $\beta'$  phase in Al-Mg-Si alloys ; Needle shape } **Widmanstätten morphology**

*$\theta$  phase in Al – Cu alloys ( $\text{Al}_2\text{Cu}$ )*



- **Polyhedral shapes:** certain crystallographic planes of the inclusion lie at cusps in the  $\gamma$ -plot

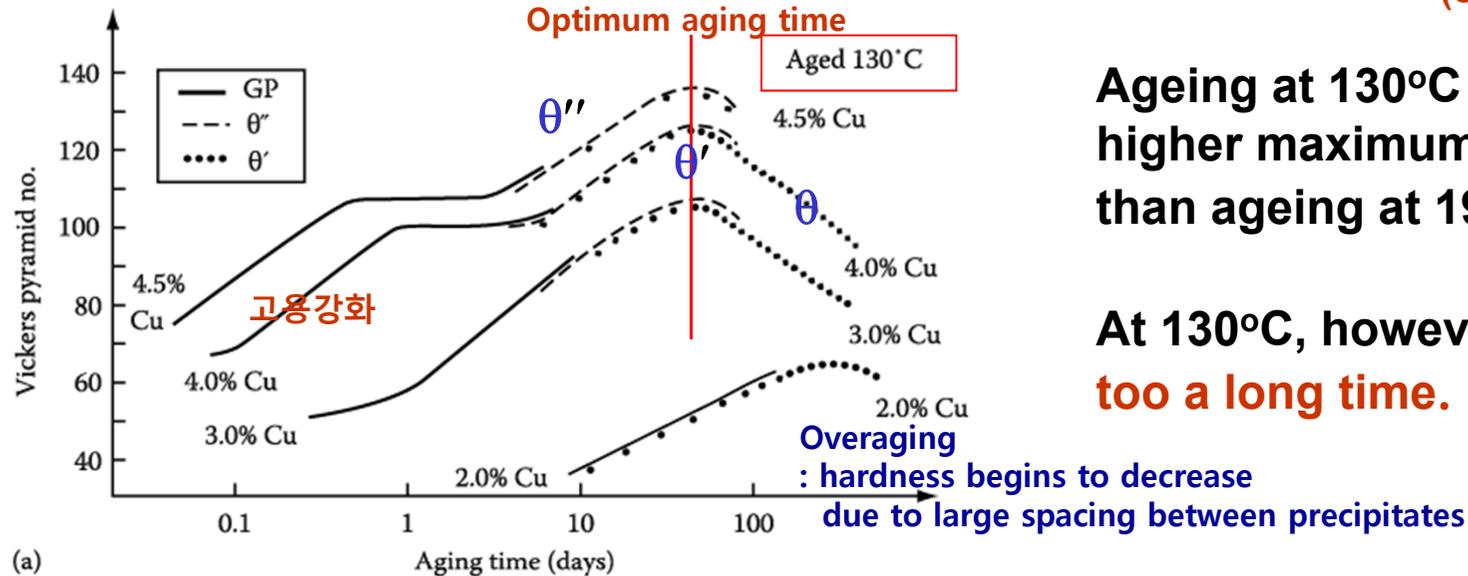
## 5.5.4. Age Hardening

Transition phase precipitation → great improvement in the mechanical properties

Coherent precipitates → highly strained matrix → dislocations~forced during deformation

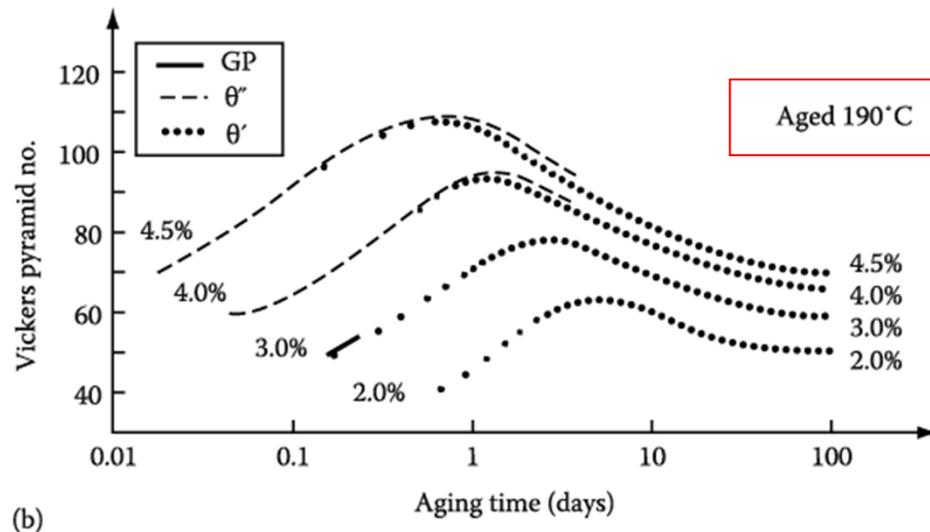
### Hardness vs. Time by Ageing

**Maximum hardness**~ largest fraction of  $\theta''$   
(coherent precipitates)



Ageing at 130°C produces higher maximum hardness than ageing at 190°C.

At 130°C, however, it takes **too a long time.**



How can you get the high hardness for the relatively short ageing time?

**Double ageing treatment**  
first below the GP zone solvus  
→ fine dispersion of GP zones  
then ageing at higher T.

Finer precipitate distribution

Fig. 5. 37 Hardness vs. time for various Al-Cu alloys at (a) 130 °C (b) 190 °C

# Precipitates on Grain Boundaries

Formation of a second-phase particle at the interfaces with two differently oriented grains

- 1) incoherent interfaces with both grains
- 2) a coherent or semi-coherent interface with one grain and an incoherent interface with the other,
- 3) coherent or semi-coherent interface with both grains

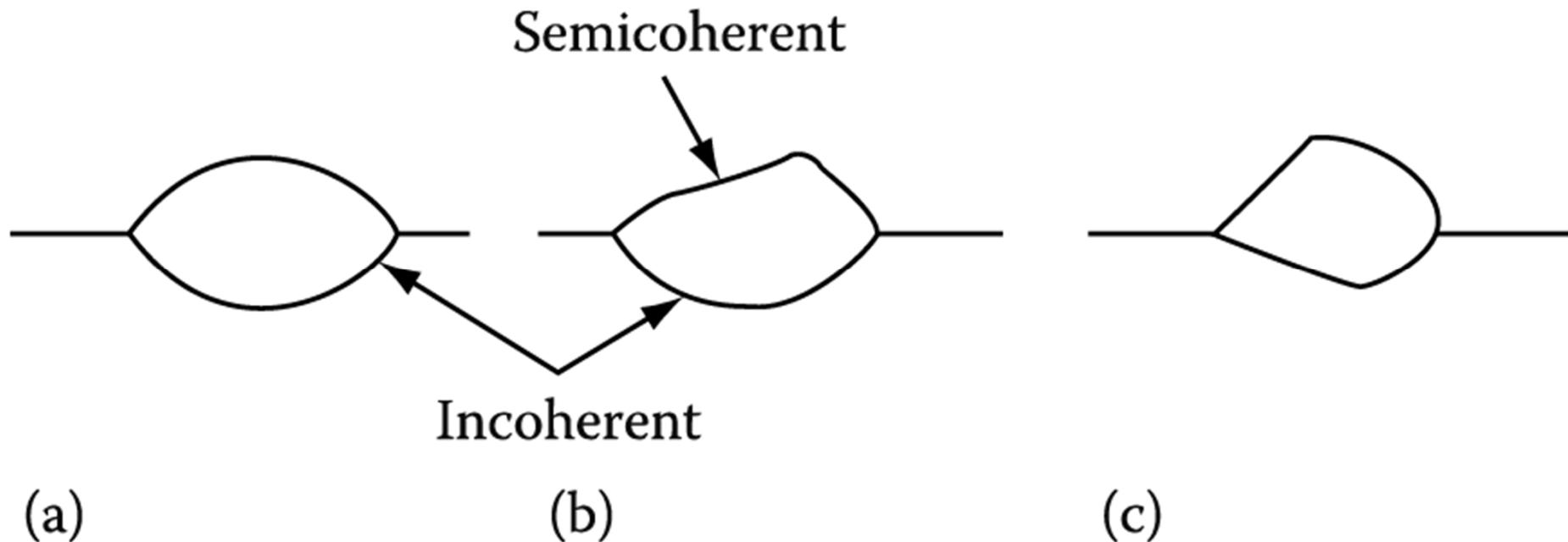


Fig. 3. 45 Possible morphologies for grain boundary precipitates. Incoherent interfaces smoothly curved. Coherent or semi-coherent interface planar.

## Precipitates on Grain Boundaries

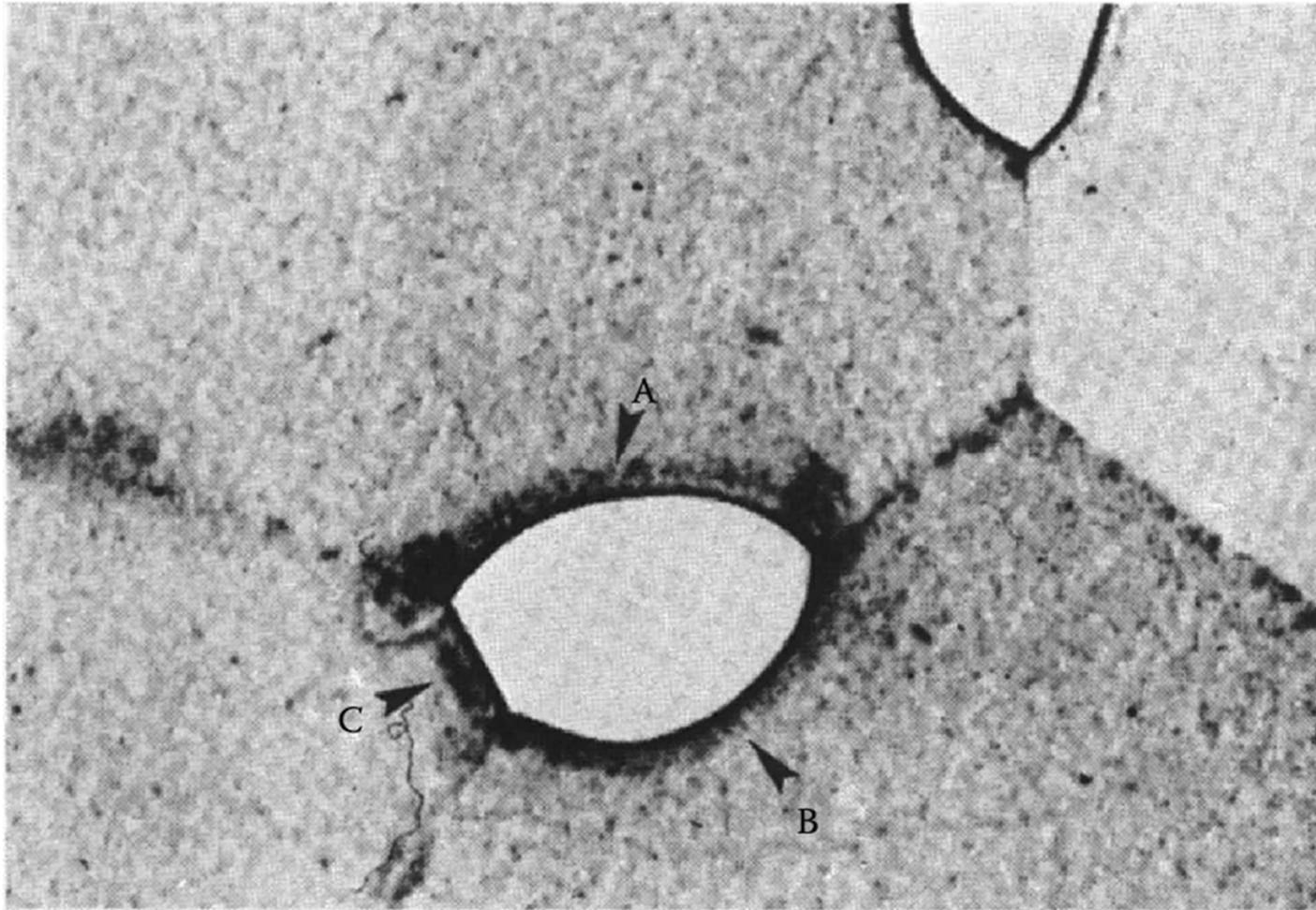
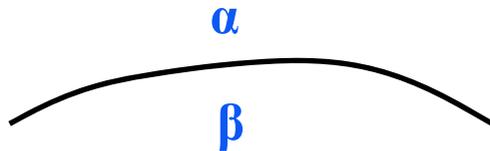


Fig. 3. 46 An  $\alpha$  precipitate at a grain boundary triple point in an  $\alpha - \beta$  Cu-In alloy. Interfaces A and B are incoherent while C is semicoherent (x 310).

A, B; Incoherent, C; Semi-coherent or coherent

# 3.4 Interphase Interfaces in Solids ( $\alpha/\beta$ ) $\sum A_i \gamma_i + \Delta G_S = \text{minimum}$

1) Interphase boundary - different two phases : different crystal structure  
different composition



Coherent/ Semicoherent/ Incoherent  
Complex Semicoherent

Fully coherent precipitates

$\gamma_{ch}$   
different composition

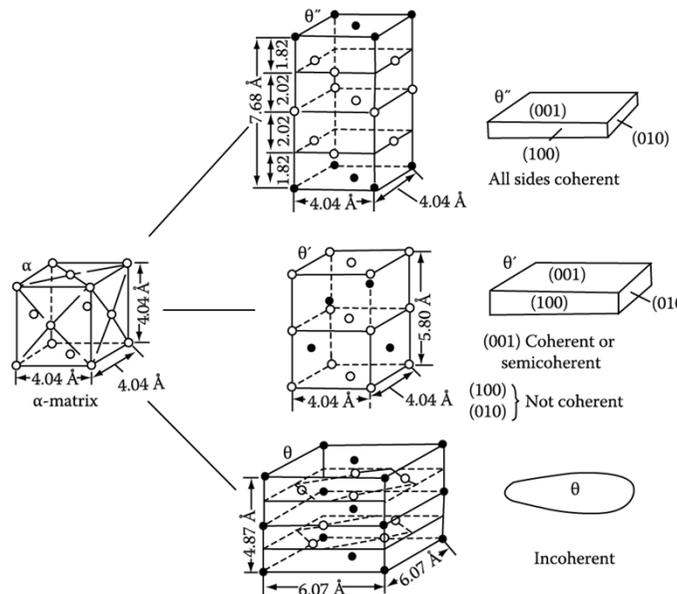
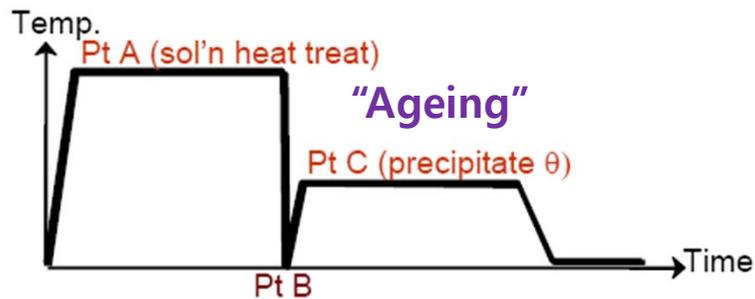


$\gamma_{ch} + \text{Lattice misfit}$   
Coherency strain energy



Incoherent inclusions  
 $\gamma_{ch} + \text{Volume Misfit } \Delta = \frac{\Delta V}{V}$   
Chemical and structural interfacial E

## 2) Second-Phase Shape: precipitate from solid solution in Al-Cu alloys



G.P. Zone



$\theta''$ , all coherent



$\theta'$ , partially coherent



$\theta$ , incoherent

# Q: How is the second-phase shape determined?

$$\sum A_i \gamma_i + \Delta G_S = \text{minimum}$$

$\gamma$  - plot + misfit strain E

Lowest total interfacial free energy

by optimizing the shape of the precipitate and its orientation relationship

Fully coherent precipitates

$\gamma_{ch}$

different composition



$\gamma_{ch} +$  *Lattice misfit*

Coherency strain energy



Incoherent inclusions

$\gamma_{ch} +$

*Volume Misfit*  $\Delta = \frac{\Delta V}{V}$

Chemical and structural interfacial E

(a) Precipitate shapes

(b) Calculation of misfit strain energy

### 3.4.3. Second-Phase Shape: Misfit Strain Effects

If misfit is small,  
Equilibrium shape of a coherent precipitate or zone **can only be predicted from the "γ-plot"**

$$\sum A_i \gamma_i$$

Misfit

$$\sum A_i \gamma_i + \Delta G_S = \text{minimum}$$

"γ-plot" + "Elastic strain energy"

#### A. Fully Coherent Precipitates

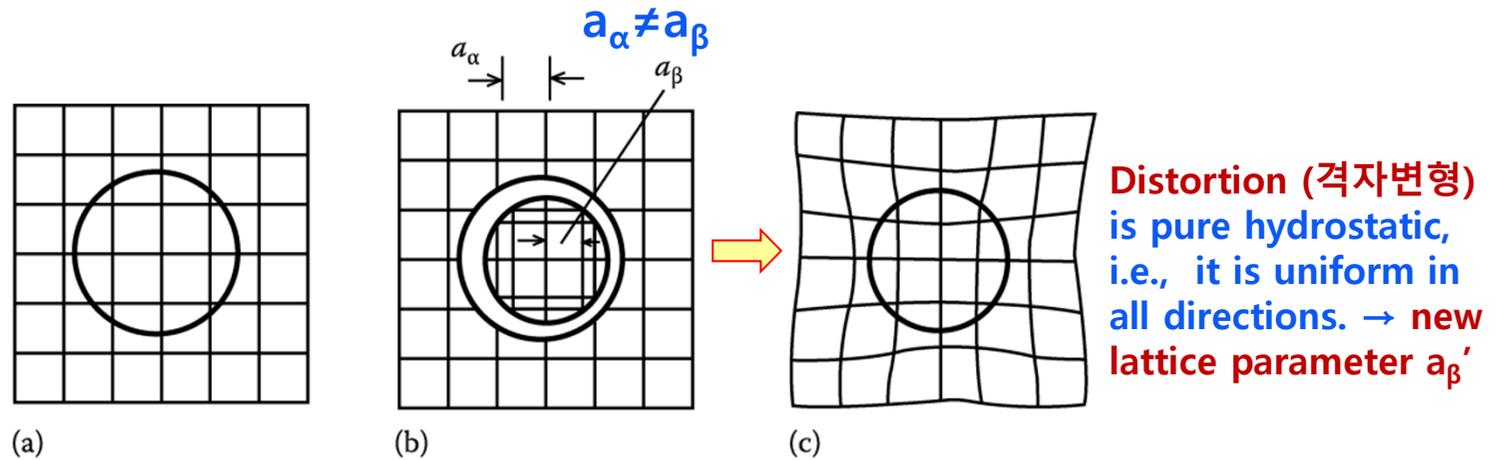


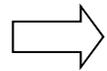
Fig. 3. 47 The origin of coherency strains. The number of lattice points in the hole is conserved.

Unconstrained Misfit

Constrained Misfit

구속되지 않은 불일치도

$$\delta = \frac{a_\beta - a_\alpha}{a_\alpha}$$



구속된 불일치도

$$\varepsilon = \frac{a'_\beta - a_\alpha}{a_\alpha}$$

① If  $E_\beta = E_\alpha, \nu = 1/3 \rightarrow \varepsilon = \frac{2}{3} \delta$

Poisson's ratio

②

In practice, different elastic constants  $E_\beta \neq E_\alpha \rightarrow 0.5\delta \leq \varepsilon \leq \delta$

if thin disc-type precipitate,

→ In situ misfit is no longer equal in all directions

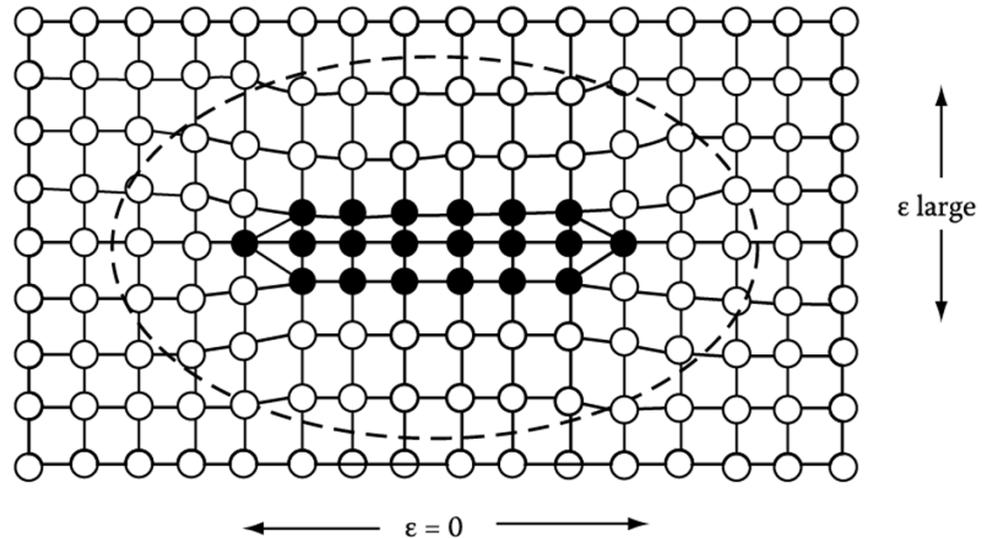


Fig. 3. 48 For a coherent thin disc there is **little misfit parallel to the plane of the disc.** **Maximum misfit is perpendicular to the disc.** → reduction in coherency strain E

\* Total elastic energy ( $\Delta G_s$ ) depends on the "shape" and "elastic properties" of both matrix and inclusion.

Elastically Isotropic Materials

&  $E_\beta = E_\alpha$

$\Delta G_s \rightarrow$  independent of the shape of the precipitate

$$\Delta G_s = 4\mu\delta^2 \cdot V \quad (\text{If } \nu=1/3)$$

here,  $\mu$  = shear modulus of the matrix,

$V$  = volume of the unconstrained hole in the matrix

$\Delta G_s \rightarrow$  dependent of the shape of the precipitate

$\Delta G_s^{\min}$ : if inclusion is hard  $\rightarrow$  sphere/ soft  $\rightarrow$  disc shape

Elastically Anisotropic Materials &  $E_\beta \neq E_\alpha$

Atom radius ( $\text{\AA}$ )    Al : 1.43    Ag : 1.44    Zn : 1.38    Cu : 1.28

Zone Misfit ( $\delta$ )                      -                      + 0.7%                      - 3.5%                      - 10.5%

GP Zone Shape

Equilibrium shape

sphere

sphere

disc

$$\sum A_i \gamma_i + \Delta G_s = \text{minimum}$$

Interfacial E effect dominant

$\delta < 5\%$

strain E effect dominant

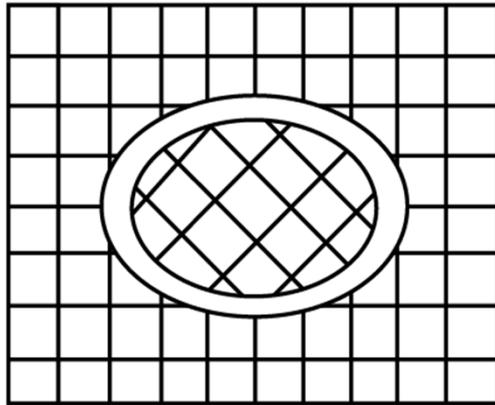
Influence of strain E ( $\delta$  = lattice misfit) on the equilibrium shape of coherent precipitation

## B. Incoherent Inclusions

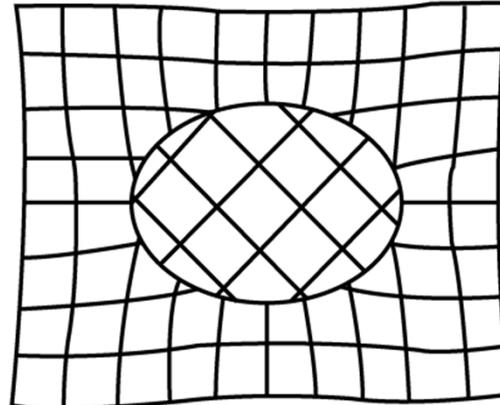
Lattice sites are not conserved. → no coherency strain,  $\Delta G_s$

But, misfit strain still arise if the inclusion is the wrong size.

$\delta$  (lattice misfit) →  $\Delta$  (volume misfit)



(a)



(b)

$$\text{Volume Misfit } \Delta = \frac{\Delta V}{V}$$

Ex) coherent spherical inclusion:  $\Delta=3\delta$

#of lattice sites within the hole is not preserved for incoherent inclusion (no lattice matching)

For spheroidal Inclusions  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

For Elliptical Inclusions  $\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{c^2} = 1$

For a homogeneous incompressible inclusion in an isotropic matrix

등방성 기지내 균질 비압축성 개재물

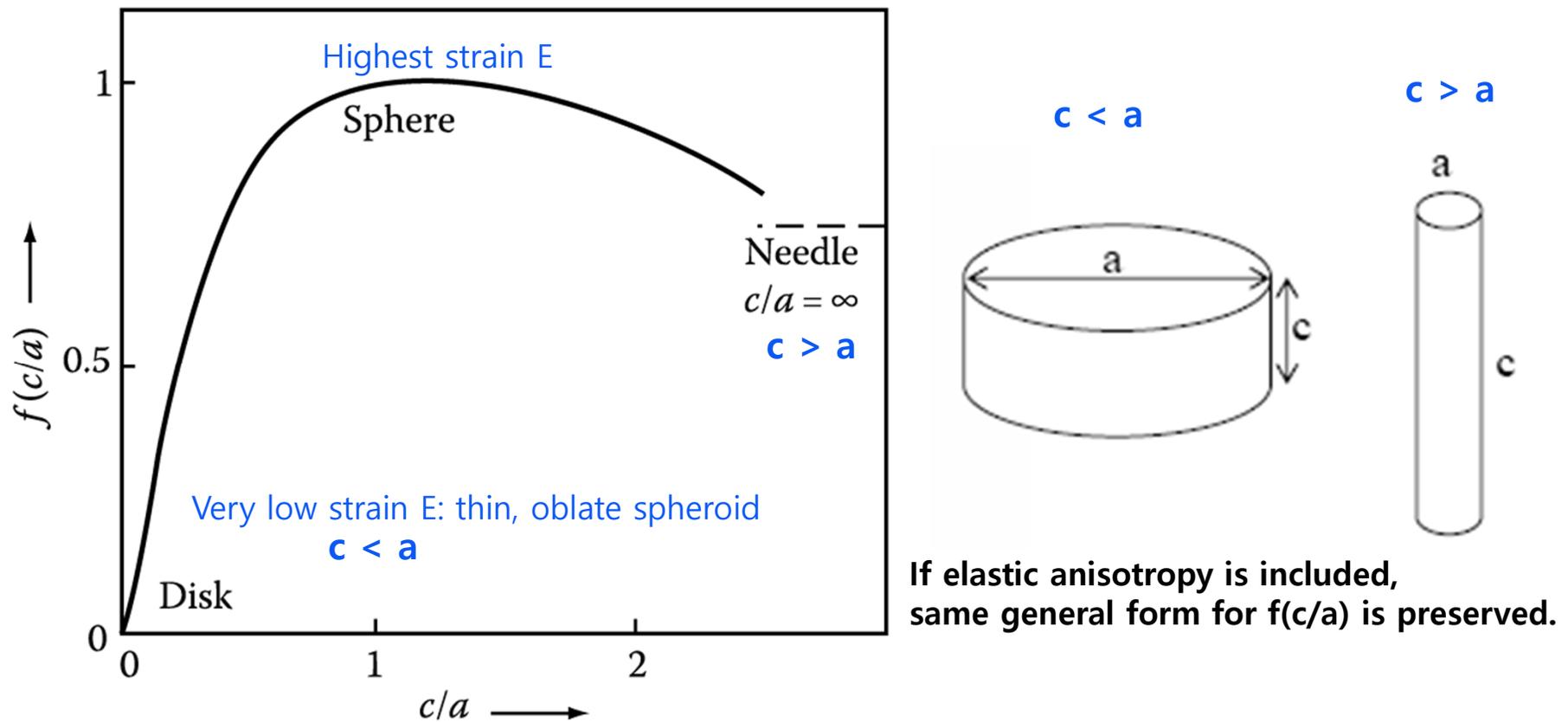
Nabarro Eq.

$$\Delta G_s = \frac{2}{3} \mu \Delta^2 \cdot V \cdot f(c/a)$$

$\mu$ : the shear modulus of the matrix

1) The elastic strain energy is proportional to the square of the volume misfit  $\Delta^2$ .

## 2) Shape effect for misfit strain $E \sim$ function $f(c/a)$



If elastic anisotropy is included, same general form for  $f(c/a)$  is preserved.

Fig. 3. 50 The variation of misfit strain energy with ellipsoid shape,  $f(c/a)$ .

$$\Delta G_s = \frac{2}{3} \mu \Delta^2 \cdot V \cdot f(c/a) \quad \Delta = \frac{V_\beta - V_\alpha}{V_\alpha} \approx 3\delta \text{ for sphere}$$

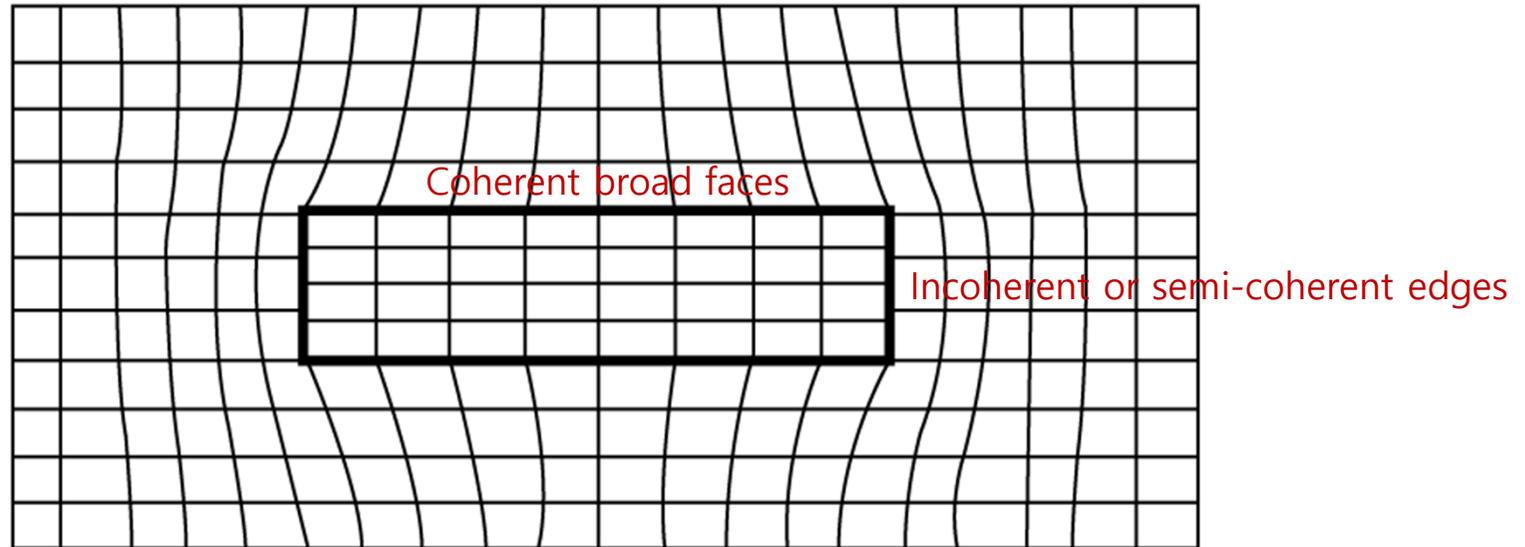
Elastic strain E                      Precipitate shape effect                       $\neq 3\delta$  for disc or needle

\* Equil. Shape of an incoherent inclusion: an oblate spheroid with  $c/a$  value that balances the opposing effects of interfacial E and strain E

( here,  $\Delta \sim$  small  $\rightarrow$  Interfacial E dominant  $\rightarrow$  roughly spherical inclusion)

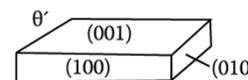
## C. Plate-like precipitates

Misfit across the broad faces → large coherency strains parallel to the plate



In situ misfit across the broad faces increases with increasing plate thickness

- greater strains the matrix and higher shear stresses at the corners of the plates
- energetically favorable for the broad faces to become semi-coherent
- the precipitate behaves as **an incoherent inclusion with comparatively little misfit strain  $E$ , ex)  $\theta'$  phase in Al-Cu alloy**



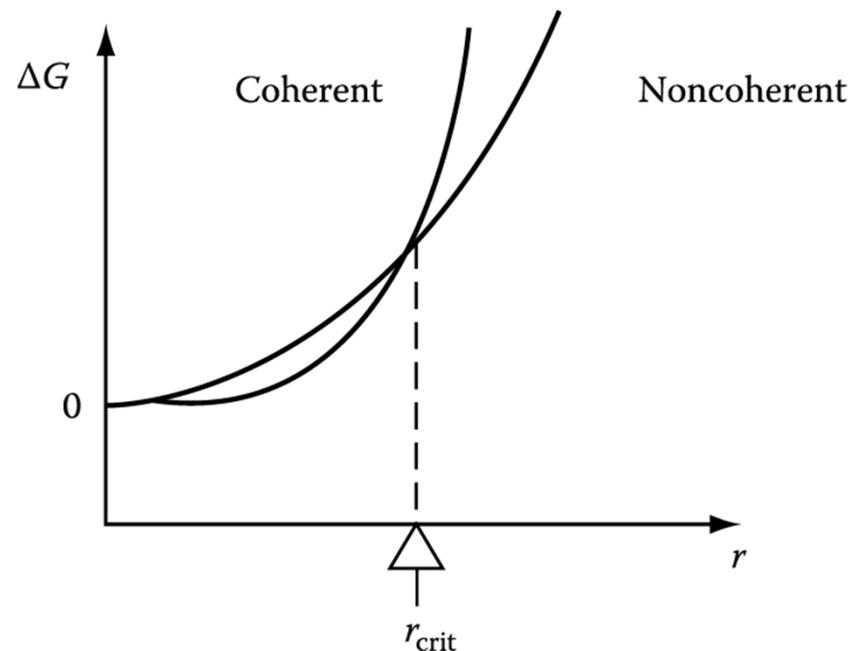
(001) Coherent or semicoherent

$\left. \begin{matrix} (100) \\ (010) \end{matrix} \right\}$  Not coherent

**Q: Which state produces the lowest total E for a spherical precipitate?**

**“Coherency loss”**

If a coherent precipitate grows, during aging for example, it should lose coherency when it exceeds  $r_{crit}$ .

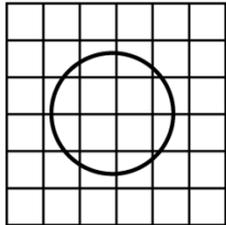


# Coherency Loss

- Precipitates with coherent interfaces = low interfacial E + coherency strain E
- Precipitates with non-coherent interfaces = higher interfacial E

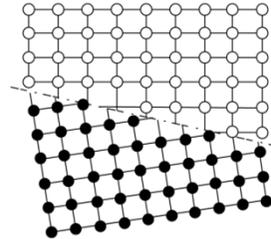
If a coherent precipitate grows, it should lose coherency to maintain minimum interfacial free E.

$$\Delta G(\text{coherent}) = 4\mu\delta^2 \cdot \frac{4}{3}\pi r^3 + 4\pi r^2 \cdot \gamma_{ch} \iff \Delta G(\text{non-coherent}) = 4\pi r^2 \cdot (\gamma_{ch} + \gamma_{st})$$



Coherency strain energy  
Eq. 3.39

Chemical interfacial E



Chemical and structural interfacial E

$$\frac{4}{3}\pi r^3 (4\pi\mu\delta^2) + 4\pi r^2 (\gamma_{ch}) = 4\pi r^2 (\gamma_{st} + \gamma_{ch})$$

coherent

$\Delta G_s$ -relaxed

$$\therefore r_{crit} = \frac{3 \cdot \gamma_{st}}{4\mu\delta^2}$$

for small  $\delta$ ,  $\gamma_{st} \propto \delta$   
(semi-coherent interface)

$$\approx \frac{1}{\delta} \quad (\delta = (d_\beta - d_\alpha) / d_\alpha : \text{misfit})$$

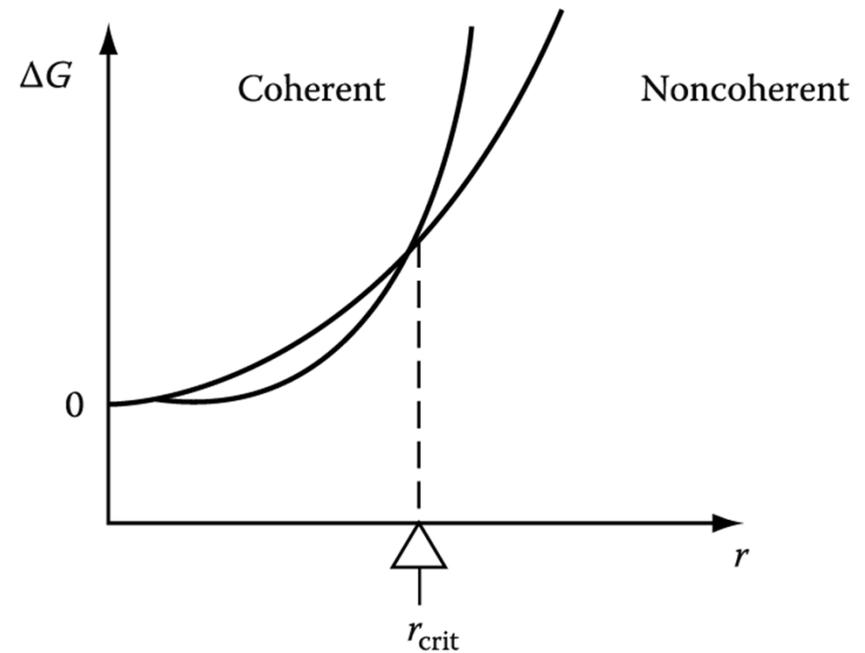
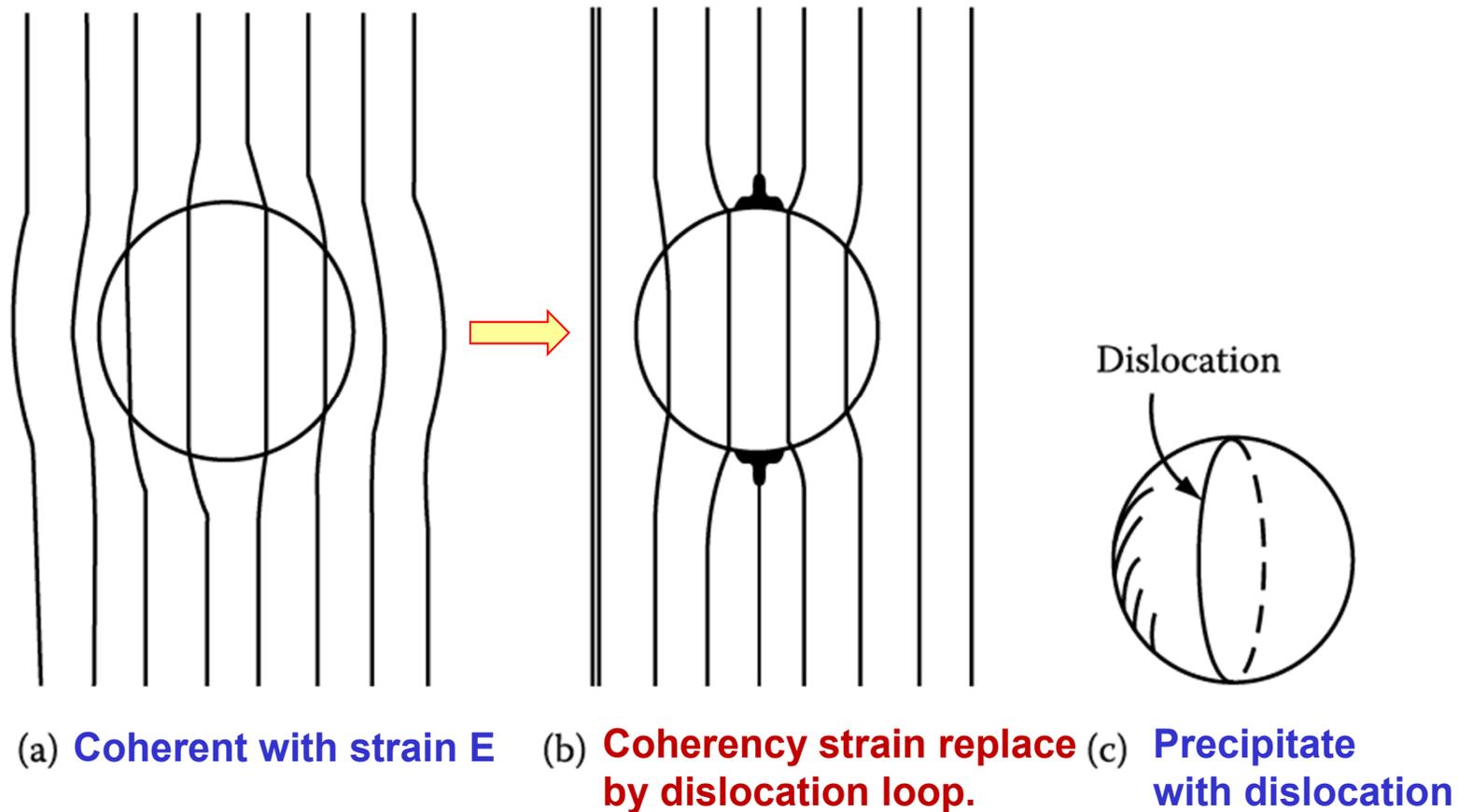


Fig. 3. 52 The total energy of matrix + precipitate vs. precipitate radius for spherical coherent and non-coherent (semicoherent or incoherent) precipitates.

If a coherent precipitate grows, during aging for example, it should lose coherency when it exceeds  $r_{crit}$ .



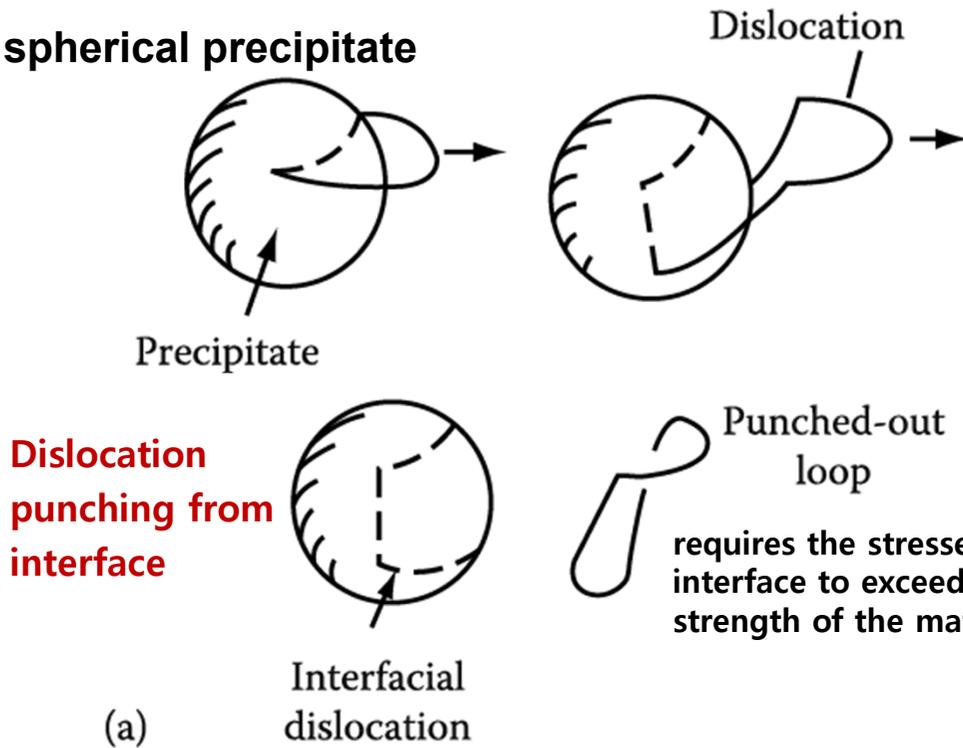
**Fig. 3.53. Coherency loss for a spherical precipitate**

In practice, this phenomena can be rather difficult to achieve.

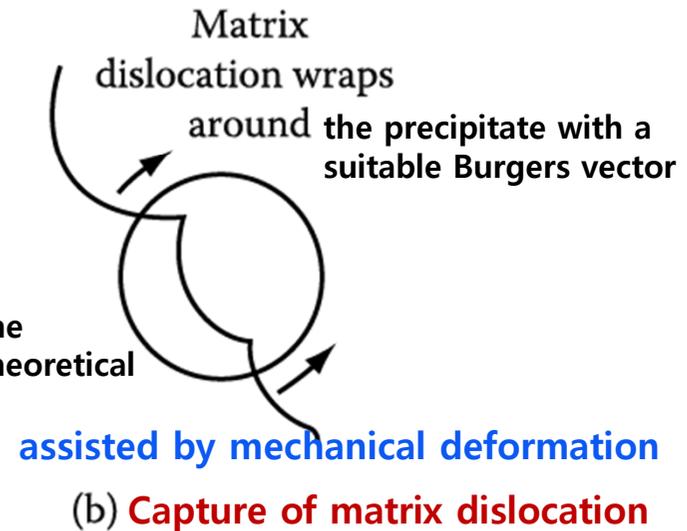
→ Coherent precipitates are often found with sizes much larger than  $r_{crit}$ .

**“Mechanisms for coherency loss”**: all require the precipitate to reach a larger size than  $r_{crit}$

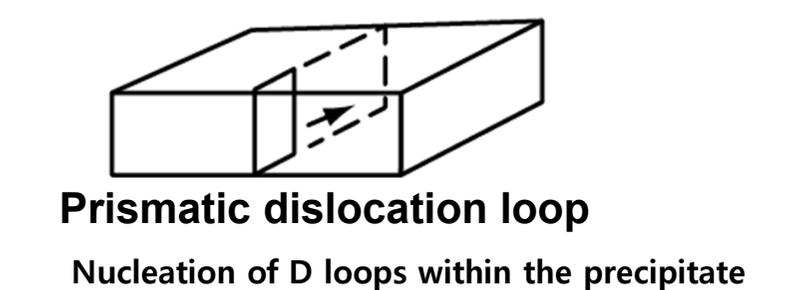
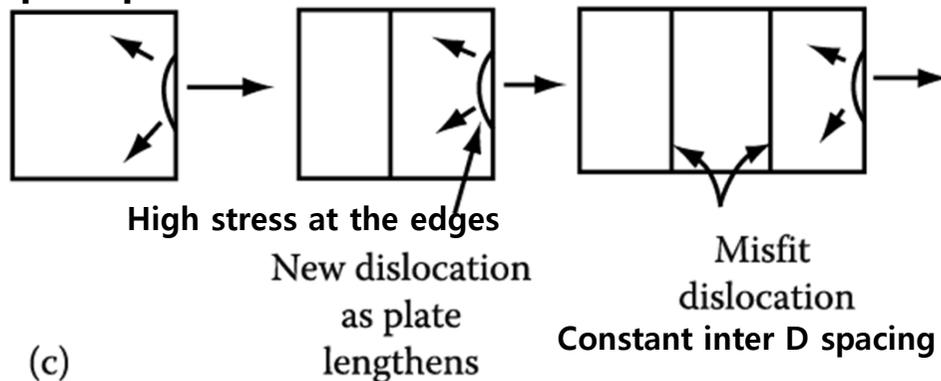
**1) spherical precipitate**



\* Punching stress ( $P_s$ )  $\sim$  independent of size, but  $P_s \propto$  constrained misfit,  $\epsilon$  ( $>\epsilon_{crit} \sim 0.05$ ),  $\rightarrow$  “precipitates with a smaller  $\epsilon$  cannot lose coherency by (a), no matter how large.”



**2) Plate precipitate**



**Nucleation of dislocation at the edge  $\rightarrow$  maintain a roughly constant inter-dislocation spacing during plate lengthening**

**(d) Vacancies can be attracted to coherent interfaces and ‘condense’ to form a prismatic dislocation loop which can expand across the precipitate**