Chapter9. Amplification of light. Lasers
Part 3

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Changhee Lee
School of Electrical and Computer Engineering
Seoul National Univ.
chlee7@snu.ac.kr

9.6 Optical-resonator theory

In a laser resonator, part of energy spills around the reflecting mirrors and is lost (***diffusion loss*** of the resonator). If $U(x,y)$ and $U'(x',y')$ represent the complex amplitudes of the radiation over the mirror surfaces, then by applying the Fresnel-Kirchhoff diffraction theory we can write

$$U'(x',y') = -\frac{ik}{4\pi}\iint U(x,y)\frac{e^{ikr}}{r}(1+\cos\theta)dxdy$$

$$r = \left[d^2 + (x'-x)^2 + (y'-y)^2\right]^{1/2}$$

$$\cos = \frac{d}{r}$$

If the mirrors are identical, the two functions U and U' will become identical except for a constant factor γ.

eigenvalue *kernel*

$$\gamma U(x',y') = \iint U(x,y)K(x,y,x',y')dxdy$$

$$K(x,y,x',y') = -\frac{ik}{4\pi}(1+\cos\theta)\frac{e^{ikr}}{r}$$

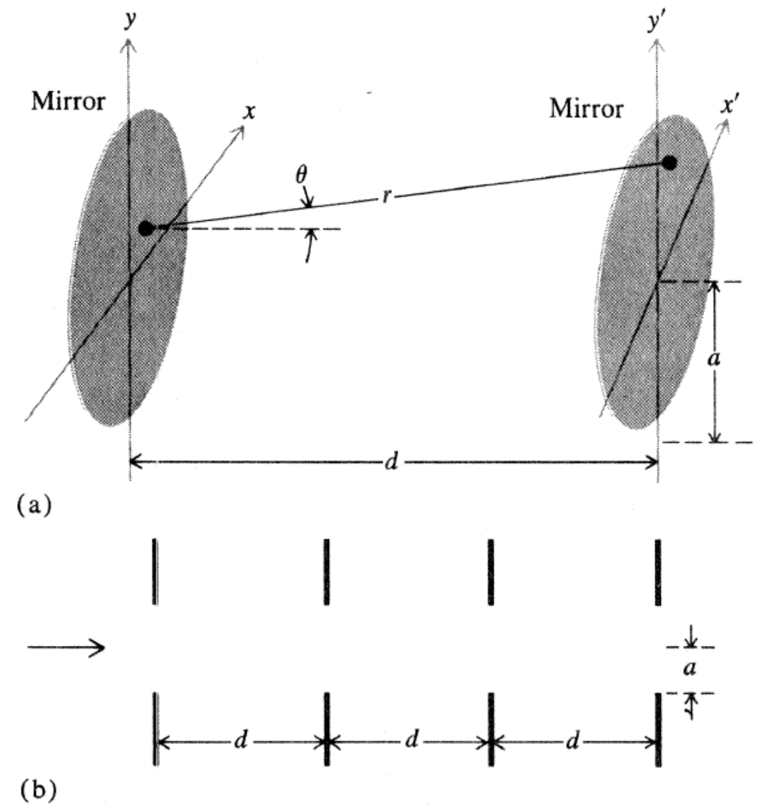


Figure 9.7. (a) Geometry of a Fabry-Perot laser cavity; (b) equivalent multiple diffraction problem.

9.6 Optical-resonator theory

There are an infinite number of solutions U_n, n=1, 2, …, each with an associated eigenvalue γ_n.

$$\gamma_n = |\gamma_n| e^{i\phi_n}$$

$$1 - |\gamma_n|^2 = \text{relative energy loss per transit due to diffraction}$$

Accurate solutions of the Fabry-Perot resonator problem are quite involved. Let's find a simple approximation by employing the same procedure as that of the Fraunhoffer diffraction case.

$$K(x, y, x', y') = Ce^{ik_1(xx'+yy')}$$

$$\gamma U(x', y') = C\iint U(x, y) e^{ik_1(xx'+yy')} dxdy$$

Thus the functions $U(x,y)$ is its own Fourier transform. The simplest of such functions is the Gaussian.

$$U(x, y) = e^{-\rho^2/w^2} = e^{-(x^2+y^2)/w^2}$$

More general functions that are their own Fourier transforms are products of *Hermite polynomials* and the Gaussians.

$$U_{pq}(x, y) = H_p\left(\frac{\sqrt{2}x}{w}\right) H_q\left(\frac{\sqrt{2}y}{w}\right) e^{-(x^2+y^2)/w^2}$$

9.6 Optical-resonator theory

The integers *p* and *q* are the order of the Hermite polynomials and each set (*p*,*q*) corresponds to a particular transverse mode of the resonator.
The lowest-order Hermite polynomial Ho is a constant, hence the simplest Gaussian mode corresponds to the set (0,0) and is called the $TEM_{0,0}$ mode. *TEM* refers to the transverse electromagnetic waves in the cavity

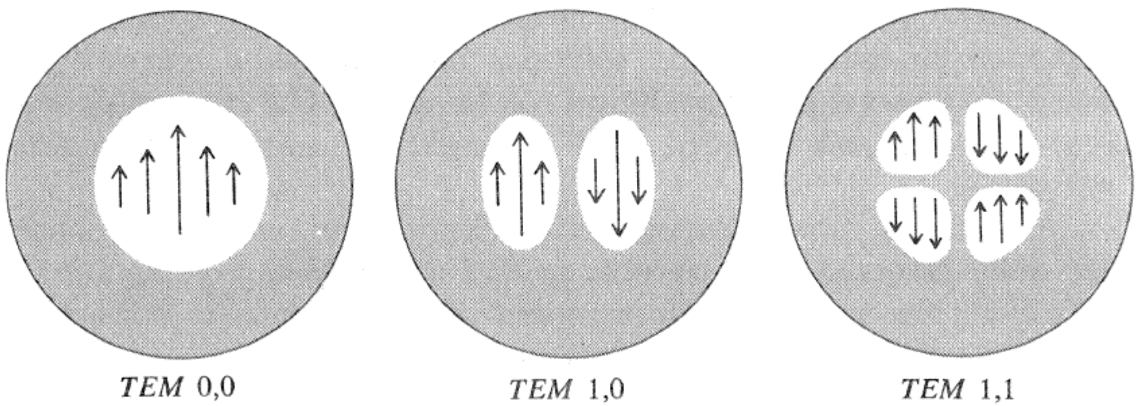


Figure 9.8. Field distributions at the mirrors for some low-order modes.

- Lasers are often designed to operate on a single transverse mode. This is usually the $TEM_{0,0}$ Gaussian mode because it has the smallest beam diameter and can be focused to the smallest spot size.
- Higher-order modes occupy a larger volume and therefore can have larger gain.

$H_n(u)$: Hermite polynomials

$$H_n(u) = e^{u^2/2}\left(u - \frac{d}{du}\right)^n e^{-u^2/2} = (-1)^n e^{u^2} \frac{d^n}{du^n} e^{-u^2}$$

$$\frac{dH_n(u)}{du} = 2nH_{n-1}(u)$$

$$H_o(u) = 1$$

$$H_1(u) = 2u$$

$$H_2(u) = -2 + 4u^2$$

$$H_3(u) = -12u + 8u^3$$

$$\vdots$$

$$H_n(u) = (-1)^n e^{u^2} \frac{d^n}{du^n} e^{-u^2}$$

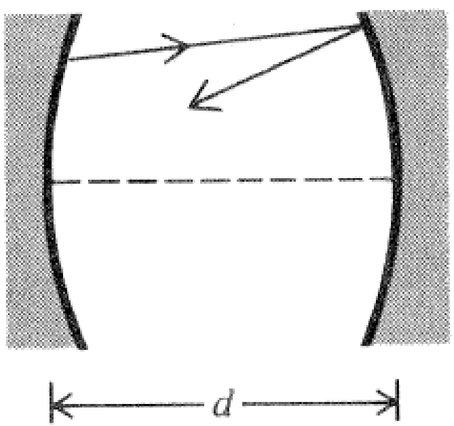
Resonator configurations, Stability

One of the most commonly used cavity configurations is known as the *confocal resonator* consisting of two identical concave spherical mirrors separated by a distance equal to the radius of curvature.

A *stable* resonator is one in which a ray inside the cavity will remain clos to the optic axis upon multiple reflections between the end mirrors.

stability criterion $0 < d < 4f$ or $0 < d < 2r$

In the confocal resonator $d = 2f = r$



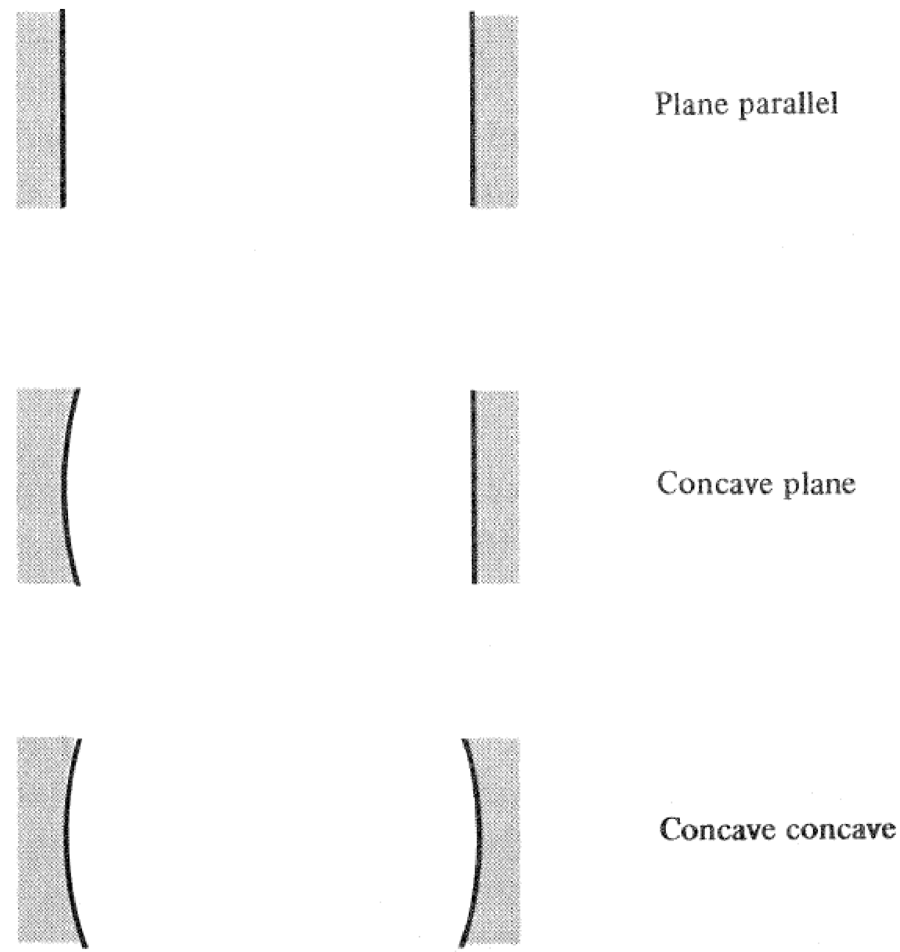


Figure 9.9. Some common laser cavities.

Diffraction loss

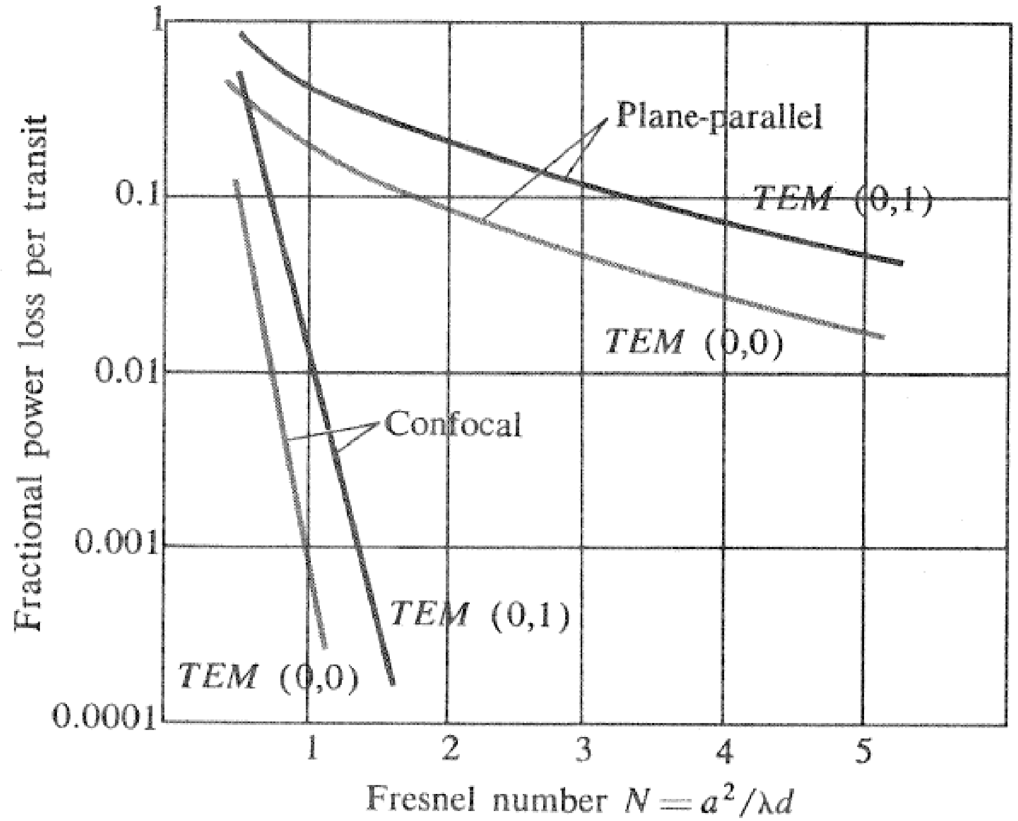


Figure 9.10. Loss curves for the first two modes in plane-parallel and confocal laser cavities.

Fresnel number $N = \dfrac{a^2}{\lambda d}$, $a =$ mirror radius, $d =$ mirror separation

With confocal spherical mirrors, the diffraction losses of low-order modes are negligibly small when N>1..

Spot size

The scale parameter w is a measure of the lateral distribution of the energy in the optical beam inside the resonator.

$$U(x,y) = e^{-\rho^2/w^2} = e^{-(x^2+y^2)/w^2}$$

The Gaussian function falls to e^{-1} when $\rho=w$; so the energy will fall to e^{-1} of its maximum value. Hence w is called the **spot size** of the dominant (0,0) mode.

$$w^2 = w_o^{\,2} + \frac{\lambda^2 z^2}{\pi^2 w_o^{\,2}}$$

For the symmetrical cavity formed by two mirrors each of radius of curvature R and separated by a distance d, the parameter w_o is given by

$$w_o^{\,2} = \frac{\lambda}{\pi}\left[\frac{d}{2}\left(R-\frac{d}{2}\right)\right]^{1/2} \qquad r_c = z + \frac{d(2R-d)}{4z}$$

In the case of confocal resonator $R=d$, the spot size at the center is $w_o = \sqrt{\dfrac{\lambda d}{2\pi}}$

and the spot size at either mirror $z=\pm d/2$ is $w = \sqrt{\dfrac{\lambda d}{\pi}}$

Spot size

At the mirrors the wave surfaces match the curvature of the mirror surface. At the center, where the spot size is minimum, the wave surface becomes planar.

Any two wave surfaces will define a cavity if the wave surfaces are replaced by mirrors that match the curvatures of the wave surfaces.

Figure 9.11. Standing wave pattern and lateral distribution of the $TEM_{0,0}$ mode of a confocal laser cavity.

9.11 Q-switching and mode locking

Q-switching is a technique by which a laser can be made to produce a pulsed output beam with extremely high (~GW) peak power, much higher than would be produced by the same laser if it were operating in a continuous wave (constant output) mode.

- Q-switching is achieved by putting some type of variable attenuator (**Q-switch**) inside the laser's optical resonator. When the attenuator is functioning, light which leaves the gain medium does not return, and lasing cannot begin.

- Initially the laser medium is pumped while the Q-switch is set to prevent feedback of light into the gain medium (producing an optical resonator with low Q). This produces a population inversion, but laser operation cannot yet occur since there is no feedback from the resonator. Since the rate of stimulated emission is dependent on the amount of light entering the medium, the amount of energy stored in the gain medium increases as the medium is pumped. Due to losses from spontaneous emission and other processes, after a certain time the stored energy will reach some maximum level; the medium is said to be **gain saturated**. At this point, the Q-switch device is quickly changed from low to high Q, allowing feedback and the process of optical amplification by stimulated emission to begin. Because of the large amount of energy already stored in the gain medium, the intensity of light in the laser resonator builds up very quickly; this also causes the energy stored in the medium to be depleted almost as quickly. The net result is a short pulse of light output from the laser (**giant pulse**) which may have a very high peak intensity.

https://en.wikipedia.org/wiki/Q-switching

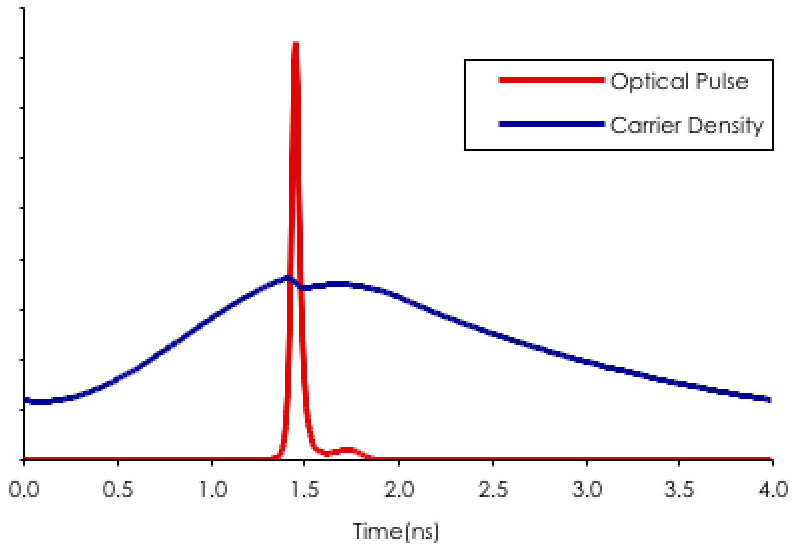
9.11 Q-switching and mode locking

- There are two main types of Q-switching:
  (1) **Active Q-switching**: The Q-switch is an externally controlled variable attenuator. This may be a mechanical device such as a shutter, chopper wheel, or spinning mirror/prism placed inside the cavity, or (more commonly) some form of modulator such as an acousto-optic device, a magneto-optic effect device or an electro-optic device - a **Pockels cell** or **Kerr cell**.

  (2) **Passive Q-switching**: the Q-switch is a saturable absorber, a material whose transmission increases when the intensity of light exceeds some threshold.

https://en.wikipedia.org/wiki/Q-switching

- **Gain-switching**: In a semiconductor laser, the optical pulses are generated by injecting a large number of carriers (electrons) into the active region of the device, bringing the carrier density within that region from below to above the lasing threshold. When the carrier density exceeds that value, the ensuing stimulated emission results in the generation of a large number of photons. However, carriers are depleted as a result of stimulated emission faster than they are injected. So the carrier density eventually falls back to below lasing threshold which results in the termination of the optical output. If carrier injection has not ceased during this period, then the carrier density in the active region can increase once more and the process will repeat itself.



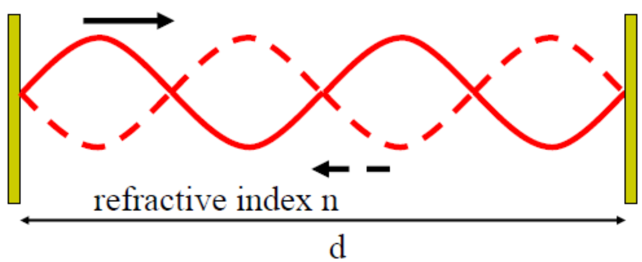
https://en.wikipedia.org/wiki/Gain-switching

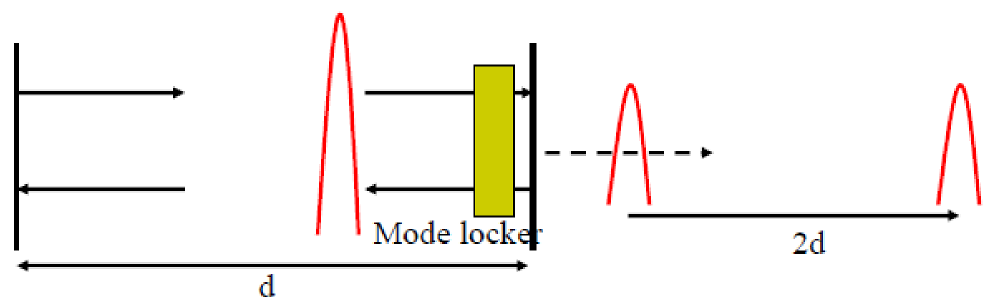
9.11 Q-switching and mode locking

Mode locking is the most important technique for the generation of repetitive, ultrashort laser pulses.

- The basis of the technique is to induce a fixed-phase relationship between the longitudinal modes of the laser's resonant cavity. The laser is then said to be '**phase-locked**' or '**mode-locked**'. Interference between these modes causes the laser light to be produced as a train of pulses. Depending on the properties of the laser, these pulses may be of extremely brief duration, as short as a few femtoseconds.
- A laser can oscillate on many longitudinal modes, with frequencies that are equally separated by the Fabry-Perot intermodal spacing $\Delta\nu_q = c/2nd$. Although these modes normally oscillate independently (they are then called *free-running modes*), external means can be used to couple them and lock their phases together. The modes can then be regarded as the components of a Fourier series expansion of a periodic function of time of period $T_F = 1/\Delta\nu_q = 2nd/c$, which constitute a periodic pulse train. The multiple monochromatic waves of equally spaced frequencies with locked phase constructively interfere.
- The mode-locking operation is accomplished by a nonlinear optical element known as the **mode locker** that is placed inside the laser cavity, typically near one end of the cavity if the laser has the configuration of a linear cavity.

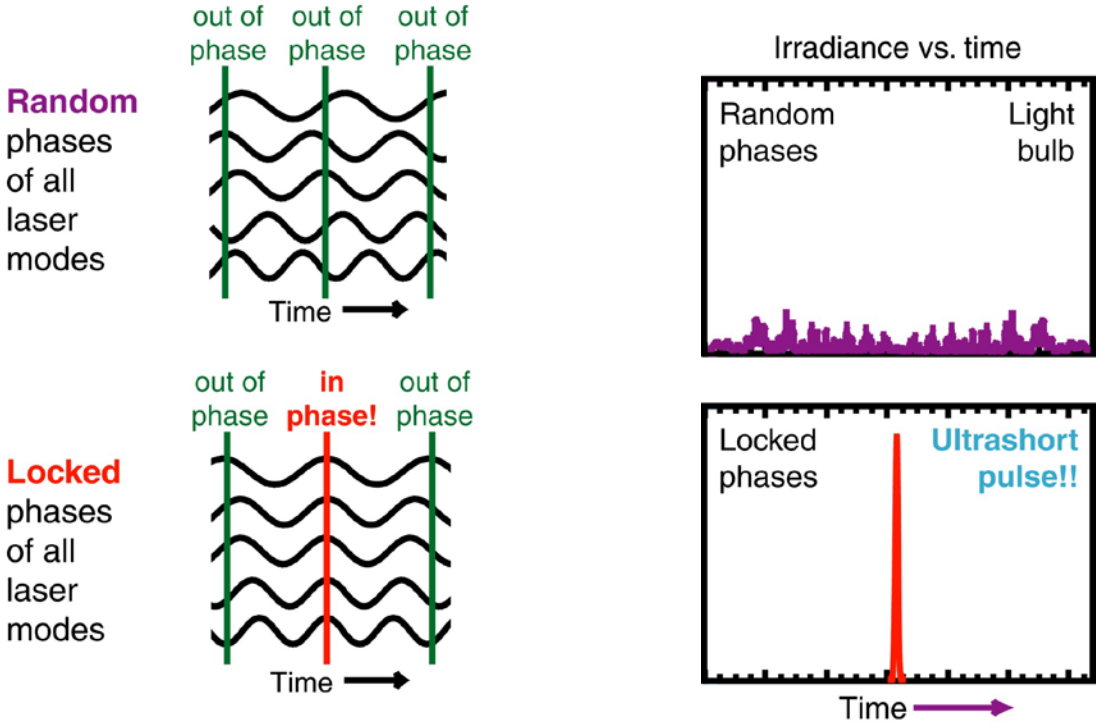
$$k \cdot 2d = \frac{2\pi n}{\lambda} \cdot 2d = \frac{2\pi n \nu}{c} \cdot 2d = 2\pi q \quad (q = 1,\ 2,\ 3, \ldots)$$



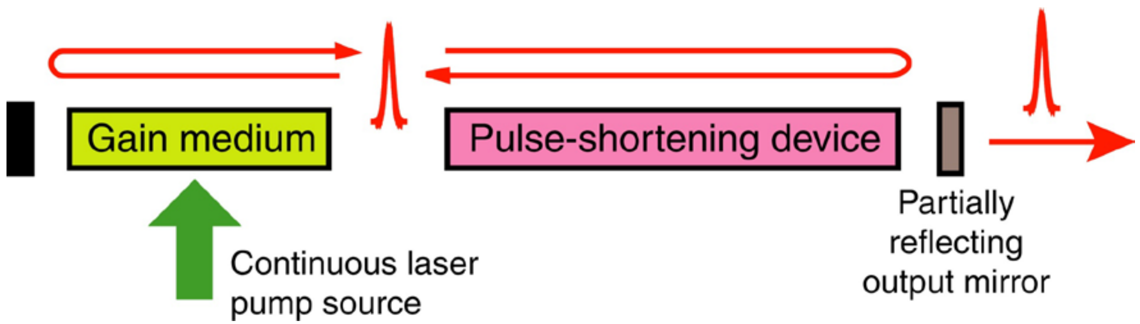


https://en.wikipedia.org/wiki/Mode-locking

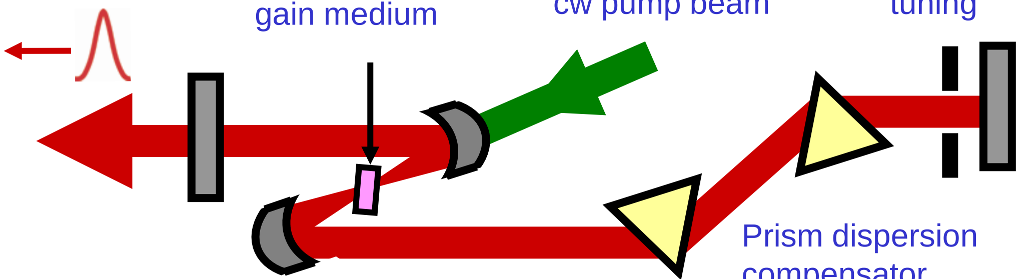
9.11 Q-switching and mode locking



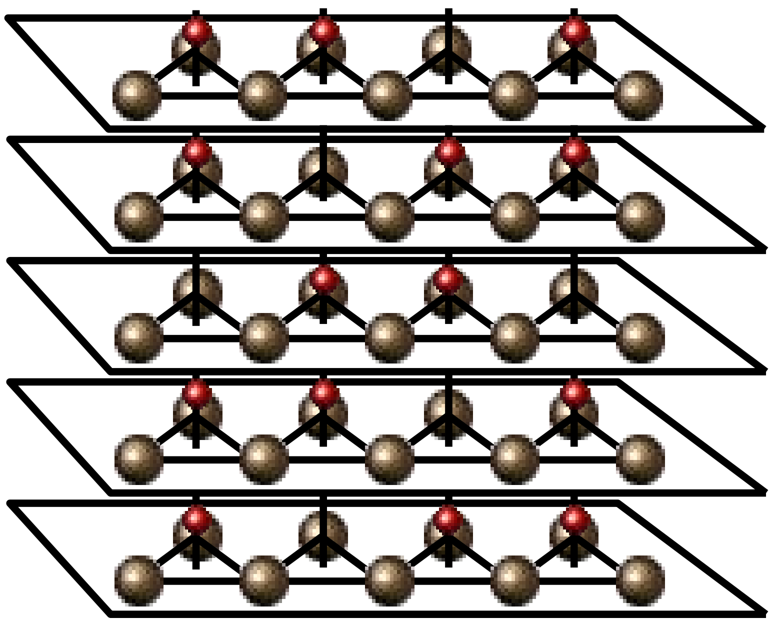
A generic ultrafast laser has a broadband gain medium, a pulse-shortening device, and two or more mirrors.



Titanium Sapphire (Ti:Sapphire)

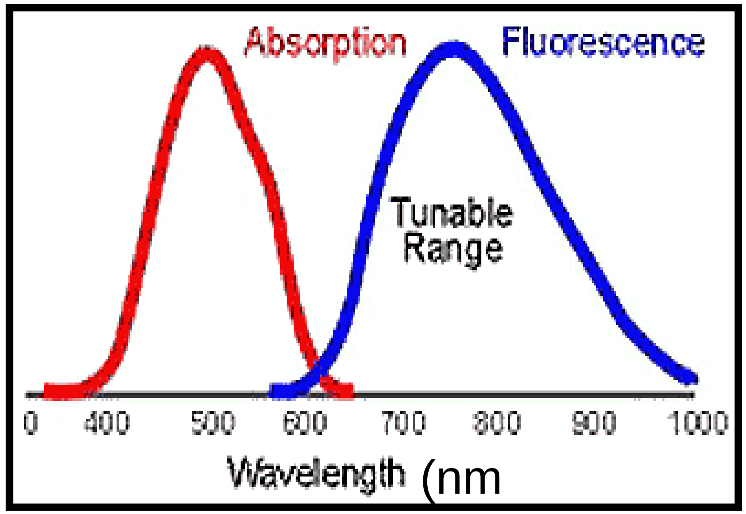


- Spectra-Physics, Tunami (<35 fs pulse length, 1 W average power)
- Coherent, Mira



Al_2O_3 lattice — oxygen, aluminum

Absorption and emission spectra of Ti:Sapphire



It can be pumped with a (continuous) Argon laser (~450-515 nm)

Ti:Sapphire lases from ~700 nm to ~1000 nm.