

# Introduction to Electromagnetism

## Static Magnetic Fields

(6-6, 6-7, 6-8, 6-9)

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# Magnetic Field Intensity

For free space:

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \quad \rightarrow \quad \frac{1}{\mu_0} \nabla \times \mathbf{B} = \mathbf{J}$$

In the presence of a magnetic material:

$$\mathbf{J}_m = \nabla \times \mathbf{M} \quad (\text{A/m}^2) \quad \leftarrow \text{Equivalent volume current density}$$

$$\rightarrow \frac{1}{\mu_0} \nabla \times \mathbf{B} = \mathbf{J} + \mathbf{J}_m = \mathbf{J} + \nabla \times \mathbf{M}$$

$$\rightarrow \nabla \times \left( \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \right) = \mathbf{J} \quad \rightarrow \quad \mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \quad (\text{A/m})$$

$$\rightarrow \nabla \times \mathbf{H} = \mathbf{J} \quad (\text{A/m}^2) \quad \leftarrow \text{Magnetic field intensity}$$

$$\rightarrow \int_S (\nabla \times \mathbf{H}) \cdot d\mathbf{s} = \int_S \mathbf{J} \cdot d\mathbf{s}$$

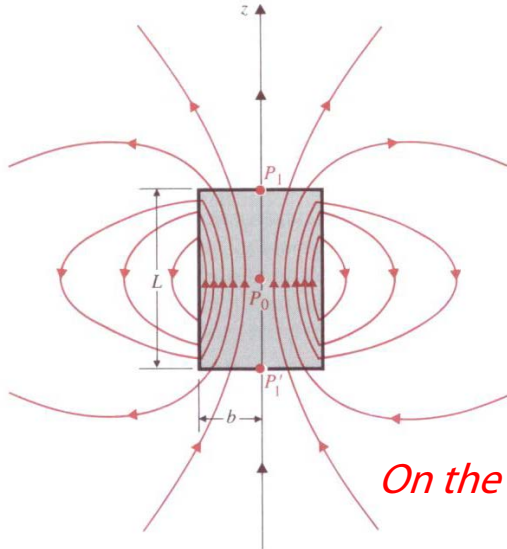
$$\rightarrow \oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{s} = I \quad (\text{A}) \quad \leftarrow \text{Ampère's circuital law}$$

# Example 6-6/6-8/6-13

For a uniform axial magnetization:  $\mathbf{M} = \mathbf{a}_z M_0$

$$\rightarrow \mathbf{J}_{ms} = \mathbf{M} \times \mathbf{a}'_n = (\mathbf{a}_z M_0) \times \mathbf{a}_r = \mathbf{a}_\phi M_0$$

$$\rightarrow \mathbf{J}_m = \nabla' \times \mathbf{M} = 0$$



Biot-Sarvart law:

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi} \left( \frac{d\mathbf{l}' \times \mathbf{R}}{R^3} \right) = \frac{\mu_0 J_{ms} dz'}{4\pi} \left( \frac{d\mathbf{l}' \times \mathbf{R}}{R^3} \right)$$

$$\rightarrow d\mathbf{l}' = \mathbf{a}_\phi b d\phi'$$

*On the z axis*  $\rightarrow \mathbf{R} = \mathbf{a}_z (z - z') - \mathbf{a}_r b \rightarrow R = [(z - z')^2 + b^2]^{1/2}$

$$\rightarrow d\mathbf{l}' \times \mathbf{R} = \mathbf{a}_\phi b d\phi' \times [\mathbf{a}_z (z - z') - \mathbf{a}_r b]$$

*Cylindrical symmetry*

$$= \mathbf{a}_r b (z - z') d\phi' + \mathbf{a}_z b^2 d\phi'$$

$$\rightarrow \mathbf{B}|_{r=0} = \frac{\mu_0}{4\pi} \int_0^L \int_0^{2\pi} \frac{M_0 d\mathbf{l}' \times \mathbf{R}}{R^3} dz' = \mathbf{a}_z \frac{\mu_0}{2} \int_0^L \frac{M_0 b^2 dz'}{[(z - z')^2 + b^2]^{3/2}} = \mathbf{a}_z \frac{\mu_0 M_0}{2} \left[ \frac{-(z - z')}{[(z - z')^2 + b^2]^{1/2}} \right]_0^L$$

$$= \mathbf{a}_z \frac{\mu_0 M_0}{2} \left[ \frac{z}{\sqrt{z^2 + b^2}} - \frac{z - L}{\sqrt{(z - L)^2 + b^2}} \right]$$

*Inside*

$$\rightarrow \mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$$

*Outside*

$$\rightarrow \mathbf{H} = \frac{\mathbf{B}}{\mu_0}$$

# Relative Permeability

Magnetic susceptibility:

$$\mathbf{M} = \chi_m \mathbf{H} \quad \leftarrow \text{For linear and isotropic}$$

$$\rightarrow \mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$$

$$\rightarrow \mathbf{B} = \mu_0(1 + \chi_m)\mathbf{H} \\ = \mu_0\mu_r\mathbf{H} = \mu\mathbf{H} \quad (\text{Wb/m}^2)$$

$$\rightarrow \mathbf{H} = \frac{1}{\mu}\mathbf{B} \quad (\text{A/m})$$

Relative permeability:

$$\rightarrow \mu_r = 1 + \chi_m = \frac{\mu}{\mu_0}$$

Analogous relations:

Electrostatics	Magnetostatics
$\mathbf{E}$	$\mathbf{B}$
$\mathbf{D}$	$\mathbf{H}$
$\epsilon$	$\frac{1}{\mu}$
$\mathbf{P}$	$-\mathbf{M}$
$\rho$	$\mathbf{J}$
$V$	$\mathbf{A}$
$\cdot$	$\times$
$\times$	$\cdot$

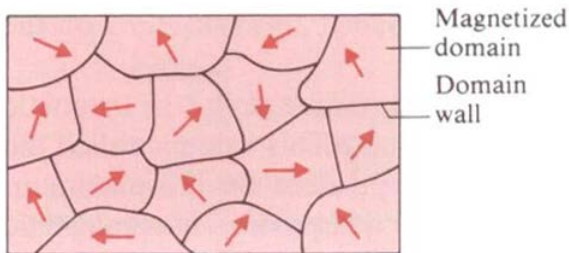
# Behavior of Magnetic Materials

## Classification of magnetic materials:

**Diamagnetic:**  $\mu_r \leq 1$  ( $\chi_m$  is a very small negative number.)  $\rightarrow \chi_m \sim -10^{-5}$   
 (bismuth, copper, lead, mercury, germanium, silver, gold, diamond, etc.)

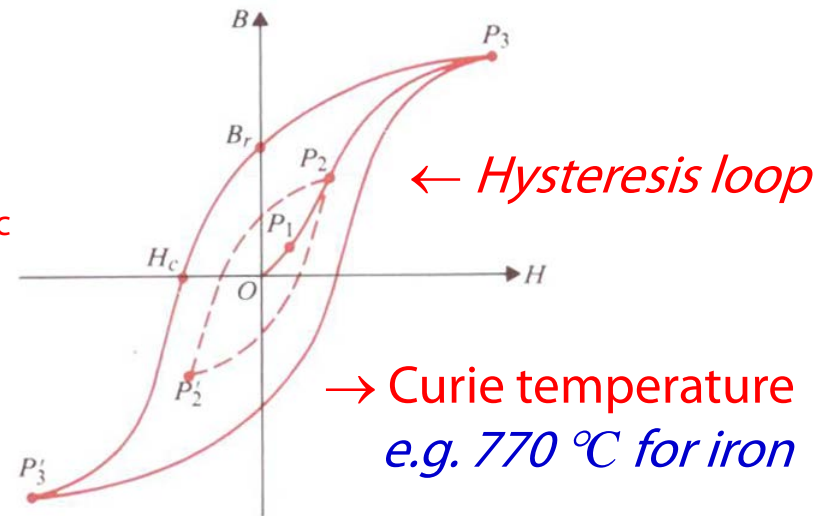
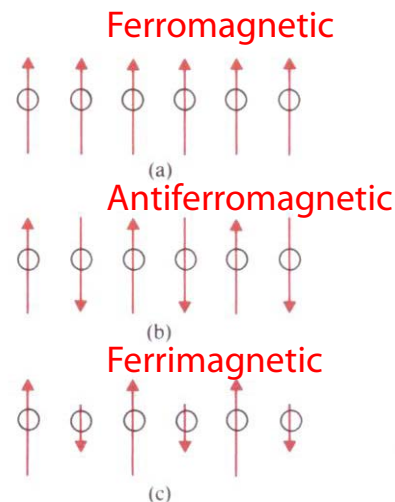
**Paramagnetic:**  $\mu_r \geq 1$  ( $\chi_m$  is a very small positive number.)  $\rightarrow \chi_m \sim +10^{-5}$   
 (aluminum, magnesium, titanium, tungsten, etc.)

**Ferromagnetic:**  $\mu_r \gg 1$  ( $\chi_m$  is a large positive number.)  $\rightarrow \chi_m > 10^3$   
 (cobalt, nickel, iron, etc.)



D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

**Domain size:**  
 $\mu\text{m} \sim \text{mm}$



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# Magnetic Circuits (1)

Two basic equations for magnetostatics:

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{H} = \mathbf{J}$$

Magnetomotive force (mmf):

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{s} = NI = \mathcal{V}_m$$

Example 6-10: ← Fringing effect neglected

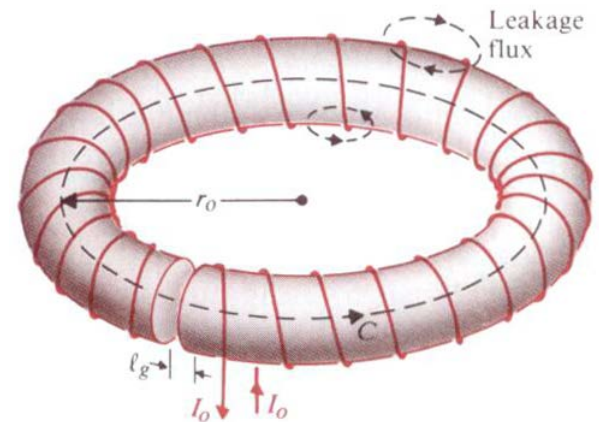
$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{s} = NI_o$$

$$\rightarrow \mathbf{B}_f = \mathbf{B}_g = \mathbf{a}_\phi B_f \quad \leftarrow \text{Why?}$$

$$\rightarrow \mathbf{H}_f = \mathbf{a}_\phi \frac{B_f}{\mu} \rightarrow \mathbf{H}_g = \mathbf{a}_\phi \frac{B_f}{\mu_0}$$

$$\rightarrow \frac{B_f}{\mu} (2\pi r_o - l_g) + \frac{B_f}{\mu_0} l_g = NI_o$$

$$\rightarrow \mathbf{B}_f = \mathbf{a}_\phi \frac{\mu_0 \mu NI_o}{\mu_0 (2\pi r_o - l_g) + \mu l_g}$$



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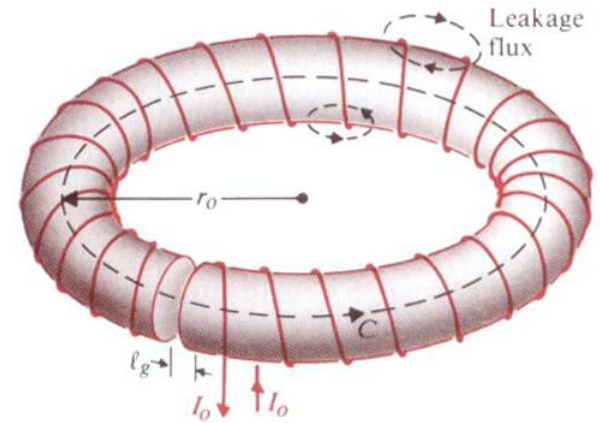
# Magnetic Circuits (2)

Magnetic flux:

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s} \cong BS$$

$$\rightarrow \Phi = \frac{NI_o}{(2\pi r_o - l_g) / \mu S + l_g / \mu_o S}$$

$$= \frac{\mathcal{V}_m}{\mathcal{R}_f + \mathcal{R}_g}$$

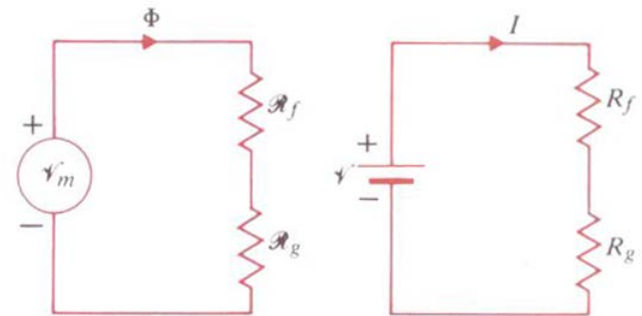


D. K. Cheng, Field and Wave Electromagnetics, 2nd ed., Addison-Wesley, 1989.

Reluctance:

$$\rightarrow \mathcal{R}_f = \frac{2\pi r_o - l_g}{\mu S} = \frac{l_f}{\mu S}$$

$$\rightarrow \mathcal{R}_g = \frac{l_g}{\mu_o S}$$



(a) Magnetic circuit.

(b) Electric circuit.

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Analogous quantities:

Magnetic Circuits	Electric Circuits
mmf, $\mathcal{V}_m (= NI)$	emf, $\mathcal{V}$
magnetic flux, $\Phi$	electric current, $I$
reluctance, $\mathcal{R}$	resistance, $R$
permeability, $\mu$	conductivity, $\sigma$

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Difficulties of magnetic circuits:

1. Leakage flux
2. Fringing effect
3. Nonlinearity:  $B/H$

# Magnetic Circuits (3)

Similarity to Kirchhoff's voltage law:

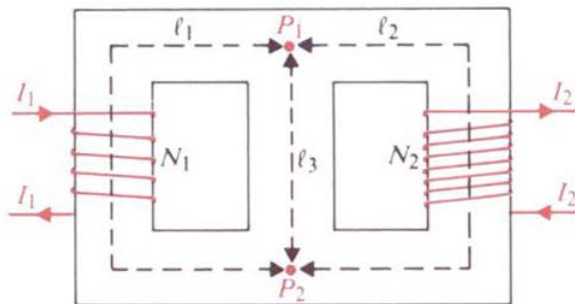
$$\Phi = \frac{V_m}{R_f + R_g} \rightarrow NI = R_f \Phi + R_g \Phi$$

$$\rightarrow \sum_j N_j I_j = \sum_k R_k \Phi_k$$

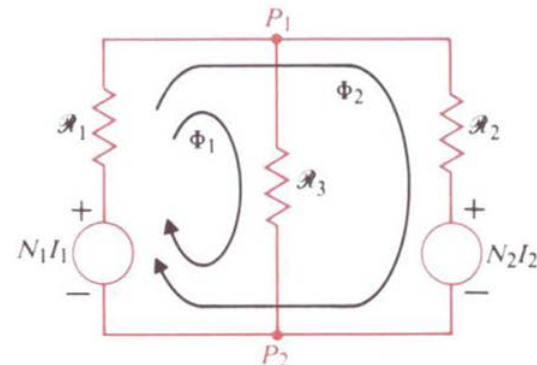
Similarity to Kirchhoff's current law:  $\leftarrow \nabla \cdot \mathbf{J} = 0$

$$\nabla \cdot \mathbf{B} = 0$$

$$\rightarrow \sum_j \Phi_j = 0$$



(a) Magnetic core with current-carrying windings



(b) Magnetic circuit for loop analysis.