Chapter 4. Multiple-beam interference

2016. 10. 11.

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4.1 Interference with multiple beams

The amplitudes of the successive transmitted rays

$$E_0 t^2,\ E_0 t^2 r^2,\ E_0 t^2 r^4,\ \cdot\ \cdot\ \cdot$$

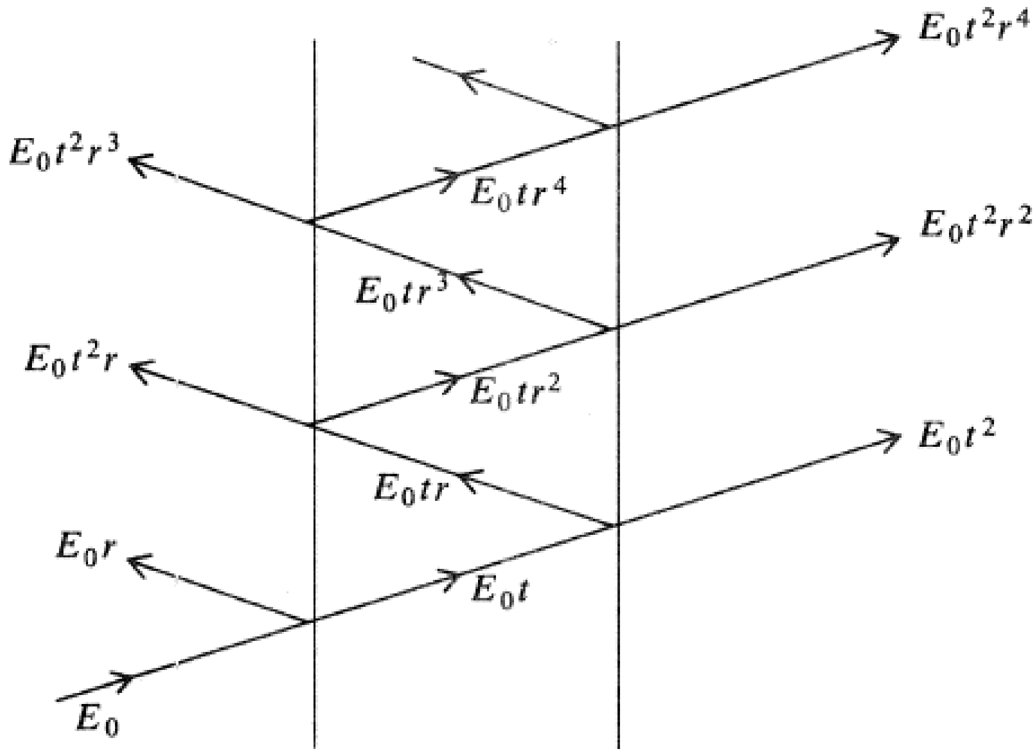


Figure 4.1. Paths of light rays in multiple reflection between two parallel mirrors. (For simplicity the mirrors are considered to be infinitely thin.)

4.1 Interference with multiple beams

The path difference between any two successive transmitted rays

$$\Delta = l_1 + l_2 = \frac{d}{\cos\theta} + l_1 \cos 2\theta = \frac{d}{\cos\theta} + \frac{d(2\cos^2\theta - 1)}{\cos\theta} = 2d\cos\theta$$

The corresponding phase difference

$$\delta = 2kd\cos\theta = \frac{4\pi}{\lambda} d\cos\theta$$

$$\delta = \frac{4\pi}{\lambda_o} nd\cos\theta$$

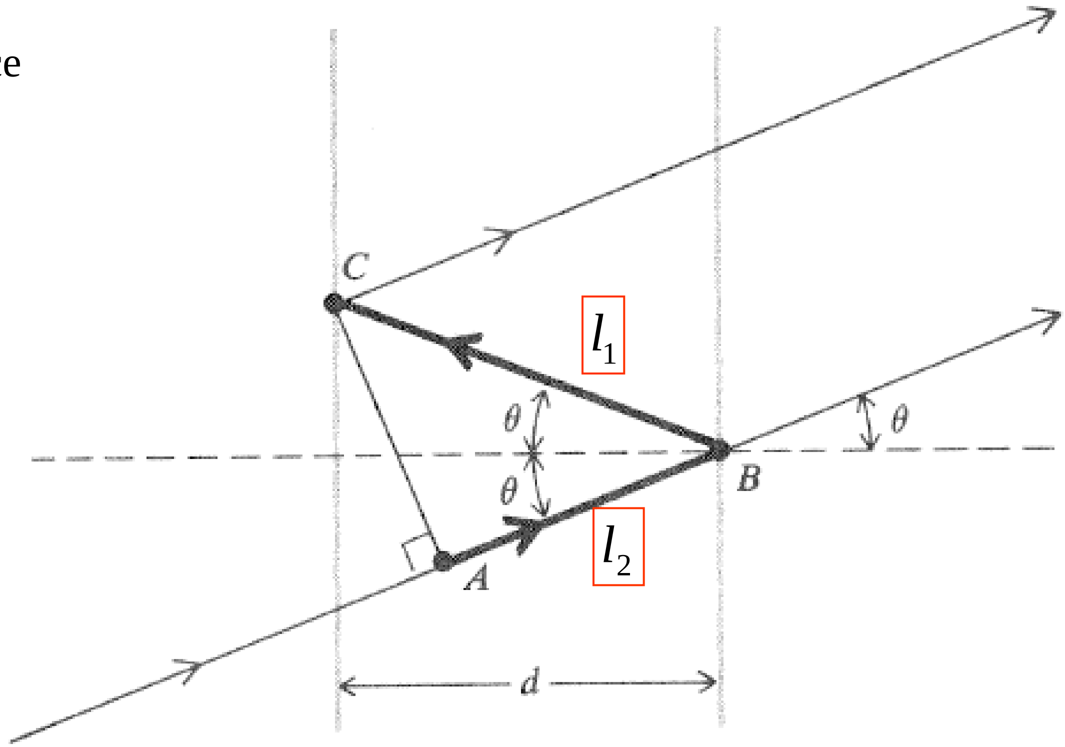


Figure 4.2. Diagram showing the path difference between two successive rays.

4.1 Interference with multiple beams

Total amplitudes of the transmitted rays

$$E_T = E_0 t^2 + E_0 t^2 r^2 e^{i\delta} + E_0 t^2 r^4 e^{i2\delta} + \cdot \quad \cdot \quad \cdot \quad = \frac{E_0 t^2}{1 - r^2 e^{i\delta}}$$

Intensity of the transmitted rays $I_T = I_0 \dfrac{|t|^4}{|1 - r^2 e^{i\delta}|^2}$

Coefficient of reflection $r = |r| e^{i\delta_r/2}$

Reflectance $R = |r|^2 = rr^*$

Coefficient of reflection $T = |t|^2 = tt^*$

$$I_T = I_0 \frac{T^2}{|1 - \mathrm{Re}^{i\Delta}|^2}, \quad \Delta = \delta + \delta_r$$

$$|1 - \mathrm{Re}^{i\Delta}|^2 = (1 - \mathrm{Re}^{i\Delta})(1 - \mathrm{Re}^{-i\Delta}) = 1 - R(e^{i\Delta} + e^{i\Delta}) + R^2 = 1 - 2R\cos\Delta + R^2$$

$$= 1 - 2R(1 - 2\sin^2\frac{\Delta}{2}) + R^2 = (1-R)^2\left[1 + \frac{4R}{(1-R)^2}\sin^2\frac{\Delta}{2}\right]$$

4.1 Interference with multiple beams

Intensity of the transmitted rays

Airy function **Coefficient of finesse**

$$I_T = I_o \frac{T^2}{(1-R)^2} \frac{1}{1+F\sin^2\frac{\Delta}{2}}, \qquad F = \frac{4R}{(1-R)^2}$$

Fringe maximum at $\frac{\Delta}{2} = N\pi, \quad 2N\pi = \frac{4\pi}{\lambda_o} nd\cos\theta + \delta_r, \quad N = \text{order of interference}$

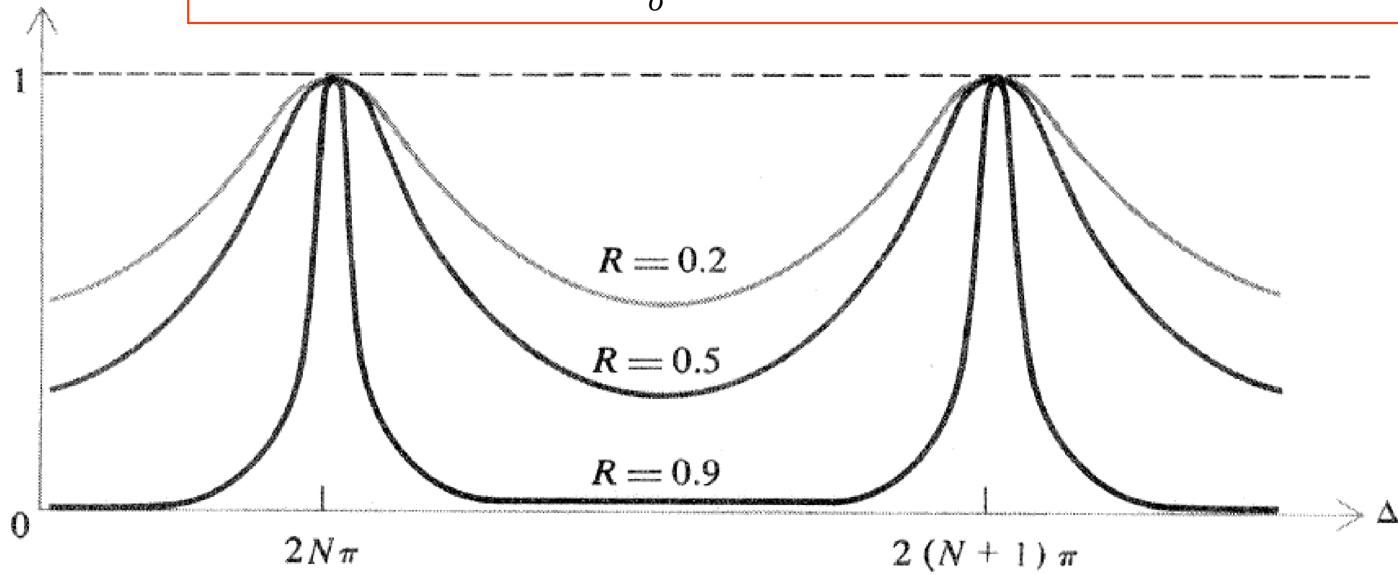


Figure 4.3. Graphs of the Airy function giving the intensity distribution of fringes in multiple-beam interference.

4.1 Interference with multiple beams

If the coefficients of reflections are different, $r_1 = |r_1| e^{i\delta_1}$ $r_2 = |r_2| e^{i\delta_2}$

$$I_T = I_o \frac{T^2}{(1-R)^2} \frac{1}{1+F\sin^2\frac{\Delta}{2}}, \qquad F = \frac{4R}{(1-R)^2}$$

Fringe maximum at $2N\pi = \frac{4\pi}{\lambda_o} nd\cos\theta + \delta_r, \qquad N = \text{order of interference}$

$$T = |t_1||t_1| \qquad R = |r_1||r_1| \qquad \delta_r = \frac{\delta_1 + \delta_2}{2}$$

$$\mathcal{I}_{\max} = \frac{I_{T(\max)}}{I_o} = \frac{T^2}{(1-R)^2}, \quad \text{at } \Delta = 0$$

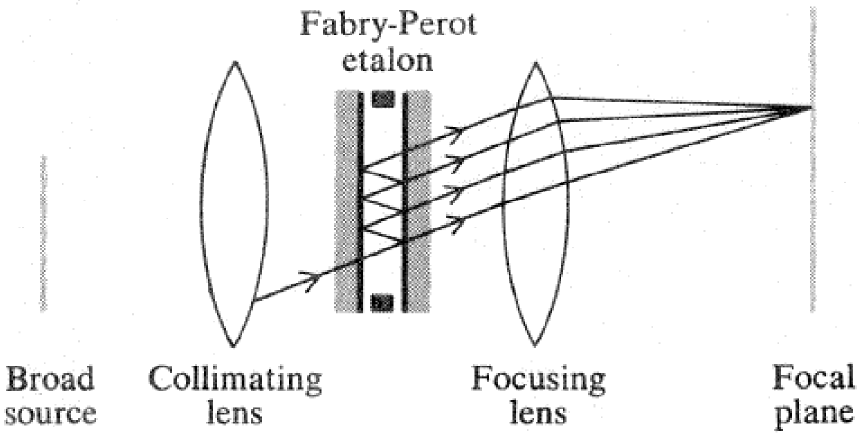
$$\mathcal{I}_{\min} = \frac{I_{T(\min)}}{I_o} = \frac{T^2}{(1+R)^2}, \quad \text{at } \Delta = \pi$$

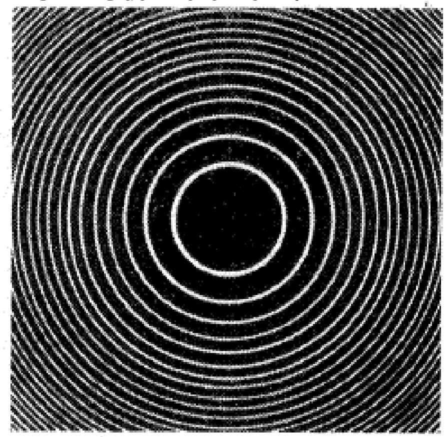
If A is the fraction of incident energy that is absorbed at each reflection, $A + R + T = 1$ $\mathcal{I}_{\max} = \left(\frac{1-A-R}{1-R}\right)^2$

If there is no absorption $R + T = 1$ $\mathcal{I}_{\max} = 1$

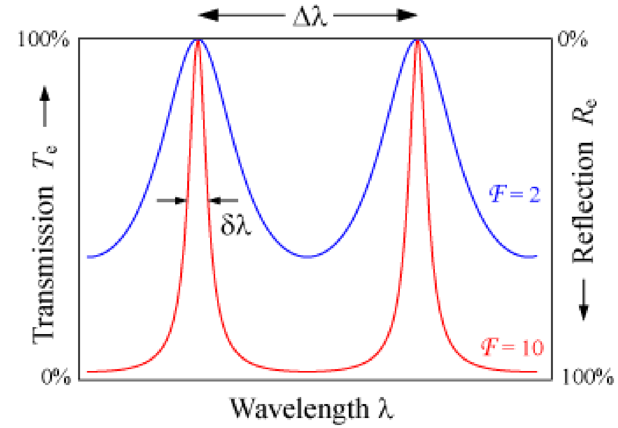
4.2 The Fabry-Perot Interferometer

Fabry-Perot etalon is made of a transparent plate with two reflecting surfaces. *Etalon* is from the French *étalon*, meaning "*measuring gauge*" or "*standard*".

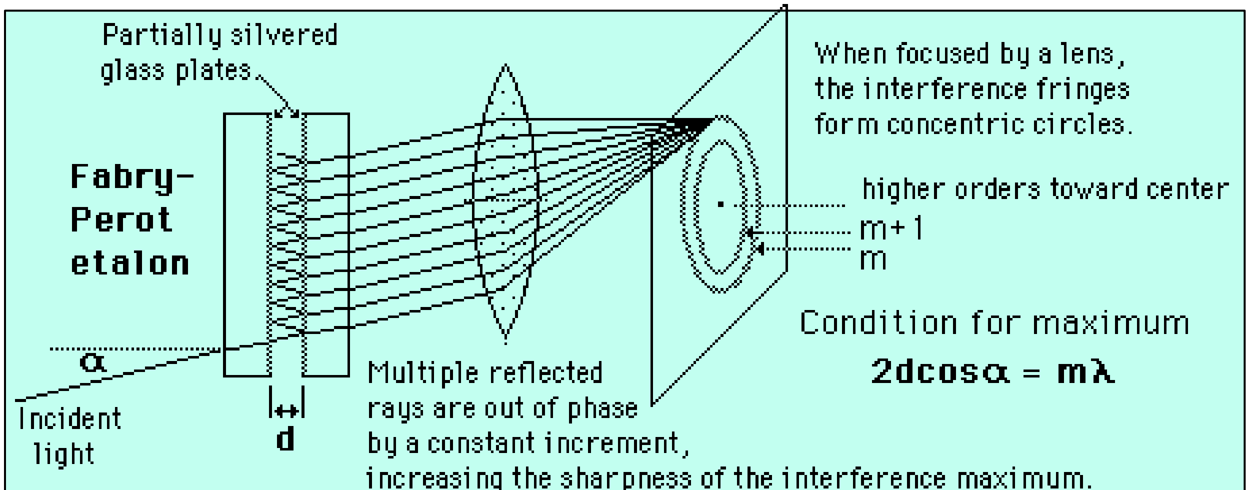




Ring system



The transmission of an etalon as a function of wavelength. A high-finesse etalon (red line) shows sharper peaks and lower transmission minima than a low-finesse etalon (blue).



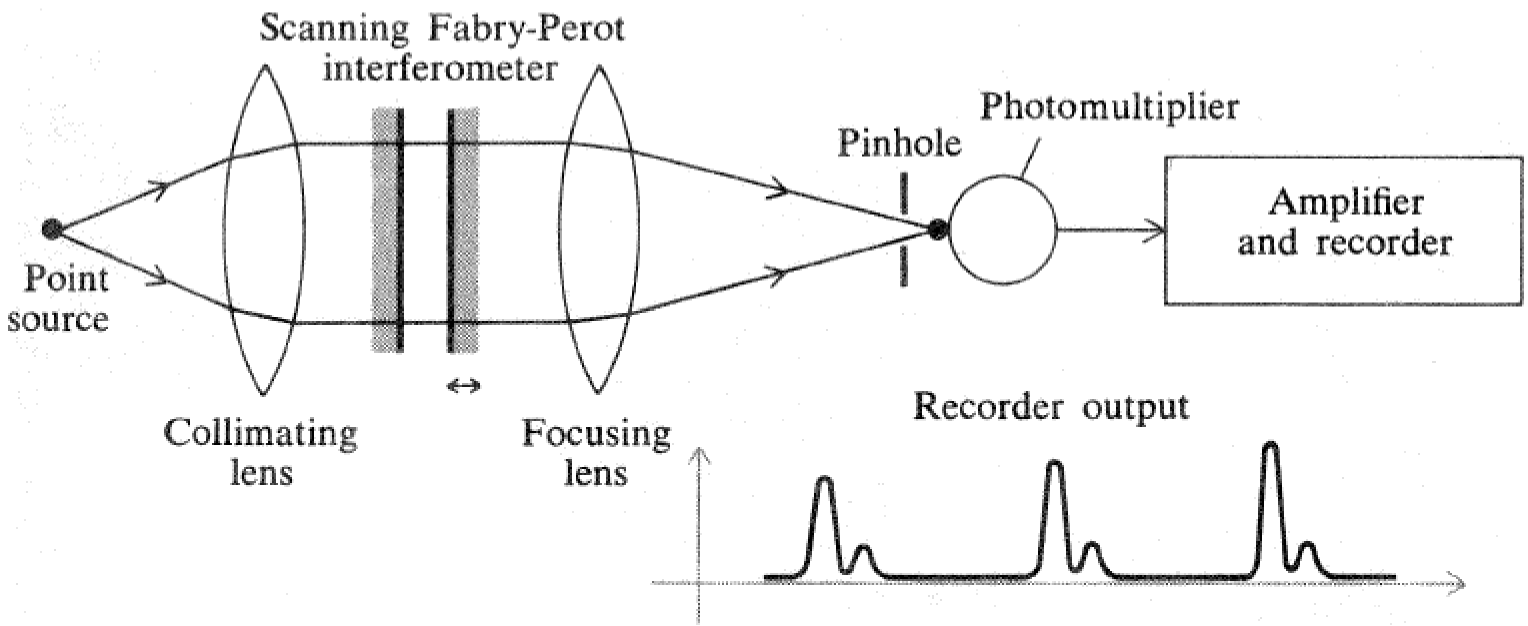
http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/fabry.html

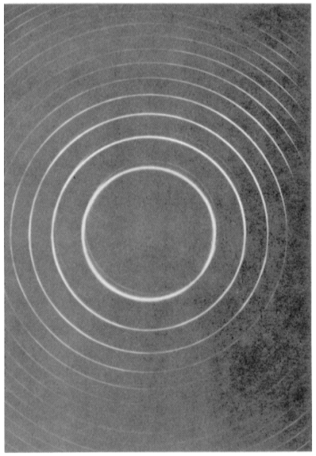
Applications: widely used in telecommunications, lasers and spectroscopy to control and measure the wavelengths of light.

https://en.wikipedia.org/wiki/Fabry%E2%80%93P%C3%A9rot_interferometer

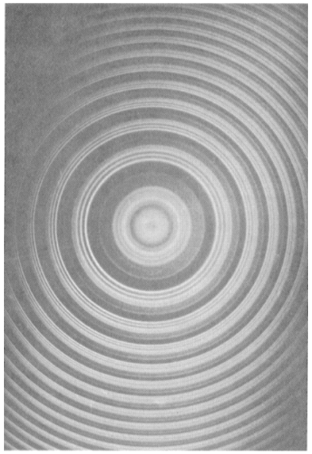
4.2 The Fabry-Perot Interferometer

Fabry-Perot interferometer is made of two parallel highly reflecting mirrors.





Fabry-Perot interference fringes from monochromatic source



Fabry-Perot interference fringes from non-monochromatic source

4.2 The Fabry-Perot Interferometer

Free spectral range of a Fabry-Perot instrument corresponds to $\Delta_{N+1} - \Delta_N = 2\pi$

$$\omega_{N+1} - \omega_N = \frac{\pi c}{nd\cos\theta},$$

$$\nu_{N+1} - \nu_N = \frac{c}{2nd\cos\theta}$$

$$\nu_{N+1} - \nu_N \cong \frac{c}{2nd} \quad \text{for small } \theta$$

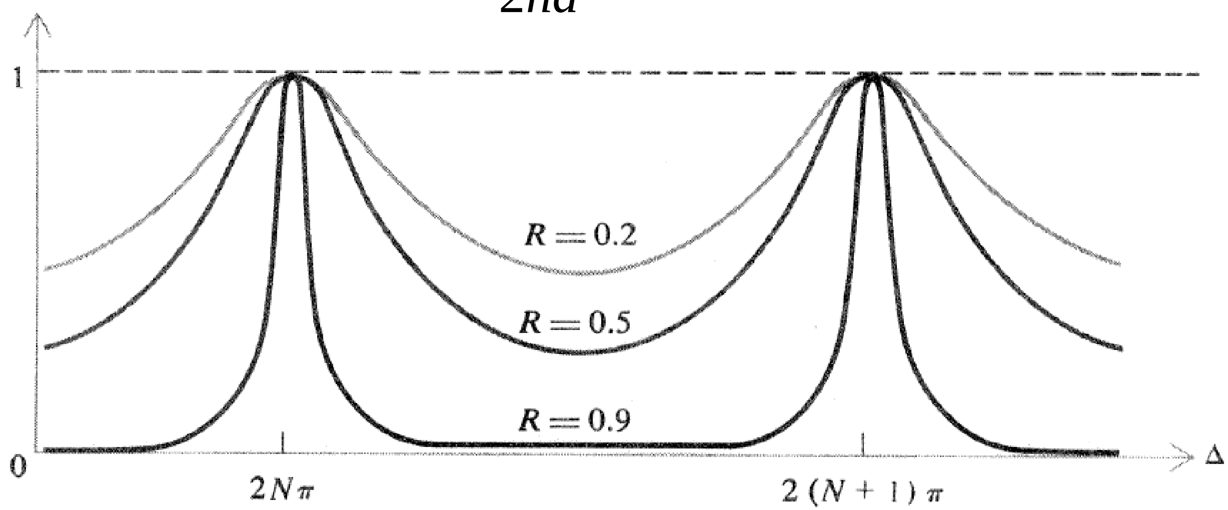


Figure 4.3. Graphs of the Airy function giving the intensity distribution of fringes in multiple-beam interference.

$$I_T = I_o \frac{T^2}{(1-R)^2} \frac{1}{1 + F\sin^2\frac{\Delta}{2}}$$

$$\Delta = \delta + \delta_r$$

$$\delta = \frac{4\pi}{\lambda_o} nd\cos\theta$$

4.3 Resolution of Fabry-Perot instruments

Suppose a spectrum consisting of two closely spaced frequencies ω and ω' is analyzed with a Fabry-Perot interferometer.

$$I_T = I_o(1+F\sin^2\frac{\Delta}{2})^{-1} + I_o(1+F\sin^2\frac{\Delta'}{2})^{-1}$$

$$\Delta \approx \delta_r + 2kd = \delta_r + \frac{2\omega d}{c}, \quad \Delta' \approx \delta_r + 2k'd = \delta_r + \frac{2\omega' d}{c} \quad \text{(assume that is } \theta \text{ small)}$$

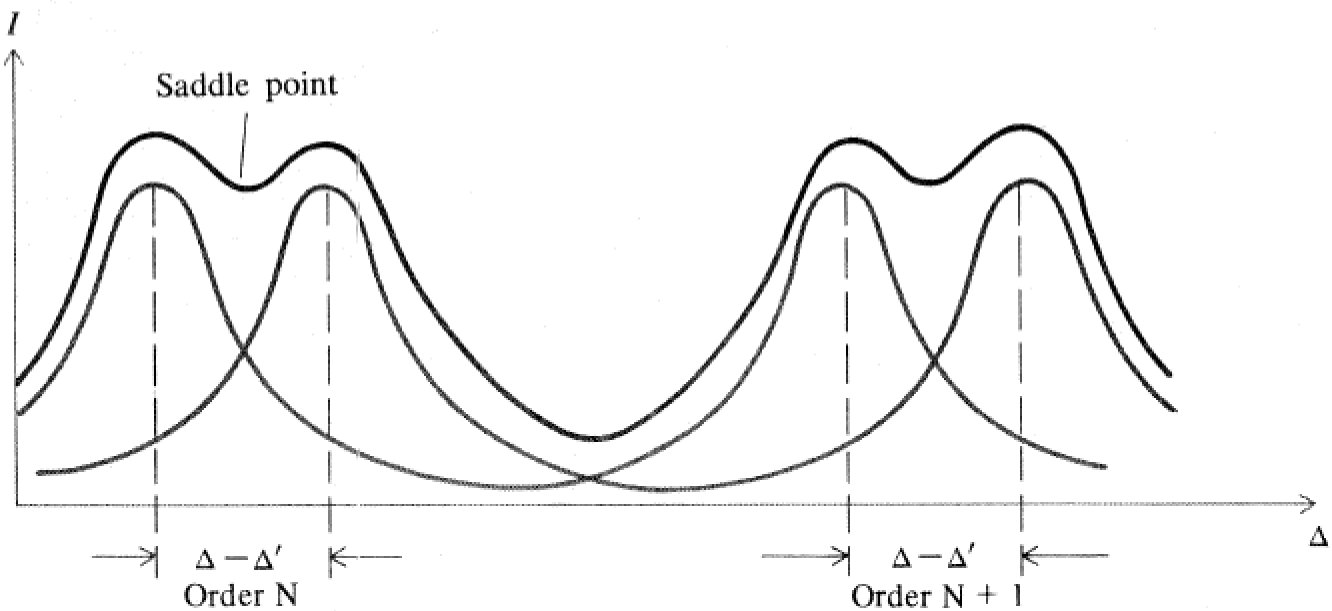


Figure 4.6. Graph of intensity distribution for two monochromatic lines in Fabry-Perot interferometry.

4.3 Resolution of Fabry-Perot instruments

Taylor criterion for resolution: Two equal lines are considered to be resolved if the individual curves cross at the half-intensity point, so that the total intensity at the saddle point is equal to the maximum intensity of either line alone.

$$I = 2I_o\left[1 + F\sin^2\left(\frac{\Delta - \Delta'}{4}\right)\right]^{-1} = I_o \quad \text{at the saddle point.}$$

$$\therefore\ F\sin^2\left(\frac{\Delta - \Delta'}{4}\right) = 1$$

$$\text{If } \Delta - \Delta' \text{ is small,}\quad \left|\Delta - \Delta'\right| = \frac{4}{\sqrt{F}} = 2\left(\frac{1-R}{\sqrt{R}}\right)$$

$$\Delta \approx \delta_r + 2kd = \delta_r + \frac{2\omega d}{c}$$

$$\delta\omega = \left|\omega - \omega'\right| = \frac{2c}{d}\frac{1}{\sqrt{F}} = \frac{c}{d}\left(\frac{1-R}{\sqrt{R}}\right)$$

Reflecting finesse

$$\mathcal{F} = \frac{\Delta_{N+1} - \Delta_N}{\left|\Delta - \Delta'\right|} = \frac{\pi}{2}\sqrt{F} = \pi\left(\frac{\sqrt{R}}{1-R}\right)$$

4.3 Resolution of Fabry-Perot instruments

Resolving power

$$\mathrm{RP} = \frac{\omega}{\delta\omega} = \frac{\nu}{\delta\nu} = \frac{\lambda}{|\delta\lambda|}$$

$$2N\pi = \Delta_N = 2kd = \frac{2\omega d}{c},$$

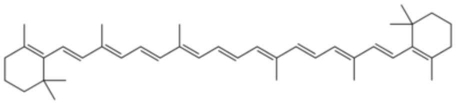
$$\therefore\ \mathrm{RP} = \frac{\omega}{\delta\omega} = \frac{\omega d}{2c}\sqrt{F} = \frac{\omega d}{2c}\frac{2}{\pi}\mathcal{F} = \frac{\omega d}{\pi c}\mathcal{F} = \frac{N\pi c}{\pi c}\mathcal{F} = N\mathcal{F}$$

$$\therefore\ \mathrm{RP} = N\mathcal{F} = N\pi\left(\frac{\sqrt{R}}{1-R}\right)$$

By increasing the order of interference, the resolving power with a given reflectance can be made as large as desired. This can easily be accomplished by increasing the mirror separation, because $N=2nd/\lambda_0$. However, the free spectral range then diminishes, and so a compromise must be chosen.

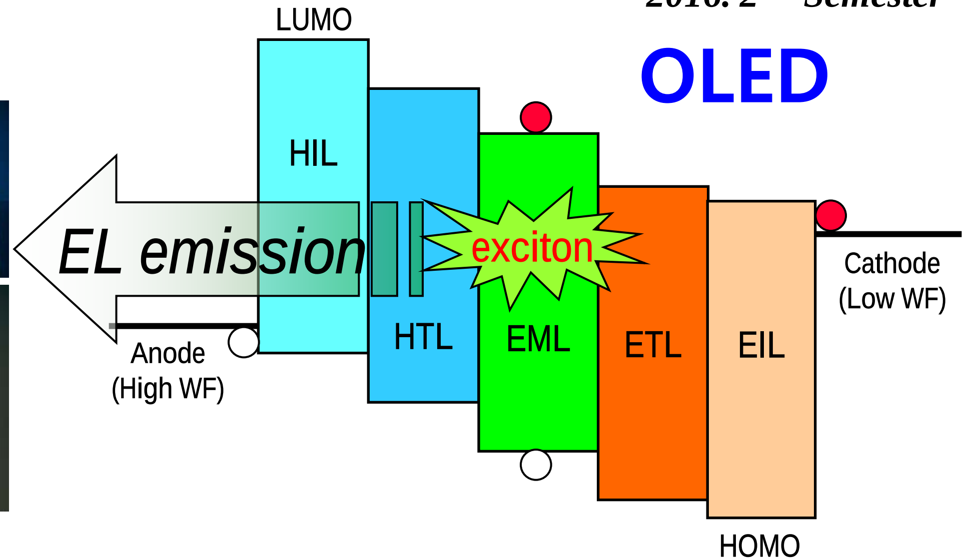
OLED?

Organic Light Emitting Diode

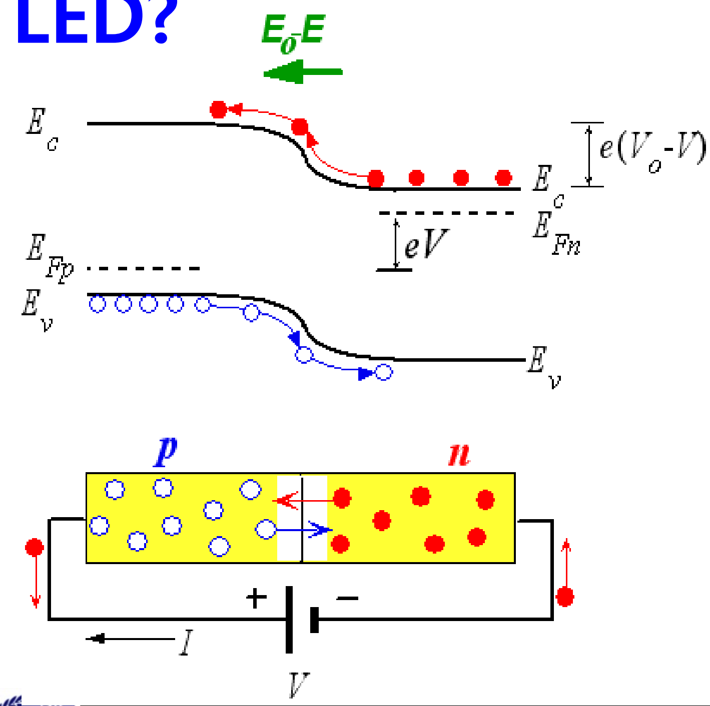


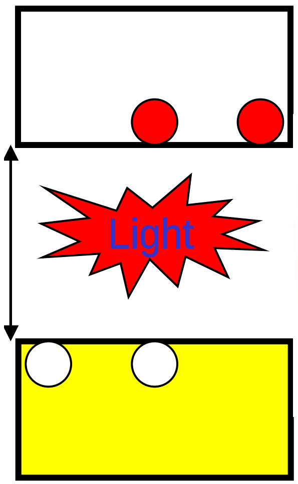
β-Carotene: can be found in yellow, orange, and green leafy fruits and vegetables such as carrots, tomatoes, sweet potatoes, oranges, etc..

OLED

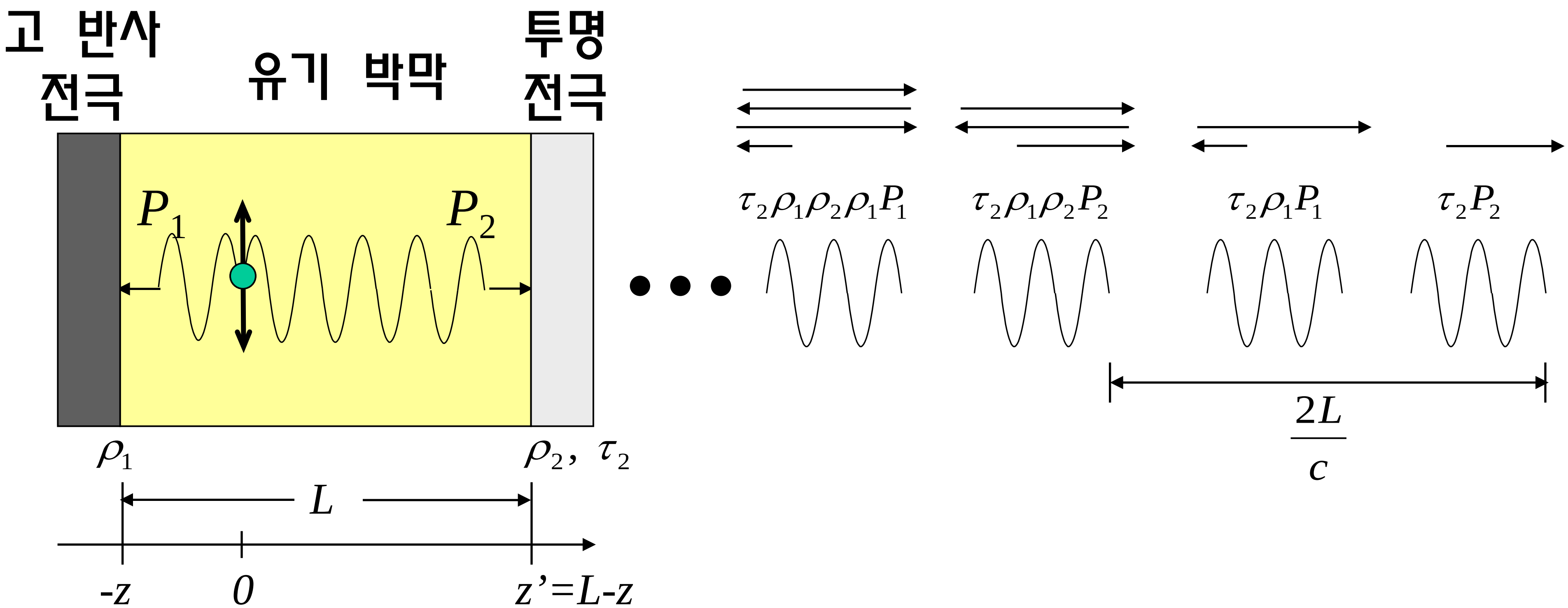


LED?





Spontaneous emission from planar microcavity



$$E_{L2}(t) = \tau_2 WP(t) + \tau_2\rho_1 WP(t - \frac{2z_1}{c}) + \tau_2\rho_1\rho_2 WP(t - \frac{2L}{c})$$

$$+ \tau_2\rho_1\rho_1\rho_2 WP(t - \frac{2z_1}{c} - \frac{2L}{c})$$

D. G. Deppe, C. Lei, C. C. Lin, and D. L. Huffaker, J. Modern Optics **41**, 325 (1994)

Spontaneous emission from planar microcavity

$$WP(\omega)$$

$$E_{L2}(\omega) = \frac{\tau_2}{2\pi}\int_{-\infty}^{\infty} WP(t)\exp(i\omega t)dt + \frac{\tau_2\rho_1}{2\pi}\int_{-\infty}^{\infty} WP(t-\frac{2z_1}{c})\exp(i\omega t)dt$$

$$+\frac{\tau_2\rho_1\rho_2}{2\pi}\int_{-\infty}^{\infty} WP(t-\frac{2L}{c})\exp(i\omega t)dt$$

$$+\frac{\tau_2\rho_1\rho_1\rho_2}{2\pi}\int_{-\infty}^{\infty} WP(t-\frac{2z_1}{c}-\frac{2L}{c})\exp(i\omega t)dt + ...$$

$$E_{L2}(\omega) = \tau_2 WP(\omega) + \tau_2\rho_1 e^{i\frac{2\omega z_1}{c}} WP(\omega) + \tau_2\rho_1\rho_2 e^{i\frac{2\omega L}{c}} WP(\omega)$$

$$+\tau_2\rho_1\rho_2\rho_1 e^{i\frac{2\omega z_1}{c}+i\frac{2\omega L}{c}} WP(\omega) + \tau_2\rho_1\rho_2\rho_1\rho_2 e^{i\frac{4\omega L}{c}} WP(\omega) + ...$$

$$= \tau_2 WP(\omega)[1 + \rho_1 e^{i\frac{2\omega z_1}{c}} + \rho_1\rho_2\rho_1 e^{i\frac{2\omega z_1}{c}+i\frac{2\omega L}{c}} + ...$$

$$+\rho_1\rho_2 e^{i\frac{2\omega L}{c}} + \rho_1\rho_2\rho_1\rho_2 e^{i\frac{4\omega L}{c}} + ...\ ]$$

$$E_{L2}(\omega) = \tau_2 WP(\omega)[1 + \rho_1 e^{i\frac{2\omega z_1}{c}}\{1 + \rho_2\rho_1 e^{i\frac{2\omega L}{c}} + ...\}$$

$$+\rho_1\rho_2 e^{i\frac{2\omega L}{c}}\{1 + \rho_1\rho_2 e^{i\frac{2\omega L}{c}} + ...\}]$$

$$1 + \rho_2\rho_1 e^{i\frac{2\omega L}{c}} + ... = \frac{1}{1 - \rho_2\rho_1 e^{i\frac{2\omega L}{c}}}$$

Spontaneous emission from planar microcavity

$$E_{L2}(\omega) = \tau_2 WP(\omega) \frac{1+\rho_1 e^{i\frac{2\omega z_1}{c}}}{1-\rho_1\rho_2 e^{i\frac{2\omega L}{c}}}$$

Emission spectrum in the forward direction

$$\left|E_{L2}(\omega)\right|^2 = \frac{(1-R_2)[1+R_1+2\sqrt{R_1}\cos(\frac{2\omega n z_1}{c})]}{1+R_1R_2-2\sqrt{R_1R_2}\cos(\frac{2\omega L}{c})}\left|WP(\omega)\right|^2$$

Interference effect

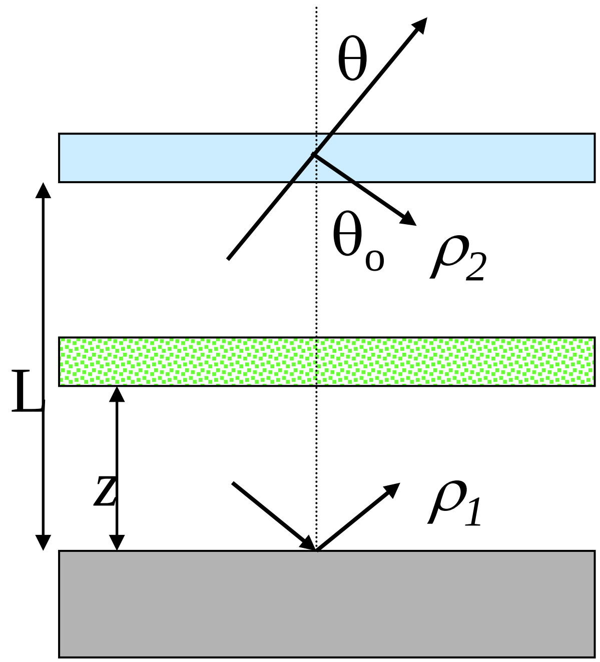
Fabry-Perot Resonator

$$= \frac{(1-R_2)[1+R_1+2\sqrt{R_1}\cos(\frac{4\pi n z_1}{\lambda})]}{1+R_1R_2-2\sqrt{R_1R_2}\cos(\frac{4\pi n L}{\lambda})}\left|WP(\omega)\right|^2$$

$$|\rho_1|^2 = R_1 \;, |\rho_2|^2 = R_2 \;, |\tau_2|^2 = 1-R_2$$

D. G. Deppe, C. Lei, C. C. Lin, and D. L. Huffaker, J. Modern Optics **41**, 325 (1994)

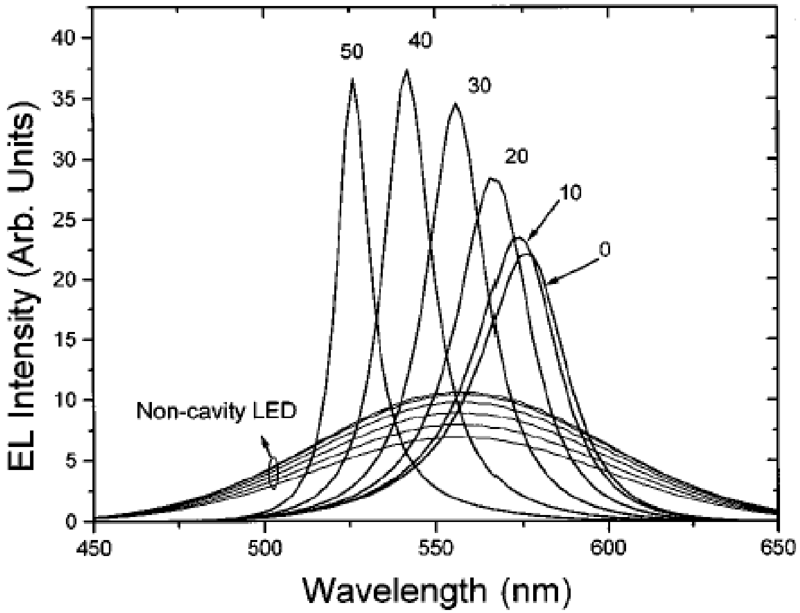
Radiation mode in top-emitting OLED

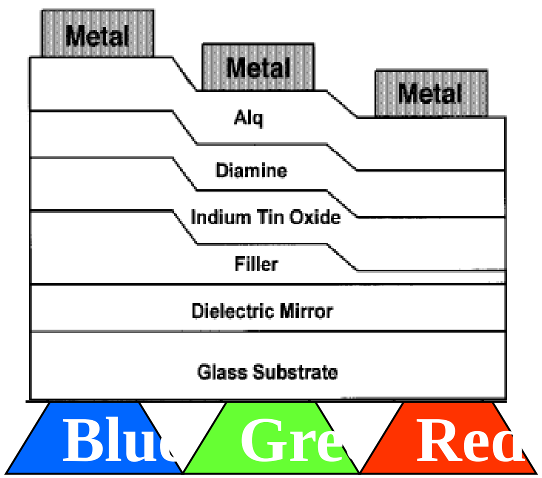


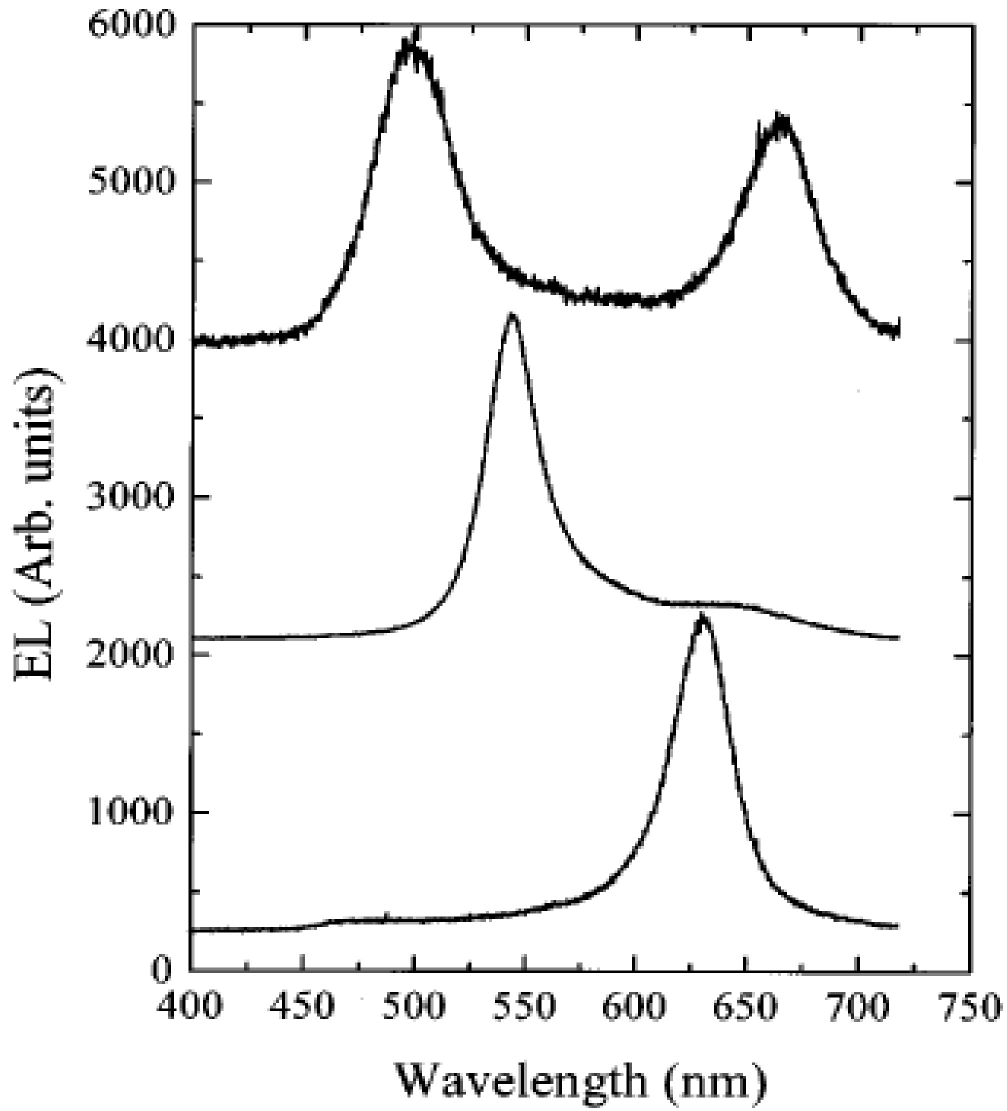
$$I_{ext}^{(s,p)}(\theta,\lambda) = \frac{\left|1 + r_1^{(s,p)} \exp(i\frac{4\pi nz\cos\theta_o}{\lambda})\right|^2}{\left|1 - r_1^{(s,p)} r_2^{(s,p)} \exp(i\frac{4\pi nL\cos\theta_o}{\lambda})\right|^2} T_2^{(s,p)} I_{\text{int}}^{(s,p)}(\lambda)$$

C. Qiu, H. Peng, H. Chen, Z. Xie, M. Wong, and H. S. Kwok, IEEE Trans. on Electron Dev. 51, 1207 (2004).

Resonant emission from microcavity







A. Dodabalapur, L. J. Rothberg, R. H. Jordan, T. M. Miller, R. E. Slusher, and J. M. Phillips, J. Appl. Phys. **80**, 6954 (1996).