Chapter 4. Multiple-beam interference
Part 2

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4.4 Theory of multilayer films

Optical surfaces having virtually any desired reflectance and transmittance characteristics may be produced by means of thin film coatings.
Examples: camera lens, high-reflecting mirrors, high-transmitting mirrors, one-way mirrors, optical filters, etc.

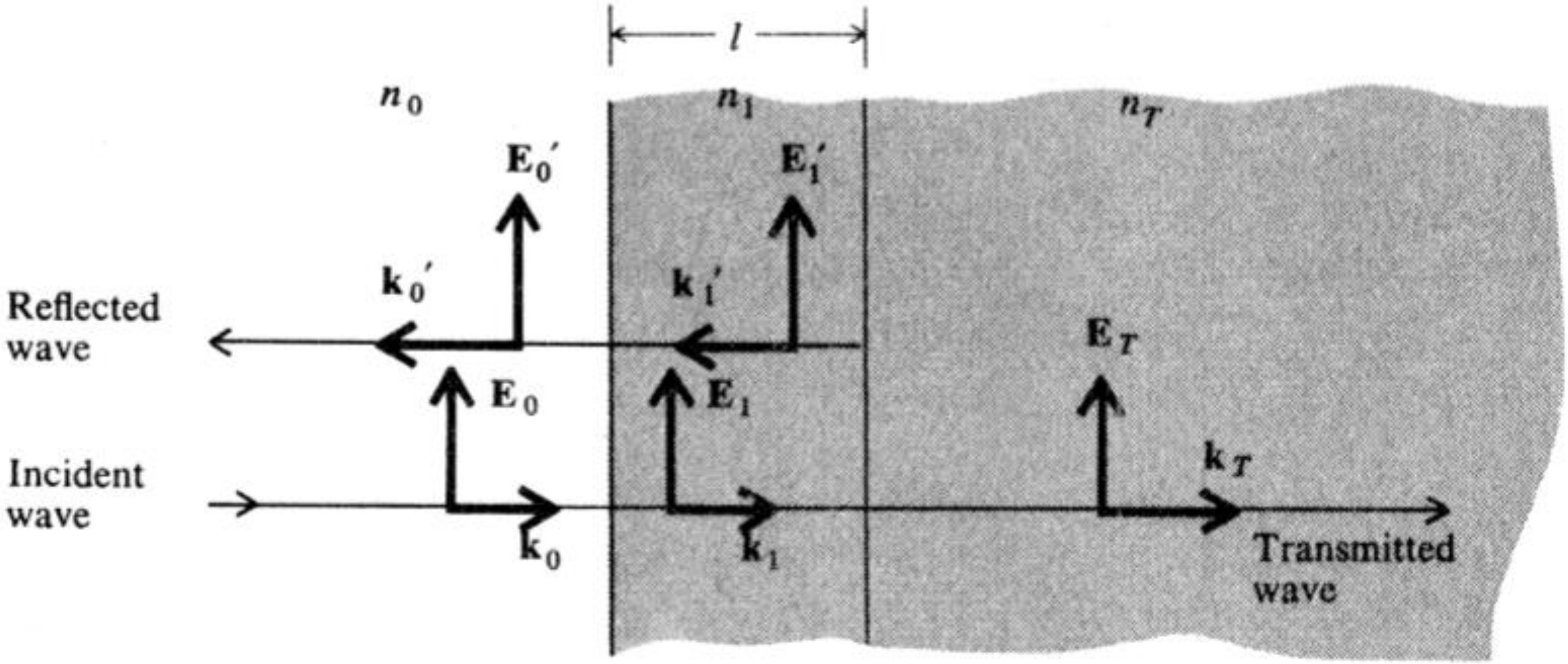


Figure 4.7. Wave vectors and their associated electric fields for the case of normal incidence on a single dielectric layer.

4.4 Theory of multilayer films

Boundary conditions

	First Interface	Second Interface
Electric:	$E_0 + E_0' = E_1 + E_1'$	$E_1 e^{ikl} + E_1' e^{-ikl} = E_T$
Magnetic:	$H_0 - H_0' = H_1 - H_1'$	$H_1 e^{ikl} - H_1' e^{-ikl} = H_T$
or	$n_0 E_0 - n_0 E_0' = n_1 E_1 - n_1 E_1'$	$n_1 E_1 e^{ikl} - n_1 E_1' e^{-ikl} = n_T E_T$

$$1 + \frac{E_0'}{E_0} = (\cos kl - i\frac{n_T}{n_1}\sin kl)\frac{E_T}{E_0}$$

$$n_0 - n_0\frac{E_0'}{E_0} = (-in_1 \sin kl + n_T \cos kl)\frac{E_T}{E_0}$$

$$\begin{bmatrix} 1 \\ n_0 \end{bmatrix} + \begin{bmatrix} 1 \\ -n_0 \end{bmatrix}\frac{E_0'}{E_0} = \begin{bmatrix} \cos kl & -i\frac{n_T}{n_1}\sin kl \\ -in_1 \sin kl & \cos kl \end{bmatrix}\begin{bmatrix} 1 \\ n_T \end{bmatrix}\frac{E_T}{E_0}$$

n_1 = index of reflection of the dielectric layer, $k = \frac{2\pi}{\lambda} = \frac{2\pi n_1}{\lambda_0}$

4.4 Theory of multilayer films

$$\begin{bmatrix} 1 \\ n_0 \end{bmatrix} + \begin{bmatrix} 1 \\ -n_0 \end{bmatrix} r = M \begin{bmatrix} 1 \\ n_T \end{bmatrix} t$$

$$r = \frac{E_0'}{E_0} \quad \text{reflection coefficient}$$

$$t = \frac{E_T}{E_0} \quad \text{transmission coefficient}$$

$$M = \begin{bmatrix} \cos kl & -i\frac{n_T}{n_1}\sin kl \\ -in_1 \sin kl & \cos kl \end{bmatrix} \quad \text{transfer matrix}$$

$$\begin{bmatrix} 1 \\ n_0 \end{bmatrix} + \begin{bmatrix} 1 \\ -n_0 \end{bmatrix} r = M_1 M_2 M_3 \cdot \cdot \cdot M_N \begin{bmatrix} 1 \\ n_T \end{bmatrix} t = M \begin{bmatrix} 1 \\ n_T \end{bmatrix} t \quad \text{for the multilayer film}$$

$$M_1 M_2 M_3 \cdot \cdot \cdot M_N = M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad \text{overall transfer matrix}$$

4.4 Theory of multilayer films

$$r = \frac{An_0 + Bn_T n_0 - C - Dn_T}{An_0 + Bn_T n_0 + C + Dn_T} \quad \text{reflection coefficient}$$

$$t = \frac{2n_0}{An_0 + Bn_T n_0 + C + Dn_T} \quad \text{transmission coefficient}$$

$$\text{reflectance} \quad R = |r|^2$$

$$\text{transmittance} \quad T = |t|^2$$

Antireflecting films

Suppose a single film of index n_1 and thickness l is placed on a glass substrate of index n_T.

$$\text{reflection coefficient} \qquad r = \frac{n_1(1-n_T)\cos kl - i(n_T - n_1^2)\sin kl}{n_1(1+n_T)\cos kl - i(n_T + n_1^2)\sin kl}$$

$$\text{reflectance} \qquad R = |r|^2 = \frac{(n_T - n_1^2)^2}{(n_T + n_1^2)^2} \quad \text{if } l = \frac{\lambda}{4},\ kl = \frac{\pi}{2}$$

$$R = 0 \quad \text{if } n_1 = \sqrt{n_T}$$

Antireflecting films

MgF_2, index n_{MgF2} =1.35, glass index n_T~1.5

R can be reduced to zero by using multilayer films of low/high/…low/high index with the thickness λ/4.

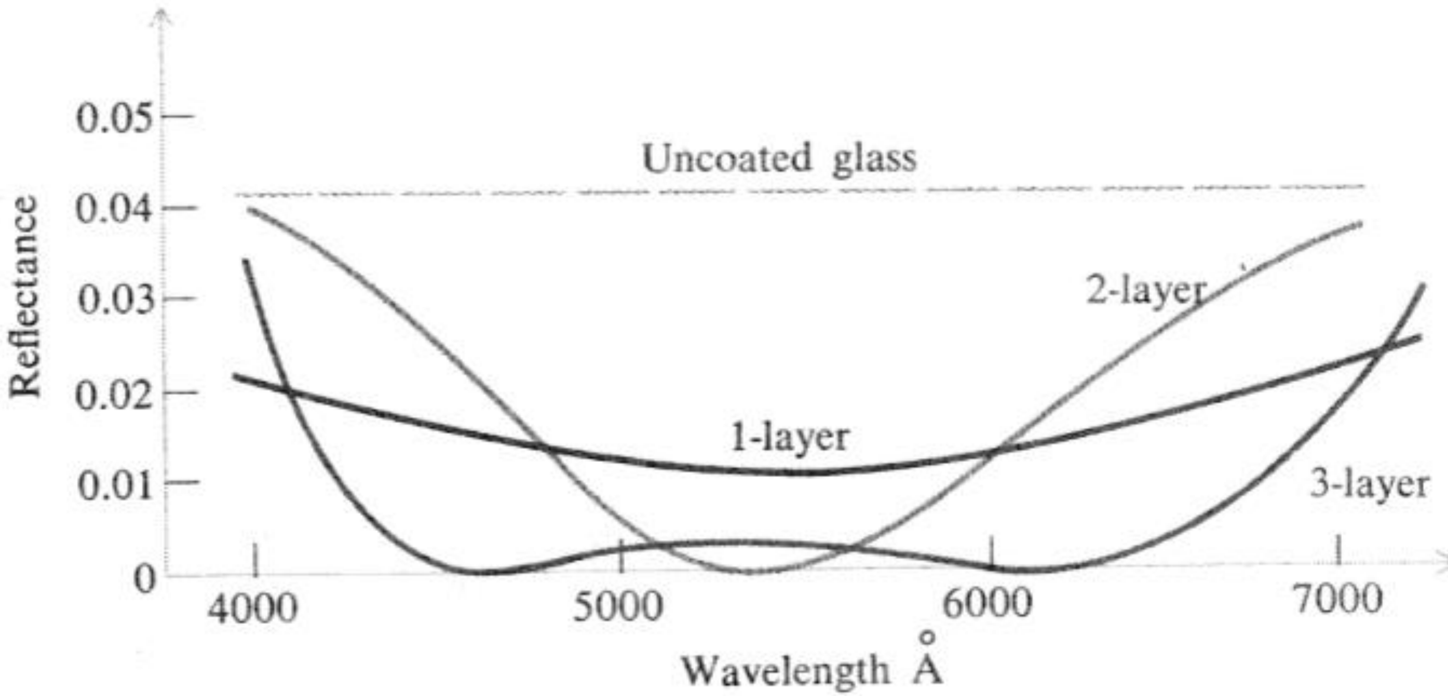
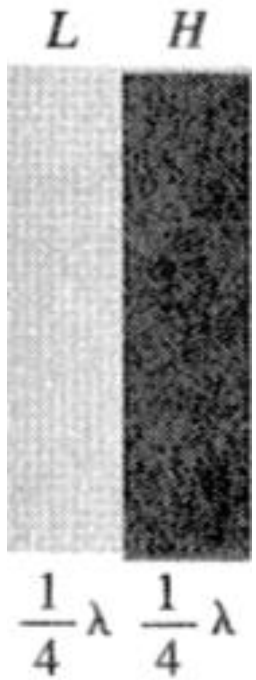


Figure 4.8. Curves of reflectance versus wavelength of antireflecting films.

Antireflecting with L/H-index films

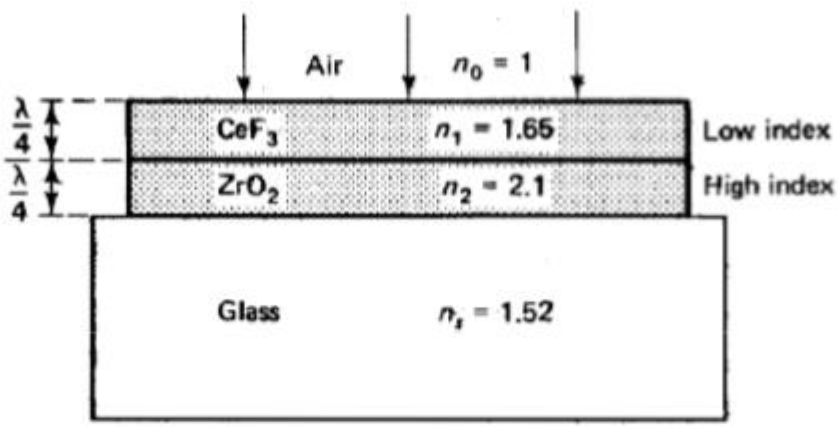


$$M=\begin{bmatrix}0 & \frac{-i}{n_L}\\ -in_L & 0\end{bmatrix}\begin{bmatrix}0 & \frac{-i}{n_H}\\ -in_H & 0\end{bmatrix}=\begin{bmatrix}\frac{-n_H}{n_L} & 0\\ 0 & \frac{-n_L}{n_H}\end{bmatrix}$$

$$r=\frac{An_0+Bn_Tn_0-C-Dn_T}{An_0+Bn_Tn_0+C+Dn_T}\quad \text{reflection coefficient}$$

$$\text{reflectance}\quad R=|r|^2$$

$$r=\frac{(\frac{-n_H}{n_L})n_0-(\frac{-n_L}{n_H})n_T}{(\frac{-n_H}{n_L})n_0+(\frac{-n_L}{n_H})n_T}=\frac{n_H^2n_0-n_L^2n_T}{n_H^2n_0+n_L^2n_T}$$

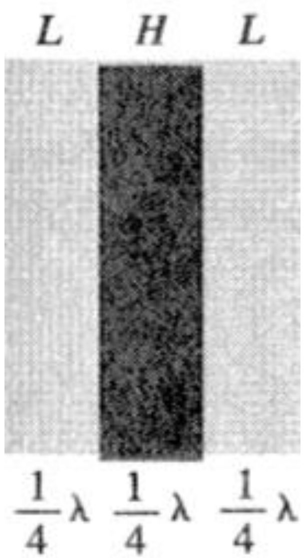


$$R=|r|^2=\left(\frac{n_H^2n_0-n_L^2n_T}{n_H^2n_0+n_L^2n_T}\right)^2$$

$$R=0\quad \text{if}\quad n_H^2n_0-n_L^2n_T=0\rightarrow\frac{n_H}{n_L}=\sqrt{\frac{n_T}{n_0}}$$

Antireflecting with L/H/L-index films

$$M = \begin{bmatrix} 0 & \frac{-i}{n_1} \\ -in_1 & 0 \end{bmatrix} \begin{bmatrix} 0 & \frac{-i}{n_H} \\ -in_H & 0 \end{bmatrix} \begin{bmatrix} 0 & \frac{-i}{n_2} \\ -in_2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-i}{n_1} \\ -in_1 & 0 \end{bmatrix} \begin{bmatrix} \frac{-n_2}{n_H} & 0 \\ 0 & \frac{-n_H}{n_2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{in_H}{n_1 n_2} \\ \frac{in_1 n_2}{n_H} & 0 \end{bmatrix}$$



$$r = \frac{(\frac{in_H}{n_1 n_2})n_T n_0 - (\frac{in_1 n_2}{n_H})}{(\frac{in_H}{n_1 n_2})n_T n_0 + (\frac{in_1 n_2}{n_H})} = \frac{n_H^2 n_T n_0 - n_1^2 n_2^2}{n_H^2 n_T n_0 + n_1^2 n_2^2}$$

$$R = |r|^2 = \left(\frac{n_H^2 n_T n_0 - n_1^2 n_2^2}{n_H^2 n_T n_0 + n_1^2 n_2^2} \right)^2$$

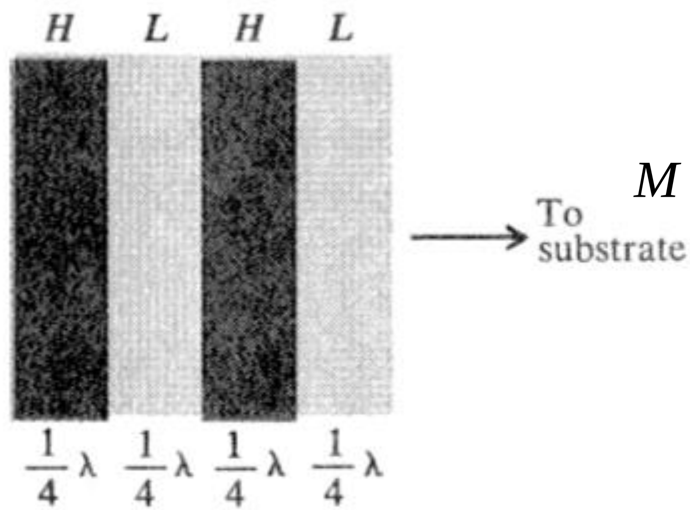
$$R = 0 \quad \text{if} \quad n_H^2 n_T n_0 - n_1^2 n_2^2 = 0 \rightarrow \frac{n_{L1} n_{L2}}{n_H} = \sqrt{n_0 n_T}$$

High-reflectance films

A high-reflectance film is made by reversing the order of deposition of the low - and high refractive index layers compared to an anti-reflecting film .

R can be high by using multilayer films of high/low/…/high/low index.

$$\begin{bmatrix} 0 & \frac{-i}{n_L} \\ -in_L & 0 \end{bmatrix}\begin{bmatrix} 0 & \frac{-i}{n_H} \\ -in_H & 0 \end{bmatrix} = \begin{bmatrix} \frac{-n_H}{n_L} & 0 \\ 0 & \frac{-n_L}{n_H} \end{bmatrix}$$



$$M = \begin{bmatrix} \frac{-n_H}{n_L} & 0 \\ 0 & \frac{-n_L}{n_H} \end{bmatrix}^N = \begin{bmatrix} (\frac{-n_H}{n_L})^N & 0 \\ 0 & (\frac{-n_L}{n_H})^N \end{bmatrix}$$

Figure 4.9. Multilayer stack for producing high reflectance. The stack consists of alternate quarter-wave layers of high and low index material. (Note: λ is the wavelength in the material.)

High-reflectance films

$$R = |r|^2 = \left[\frac{(\frac{-n_H}{n_L})^N - (\frac{-n_L}{n_H})^N}{(\frac{-n_H}{n_L})^N + (\frac{-n_L}{n_H})^N} \right]^2 = \left[\frac{(\frac{n_H}{n_L})^{2N} - 1}{(\frac{n_H}{n_L})^{2N} + 1} \right]^2$$

$R \to 1$ for large N

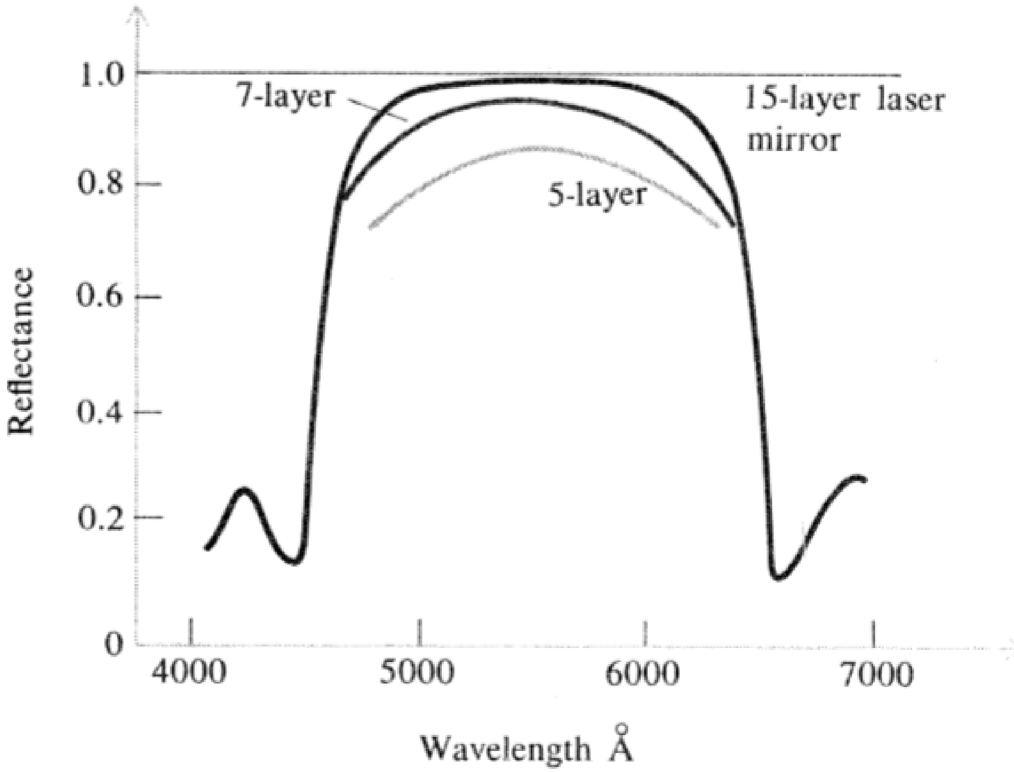


Figure 4.10. Reflectance curves for some multilayer high-reflectance films.

The reflectance approaches unity for large *N*.

Example:

- An 8-layer stack (N=4) of ZnS (n_H=2.3) and MgF_2 (n_L=1.35) gives a reflectance of ~0.97, which is higher than the reflectance of pure Ag in the visible region of the spectrum.
- A 30-layer stack results in a reflectance of better than 0.999.

Fabry-Perot Interference Filter

A Fabry-Perot type of filter (*Fabry-Perot etalon with a very spacing*) consists of a layer of dielectric having a **thickness of λ/2** (for some wavelength λ_0) and bounded on both sides by partially reflecting surfaces.

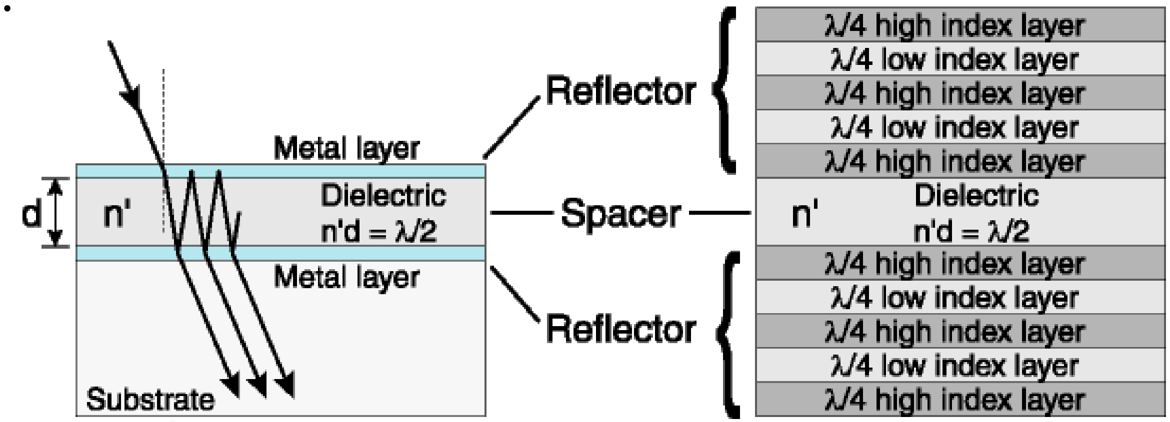
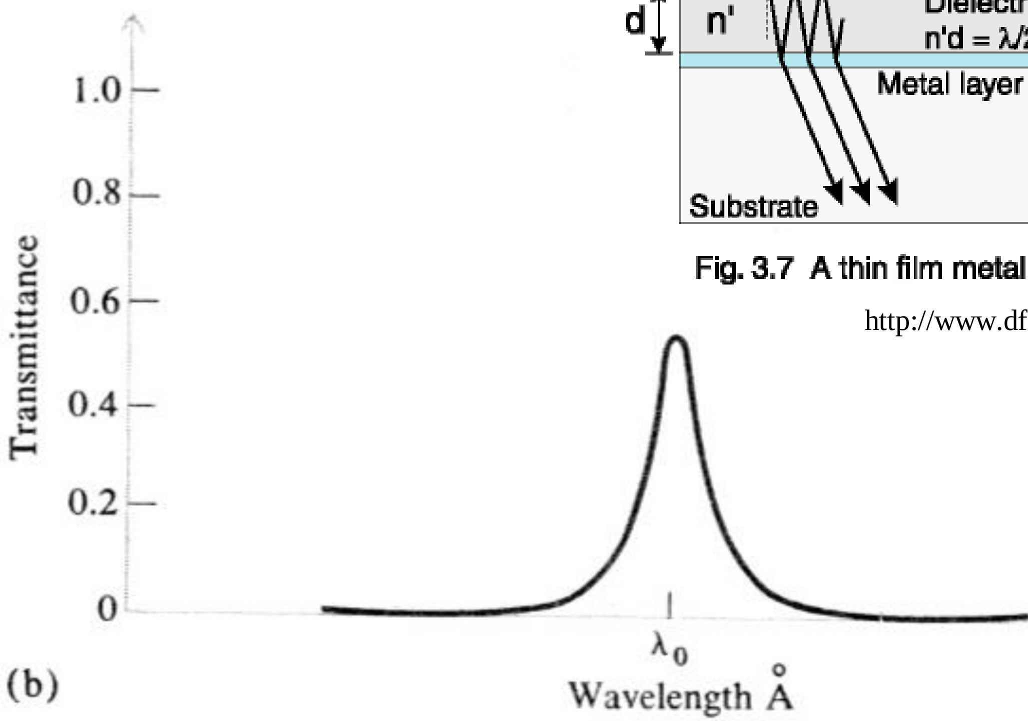


Fig. 3.7 A thin film metal interference filter and an all dielectric interference filter.

http://www.dfisica.ubi.pt/~hgil/fotometria/HandBook/ch03.html



(b)

(a)

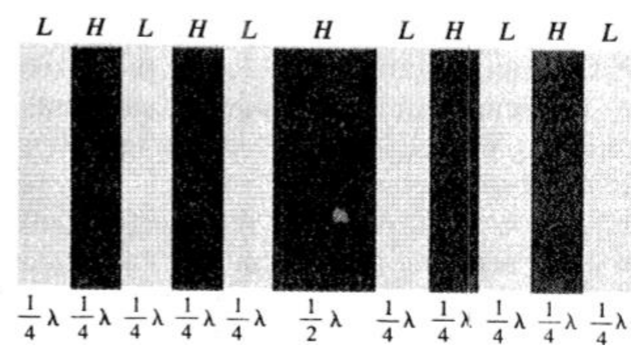
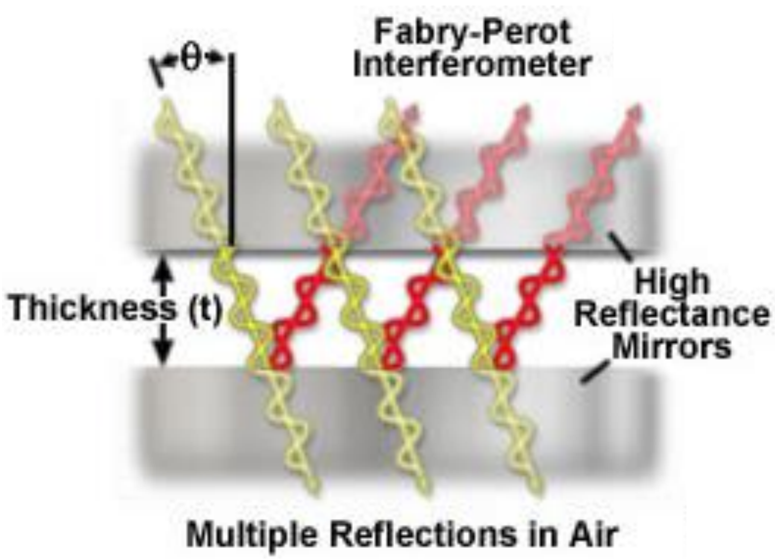
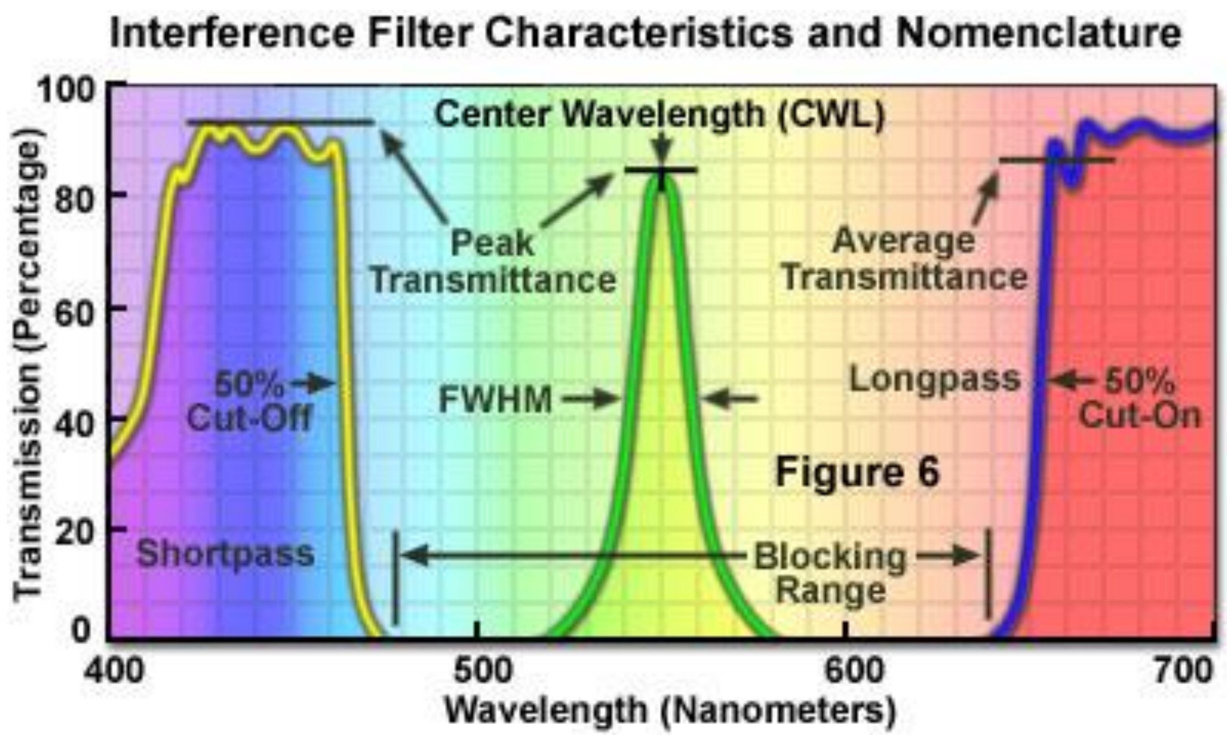


Figure 4.11. Multilayer Fabry-Perot interference filter.

Fabry-Perot Interference Filter

A Fabry-Perot type of filter (*Fabry-Perot etalon with a very spacing*) consists of a layer of dielectric having a **thickness of λ/2** (for some wavelength λ_0) and bounded on both sides by partially reflecting surfaces.





http://www.olympusmicro.com/primer/techniques/fluorescence/interferencefilterintro.html

Homework set #3.

Solve Problems 4.1, 4.2, 4.3, 4.4, 4.5, 4.7, 4.8, 4.10

Due date: 2016. 10. 20 (목)