Topics in Ship Structural Design (Hull Buckling and Ultimate Strength)

Lecture 7 Elastic Buckling of Stiffened Panels

Reference : Ship Structural Design Ch.13

NAOE

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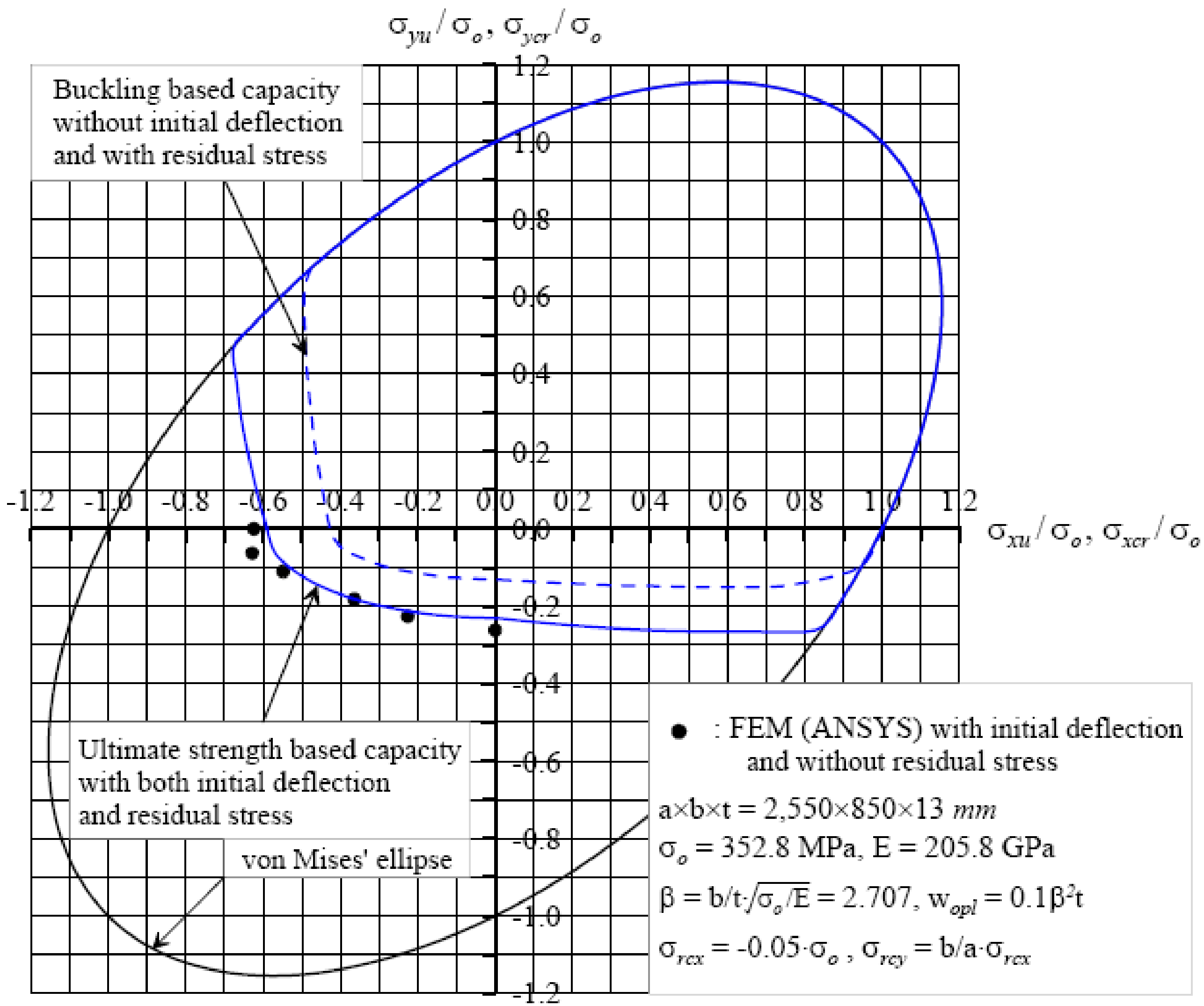
Comparison between Buckling and Ultimate Strength of plating

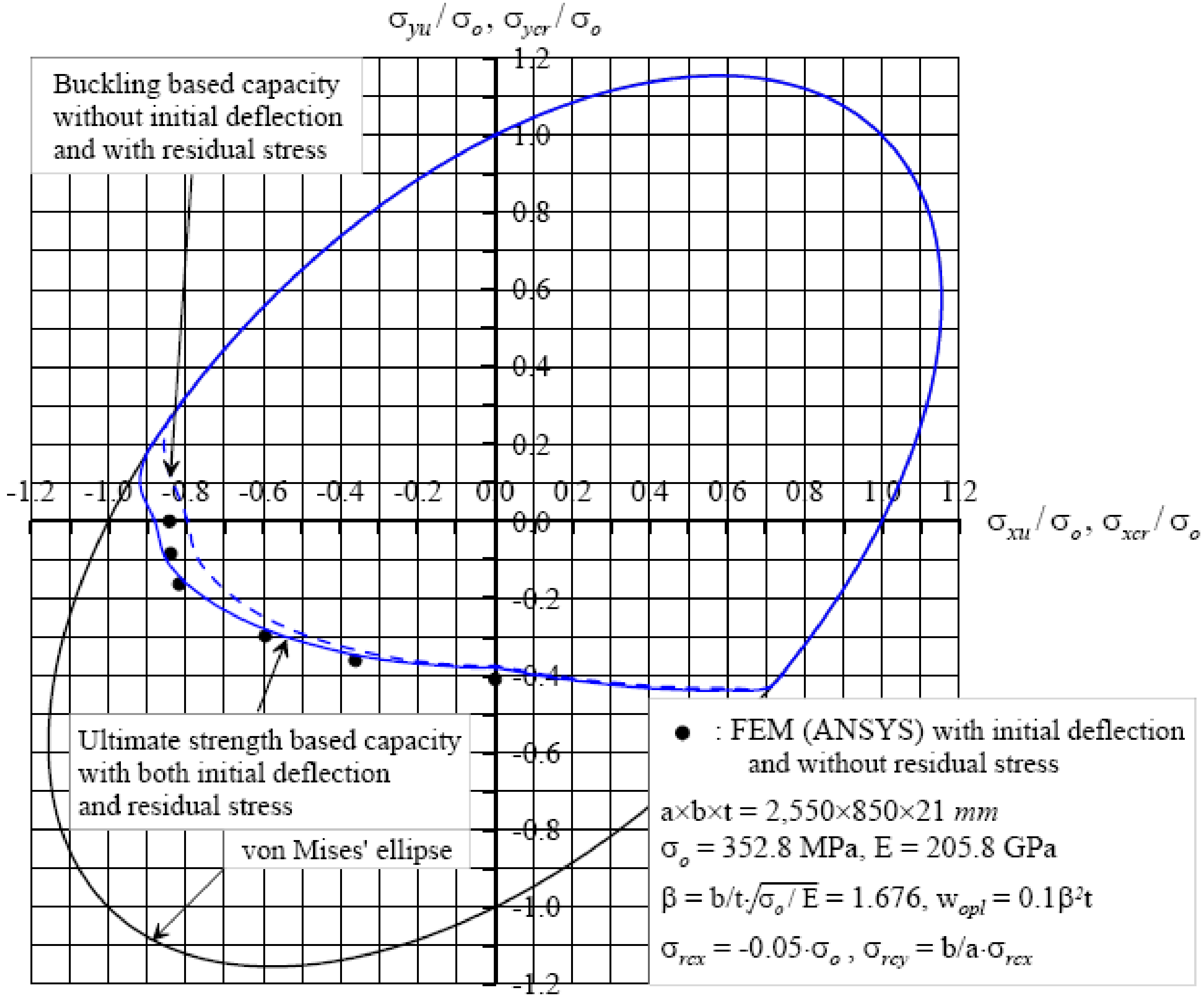
❖ Plate capacity interactions between biaxial compression

- FEA, Buckling, Ultimate strength, a / b = 3, t = **13**mm



Comparison between Buckling and Ultimate Strength of plating

- Plate capacity interactions between biaxial compression
  - FEA, Buckling, Ultimate strength, a / b = 3, t = **21**mm



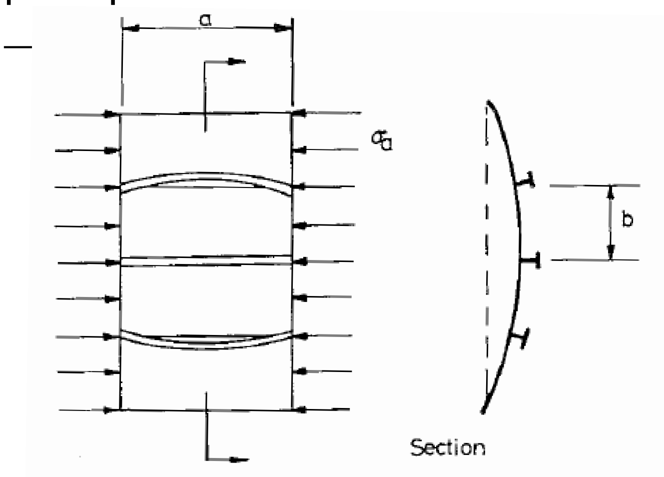
Two types of buckling

- Stiffened panels can buckle in essentially two different ways: overall buckling and local buckling.
- Overall buckling : stiffeners buckles along with the plating
- Local buckling :

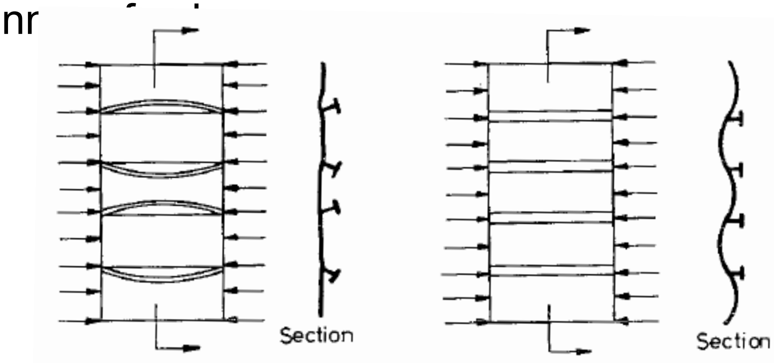
  1) stiffeners buckle prematurely because of inadequate rigidity

  2) plate panels buckle between the stiffeners → shedding extra load into stiffeners

  – ie manr



Overall buckling



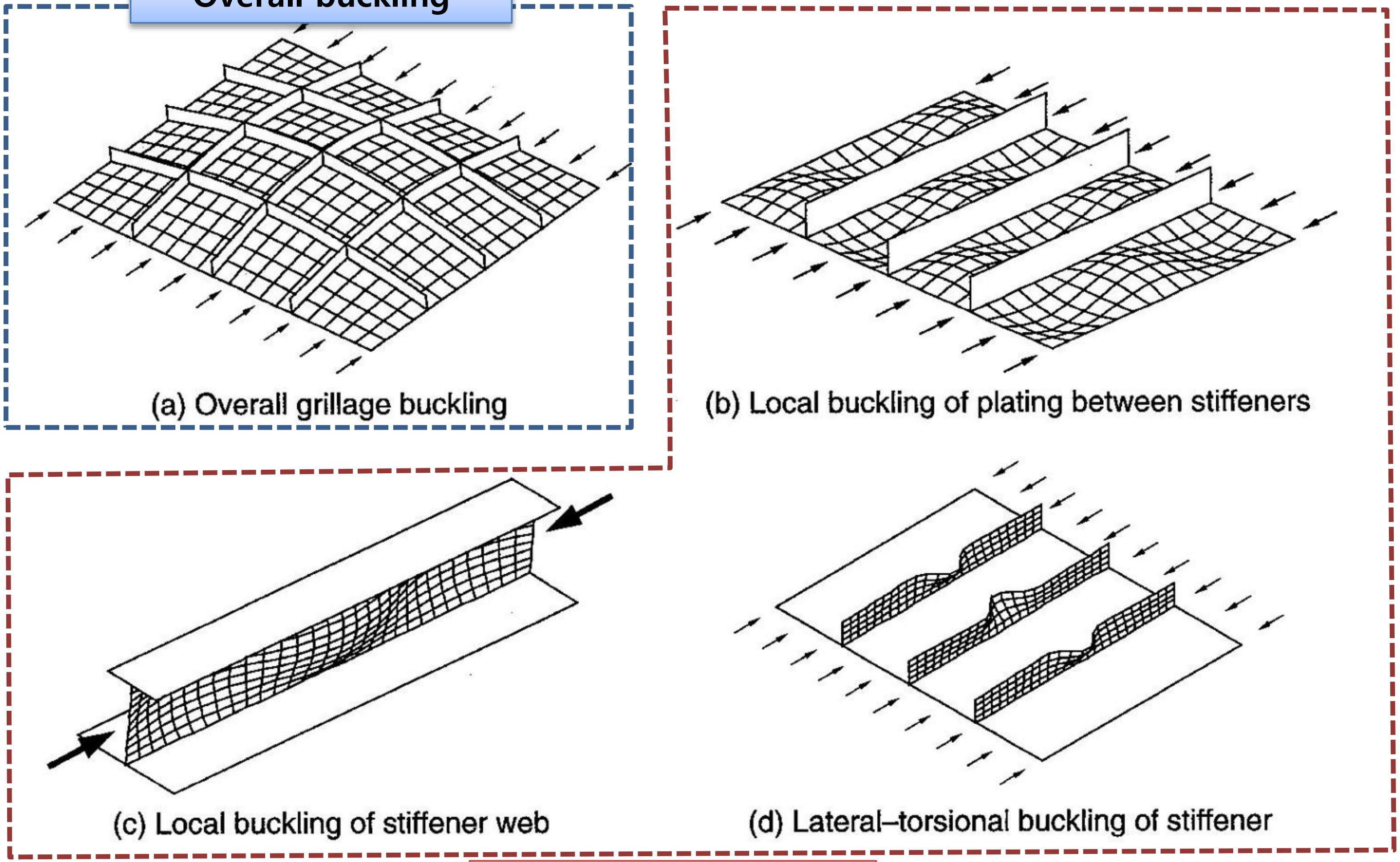
Torsional buckling (tripping) of stiffeners

Plate buckling

Local buckling

- For most ship panels, the buckling is inelastic. "failure" instead of "buckling"
- Nevertheless, elastic buckling analysis gives a good indication of the likely modes of failure and a foundation for the more complex question of the inelastic buckling and ultimate strengths of stiffened panels.

Overall buckling



Local buckling

Schematics of various types of stiffed panel buckling

General

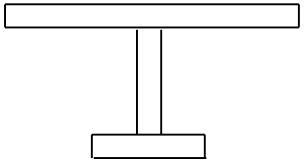
- **STEP 1 :** Minimum flexural rigidity of stiffeners to avoid overall buckling

→ The minimum value of γ_x to ensure stiffener buckling does **not** precede plate buckling

γ_x :: flexural rigidity of stiffener + plating /the flexural rigidity of the plating

$$\gamma_x = \frac{EI_x}{Db} = \frac{12(1-v^2)I_x}{bt^3}$$

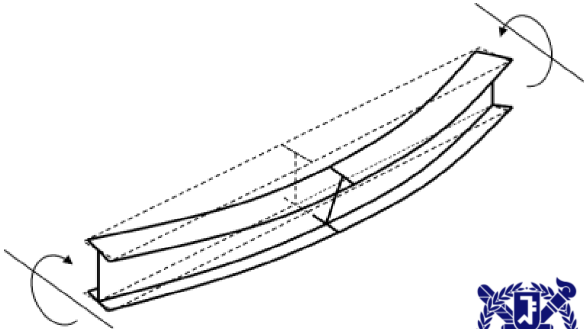
- **STEP 2 :** Calculation of Column Buckling stress



→ Buckling of a column composed of stiffener and plating of effective breadth

- **STEP 3 :** Calculation of stiffener-tripping stress

→ Flexural-torsional buckling of a stiffener with rotational restraint by plating



Idealization of continuous stiffened panel

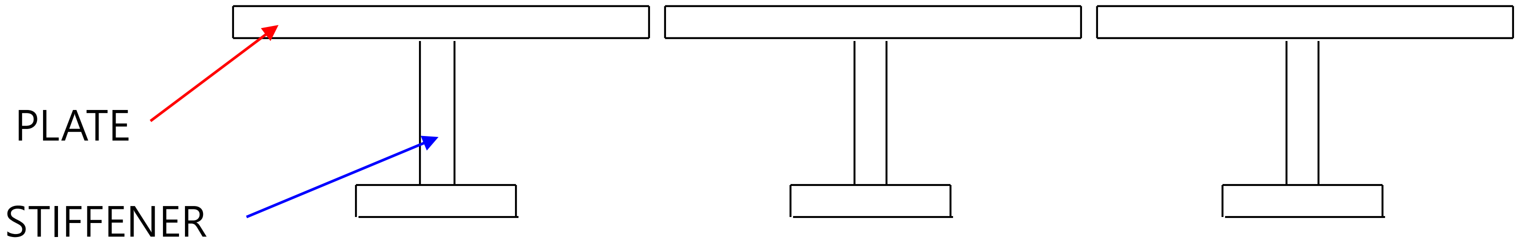


Plate-Stiffener (BEAM) Combination model

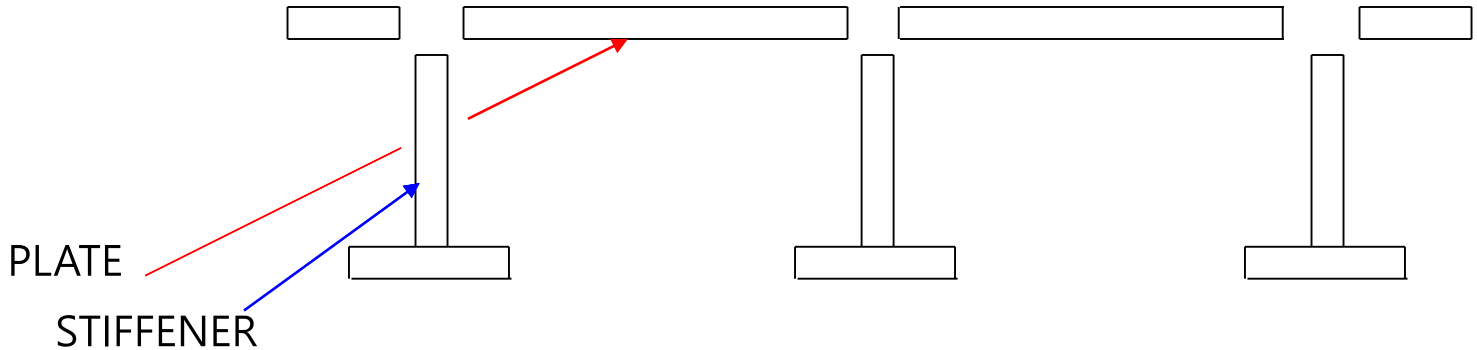


Plate-Stiffener Separation model

Orthotropic plate model

General

- Analytical methods of solving buckling problems include two principal types:
  discrete beam : more versatile and accurate, but more computation
  orthotropic plate : only when the stiffeners are very closely spaced.

- In this lecture, it is assumed that
  - ✓ the edges of the panel are simply supported
  - ✓ individual elements of the stiffeners are not subject to instability

- First principle in regard to stiffener
  - ✓ stiffeners should be sufficiently rigid and stable (∵ stiffener buckling = overall buckling and the plating is left with almost no lateral rigidity
  - ✓ Substantial lateral load ship panels must carry requires sufficiently large stiffness and rigidity.

- It is best to first perform an elastic buckling analysis because:
  - ✓ relatively simple, consisting mostly of explicit formulas
  - ✓ for slender panels, it may be one of the governing failure modes
  - ✓ it indicates whether an inelastic analysis is required

Minimum flexural rigidity to avoid overall buckling

- Some parameters
  - ✓ the ratio of the flexural rigidity of the combined section to the flexural rigidity of the plating

$$\gamma_x = \frac{EI_x}{Db} = \frac{12(1-v^2)I_x}{bt^3}$$

  - ✓ the panel aspect ratio

$$\Pi = \frac{L}{B} = \frac{a}{B}$$

  - ✓ the area ratio

$$\delta_x = \frac{A_x}{bt}$$

Minimum flexural rigidity to avoid overall buckling

- The minimum value of γ_x (the less one of the two values) to ensure stiffener buckling does **not** precede plate buckling $\gamma_x > \gamma_{min}$

(1) panel with one central longitudinal stiffener (whichever is less)

$$\gamma_x = 48.8 + 112\delta_x(1 + 0.5\delta_x)$$

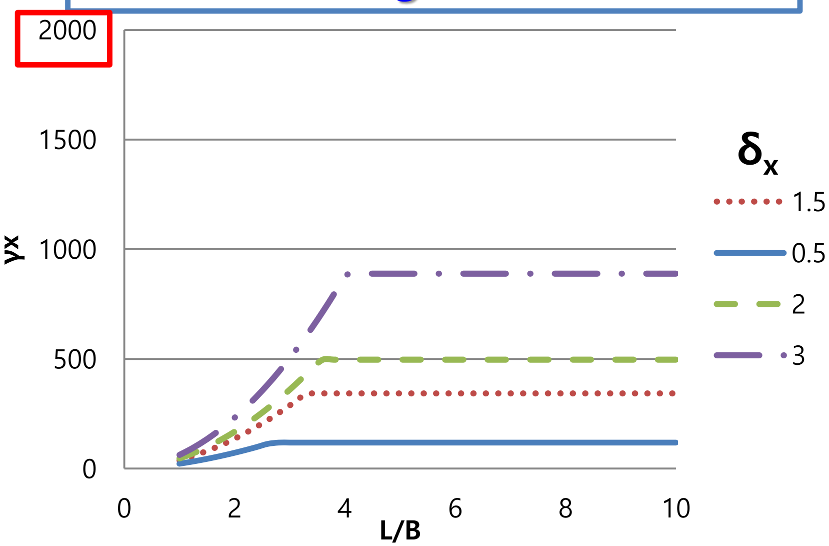
$$\gamma_x = 22.8\Pi + (2.5 + 16\delta_x)\Pi^2 - 10.8\sqrt{\Pi}$$

(2) panel with two equally spaced longitudinal stiffeners(whichever is less)

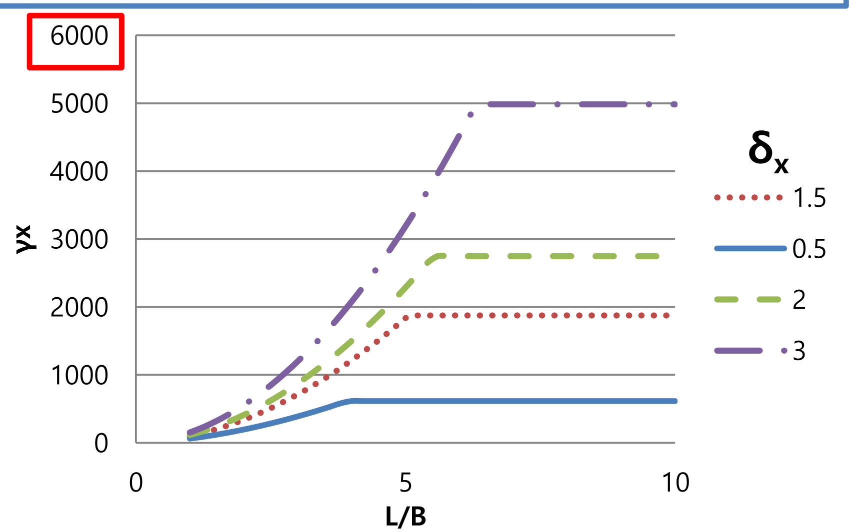
$$\gamma_x = 43.5\sqrt{\Pi^3} + 36\Pi^2\delta_x$$

$$\gamma_x = 228 + 610\delta_x + 325\delta_x^2$$

one central longitudinal stiffener



two equally spaced longitudinal stiffener



Minimum flexural rigidity to avoid overall buckling

- A more general solution, valid for any number of stiffeners, has been presented by Klitchiff

$$\gamma_x = \delta_x (1 + N_B^2 \Pi^2)^2 + \frac{4}{\pi} \Pi (1 + N_B^2 \Pi^2) \sqrt{2 + N_B^2 \Pi^2}$$

where *N_B=number of panels=1+number of longitudinal stiffeners*

Homework 6-1 Plot Klitchiff's curve versus L/B for different N_B and δ_x and compare with the previous two curves.

Calculation of overall buckling stress

- An alternative approach is to calculate the overall buckling stress $(\sigma_a)_{cr}$ and compare it to the plate buckling stress σ_0.

$$(\sigma_a)_{cr} \geq \sigma_0 \qquad \sigma_0 = 3.62E\left(\frac{t}{b}\right)^2$$

- In short panels, the equivalent slenderness ratio of each column is the actual slenderness of the section or in terms of nondimensional parameter.

$$\left(\frac{L}{\rho}\right)_{eq} = \frac{a}{\rho} = \frac{a}{\sqrt{I_x/(A_x+bt)}} \qquad \left(\frac{L}{\rho}\right)_{eq} = \frac{a}{t}\sqrt{\frac{12(1-v^2)(1+\delta_x)}{\gamma_x}}$$

- In long panels, the stiffeners receive some lateral restraint from the sides of the panel→ bukling in more than one half wave.

the equivalent slenderness ratio < the value given by the former formula

$$\left(\frac{L}{\rho}\right)_{eq} = C_\Pi \frac{a}{\rho} = \frac{C_\Pi a}{\sqrt{I_x/(A_x+bt)}}$$

where Cπ is given by whichever is less

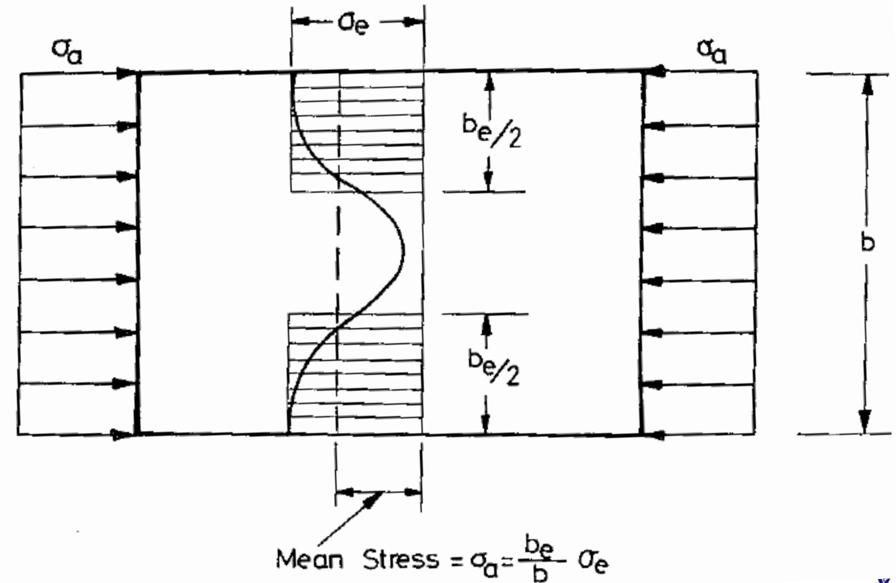
$$C_\Pi = \frac{1}{\Pi}\sqrt{\frac{\gamma_x}{2(1+\sqrt{1+\gamma_x})}} \qquad C_\Pi = 1 \qquad \boxed{\gamma_x = \frac{EI_x}{Db} = \frac{12(1-v^2)I_x}{bt^3}}$$

Calculation of overall buckling stress

- Then $$(\sigma_a)_{cr} = \frac{\pi^2 E}{(L/\rho)_{eq}^2}$$
- Using the adjective "slender" to describe a panel in which the overall buckling stress calculated from elastic theory is less than the yield stress.
$$\frac{\pi^2 E}{(L/\rho)_{eq}^2} < \sigma_Y$$
- Normally, plate buckling precedes overall buckling → When overall buckling occurs the plate flange of the stiffener will not be fully effective over the width b.(due to out-of-plane deformation, caused by buckling)

- The buckled center portion is discounted completely and the original b is replaced by b_e

$$\sigma_e = \frac{b}{b_e}\sigma_a$$



Calculation of overall buckling stress

- The effective width is taken to be the width at which the equivalent plate would buckle at an applied stress of σ_e

$$\sigma_e = k\frac{\pi^2 D}{b_e^2 t}$$

for the original plate

$$(\sigma_a)_{cr} = k\frac{\pi^2 D}{b^2 t}$$

- if it is assumed that *k* is the same in both cases then

$$\frac{b_e}{b} = \sqrt{\frac{(\sigma_a)_{cr}}{\sigma_e}}$$

- For long plate (a/b>1)

$$\boxed{(\sigma_a)_{cr} = 4\frac{\pi^2 D}{b^2 t}}$$

$$\frac{b_e}{b} = 1.9\frac{t}{b}\sqrt{\frac{E}{\sigma_e}}$$

- The effective width would reach its smallest possible value when σ_a reached yield σ_{yield}.

$$\left(\frac{b_e}{b}\right)_{\min} = \frac{1.9}{\beta}$$

$$\boxed{\beta = \frac{b}{t}\sqrt{\frac{\sigma_Y}{E}}}$$

13.1 Longitudinally Stiffened Panels

Calculation of overall buckling stress

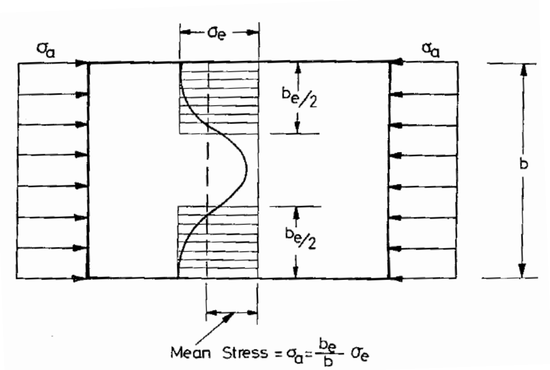
- The critical value is given by $$(\sigma_e)_{cr} = \frac{\pi^2 E}{(L/\rho_e)^2_{eq}}$$
- The axial stress in the stiffener is larger than the external applied stress.

  $(\sigma_a)_{cr}$ axial stress corresponding to ***$(\sigma_e)_{cr}$***

$$\sigma_a(bt + A_x) = \sigma_e(b_e t + A_x) \qquad (\sigma_a)_{cr} = \left(\frac{b_{et} + A_x}{bt + A_x}\right)\frac{\pi^2 E}{(L/\rho_e)^2_{eq}}$$

- A suitable procedure

  1. Assume some initial value of b_e
  2. Calculate δ_{xe} and I_{xe}, and then evaluate $(L/\rho_e)_{eq}$
  3. Calculate $(\sigma_e)_{cr}$ ⇐ $\boxed{(\sigma_e)_{cr} = \frac{\pi^2 E}{(L/\rho_e)^2_{eq}}}$
  4. Using this value, recalculate b_e from ⇐ $\boxed{\frac{b_e}{b} = 1.9\frac{t}{b}\sqrt{\frac{E}{\sigma_e}}}$
  5. Repeat from step 2 until b_e has converged
  6. Calculate $(\sigma_a)_{cr}$ ⇐ $\boxed{(\sigma_a)_{cr} = \left(\frac{b_{et} + A_x}{bt + A_x}\right)\frac{\pi^2 E}{(L/\rho_e)^2_{eq}}}$



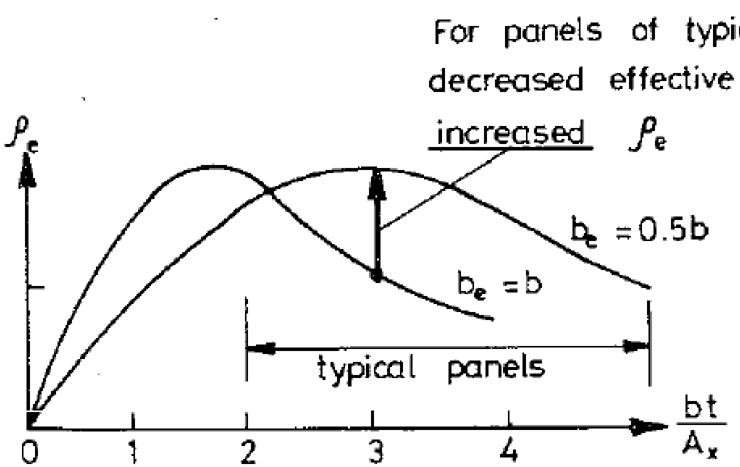
- a single nonlinear equation for b_e

$$\frac{\pi d}{1.9at} b_e \sqrt{\frac{A_w}{12}(A_w + 4A_f) + \left(\frac{A_w}{3} + A_f\right) b_e t} = A_w + A_f + b_e t$$

Calculation of overall buckling stress

- $(\sigma_e)_{cr}$ depends on ρ_e depends on depends on b_e.
- When $bt > 2A_x$ (usual case), decrease of b_e → increase ρ_e → increase in $(\sigma_a)_{cr}$
- The lowest of $(\sigma_a)_{cr}$ corresponds to $b_e = b$, when plating in fully effective.

$$(\sigma_e)_{cr} = \frac{\pi^2 E}{(L/\rho_e)^2_{eq}}$$



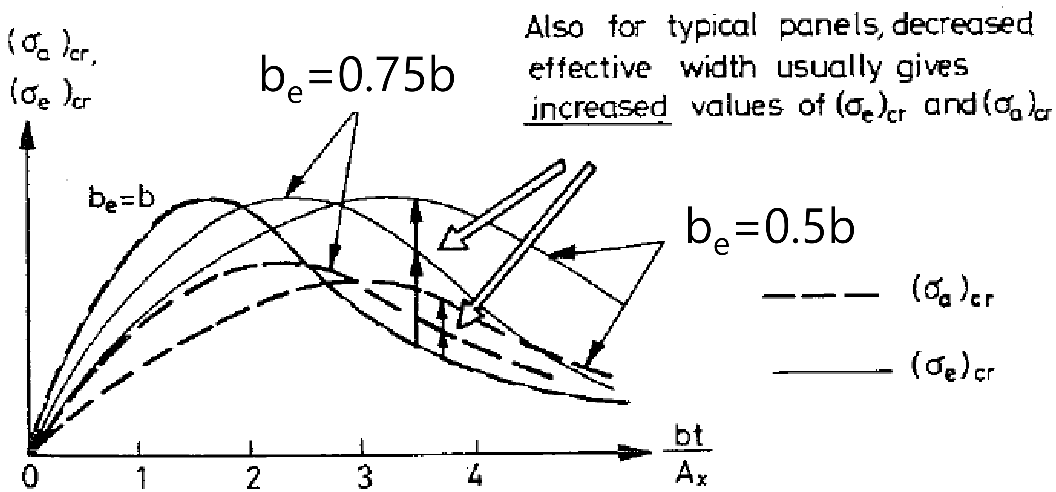
Large stiffener

(a)

Small stiffener

$$(\sigma_e)_{cr} = \frac{\pi^2 E}{(L/\rho_e)^2_{eq}}$$

$$(\sigma_a)_{cr} = \left(\frac{b_{et} + A_x}{bt + A_x}\right)\frac{\pi^2 E}{(L/\rho_e)^2_{eq}}$$



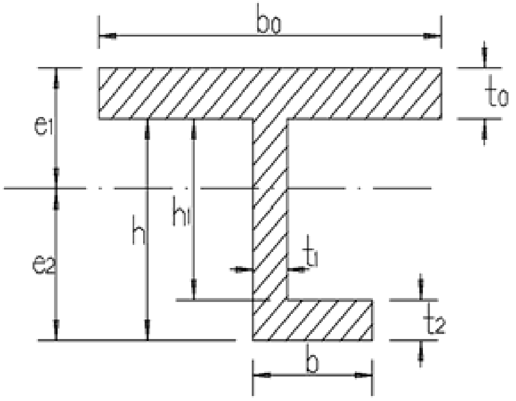
Large stiffener

(b)

Small stiffener

Calculation of overall buckling stress

Homework 6-2 illustrates the previous graph using the following formula



$$A = b_0 t_0 + b t_2 + h_1 t_1$$

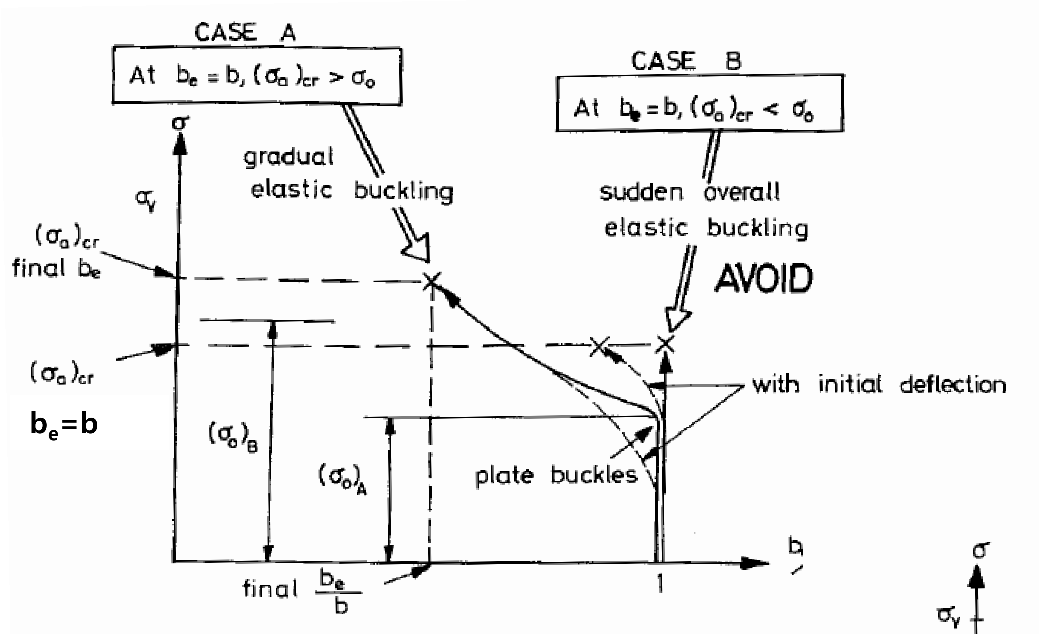
$$I = \frac{b_0 t_0^3}{3} + \frac{b h^3}{3} - \frac{(b - t_1) h_1^3}{3} - A(e_1 - t_0)^2$$

$$e_1 = t_0 + \frac{b h^2 - (b - t_1) h_1^2 - b_0 t_0^2}{2A}$$

13.1 Longitudinally Stiffened Panels

Calculation of overall buckling stress

- Possible modes of elastic buckling of stiffened panels



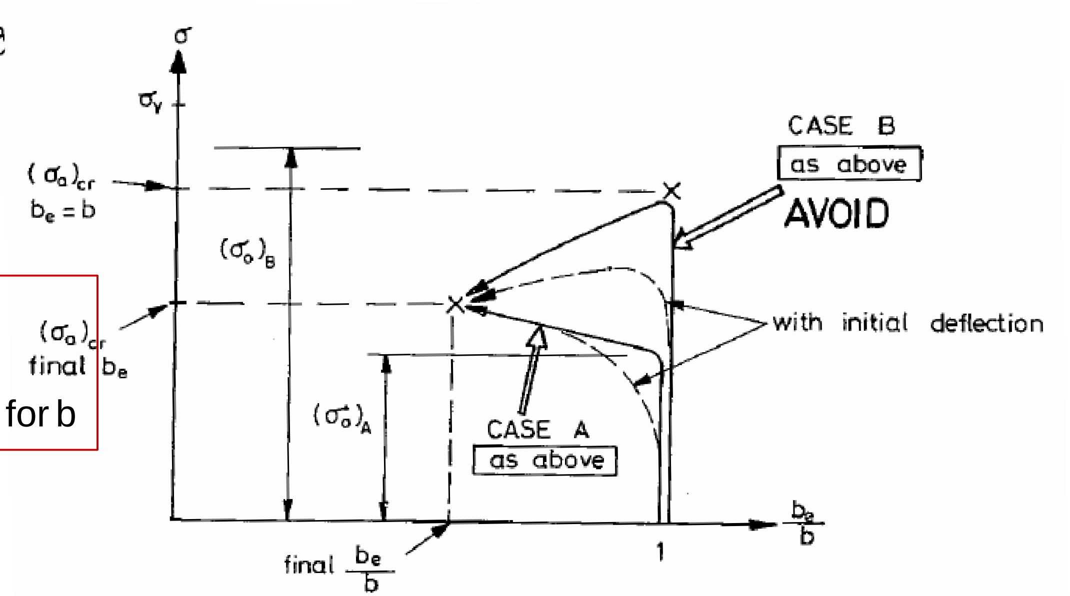
(a) SMALL STIFFENERS $\left(A_X < \frac{bt}{2}\text{, say}\right)$

For small stiffener

$(\sigma_a)_{cr}$ for b_e > $(\sigma_a)_{cr}$ for b

For large stiffener

$(\sigma_a)_{cr}$ for b_e < $(\sigma_a)_{cr}$ for b

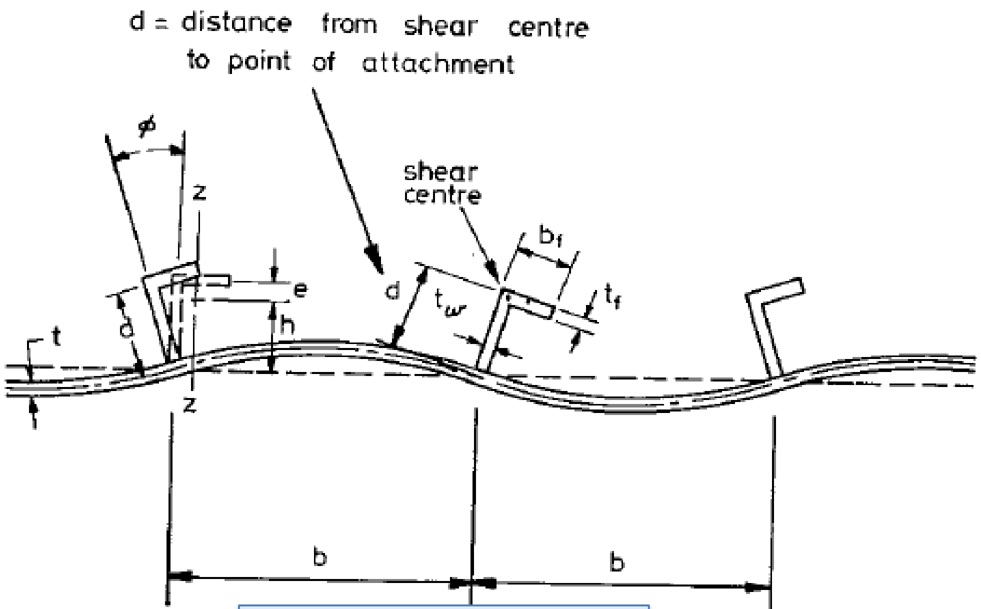


(b) LARGE STIFFENERS $\left(A_X > \frac{bt}{2}\text{, say}\right)$

13.1 Longitudinally Stiffened Panels

Local buckling of stiffener (tripping)

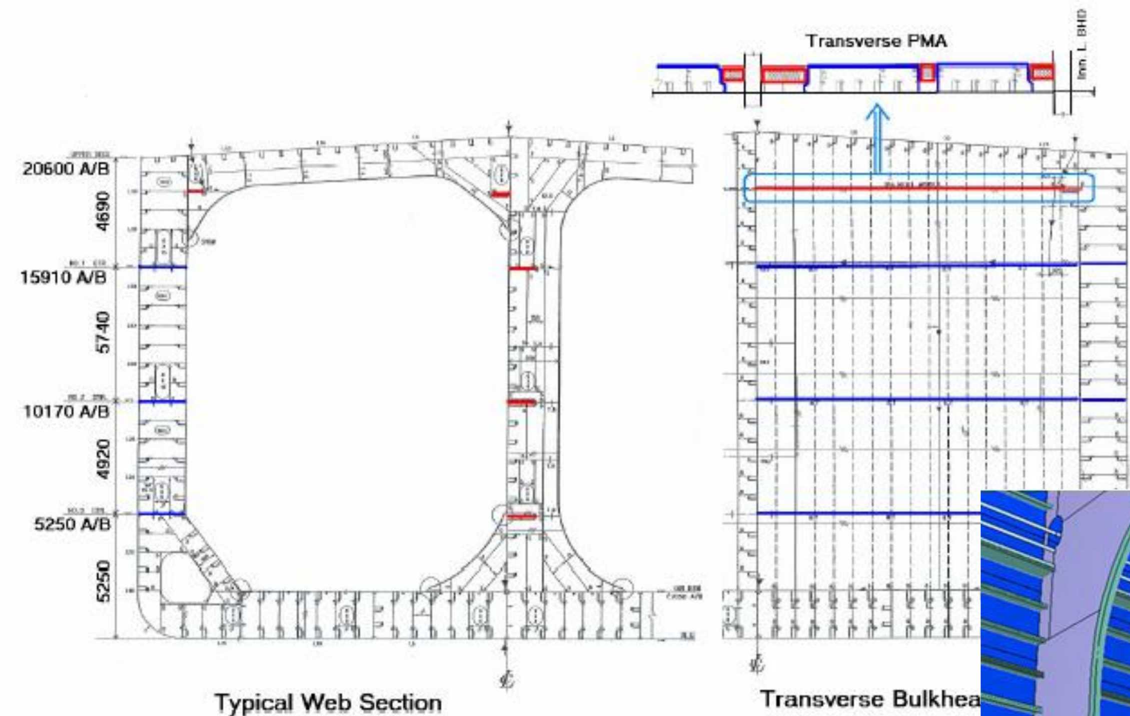
- A stiffener may buckle by twisting about its line of attachment to the plating, "tripping". The direction of the tripping alternates as shown.
- Tripping and plat buckling do interact but they can occur in either order.
- Tripping failure is regarded as collapse, the tripping leads no stiffening and overall buckling follows immediately. Elastic tripping is a quite sudden phenomenon → a most undesirable mode of buckling
- Open sections used in ship panels have relatively little torsional rigidity.
- Closed cross section : difficulties in fabrication, inspection and control of corrosion.



Stiffener tripping

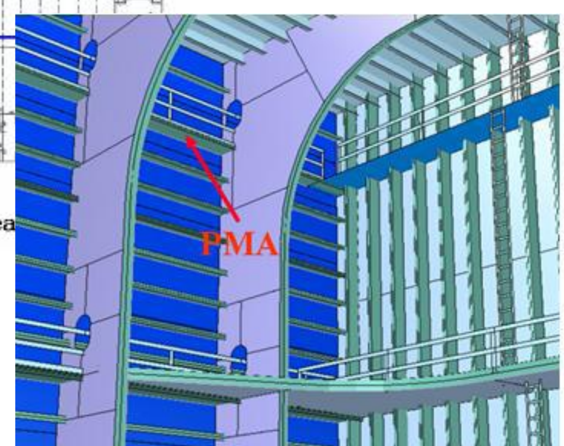
Permanent Means of Access (PMA)

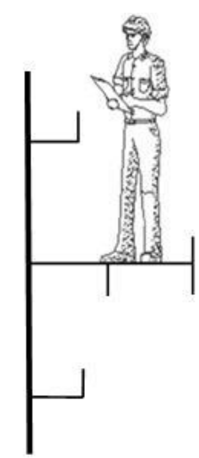
- IMO introduced PMA regulations for securing access to cargo holds and ballast tanks in oil tankers and bulk carriers.
- Overall and close-up inspections and thickness measurements of the critical hull structural parts.



Typical Web Section

Transverse Bulkhea





Ultimate Strength Assessment of PMA

❖ **Objective :** Ultimate strength assessment of PMA using nonlinear FE analysis
Establish evaluation procedure for CSR Tanker design

❖ **Research :**

- No specified rule for PMA structure in CSR
- Local support member scantling rule in CSR → over scantling
- Establish a ultimate strength assessment procedure.
- Propose a evaluation criteria based on CSR cargo hold analysis.

Rule for Local Support Members in CSR

SECTION 10- BUCKLING AND ULTIMATE STRENGTH
2.2 Plates and Local Support Members
2.2.1 Proportions of plate panels and local support members

(b) stiffener web plate

$$t_{w-net} \geq \frac{d_w}{C_w}\sqrt{\frac{\sigma_{yd}}{235}}$$

(c) flange/face plate

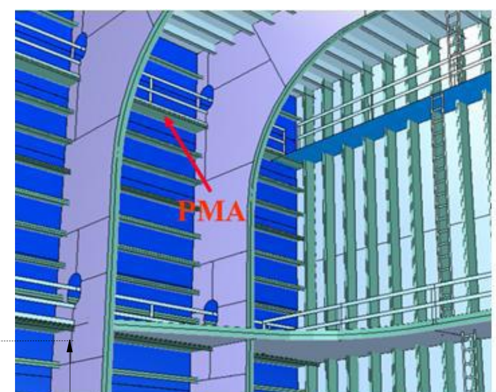
$$t_{f-net} \geq \frac{b_{f-out}}{C_f}\sqrt{\frac{\sigma_{yd}}{235}}$$

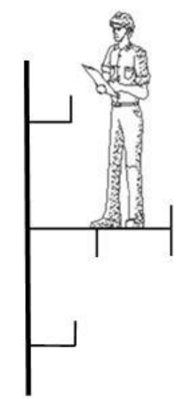
CSR req. : $h \times t_w + b_f \times t_f$

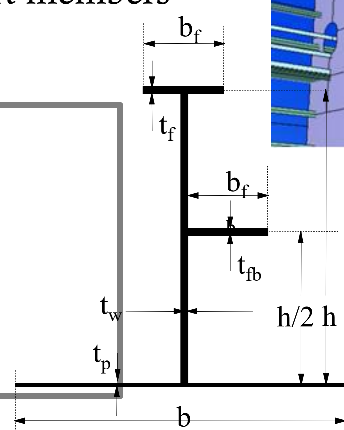
1150x20.5+288x16.5

Proposed :

1150x12.0+150x15.0

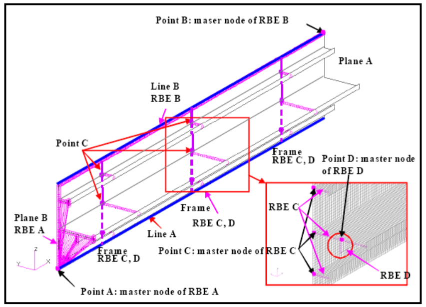




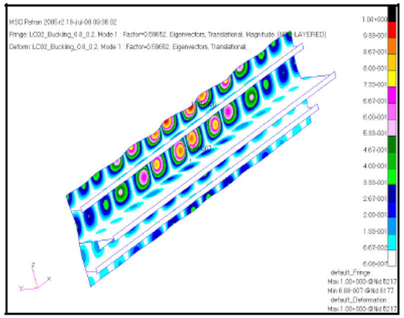


Ultimate Strength Assessment of PMA

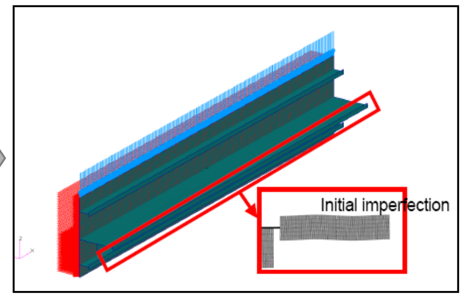
FE Modeling & Boundary Condition



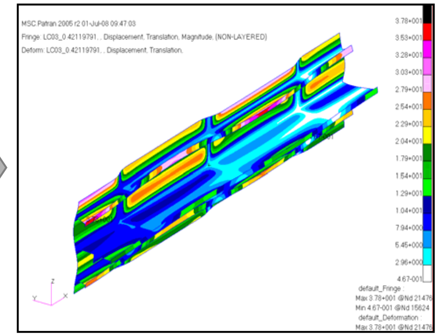
Linear Buckling Analysis



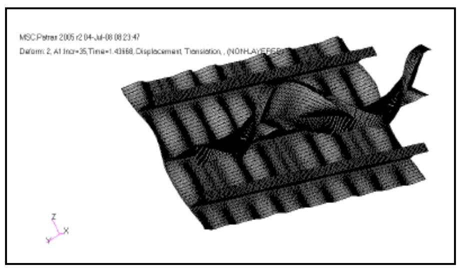
Initial Imperfection



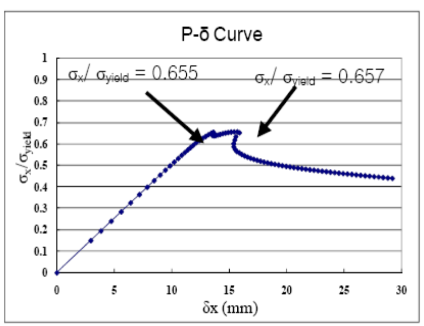
Nonlinear FE Analysis



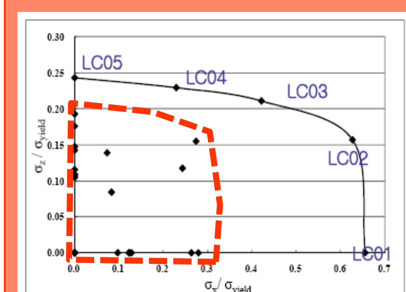
Check Failure Mode



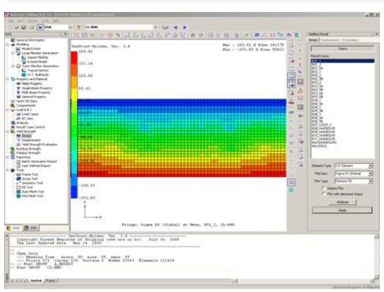
Ultimate Strength from P-δ curve



Compare Capacity Curve with Stress



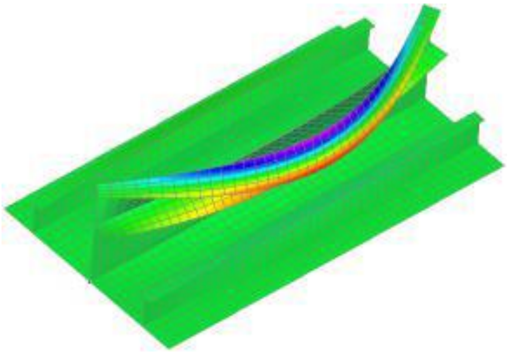
CRS Cargo Hold Analysis



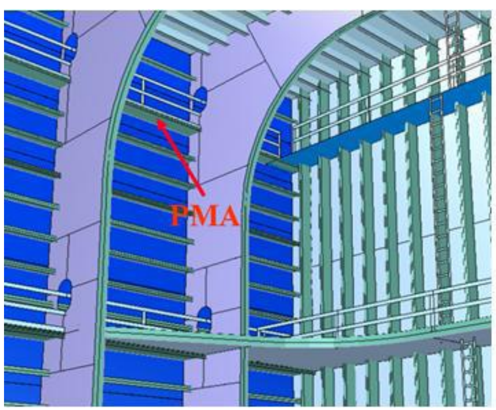
❖ **Result** : **Proposed scantling satisfies required strength.**

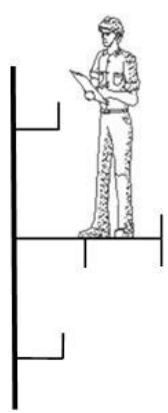
Elastic Lateral Torsional Buckling of PMA I

- **Objective :** **Establish analytical method to predict lateral torsional buckling of PMA**
- **Research :**
  - **Rayleigh – Riz method**
  - **Assumption of deflection and strain of PMA section**
  - **Total strain energy and work done during buckling**
  - **Solve an eigen-value problem**

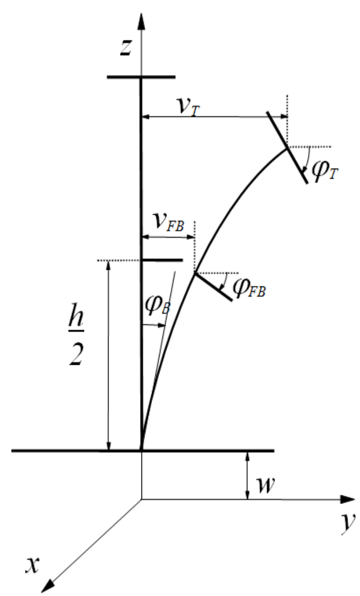


PMA Structure





Assumption of sectional displacement



$$u_w = (z_o - z)w_{,x} + a_o v_{T,x}$$

$$v_w = v(x,z)$$

$$w_w = w(x)$$

$$u_f = (z_o - h_w)w_{,x} + (a_o - y)v_{T,x}$$

$$v_f = v_T(x)$$

$$w_f = w - y\varphi_T$$

$$u_{fb} = (z_0 - \frac{h_w}{2})w_{,x} + (a_1 - y)v_{FB,x}$$

$$v_{fb} = v_{FB}(x)$$

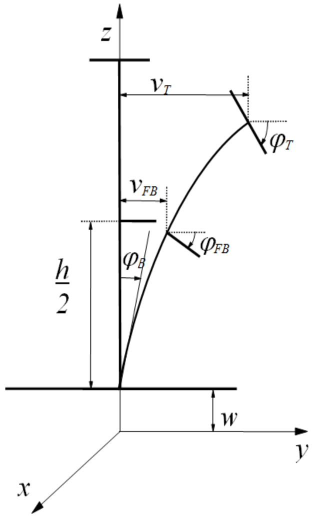
$$w_{fb} = w - y\varphi_{FB}(x)$$

Elastic Lateral Torsional Buckling of PMA II

Flexural Out-of-Plane Displacement of web plate

$$v_w = \varphi_B(x) f_1(z) + v_T(x) f_2(z) + \varphi_T(x) f_3(z) + v_{FB}(x) f_4(z) + \varphi_{FB}(x) f_5(z)$$

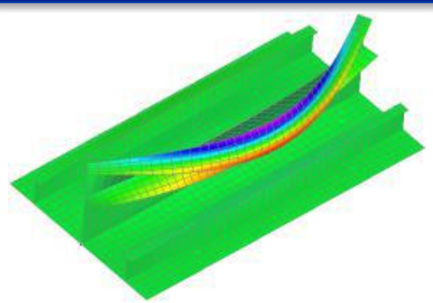
Compatibility constraints



$$\begin{cases} \varphi_T = (v_{w,z})_{z=h_w} \\ \varphi_B = (v_{w,z})_{z=0} \\ v_T = (v_w)_{z=h_w} \\ v_B = (v_w)_{z=0} = 0 \\ v_{FB} = (v_w)_{z=h_m} \\ \varphi_{FB} = (v_{w,z})_{z=h_m} \end{cases}$$

$$\begin{cases} f_1 = h_w\left[\dfrac{z}{h_w} - 6\left(\dfrac{z}{h_w}\right)^2 + 13\left(\dfrac{z}{h_w}\right)^3 - 12\left(\dfrac{z}{h_w}\right)^4 + 4\left(\dfrac{z}{h_w}\right)^5\right] \\ f_2 = 7\left(\dfrac{z}{h_w}\right)^2 - 34\left(\dfrac{z}{h_w}\right)^3 + 52\left(\dfrac{z}{h_w}\right)^4 - 24\left(\dfrac{z}{h_w}\right)^5 \\ f_3 = h\left[-\left(\dfrac{z}{h_w}\right)^2 + 5\left(\dfrac{z}{h_w}\right)^3 - 8\left(\dfrac{z}{h_w}\right)^4 + 4\left(\dfrac{z}{h_w}\right)^5\right] \\ f_4 = 16\left(\dfrac{z}{h_w}\right)^2 - 32\left(\dfrac{z}{h_w}\right)^3 + 16\left(\dfrac{z}{h_w}\right)^4 \\ f_5 = h\left[-8\left(\dfrac{z}{h_w}\right)^2 + 32\left(\dfrac{z}{h_w}\right)^3 - 40\left(\dfrac{z}{h_w}\right)^4 + 16\left(\dfrac{z}{h_w}\right)^5\right] \end{cases}$$

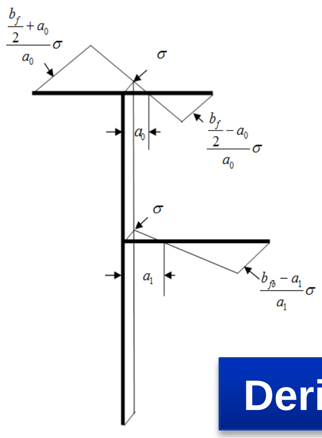
Lengthwise Displacement



$$\begin{Bmatrix} \varphi_B \\ v_T \\ \varphi_T \\ v_{FB} \\ \varphi_{FB} \\ w \end{Bmatrix} = \begin{Bmatrix} \bar{\varphi}_B \\ \bar{v}_T \\ \bar{\varphi}_T \\ \bar{v}_{FB} \\ \bar{\varphi}_{FB} \\ \bar{w} \end{Bmatrix} \sin\frac{m\pi x}{l} = \{\delta\}\sin\lambda x$$

Elastic Lateral Torsional Buckling of PMA III

Derivation of Strain Energy



$$U=\frac{1}{2}E\int_0^l Iw_{,xx}{}^2dx+\frac{1}{2}E\int_0^l I_{zf}v_{T,xx}{}^2dx+\frac{1}{2}E\int_0^l 2I_{zyf}w_{,xx}v_{T,xx}dx+\frac{1}{2}G\int_0^l J_f\varphi_{T,x}^2dx$$

$$+\frac{1}{2}E\int_0^l I_{zfb}v_{FB,xx}{}^2dx+\frac{1}{2}E\int_0^l 2I_{zyfb}w_{,xx}v_{FB,xx}dx+\frac{1}{2}G\int_0^l J_{fb}\varphi_{FB,x}^2dx+u_{wo}$$

$$=\frac{1}{2}\{\delta\}^T[K_L]\{\delta\}$$

$$\begin{cases}
I=\int_{A_w}(z_o-z)^2dA+\int_{A_f}(z_o-h_w)^2dA+\int_{A_{fb}}(z_o-\frac{h_w}{2})^2dA \\
\quad =t_w(h_wz_0^2-h_w^2z_0+\frac{h_w^3}{3})+t_fb_f(z_0-h_w)^2+t_{fb}b_{fb}(z_0-\frac{h_w}{2})^2 \\
I_{zf}=\int_{A_f}(y-a_o)^2dA=\left(\frac{b_f^3}{12}+a_0^2b_f\right)t_f \\
I_{zfb}=\int_{A_w}a_1^2dA+\int_{A_{fb}}(y-a_1)^2dA=a_1^2h_wt_w+\left(\frac{b_{fb}^3}{3}-a_1b_{fb}^2+a_1^2b_{fb}\right)t_{fb} \\
I_{zyf}=\int_{A_f}(z_o-h_w)(a_o-y)dA=a_0(z_0-h_w)b_ft_f \\
I_{zyfb}=\int_{A_w}(z_o-z)a_1dA+\int_{A_{fb}}(z_0-\frac{h_w}{2})(a_1-y)dA \\
\quad =a_1(z_0h_w-\frac{h_w^2}{2})t_w+(z_0-\frac{h_w}{2})(a_1-\frac{b_{fb}}{2})t_{fb}b_{fb} \\
J_f=G\frac{b_ft_f^3}{3} \\
J_{fb}=G\frac{b_{fb}t_{fb}^3}{3}
\end{cases}$$

Derivation of Work Done during Buckling

$$V=\frac{1}{2}\int_{A_f}\sigma\int_0^l\left(v_{T,x}^2+(w-y\varphi_{T,x})^2\right)dxdA+\frac{1}{2}\int_{A_w}\sigma\int_0^l(w_{,x}^2+v_{w,x}^2)dxdA$$

$$+\frac{1}{2}\int_{A_{fb}}\sigma\int_0^l\left(v_{FB,x}^2+(w-y\varphi_{FB,x})^2\right)dxdA$$

$$=\frac{1}{2}\{\delta\}^T[\overline{K}_G]\{\delta\}$$

$$g_{11}=\int_{A_w}\sigma f_1^2dA,\quad g_{12}=\int_{A_w}2\sigma f_1f_2dA,\quad g_{13}=\int_{A_w}2\sigma f_1f_3dA,$$

$$g_{14}=\int_{A_w}2\sigma f_1f_4dA,\quad g_{15}=\int_{A_w}2\sigma f_1f_5dA,\; g_{22}=\int_{A_f}\sigma dA+\int_{A_w}\sigma f_2^2dA,$$

$$g_{23}=\int_{A_w}2\sigma f_2f_3dA,\quad g_{24}=\int_{A_w}2\sigma f_2f_4dA,\quad g_{25}=\int_{A_w}2\sigma f_2f_5dA,$$

$$g_{33}=\int_{A_f}\sigma y^2dA+\int_{A_w}\sigma f_3^2dA,\quad g_{34}=\int_{A_w}2\sigma f_3f_4dA,\quad g_{35}=\int_{A_w}2\sigma f_3f_5dA$$

$$g_{44}=\int_{A_{fb}}\sigma dA+\int_{A_w}\sigma f_4^2dA,\quad g_{45}=\int_{A_w}2\sigma f_4f_5dA$$

$$g_{55}=\int_{A_{fb}}\sigma y^2dA,\quad g_{56}=\int_{A_{fb}}\sigma(-y)dA$$

$$g_{66}=\int_{A_f}\sigma dA+\int_{A_w}\sigma dA+\int_{A_{fb}}\sigma dA$$

6 X 6 Eigenvalue Problem

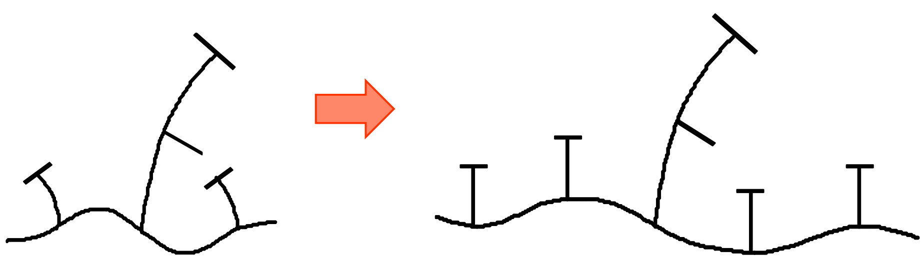
$$\left|[K_L]-[K_G]\right|=0$$

"At the limit of stability, the second variation of total potential energy is zero"

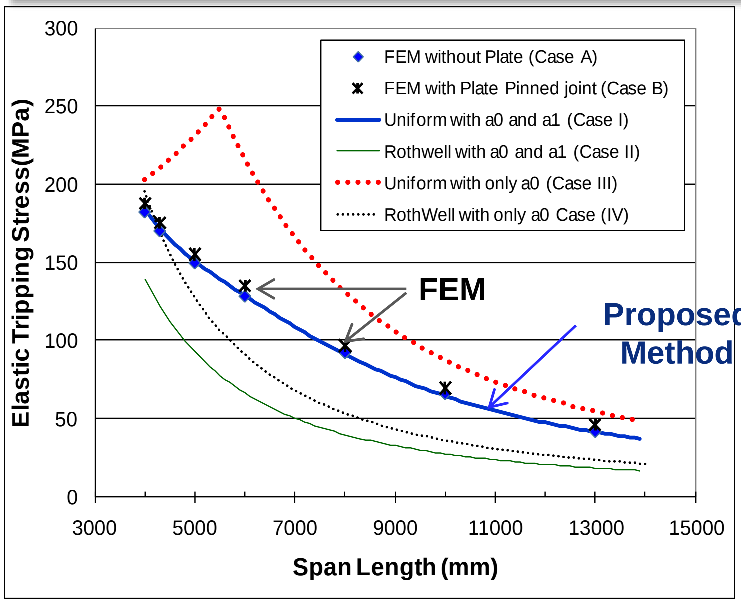
Elastic Lateral Torsional Buckling of PMA IV

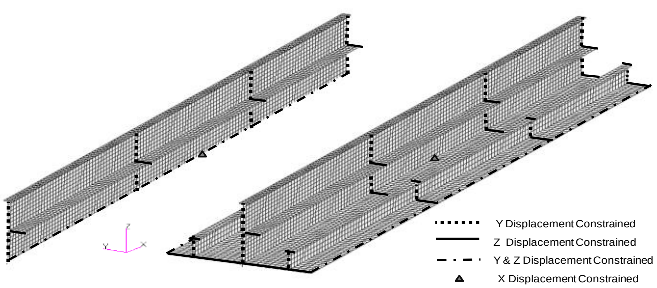
Extended Plate Rotational Restraint

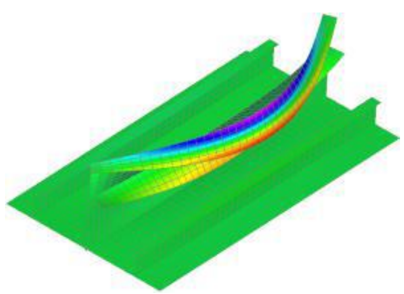
$$\Pi_{po} = \frac{1}{2} k_{spring} \bar{\varphi}_B^{2} = \begin{cases} \dfrac{l}{b_p} D_p \bar{\varphi}_B^{2} & \text{for Case X} \\ \dfrac{l}{4b_p} D_p \bar{\varphi}_B^{2} & \text{for Case Y} \end{cases}$$



Comparison with FE analysis Results

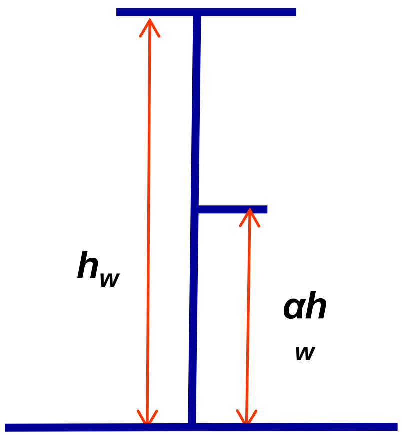


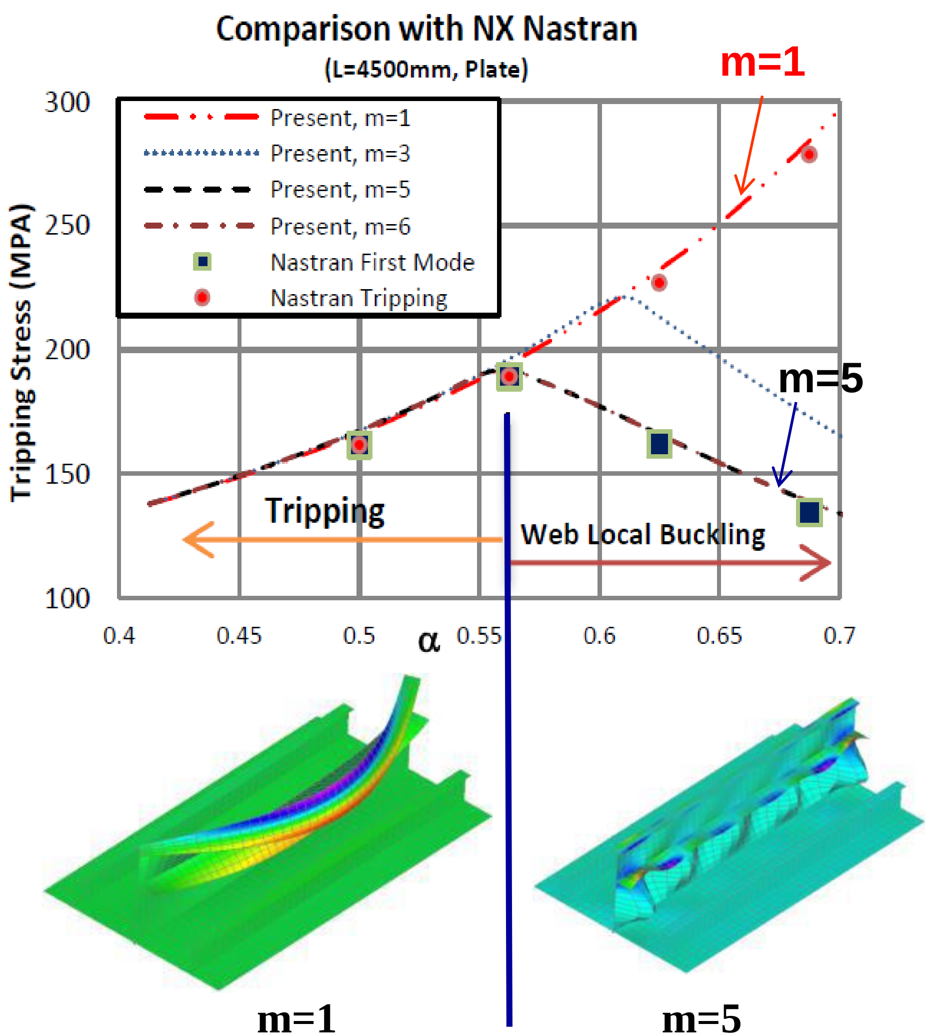




Elastic Lateral Torsional Buckling of PMA V

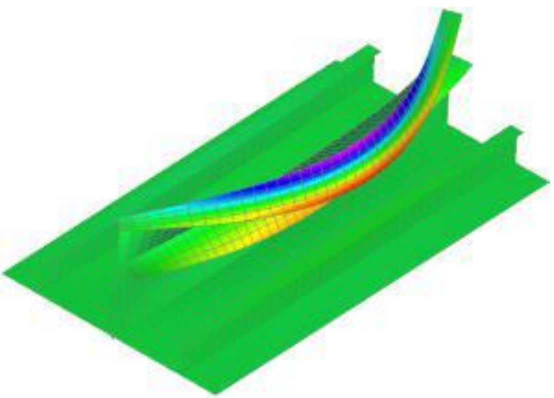
Tripping v.s. web plate local buckling for varying mid-flat-bar location





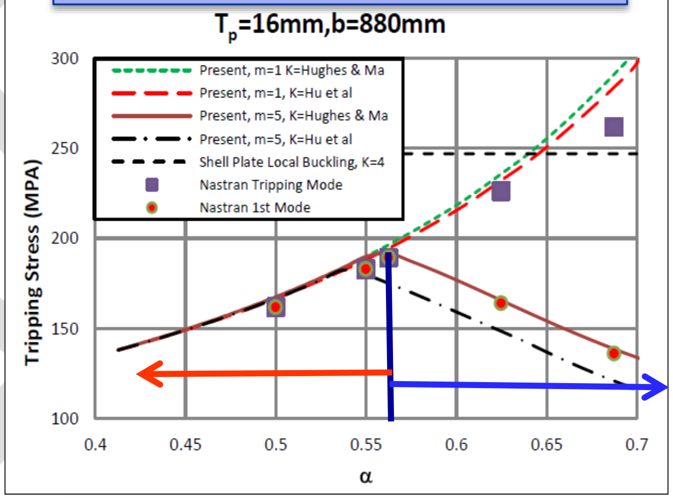
Elastic Lateral Torsional Buckling of PMA VI

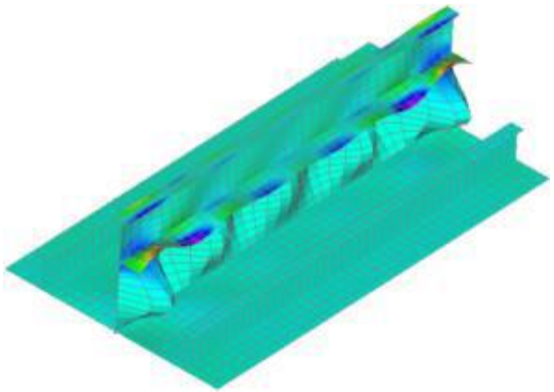
Web Local Buckling v.s. Bottom Plate Buckling



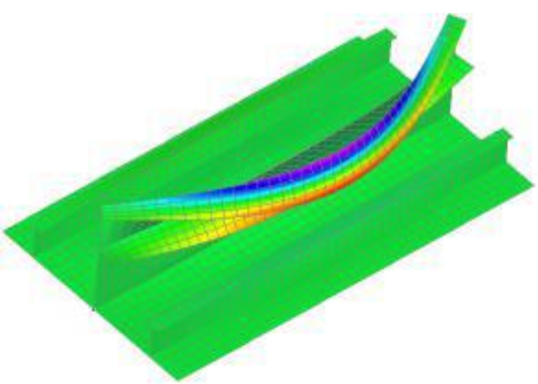
Lateral Torsional Buckling

Thick bottom plate (16t)



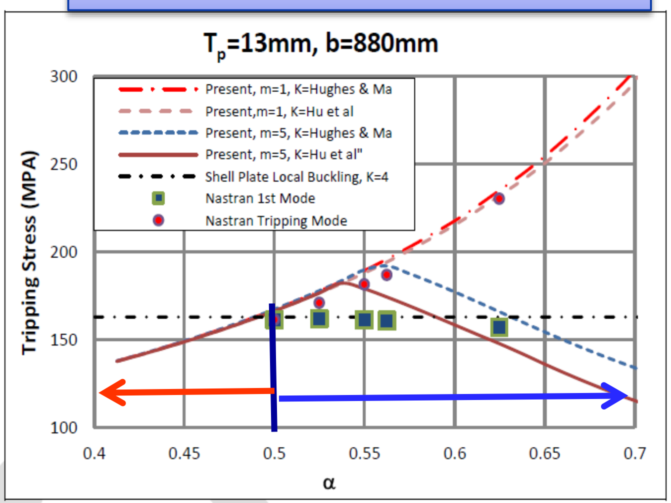


Web Local Buckling



Lateral Torsional Buckling

Thin bottom plate (13t)



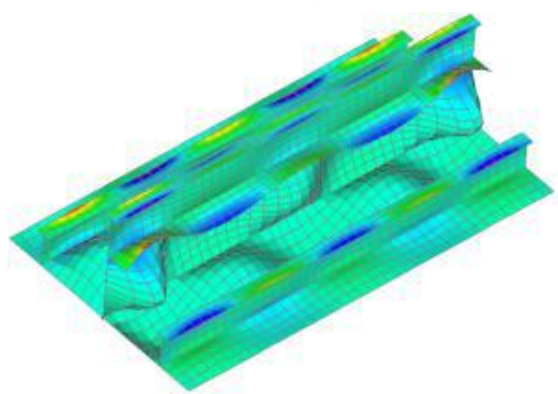


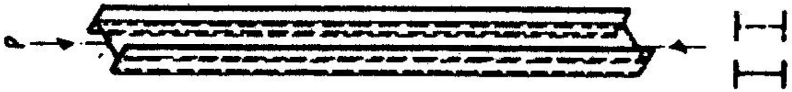
Plate Buckling

13.1 Longitudinally Stiffened Panels

Types of stiffener buckling and method of analysis

- Flexural-torsional buckling : torsional buckling of a stiffener may also be caused by a bending mo ments → compression in the flange.

- Flexural buckling of columns : Bending about the axis of least resistance

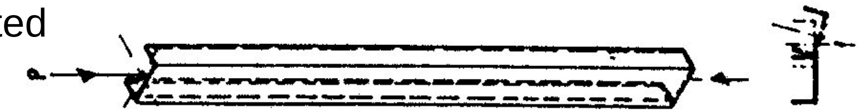


Flexural buckling

- Torsional buckling of columns : Twisting withou bending.

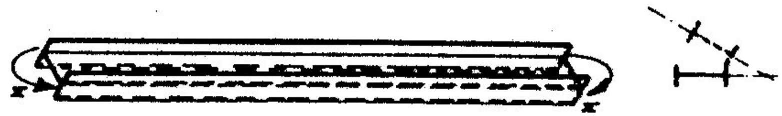
Torsional buckling

- Flexural-torsional buckling of columns : subjected to compression → simultaneous twisting and bending.



Flexural-torsional buckling

- Lateral-torsional buckling of beams : subjected to bending, → simultaneous twisting and bending.



Lateral-torsional buckling

- Local buckling : Buckling of a thin-walled part of the cross-section (plate-buckling, shell-buckling)

Types of stiffener buckling and method of analysis

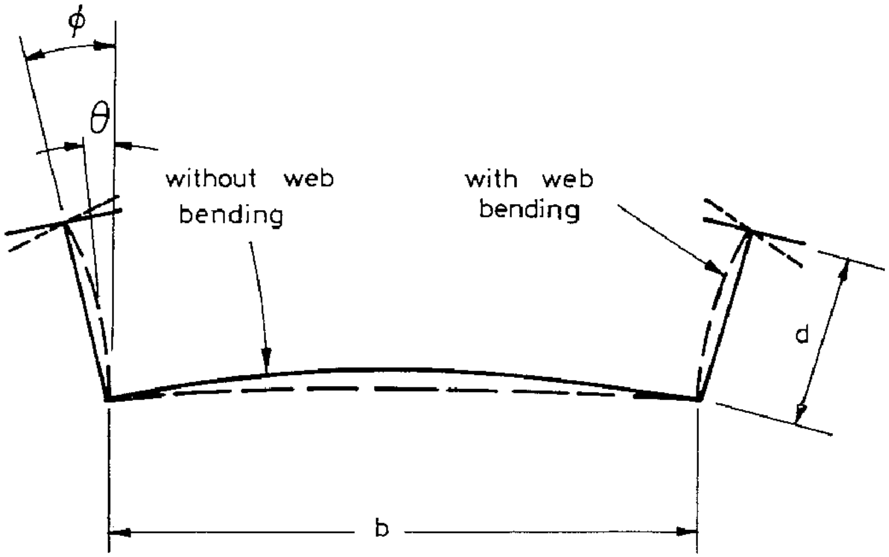
- **Flexural-torsional buckling** : torsional buckling of a stiffener may also be caused by a bending moments
  → compression in the flange.
  → nonlinear coupling between the flexural and torsional response

❖ Two different methods for dealing nonlinear elastic buckling

- "folded plate" analysis based on finite difference methods
  - simpler
  - is restricted to certain boundary conditions and simple forms of structural geometry
  - basically limited to elastic buckling of bifurcation type
- nonlinear frame (finite element) analysis
  - necessarily computer-based, but quite economical
  - is much more general and can deal with other forms of nonlinearity

Stiffener buckling due to axial compression

- A stiffener acts essentially as a column, but tripping or torsional buckling differs from that of a column in three ways
  - the rotation occurs about an enforced axis-the line of attachment to the plating.
  - the plate offers some restraint against this rotation.
  - it is not necessarily rigid body rotation.



Effect of web bending

Stiffener buckling due to axial compression

- The governing differential equation for the rotation is:

$$EI_{SZ} d^2 \frac{d^4\phi}{dx^4} - (GJ - \sigma_a I_{SP}) \frac{d^2\phi}{dx^2} + K\phi = 0$$

where

- I_{sz} = moment of inertia of the stiffener
- d = stiffener web height
- I_{sp} = polar moment of inertia of the stiffener about the center of rotation
- K_ϕ = distributed rotational restraint which plating exerts on the stiffener.
- $\Phi(x)$ = a buckled shaped in which the rotation Φ varies sinusoidally in m half waves over the length a.
- $\sigma_{a,T}$ = the elastic tripping stress, the minimum value of applied in-plane stress σ_a that would cause tripping

Stiffener buckling due to axial compression

- $\sigma a,T$ satisfies the following, where m is a positive integer.

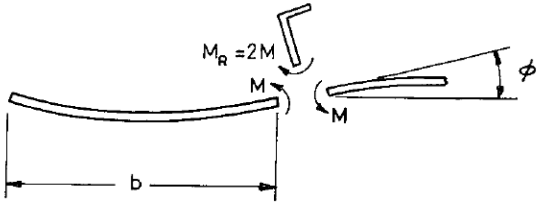
$$EI_{SZ}d^2\frac{m^4\pi^4}{a^4}+(GJ-\sigma_a I_{sp})\frac{m^2\pi^2}{a^2}+K_\phi(\sigma_{a,}m)=0$$

- K_ϕ is a function of σ_a and m.
- K_ϕ is dependent on σ_a because plate buckling can diminish or eliminate K_ϕ.
- K_ϕ is dependent on m.

- The rotational restraint coefficient is offered by the plating
  → to be defined by the plate's flexural rigidity which causes a total distributed restraining moment M_R=2M along the line of the stiffener attachment.

- If a>>b, unit strip of plating across the span b, ϕ = ½Mb/D.

$$K_\phi=\frac{M_R}{\phi}=\frac{4D}{b}$$



- This assumes that the buckled displacement of the stiffener is entirely due to rigid body rotation.

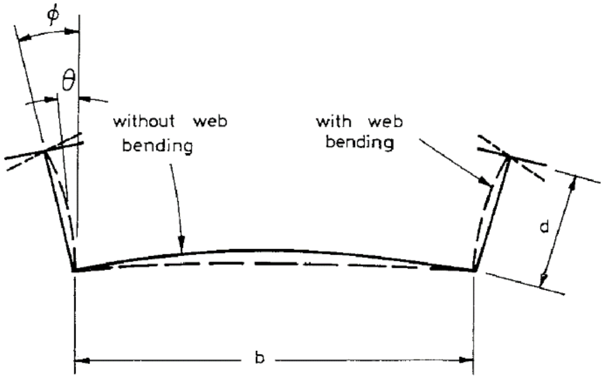
Stiffener buckling due to axial compression

- In practice some of the sideways displacement of the stiffener flange occurs due to bending of the web. Even when the stiffener web is slender.

- C_r is the factor by which the plate rotational restraint is reduced due to web bending.

$$\theta = C_r \phi \qquad C_r = \frac{1}{1+(2/3)(t/t_w)^3(d/b)}$$

where (for σ_a=0)



- C_α is a correction factor because of the effect of plate aspect ratio.

$$C_\alpha = 1+\frac{m^2}{\alpha^2} \qquad K_\phi = \frac{4D}{b} C_r C_\alpha$$

$$C_r = \frac{1}{1+0.4(t/t_w)^3(d/b)}$$

Stiffener buckling due to axial compression

- C_α : correction factor for aspect ratio and plate buckling effects because plate stiffness decreases as the applied stress σ_a approaches plate buckling stress σ_0.

$$C_\alpha = 1\text{-}\left(\frac{2\sigma_a}{\sigma_0} - 1\right)\frac{m^2}{\alpha^2}$$

- $C_\alpha = 1-(m/\alpha)^2$ when $\sigma_a = \sigma_0$. if $m=\alpha$ (aspect ratio), $C_\alpha = 0$. and $K_\phi = 0$.
- Tripping occurs in a single half-wave, m=1 → the same as number of half-wave for square or short panel. $\alpha \approx 1$ → complete loss of rotational restraint.
- For stiffened panels of usual proportions, tripping occurs in a single half-wave, and hence it is mainly square or short panels in which the loss of stiffness can occur.

$$K_\phi = \frac{4D}{b}\left[\frac{1}{1+0.4\left(\frac{t}{t_w}\right)^3 \frac{d}{b}}\right]\left[1-\left(\frac{2\sigma_a}{\sigma_0}-1\right)\frac{m^2}{\alpha^2}\right] \qquad m \geq 2$$

$$\boxed{C_r = \frac{1}{1+0.4(t/t_w)^3(d/b)}}$$

$$\boxed{C_\alpha = 1\text{-}\left(\frac{2\sigma_a}{\sigma_0} - 1\right)\frac{m^2}{\alpha^2}}$$

Stiffener buckling due to axial compression

- by substituting K_ϕ and solving for σ_a.

$$\boxed{K_\phi = \frac{4D}{b}\left[\frac{1}{1+0.4\left(\frac{t}{t_w}\right)^3 \frac{d}{b}}\right]\left[1-\left(\frac{2\sigma_a}{\sigma_0}-1\right)\frac{m^2}{\alpha^2}\right]} \Rightarrow EI_{SZ}d^2\frac{m^4\pi^4}{a^4}+(GJ-\sigma_a I_{sp})\frac{m^2\pi^2}{a^2}+K_\phi(\sigma_a,m)=0$$

$$\Rightarrow \sigma_{a,T} = \underset{m=1,2,\dots}{Minimum,}\left\{\frac{1}{I_{sp}+\frac{2C_r b^3 t}{\pi^4}}\left[GJ+\frac{m^2\pi^2}{a^2}EI_{sz}d^2+\frac{4DC_r}{\pi^2 b}\left(\frac{a^2}{m^2}+b^2\right)\right]\right\} \qquad \boxed{C_r = \frac{1}{1+0.4(t/t_w)^3(d/b)}}$$

- Here, regard m continuous variable. Find m which gives the lowest $\sigma_{a,T}$

$$\frac{d\sigma_{a,T}}{dm}=0 \qquad m \cong \frac{a}{\pi}\sqrt[4]{\frac{4DC_r}{EI_{SZ}d^2 b}}$$

- m can be predicted and trial-and-error approach can be avoided.

Stiffener buckling due to axial compression

After estimating m,

$$m \cong \frac{a}{\pi}\sqrt[4]{\frac{4DC_r}{EI_{SZ}d^2b}}$$

(a) Solution when m≥2,

$$\sigma_{a,T} = \frac{1}{I_{sp}+\frac{2C_r b^3 t}{\pi^4}}\left(GJ + 4\sqrt{\frac{DC_r EI_{SZ}d^2}{b}} + \frac{4DC_r b}{\pi^2}\right)$$

$$\sigma_{a,T} = \underset{m=1,2,...}{Minimum,}\left\{\frac{1}{I_{sp}+\frac{2C_r b^3 t}{\pi^4}}\left[GJ + \frac{m^2\pi^2}{a^2}EI_{sz}d^2 + \frac{4DC_r}{\pi^2 b}\left(\frac{a^2}{m^2}+b^2\right)\right]\right\}$$

(b) Solution when m=1,

ex) estimated m≈1.3 → m=1

estimated m≈1.7 → m=1 or m=2

$$m = 1$$

$$\sigma_{a,T} = \frac{1}{I_{sp}+\frac{2C_r b^3 t}{\pi^4}}\left(GJ + \frac{\pi^2 EI_{SZ}d^2}{a^2} + \frac{4DC_r}{\pi^2 b}(a^2+b^2)\right)$$

$$\sigma_{a,T} = \underset{m=1,2,...}{Minimum,}\left\{\frac{1}{I_{sp}+\frac{2C_r b^3 t}{\pi^4}}\left[GJ + \frac{m^2\pi^2}{a^2}EI_{sz}d^2 + \frac{4DC_r}{\pi^2 b}\left(\frac{a^2}{m^2}+b^2\right)\right]\right\}$$

(c) Lower bound solution for quick check

- ignore plate restraint effect C_α =0, i.e. $\sigma_a = \sigma_0 \rightarrow K_\phi = 0$
- ignore GJ term since it is small.

$$C_\alpha = 1\text{-}\left(\frac{2\sigma_a}{\sigma_0}-1\right)\frac{m^2}{\alpha^2}$$

$$\sigma_{a,T} = \frac{1}{I_{sp}}\left[\pi^2 E\left(\frac{d}{a}\right)^2 I_{SZ}\right]$$

Stiffener buckling due to axial compression

- I_{sz} and I_{sp} can be expressed in terms of stiffener areas

$$I_{sp} = d^2\left(A_f + \frac{A_w}{3}\right) \qquad I_{sz} = b_f^2 \frac{A_f\left(\frac{A_x}{3} - \frac{A_f}{4}\right)}{A_x} \cong \frac{A_f A_w b_f^2}{3A_x}$$

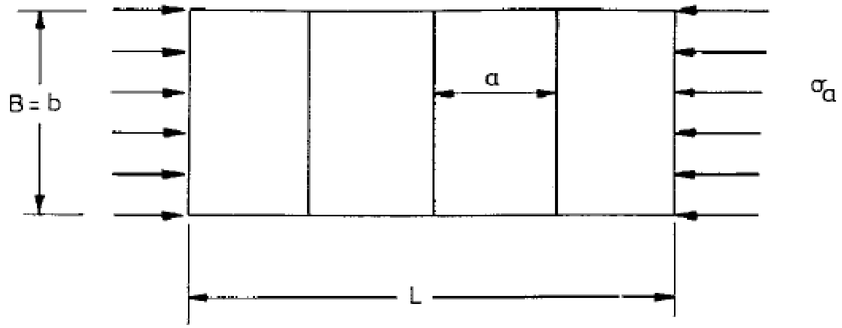
- Also, by defining a fractional flange area $f=A_f/A_x$, it becomes

$$\sigma_{a,T} = \frac{\pi^2 Ef(1-f)}{1+2f}\left(\frac{b_f}{a}\right)^2$$

- a : adverse effect of panel length
- b_f : strengthening effect, but cannot be increased indefinitely due to possibility of flange buckling.
- b_f/t_f <14 for mild steel to avoid the flange buckling.

Transversely Stiffened Panels

- The strength of the stiffened panel is determined by the buckling strength of the plate between stiffeners.



- The required minimum value of γ_y given is

$$\gamma_y = \frac{(4N_L^2-1)\left[\left(N_L^2-1\right)^2-2\left(N_L^2+1\right)\kappa+\kappa^2\right]}{2\left(5N_L^2+1-\kappa\right)\Pi^4} \qquad \gamma_y = \frac{EI_y}{Da} \qquad \kappa = \alpha^2\Pi^2 \qquad \Pi = \frac{L}{B} = \frac{L}{b}$$

where

N_L= number of plate panels= L/a, N_{L-1} = number of stiffeners

EI_y= flexural rigidity of one transverse stiffener,

including a plate flange of full width a.

Homework 6-1 due date 21th November

- Homework 6-1 Analyze the effect of a, b, d, b_f, t_f, t_w, h, t_{plate} using ANOVA table and an orthogonal array. Recommend to use Minitab.
  a=2400±10%, b=800±10%, d=450mm, b_{f},=250mm

t_f,= t_w=t_{plate}=15 mm±10%,

$$\sigma_{a,T} = \underset{m=1,2,\ldots}{Minimum,}\left\{ \frac{1}{I_{sp} + \frac{2C_r b^3 t}{\pi^4}} \left[GJ + \frac{m^2\pi^2}{a^2} EI_{sz} d^2 + \frac{4DC_r}{\pi^2 b}\left(\frac{a^2}{m^2} + b^2\right)\right]\right\}$$

$$I_{sp} = d^2\left(A_f + \frac{A_w}{3}\right)$$

$$I_{sz} = b_f^2 \frac{A_f\left(\frac{A_x}{3} - \frac{A_f}{4}\right)}{A_x} \cong \frac{A_f A_w b_f^2}{3A_x}$$

$$J = \frac{t_f^3 b_f}{3} + \frac{t_w^3 d}{3}$$

Homework 6-1

- Orthogonal array for 8 parameters with 3 levels

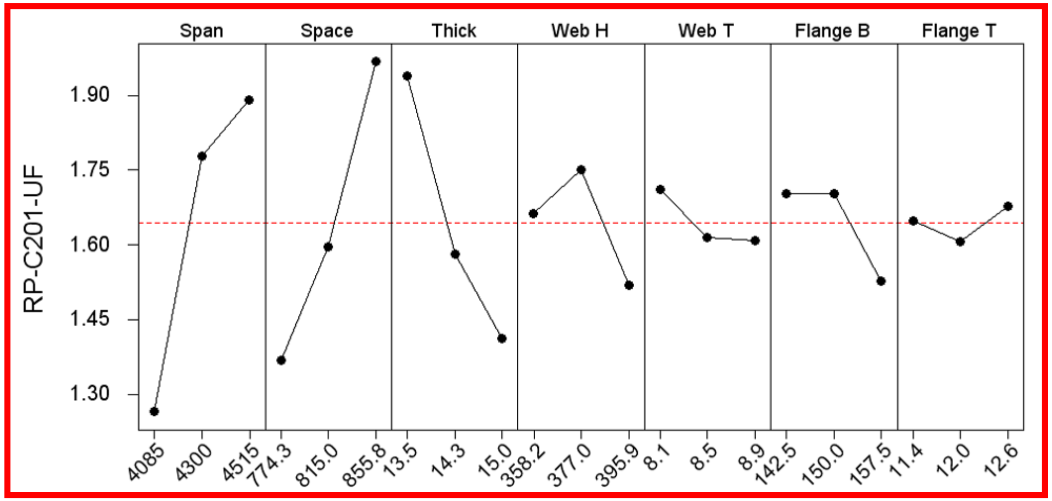
Experiment Number	Column 1	2	3	4	5	6	7	8
1	1	1	1	1	1	1	1	1
2	1	1	2	2	2	2	2	2
3	1	1	3	3	3	3	3	3
4	1	2	1	1	2	2	3	3
5	1	2	2	2	3	3	1	1
6	1	2	3	3	1	1	2	2
7	1	3	1	2	1	3	2	3
8	1	3	2	3	2	1	3	1
9	1	3	3	1	3	2	1	2
10	2	1	1	3	3	2	2	1
11	2	1	2	1	1	3	3	2
12	2	1	3	2	2	1	1	3
13	2	2	1	2	3	1	3	2
14	2	2	2	3	1	2	1	3
15	2	2	3	1	2	3	2	1
16	2	3	1	3	2	3	1	2
17	2	3	2	1	3	1	2	3
18	2	3	3	2	1	2	3	1

A sample of the use of ANOVA table and Orthogonal Array

Selection of Stiffened Plate Parameters
Orthogonal Array L27, 3^7

Span	Space	t_p	h_w	t_w	b_f	t_f
3870.0	733.5	12.8	339.3	7.7	135.0	10.8
3870.0	733.5	12.8	339.3	8.5	150.0	12.0
3870.0	733.5	12.8	339.3	9.4	165.0	13.2
3870.0	815.0	14.3	377.0	7.7	135.0	10.8
3870.0	815.0	14.3	377.0	8.5	150.0	12.0
3870.0	815.0	14.3	377.0	9.4	165.0	13.2
3870.0	896.5	15.7	414.7	7.7	135.0	10.8
3870.0	896.5	15.7	414.7	8.5	150.0	12.0
3870.0	896.5	15.7	414.7	9.4	165.0	13.2
4300.0	733.5	14.3	414.7	7.7	150.0	13.2
4300.0	733.5	14.3	414.7	8.5	165.0	10.8
4300.0	733.5	14.3	414.7	9.4	135.0	12.0
4300.0	815.0	15.7	339.3	7.7	150.0	13.2
4300.0	815.0	15.7	339.3	8.5	165.0	10.8
4300.0	815.0	15.7	339.3	9.4	135.0	12.0
4300.0	896.5	12.8	377.0	7.7	150.0	13.2
4300.0	896.5	12.8	377.0	8.5	165.0	10.8
4300.0	896.5	12.8	377.0	9.4	135.0	12.0
4730.0	733.5	15.7	377.0	7.7	165.0	12.0
4730.0	733.5	15.7	377.0	8.5	135.0	13.2
4730.0	733.5	15.7	377.0	9.4	150.0	10.8
4730.0	815.0	12.8	414.7	7.7	165.0	12.0
4730.0	815.0	12.8	414.7	8.5	135.0	13.2
4730.0	815.0	12.8	414.7	9.4	150.0	10.8
4730.0	896.5	14.3	339.3	7.7	165.0	12.0
4730.0	896.5	14.3	339.3	8.5	135.0	13.2
4730.0	896.5	14.3	339.3	9.4	150.0	10.8

DNV RP-C201 U.F v.s. Scantlings



DNV PULS v.s. Scantlings