Topics in Ship Structural Design
(Hull Buckling and Ultimate Strength)

Lecture 8 Ultimate Strength of Stiffened Panels

Reference : Ship Structural Design Ch.14
Ultimate Limit State Design of Steel-Plated Structures Ch. 6
PULS Manual

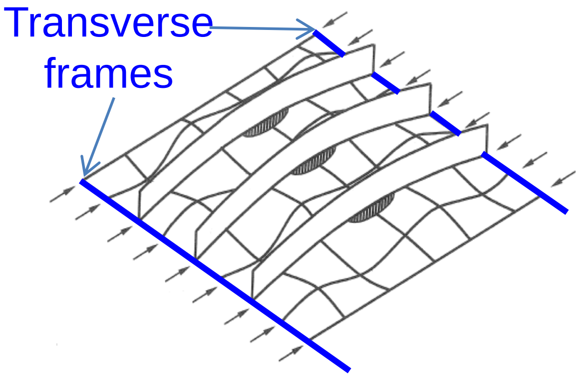
NAOE
Jang, Beom Seon

14.1 boundary conditions, load types, and collapse modes

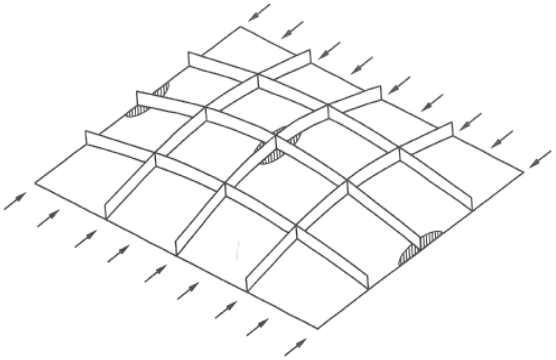
General

- Stiffened panels in ship structures are usually sufficiently sturdy that $(\sigma_a)_{cr} > \sigma_Y$. → the mode of compressive collapse is a complicated inelastic process. "failure" or "collapse" instead of buckling.
- Two possible levels of collapse
  - ✓ Interframe collapse : collapse of stiffened panels between transverse frames
  - ✓ Gross panel collapse, involving both longitudinal and transverse stiffener.
- Cross-stiffened plate to be designed such that interframe collapse occurs before gross panel collapse (more catastrophic).

  → only interframe collapse is dealt in this chapter.



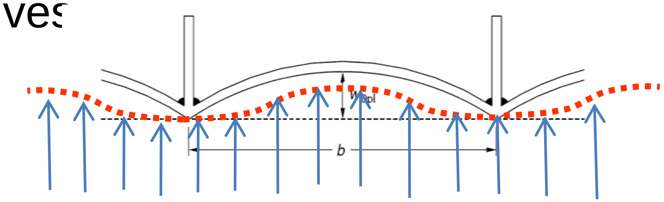
Interframe collapse



Gross panel collapse

Boundary conditions for ultimate strength analysis

- Regions having a large lateral load, each panel receives some rotational restraint from the adjacent panel → larger ultimate strength than simply supported.

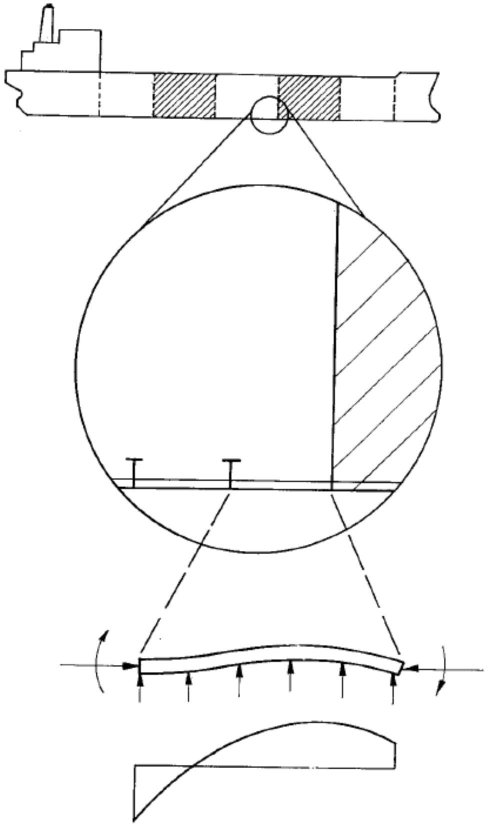


- However, there are many situations in which a bottom panel is far from being clamped.

- Although some other panels may have rotational restraint → impractical for panels to differ from one frame to another

- The safest and best procedure is to regard the panel as simply supported. However, the lateral load should not be ignored since it decreases panel's ultimate strength.

- Although the panel ends are being taken as simply supported the theory presented in this chapter does not require that M_0 and δ_0 be the simply supported values. → largest values which will occur in worst case.



Occurrence of nonclamped conditions in bottom panels

Basic load types and associated mechanisms of collapse

- For longitudinally stiffened panels with simply supported edges there are three basic types of loads:

1. Lateral load causing negative bending of the panel .
2. Lateral load causing positive bending of the panel .
3. In-plane compression .

More focus on lateral pressure to induce yielding rather than buckling.

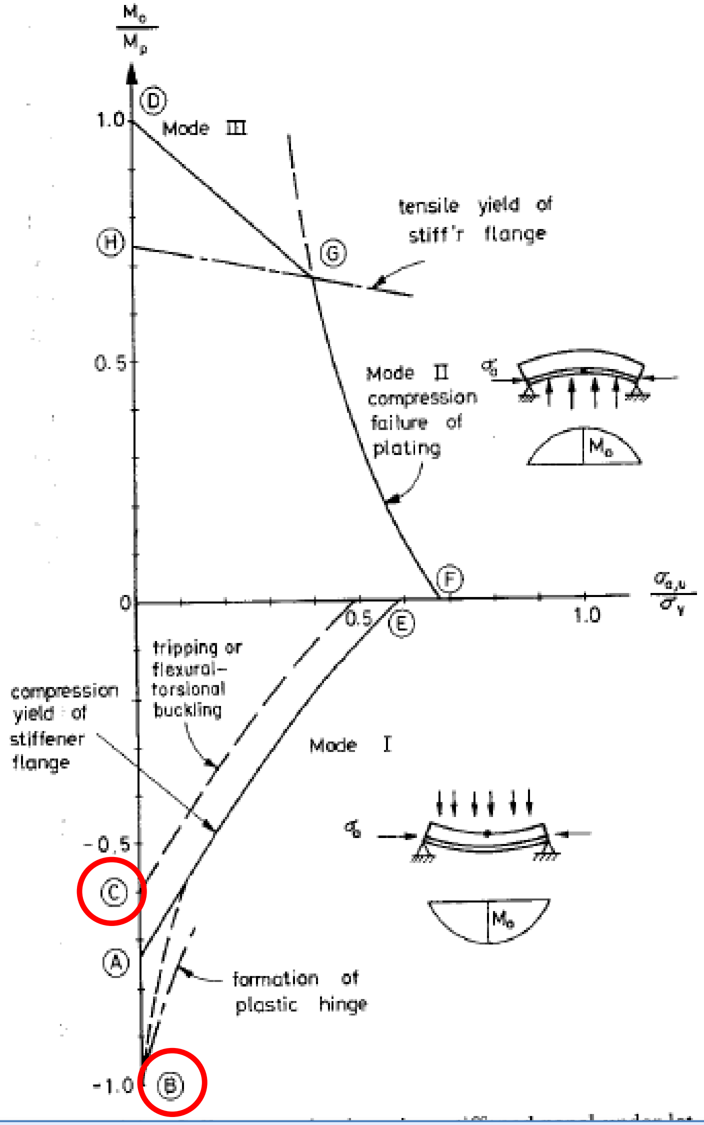
Notes:

bending moment (+) : compression in the plating

in-plane loads (+) : compression.

Negative Bending

- Plating is in tension and hence cannot buckle.
- Since the neutral axis is close to the plating, stiffener flange reaches yield first and spread through the stiffener.
- Collapse eventually occurs when the bending moment reaches M_p. The value at which a plastic hinge is formed. (Point B)
- If stiffener buckling occurs, it can not reach the full value of M_p (Point C)

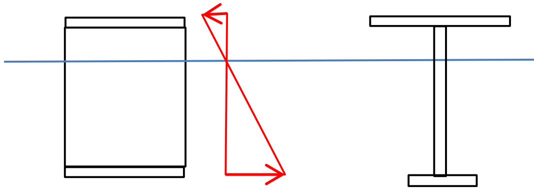


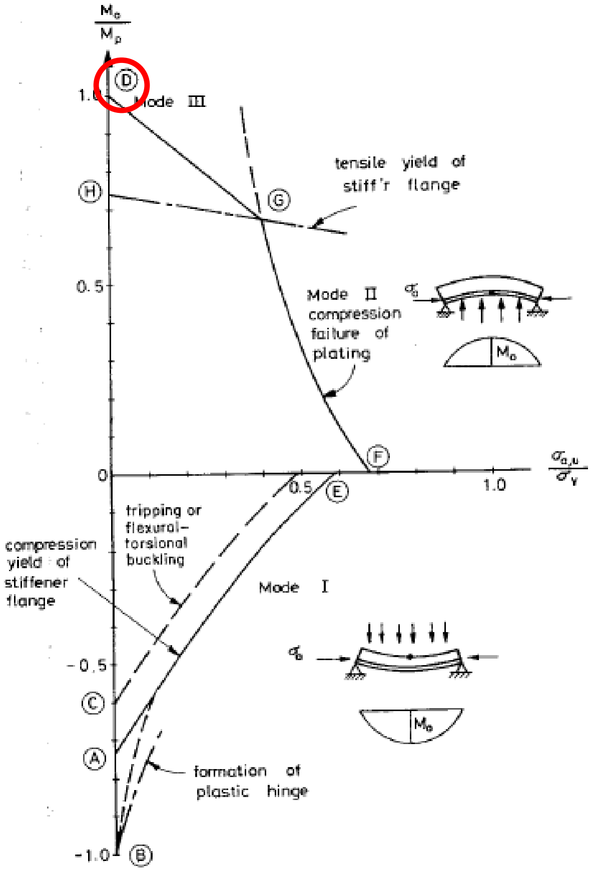
Collapse mechanisms in a stiffened panel under lateral and in-plane loads

Positive Bending

- Plating is in compression,
- Since the neutral axis is very close to the plating → the plate will not be heavily stressed until the load becomes very large and collapse is imminent (임박한).
- In fact, the neutral axis lies within the plating thickness → some portion of the plate thickness is in tension → this stabilizes the plate and prevents from buckling.
- The entire stiffener has yielded in tension → the plate begins to yield in compression

  → Collapse finally occurs as the result of a plastic hinge.

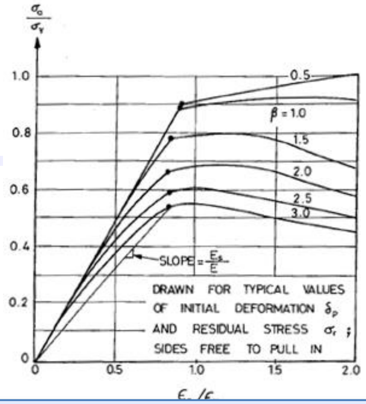




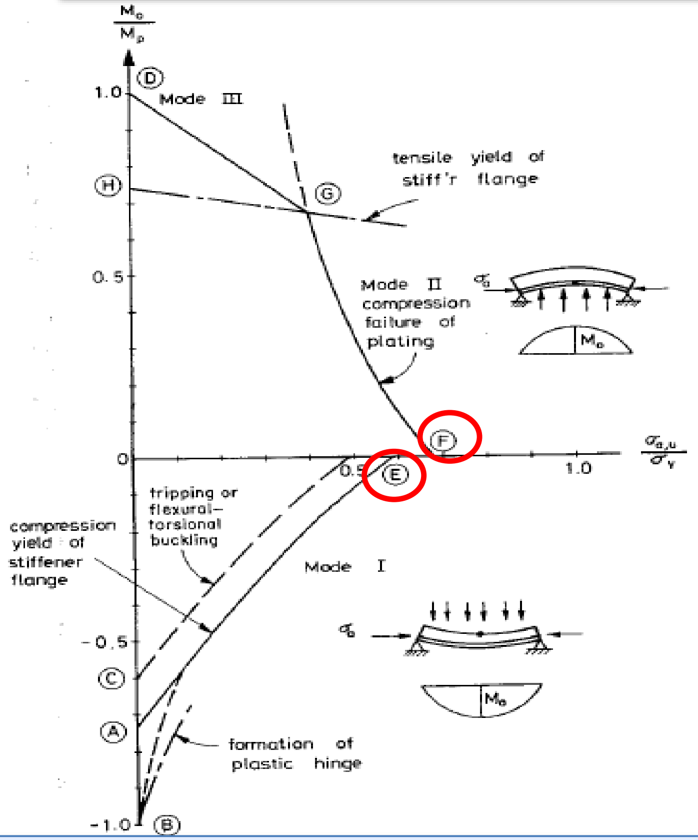
Collapse mechanisms in a stiffened panel under lateral and in-plane loads

Uniform In-Plane Compression

- Uniform compression → a uniform shortening of the panel, ex) hull girder stress
- For typical ship panels, the plate's response becomes inelastic prior to collapse.
- Plate portion is regarded as being made of a different material having a reduced elastic modulus with initial eccentricity.
- "Collapse" is considered to occur when the stress in the extreme fiber of the compression flange of the column reaches the failure value.
- Two different values of in-plane collapse load, depending on which flange (plate or stiffener) is the compression flange depending on the direction of the buckling (Point E and F).



Plate's load versus end shortening



Collapse mechanisms in a stiffened panel under lateral and in-plane loads

Combined Loads

- With lateral load, the plate-stiffener “column” becomes a “beam column”.

  → collapse still occurs as the result of “failure of a flange”, but the bending moment and deflection caused by the lateral load must be taken into account.

- Four modes of collapse
  - ✓ Tensile failure of plating
  - ✓ Tensile failure of stiffener flange
  - ✓ Compressive failure of plating
  - ✓ Compressive failure of stiffener flange

Q: which mode never occur?

A: Tensile failure of buckling doesn’t since neutral axis is so close to the plating

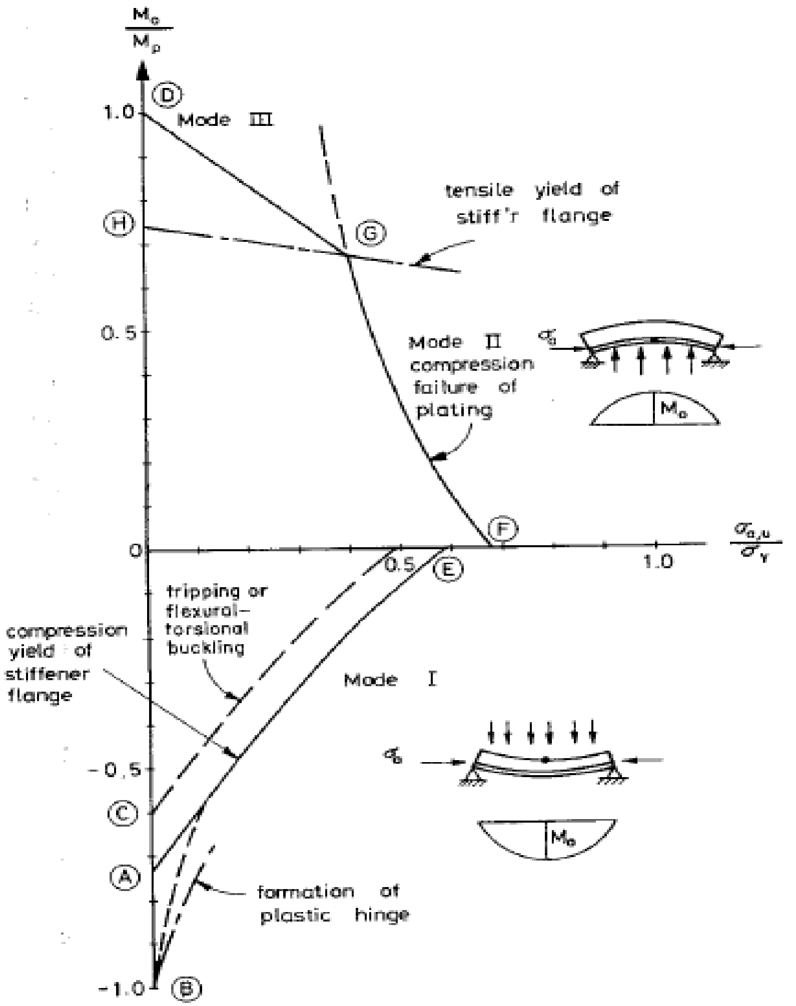


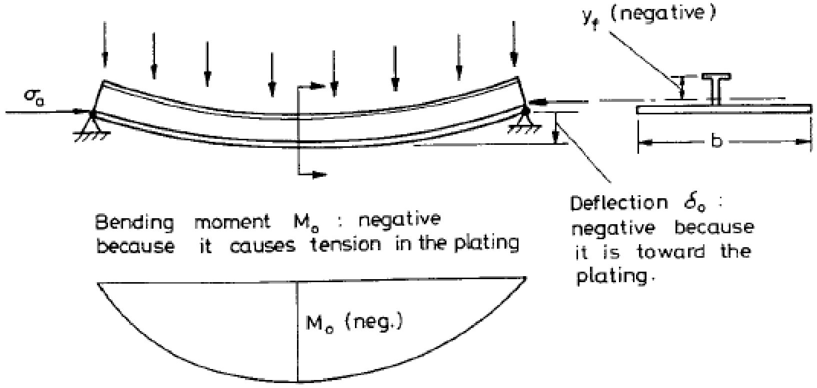
Figure 14.2 Collapse mechanisms in a stiffened panel under lat-

Collapse mechanisms in a stiffened panel under lateral and in-plane loads

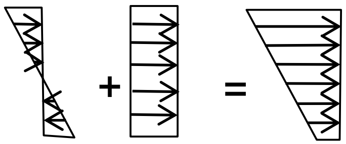
Compression + Negative Bending

❖ Mode I :Compression failure of the stiffener

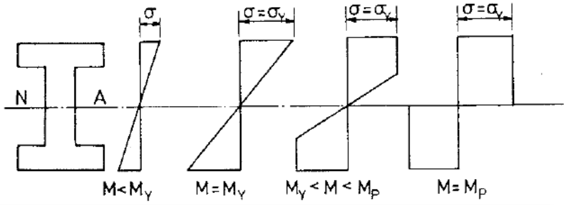
- Collapse occurs as the result of compressive failure of the stiffener flange. Either by compressive yield or by buckling.
- **When there is considerable amount axial compressive stress σ_a →** Compressive yielding commences sooner and progresses more rapidly, whereas plate yielding is delayed.
  - ✓ it cannot reach a plastic hinge condition (which forms under purely lateral load)
  - ✓ after the yielding has penetrated through the stiffener flange, the stiffener reaches its limit of stress absorption,
  - ✓ the effective moment of inertial of the section becomes very small, since only plating is effective
- Once the stiffener flange is fully yielded collapse occurs soon afterward. Sudden collapse.



In-plane compression and negative bending



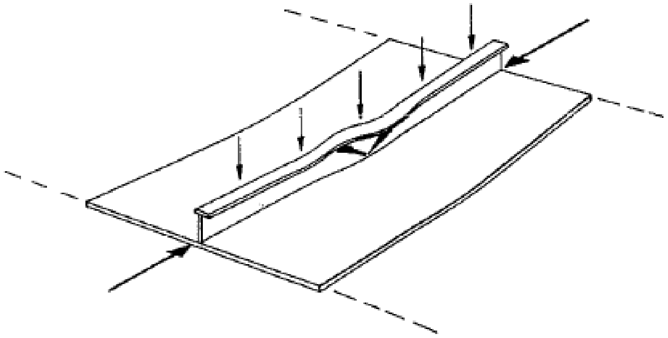
Increased compressive stress



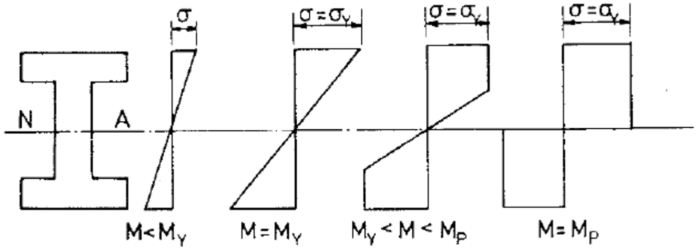
Plastic hinge formation

Compression + Negative Bending

- Precise cause of the failure is the formation of a local plastic mechanism in the flange.
- It is quite local because the flange becomes fully plastic at the point where the bending moment is a maximum → large local sideways deflection and rotation → Not tripping due to local plastic mechanism rather than overall flange buckling.
- **When the loading is purely lateral,**
  - ✓ The plate-stiffener combination can reach a plastic hinge condition
  - ✓ Curve between stiffener flange yield → plastic hinge collapse point is very steep.
  - ✓ A small underestimate of axial load → serious overestimate of the collapse load
  - ✓ It is best to use the "stiffener flange yield" mechanism. (Mode I)



Local plastic collapse mechanism in a stiffener flange

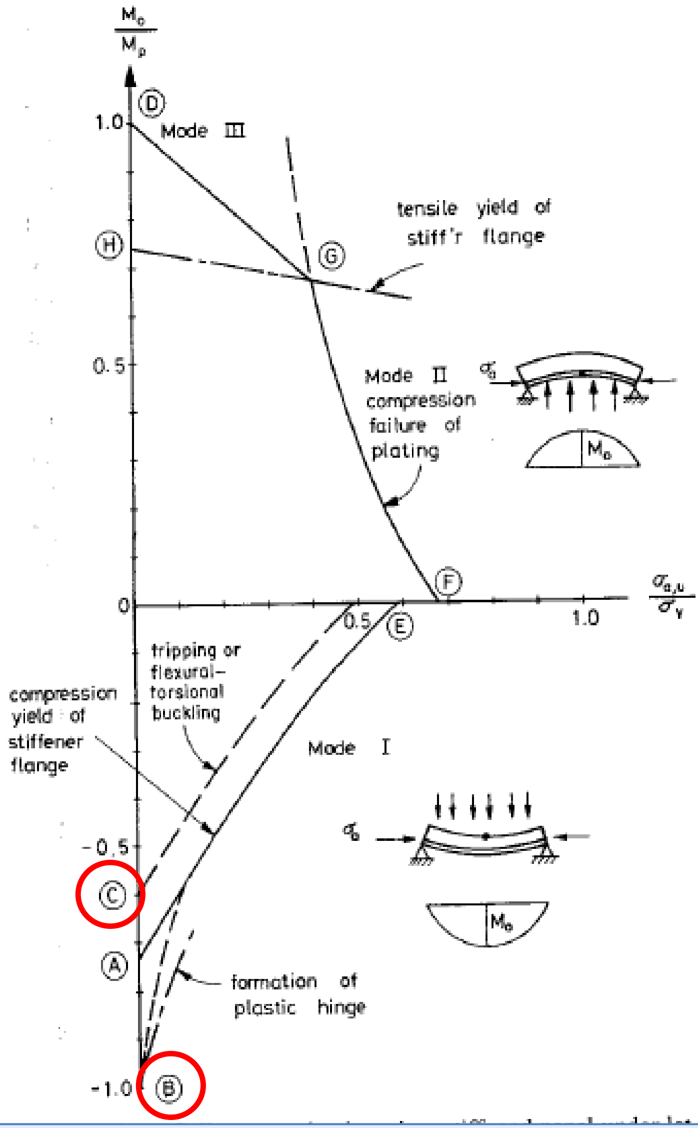


Plastic hinge formation

Compression + Negative Bending

- Alternatively, the stiffener may fail by tripping.
- If the amount of bending is small, the elastic tripping stress is given by the formulas of section 13.1
- If not, the stiffener may undergo flexural-torsional buckling → but this require the fully nonlinear FE method.
- A simple alternative suggested by Smith.
- Max. Compressive stress in the flange must be kept below $\sigma_{a,}$

  → $(\sigma_{a,})_T$ from elastic theory should be well above σ_Y



Collapse mechanisms in a stiffened panel under lateral and in-plane loads

Compression + Negative Bending

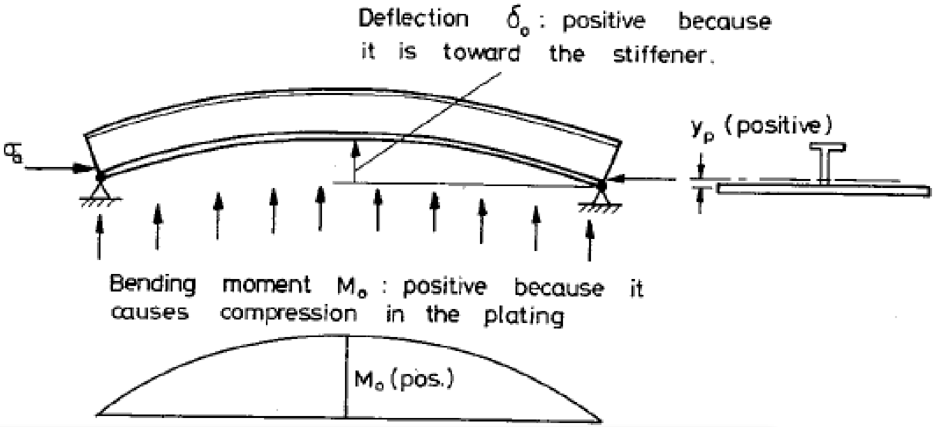
- Stiffener may fail by tripping.
- True collapse value require the fully nonlinear FE method.

❖ **Simple alternative suggested by Smith**

- Max. compressive stress in the flange < σ_{YS} in order to avoid a local plastic mechanism
- Stiffener buckling can be avoided simply by requiring that $\sigma_{a,T}$ from the elastic theory > σ_{YS} .

Compression + Positive Bending

- **Mode II : Compression Failure of Plating**
  - With small and moderate lateral loads, collapse occurs when the stress in the plating reaches the failure value.
  - Secant modulus to represent the plate's stress-strain relationship
- **Mode III : Compression Failure of Stiffener and Plating**
  - With large lateral loads, the bending causes large tensile stresses in the stiffener and these are reduced by the in-plane applied compressive stress.
  - Final collapse happens due to a combination of compressive failure of the plating and tensile yielding of the stiffener.



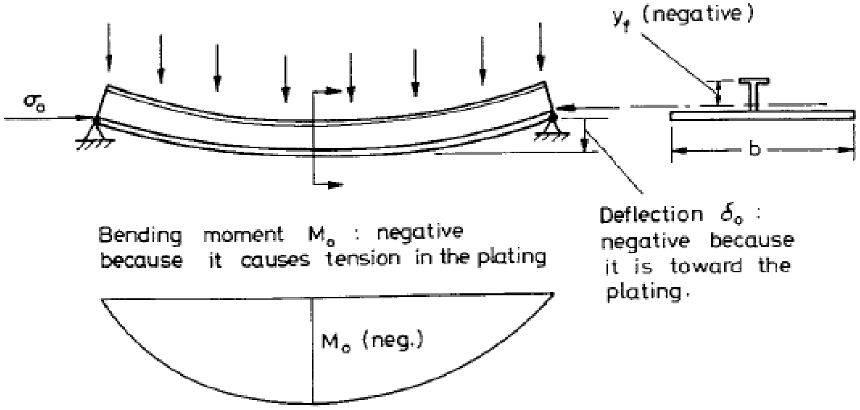
In-plane compression and positive bending

Mode I : Compression failure of the stiffener

- Collapse occurs due to compression failure of the stiffener flange.
- The failure value σ_F is either the yield stress σ_Y or the elastic tripping stress $\sigma_{a,T}$ (Section 13) whichever is less.

$$\sigma_{\mathrm{F}} = \mathrm{MIN}\left\{\sigma_{\mathrm{Ys}}, \sigma_{a,T}\right\}$$

- Bending Moment and deflection due to lateral load + initial eccentricity
- As the yield zone penetrates from the outer surface to the mid-thickness of the flange, there is some linearity but negligible.
- The failure process can be regarded as entirely elastic and can be completely described by the beam-column theory.
- The plate flange in tension → $b_e=b$



In-plane compression and negative bending

Mode I : Compression failure of the stiffener

- The total stress in the mid-thickness of the flange is

$$\sigma_{\mathrm{f}} = \sigma_a + \frac{M_0 y_f}{I} + \frac{\sigma_a A\left(\delta_0 + \Delta\right) y_f}{I}\phi$$

- $\sigma_{a,u}$: ultimate value of applied stress σ_a, at which $\sigma_f = \sigma_F$.

$$R = \frac{\sigma_{a,u}}{\sigma_F},\ \lambda = \frac{a}{\pi\rho}\sqrt{\frac{\sigma_F}{E}},\ \eta = \frac{(\delta_0 + \Delta) y_f}{\rho^2},\ \mu = \frac{M_0 y_f}{I\sigma_F}$$

$$(1\text{-R} - \mu)\left(1 - \lambda^2 R\right) = \eta R \qquad \mathrm{R} = \frac{\zeta}{2} - \sqrt{\frac{\zeta^2}{4} - \frac{1-\mu}{\lambda^2}}$$

$$\text{where} \quad \zeta = 1\text{-}\mu + \frac{1+\eta}{\lambda^2}$$

- Compression yield of stiffener flange

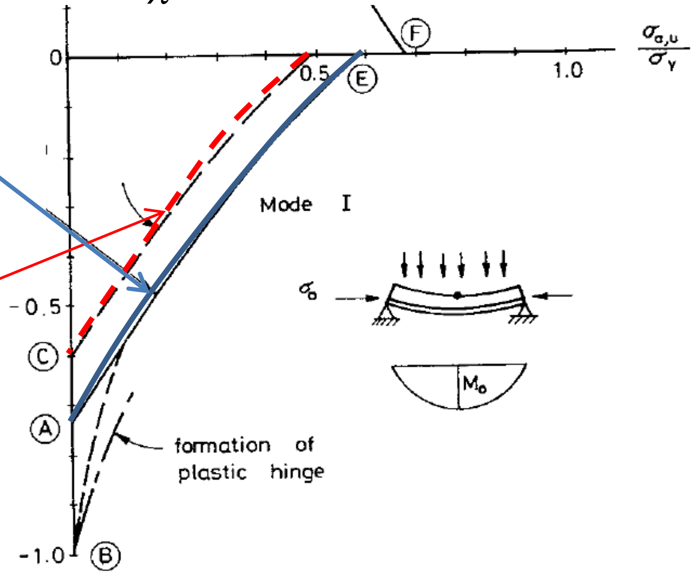
$$If\ \ \sigma_F = \sigma_{YS},\ \left(\frac{\sigma_{a,u}}{\sigma_{YS}}\right)_I = R$$

- Tripping or flexural torsional buckling

$$If\ \ \sigma_F = \sigma_a,\ \left(\frac{\sigma_{a,u}}{\sigma_{YS}}\right)_I = R\frac{\sigma_{a,T}}{\sigma_{YS}}$$

- A, I : Cross-sectional area and moment of inertia of the beam-column
- M_0, δ_0 : max. bending moment and ma. Deflection due to the lateral load
- Δ : initial eccentricity of the beam column.
- M_0: distance from the centroid axis of the cross section.
- Φ : Magnification factor due to the axial compressive stress σ_0

$$\phi = \frac{1}{1 - \frac{\sigma_a}{\sigma_E}},\ \sigma_E = \frac{\pi^2 EI}{Aa^2}$$

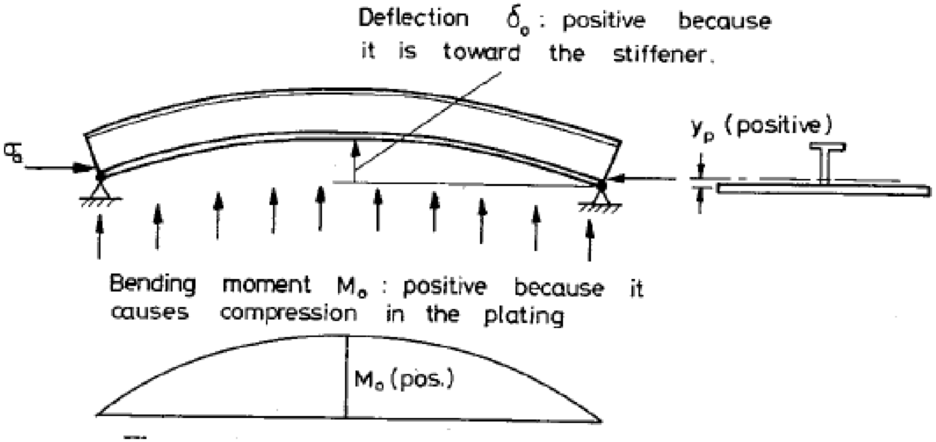


Mode II : Compression failure of the plating

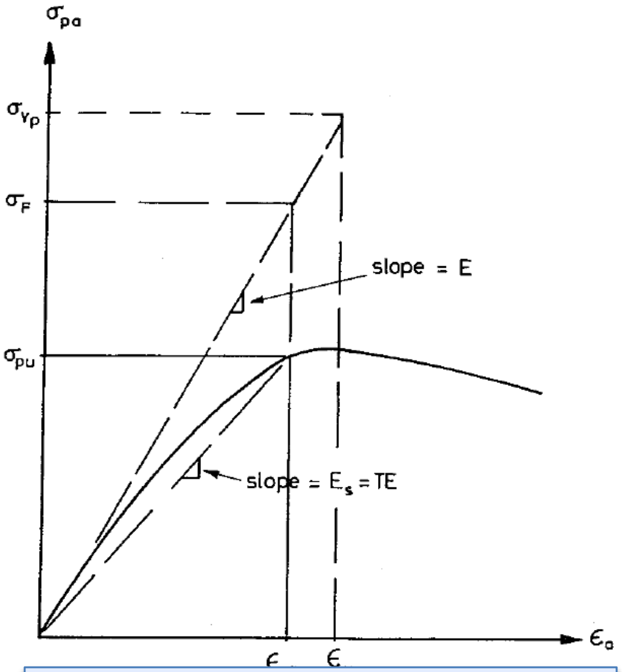
- If bending is small or moderate in magnitude, collapse occurs due to compression failure of the plate flange.

- If the plating remained perfectly elastic, with no buckling or nonlinearity, the total applied stress acting on the plating , σ_{pa}, would be given by the usual beam-column formula

$$\sigma_{\mathrm{pa}} = \sigma_a + \frac{M_0 y_p}{I} + \frac{\sigma_a A\left(\delta + \delta_0\right) y_p}{I} \phi$$



In-plane compression and positive bending

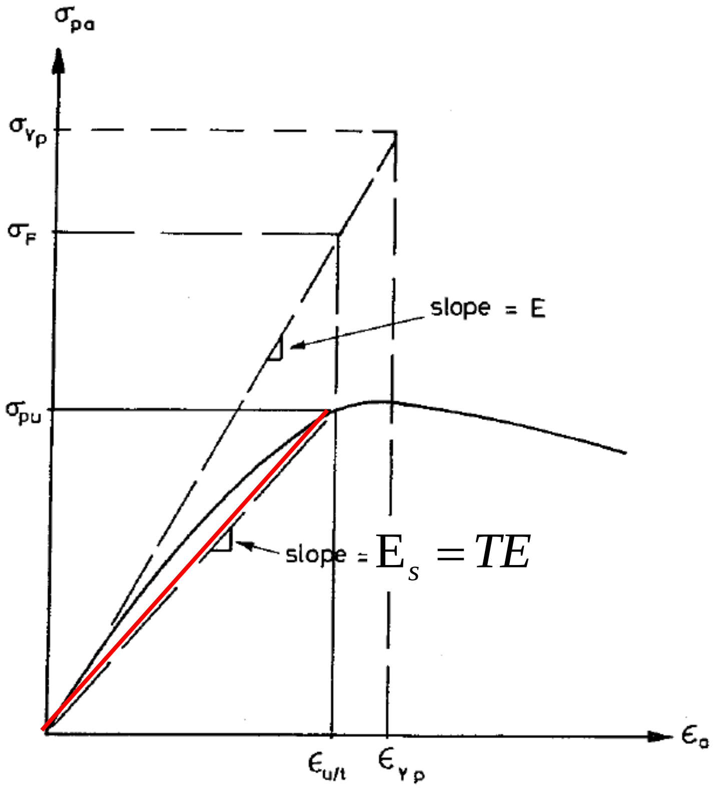


Typical curve of load versus end shortening for welded steel plating

Mode II : Compression failure of the plating

- The compressive collapse of welded plating is a complex inelastic process due to residual stress distribution
- The slope of the curve begins to decrease well before the “plate failure”
- σ_{pu}: true value of applied stress which corresponds to plate failure.
- The average plate stiffness is significantly less than the elastic or material stiffness E.
- The average stiffness over the entire range of loading is given by the secant modulus.

$$E_s = TE$$



Typical curve of applied load versus end shortening for welded steel plating

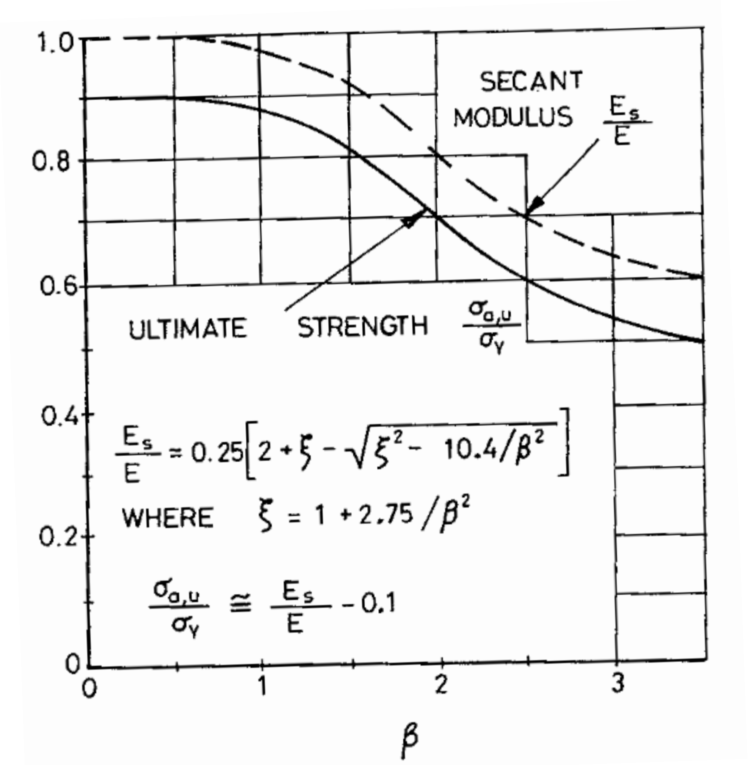
Mode II : Compression failure of the plating

- The mathematical expression for the secant modulus (Ch. 12) was given in which for convenience is

$$T = \frac{E_s}{E} = 0.25\left(2 + \xi - \sqrt{\xi^2 - \frac{10.4}{\beta^2}}\right)$$

where $\xi = 1 + \frac{2.75}{\beta^2}$

$$\frac{\sigma_{pu}}{\sigma_{Yp}} = T - 0.1 = 0.25\left(1.6 + \xi - \sqrt{\xi^2 - \frac{10.4}{\beta^2}}\right)$$



- Plate flange has a different elastic modulus of the plate flange than stiffener flange → equivalent section which has a uniform elastic modulus.
- The area of plate flange is reduced $bt = b_{tr}t = Tbt$, $b_{tr} = Tb$
- Total area of the transformed section $A_{tr} = A_s + b_{tr}t$

Mode II : Compression failure of the plating

- Axial stress in the transformed section (tr) is

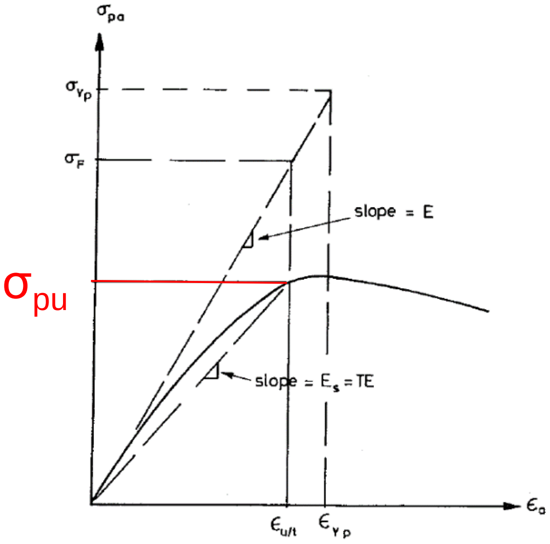
$$\sigma_{a,tr} = \sigma_a \frac{A}{A_{tr}} = \sigma_a \frac{A_s + bt}{A_s + b_{tr} t}$$

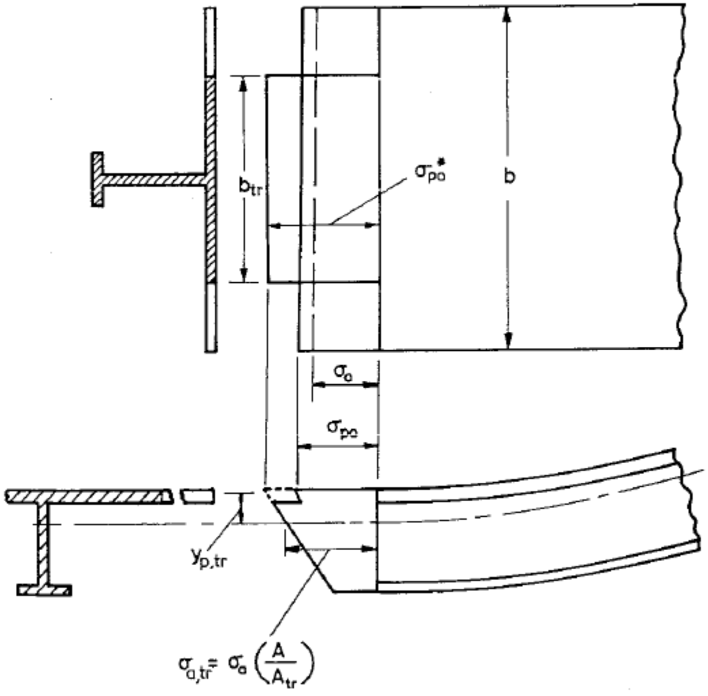
- We apply the ordinary beam column theory to the transformed section we obtain

$$\sigma_{pa} = T\sigma_{pa}^{*}$$

$$\sigma_{pa} = T\left(\sigma_{a,tr} + \frac{M_0 y_{p,tr}}{I_{tr}} + \frac{\sigma_{a,tr} A_{tr} (\Delta + \delta_0) y_{p,tr}}{I_{tr}} \phi \right)$$

- Collapse occur when σ_{pa} reaches σ_{pu}.





Use of transformed section to represent combined stiffener-and-plate as an equivalent elastic beam column

Mode III : Combined failure of stiffener and plating

- Large bending moment

→ tensile yielding in the stiffener flange & compressive failure of the plating

→ further reduction in the effectiveness of section

- As stiffener reaches full yield, neutral axis moves toward the plating and eventually enters the plating → partially in tension

  → it stabilizes the plating such that it can absorb the full compressive yield stress

- For a purely lateral load collapse does not occur until the panel has reached a plastic hinge condition at plating.
- Mode III : straight line between Point D & G

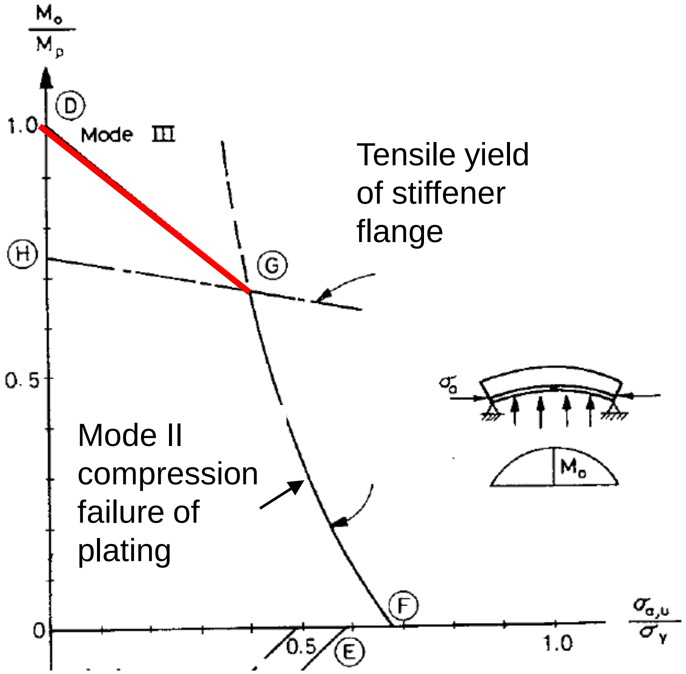
  Actual collapse points from experiments

  above line DG

- Analysis only for calculating Point G

  flange yield stress

  = ultimate stress of plating of Mode II



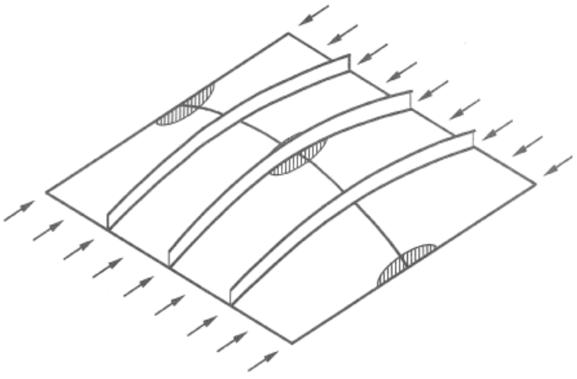
Tensile yield of stiffener flange

Mode II compression failure of plating

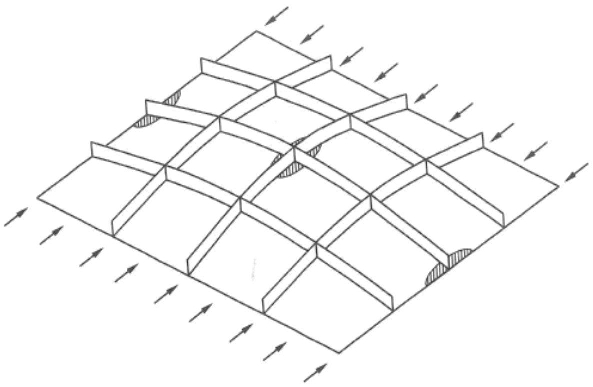
$$-\sigma_{Ys} = \left(\sigma_{a,tr}\right)_{ult} + \frac{M_0 y_{f,tr}}{I_{tr}} + \frac{\left(\sigma_{a,tr}\right)_{ult} A_{tr}(\delta_0 + \Delta) y_{f,tr}}{I_{tr}} \phi + \frac{\left(\sigma_{a,tr}\right)_{ult} A_{tr} \Delta_p y_{f,tr}}{I_{tr}}$$

6. Post buckling and ultimate strength behavior of stiffened panels and grillages

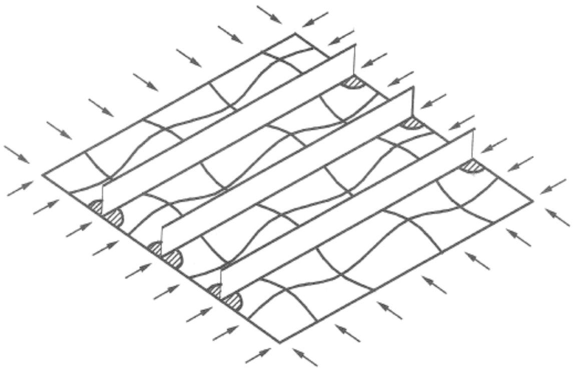
Collapse Modes of Stiffened plate



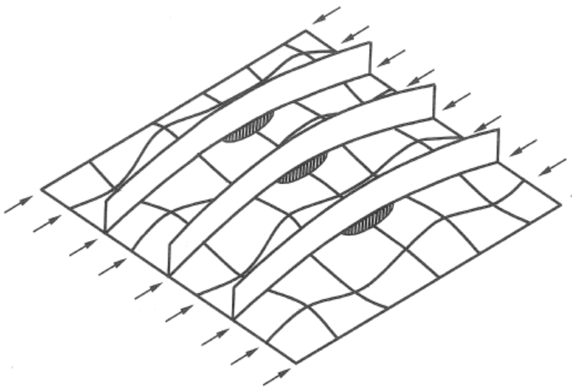
Mode I-1: Overall collapse of a uniaxially stiffened panel as a unit



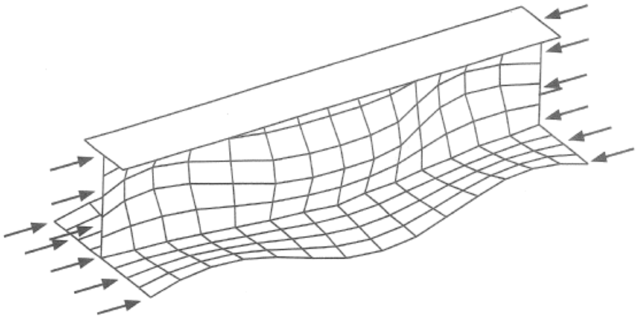
Mode I-2: Overall collapse of a cross-stiffened panel as a unit



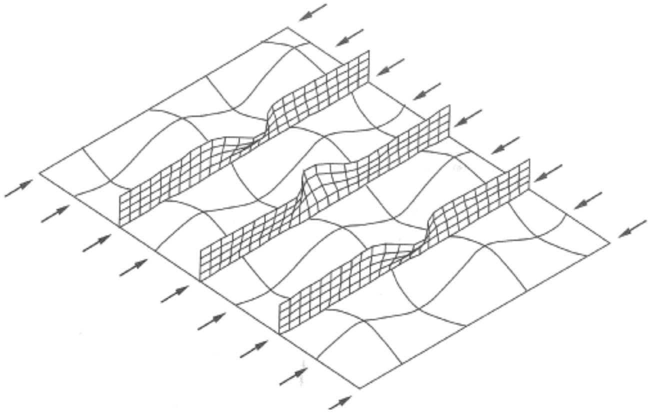
Mode II: Plate induced failure, biaxially compressive collapse, yielding at the corners of plating between stiffeners



Mode III: Plate induced failure – Beam-column type collapse, yielding of plate-stiffener combination at mid-span



Mode IV: Stiffener induced failure - local buckling of the stiffener web

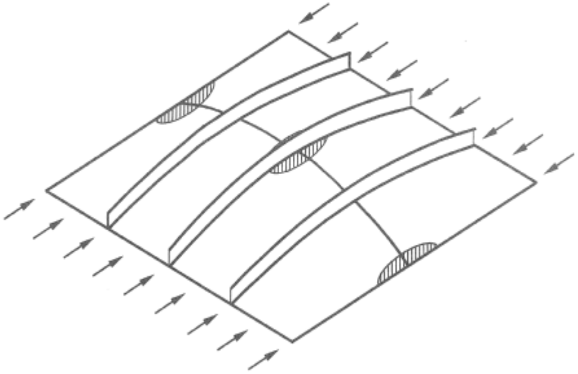


Mode V: Stiffener induced failure - lateral-torsional buckling of stiffener

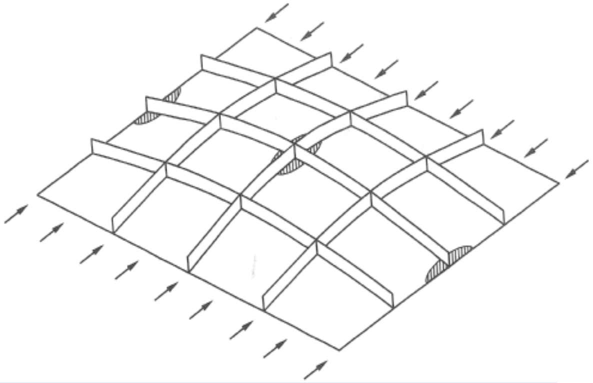
ULS for Collapse Mode I

❖ Mode I Overall collapse

- When the stiffeners are relative weak
- Stiffeners buckle together with plating as a unit
- The stiffened panel can normally sustain further loading even after overall buckling in the elastic regime occurs.
- Ultimate strength is eventually reached by formation of a large yield region inside the panel and/or along the panel edges.
- Mode I : beam-column type failure.
- Mode II : resembles that of ‘orthotropic plate’



Mode I-1: Overall collapse of a uniaxially stiffened panel as a unit

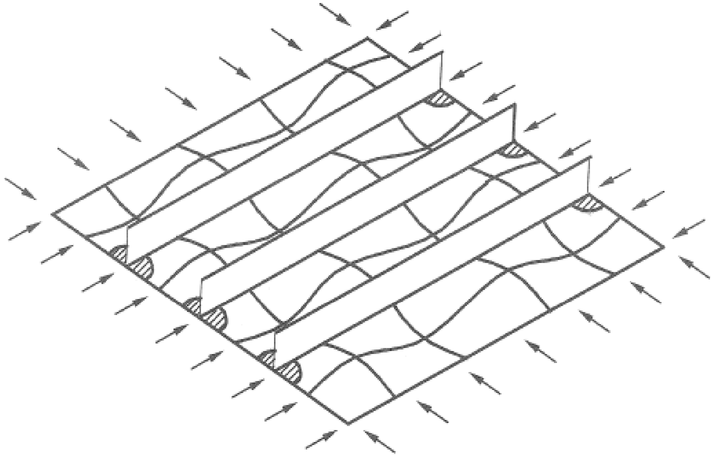


Mode I-2: Overall collapse of a cross-stiffened panel as a unit

ULS for Collapse Mode II

❖ **Mode II Plate induced failure - yielding at the corners of plating between Stiffeners**

- Panel collapses by yielding along the plate-stiffener intersection at the panel edge with no stiffener failure.
- The most highly compressed in both directions, intersection of two straight stiffeners.
- When predominantly subjected to biaxial compressive loads and/or when the plating is stocky, i.e. thick plating.

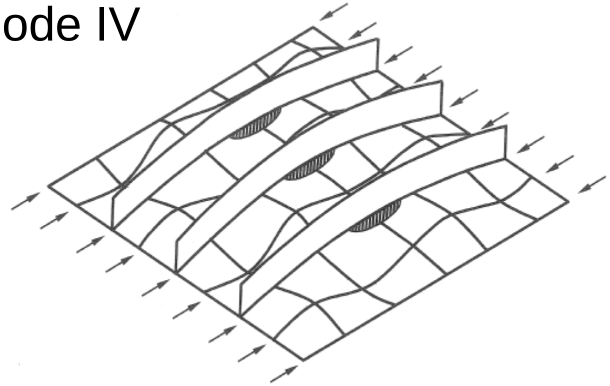


Mode II: Plate induced failure, biaxially compressive collapse, yielding at the corners of plating between stiffeners

ULS for Collapse Mode III

- **Mode III Plate induced failure - yielding of plate-stiffener combination at mid-span**
  - Ultimate strength is reached by yielding of the plate-stiffener combination at mid-span.
  - When the stiffener size are intermediate.
  - Beam-column type collapse , Perry-Robertson formula
    - ✓ Stiffener-induced failure or plate induced failure depending on the yield at tip of the stiffener or the plating

      → too pessimistic when the stiffeners are relatively small.
    - ✓ Stiffener can resist the further loading even after the first yielding occurs at the extreme fiber of the stiffener.
  - Local web buckling of stiffener web → Mode IV
  - Lateral-torsional buckling → Mode V



Mode III: Plate induced failure –
Beam-column type collapse,
yielding of plate-stiffener combination at mid-span

ULS for Collapse Mode III

- **Mode III Plate induced failure - yielding of plate-stiffener combination at mid-span**
  - **Combined Longitudinal Axial Load and Lateral Pressure**
  - Effectiveness of plating is evaluated by considering
    - ✓ Initial deflection
    - ✓ Residual stress
  - Subjected to axial load and lateral pressure.
  - For relatively weak stiffener, Perry-Robertson approach

    ultimate axial compressive stress < bare plate.

    while orthotropic plate approach < real panel ultimate strength

Lower limit of the panel ultimate strength

= weighted average of bare ultimate strength(σ_{xu}^{GB}) and the orthotropic plate ultimate strength (σ_{xu}^{GO}).

$$|\sigma_{xu}^{\mathrm{III}}| \geq \left| \frac{t\sigma_{xu}^{GB} + t_{xeq}\sigma_{xu}^{GO}}{t + t_{xeq}} \right| \qquad t_{xeq} = t_{plate} + \frac{n_{stiff} A_{stiff}}{B}$$

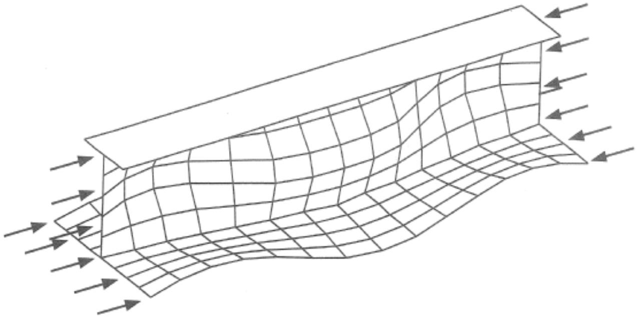
ULS for Collapse Mode IV

❖ Mode IV Stiffener induced failure - local buckling of the stiffener web

- When the ratio of stiffener web height to stiffener web thickens is large.
- When the type of the stiffener flange is inadequate to remain straight.
- Once web buckling occurs, the bucked or collapsed plating may be left with little stiffening

→ ULS is immediately reached immediately after web local buckling

- Ultimate strength = local buckling strength considering the rotational restraints along plate-stiffener and stiffener web-flange junctions.



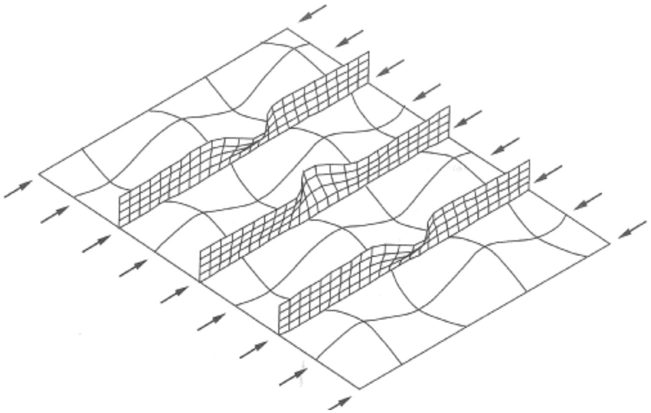
Mode IV: Stiffener induced failure - local buckling of the stiffener web

ULS for Collapse Mode V

❖ **Mode V Stiffener induced failure - lateral-torsional buckling of stiffener**

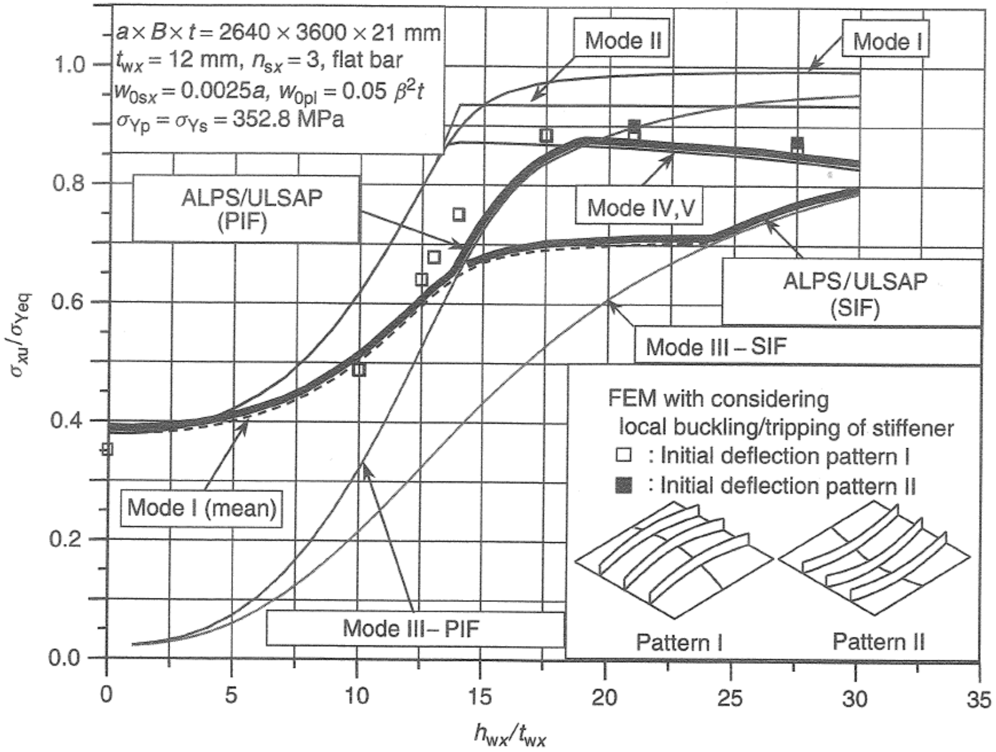
- Ultimate strength is reached subsequent to lateral-torsional buckling (tripping).
- When the torsional rigidity of the stiffener is small or the stiffener flange is weak.
- Once web buckling occurs, the bucked or collapsed plating may be left with little stiffening

→ The stiffened panel is considered to collapse if tripping occurs.



Mode V: Stiffener induced failure - lateral-torsional buckling of stiffener

Verification example

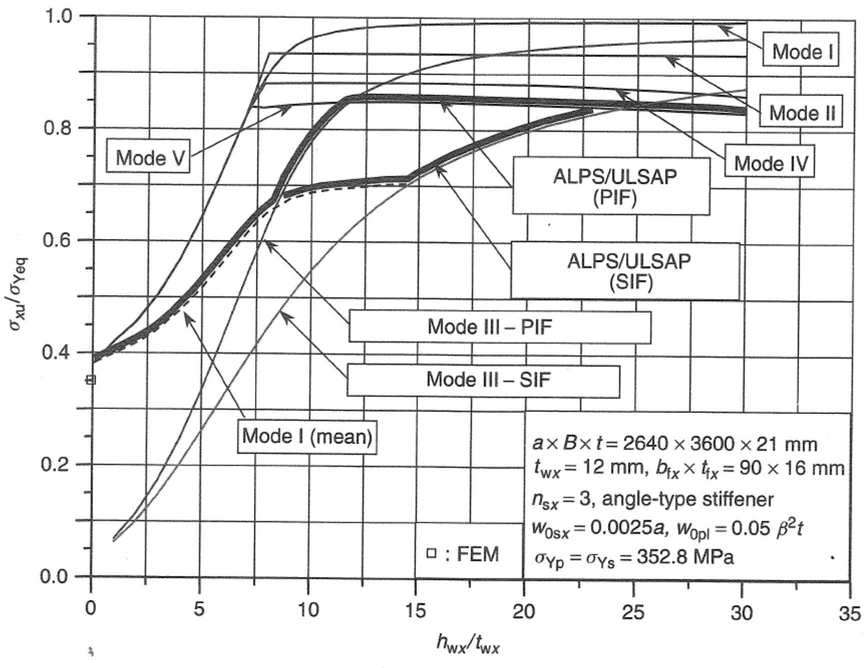


Ultimate strengths of longitudinally stiffened panels with three flat bars under uniaxial compression

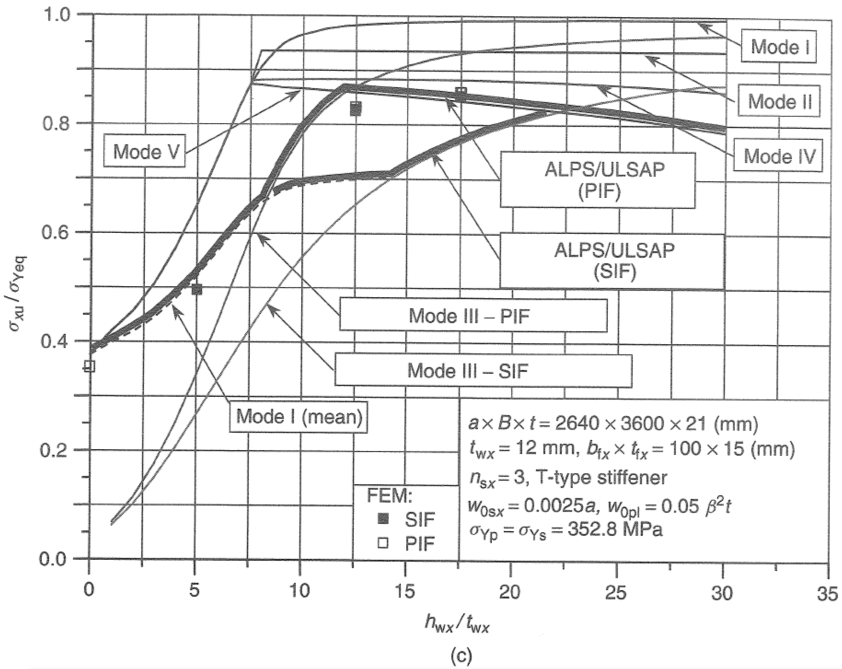
$$|\sigma_{xu}^{\mathrm{III}}| \geq \left| \frac{t\sigma_{xu}^{GB} + t_{xeq}\sigma_{xu}^{GO}}{t + t_{xeq}} \right|$$

- Mode I: Overall collapse
- Mode I (mean) : bare plate and the orthotropic plate ultimate strength
- Mode III: beam column type collapse, Plate induced failure or Stiffener induced failure
- Mode IV, V : Stiffener web buckling, tripping

Verification example



Ultimate strengths of longitudinally stiffened panels with three angle bars under uniaxial compression



Ultimate strengths of longitudinally stiffened panels with three Tee-stiffeners under uniaxial compression

PULS – Nonlinear Plate theory

❖ Elastic Buckling Analyses and Plasticity correction

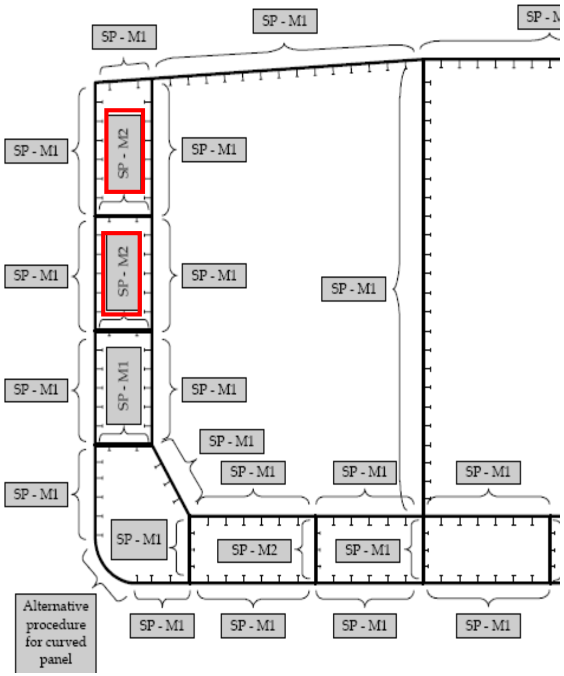
- Uni-axial buckling of plating

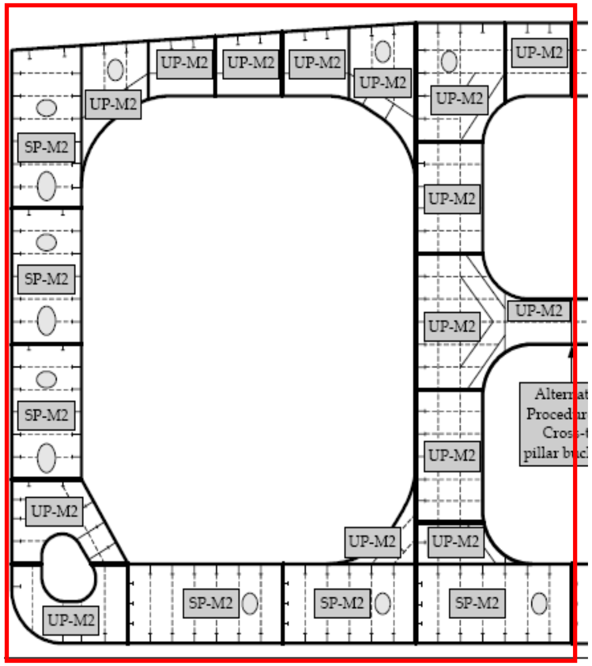
❖ Advanced Buckling Analyses

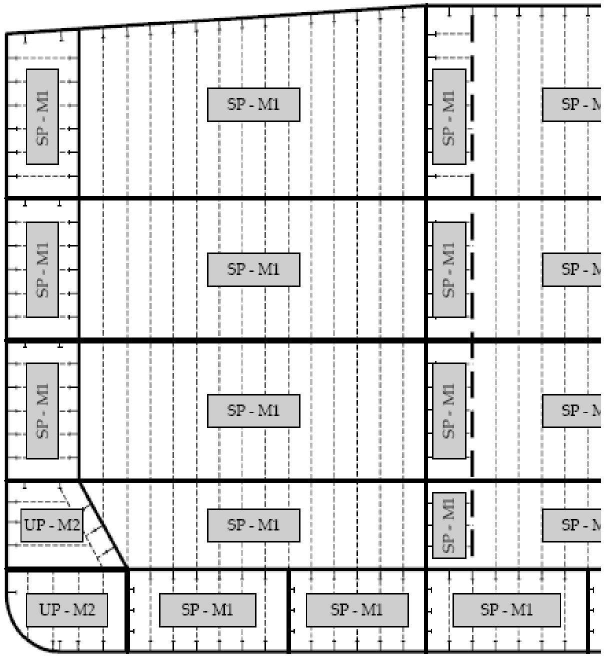
- Geometric nonlinearity
- Material nonlinearity
- initial imperfection
- Welding residual stress
- Interactions between Plate, Stiffener, girders
- Interaction between bi-axial compression, shear lateral pressure
- boundary condition effect

Structural Modeling and Capacity Assessment Method

- SP – M1 : Stiffened panel – Method 1 (with Stress redistribution after local yielding)
- SP – M2 : Stiffened panel – Method 2 (without Stress redistribution after local yielding)
- UP – M2 : Unstiffened panel – Method 2 (without Stress redistribution after local yielding)







UNSTIFFENED PLATE

❖ UNSTIFFENED PLATE (U3)

i) Elastic buckling is accepted

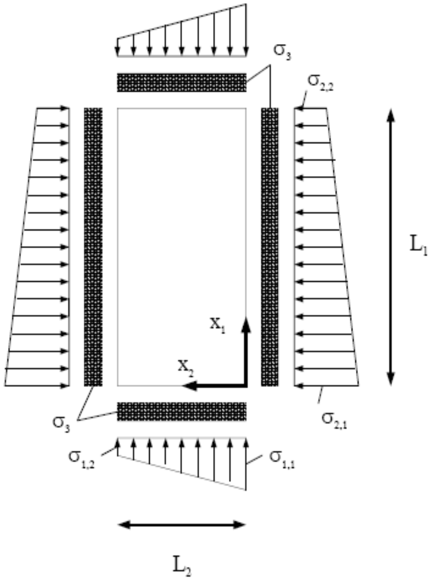
ii) Permanent buckles are not accepted

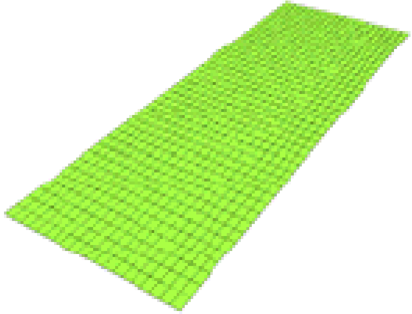
iii) Local eigen value Buckling (LED) calculation

iv) nonlinear postbuckling analysis,
Calculation of Ultimate Capacity (UC)

v) Buckling load = Min. (UC, LED)

vi) Default imperfection to be considered





STIFFENED PLATE (S3)

- **STIFFENED PLATE (S3)**
- **General**
  - ✓ The stresses in the hard corners are calculated as the sum of the direct applied membrane stresses added to the second order membrane stress due to buckling.
  - ✓ The second order membrane stresses have contributions from the local buckling of the plate between stiffeners-sideways/torsional buckling of the stiffeners and global buckling of the stiffeners
  - ✓ Bending stresses across any component plate thickness are not included in the limit state yield criteria.

i) Elastic local buckling of a panel is accepted (Local Eigen Value)

- plate buckling, torsional stiffener buckling, stiffener web buckling

ii) Permanent buckles are not accepted (Ultimate strength evaluation)

- By ensuring the maximum membrane stresses within a panel to stay below the yield stress condition
- Six limit states

ii) Global buckling is not accepted (Overall Elastic Eigen value)

- This principle ensures the panel as a whole to have sufficient out-of plane bending stiffness to avoid global buckling

STIFFENED PLATE (S3)

- The **ultimate limit state** calculation procedure for the S3 element can be split into three levels:
  - i) Local level: Establishment of orthotropic macro material coefficients and assessment of local eigenvalues. Non-linear analysis.

  - ii) Global level: Eigenvalue calculation of the global/overall mode (GEB), and nonlinear global deflection analysis, including knock down effects from local buckling, postbuckling and local imperfections/residual stresses.

  - iii) Ultimate limit state: Global non-linear analysis with explicit solution of different limit state functions for identifying the most critical failure hot spot location and corresponding loads acting on the panel.

STIFFENED PLATE (S3)

❖ General available results

i) Local eigenvalue (LEB) (SLS : Service Limit State)
ii) Global eigenvalue (GEB)
iii) Ultimate capacity stress (UC)
iv) Buckling load = minimum of eigenvalues and ultimate load
v) Usage factor η with reference to ultimate load and buckling load

i) and ii). The eigenvalue and the corresponding eigenmode are ideal elastic buckling stresses with associated buckling shape for a stiffened panel with perfect flat geometry.

iii) The ultimate stress (UC) is the maximum nominal stress the panel can carry for the defined proportional load history.

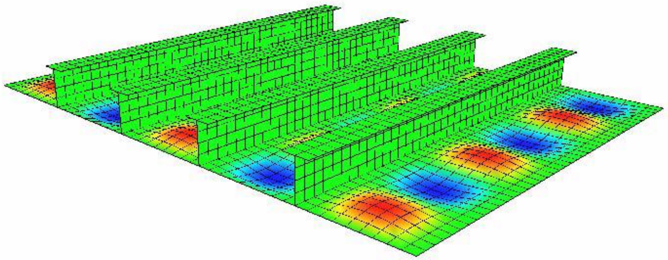
iv) The buckling load is defined as the minimum of the eigenvalues and the ultimate load(ii).

v) The UC usage factor describes the margin between the applied loads and the corresponding ultimate capacity stresses.

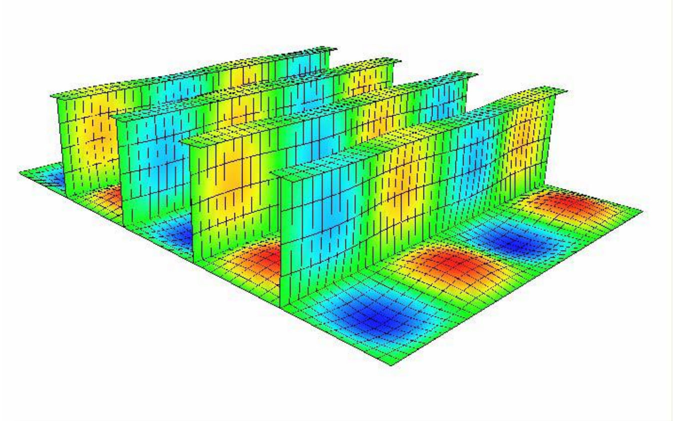
Local level: Elastic Eigenvalue and reduced stiffness properties

❖ Three categories of local buckling modes

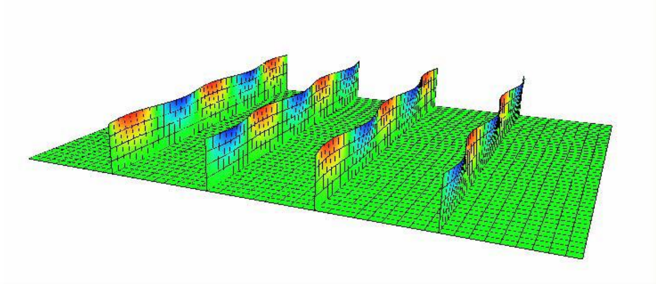
- the eigenvalue and the postbuckling strength of panels buckling in local modes
- Local geometrical imperfection effects and residual stress effects are implicitly considered in a set of orthotropic macro material coefficients.
- The local buckling is accounted for on the overall strength by reducing the orthotropic macro material coefficients in an orthotropic panel



i) plate buckling



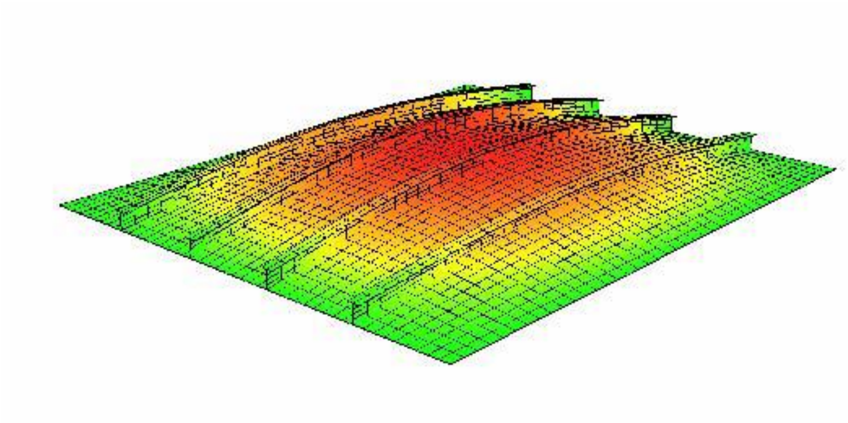
ii) torsional stiffener buckling :
interaction with plate buckling and some torsional stiffener buckling



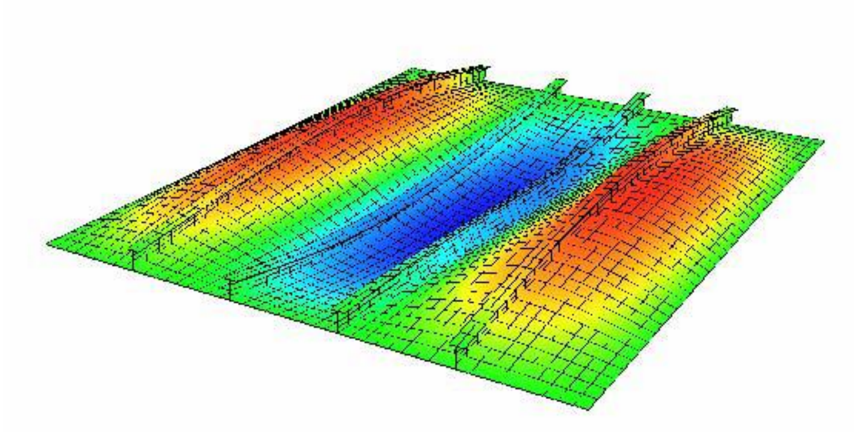
iii) stiffener web plate buckling
tall profiles and flat bar profiles

Global level: Overall elastic eigenvalue of panel; GEB

- Overall mode lifting the stiffeners out-of-plane together with the continuous plating assuming lateral support along all four outer edges
- Using the orthotropic plate theory with modified orthotropic macro material coefficients accounting for the local buckling effects.



Example for an axially compressed panel



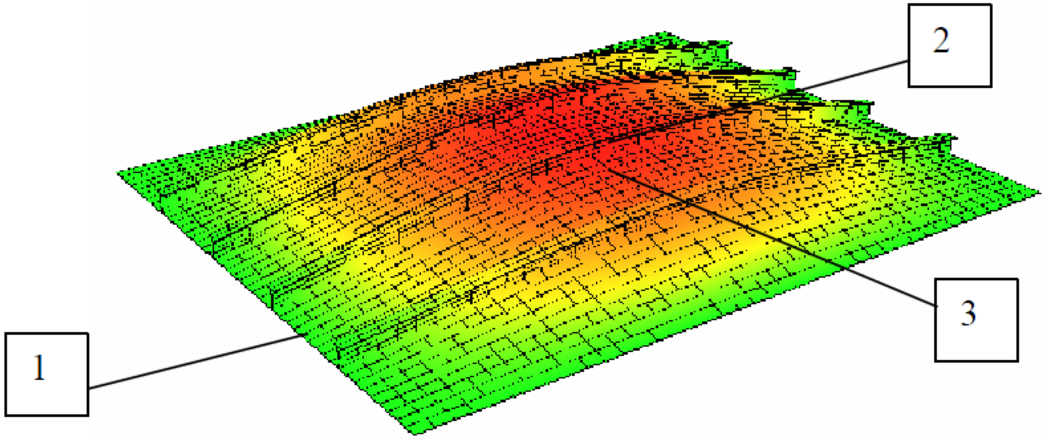
Example for a transversely compressed and shear loaded panel

Local stress limit states: Ultimate strength evaluation

- **i = 1; Plate criterion: Stress control along plate edges**
  - based on max edge stresses along supported edges
  - typical: transverse load when local buckling dominates

- **i = 2; Stiffener tension criterion: Stress control at midspan $x_1 = L_1/2$**
  - for global panel deflecting towards stiffener flange, tension criterion
  - kick in for transverse compressive loads for panel with small stiffeners, i.e. large global effects

- **i = 3; Plate compression criterion: Stress control at midspan $x_1 = L_1/2$;**
  - in plating for global panel deflecting towards stiffener flange, compression criterion (PI collapse)

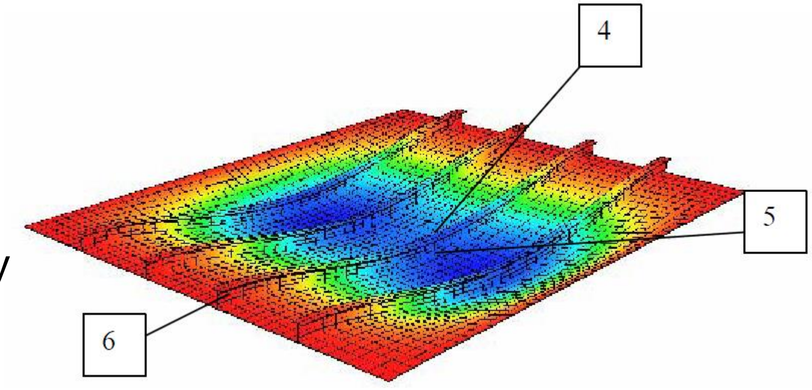


Local stress limit states: Ultimate strength evaluation

- **i = 4; Stiffener compression criterion: Stress control at midspan**
  - in stiffener flange for global panel deflecting towards plating, compression criterion (SI collapse) (typical for pure axial load)
- **i = 5; Plate tension criterion: Stress control in plate; at midspan**
  - in plating for global panel deflecting towards plating, tension criterion

- **i = 6; Stiffener bending stress criterion at support: Stress and capacity control at support $x_1 = 0$;**
  - compressive or tension criterion, kicks in for cases with lateral pressure.
  - Control the bending and shear capacity of the stiffeners under the influence of combined lateral load and in-plane loads.

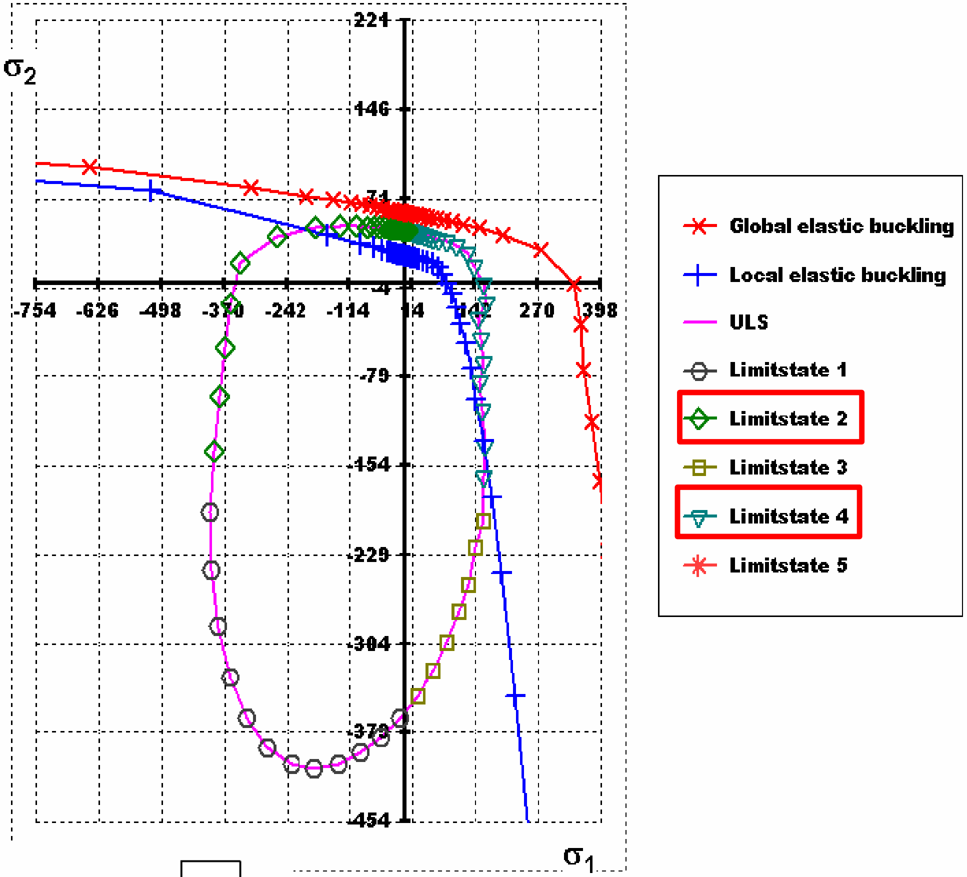
- Yielding in the stiffener flange at the transverse frames is accepted, since stiffeners have significant strength reserves after first yield when subjected to lateral pressure.
- The panel is loaded until the plastic capacity of the stiffeners is reached.

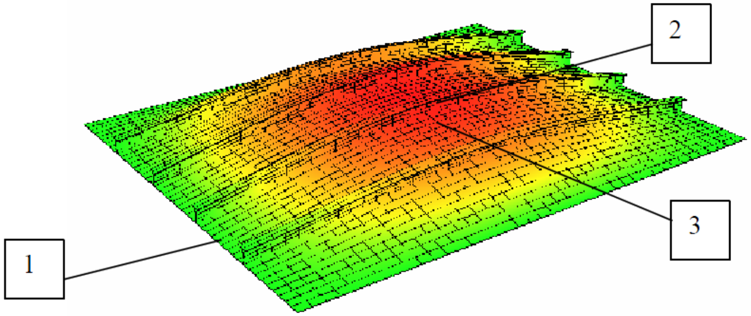


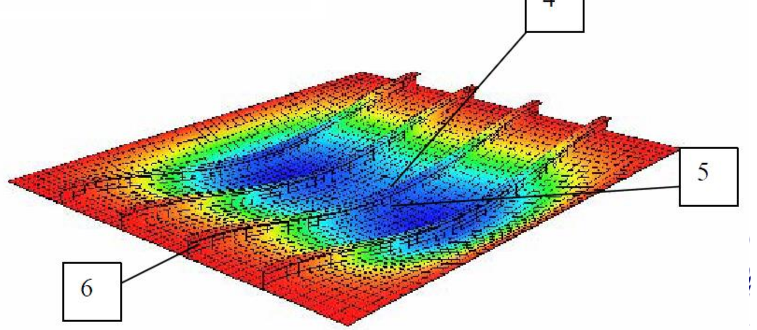
CSR Rule - PULS

Example of Capacity Curve I

- ❖ Bi-axial loading, all quadrants, thin plate

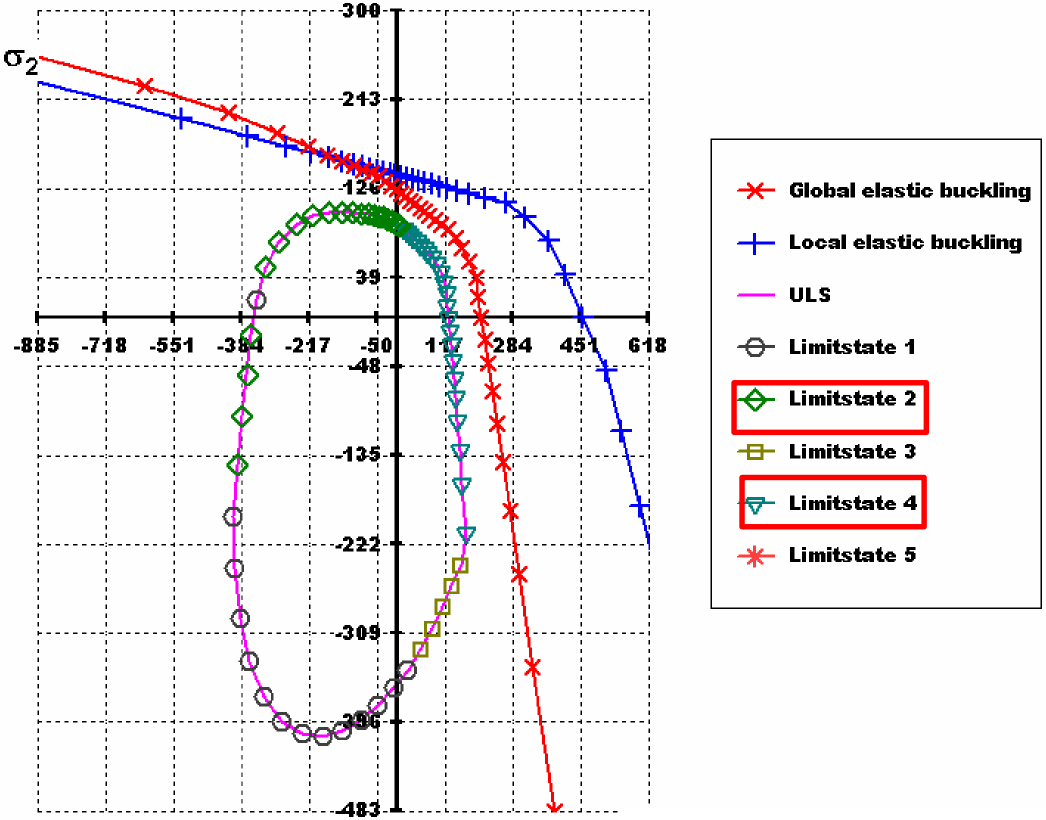


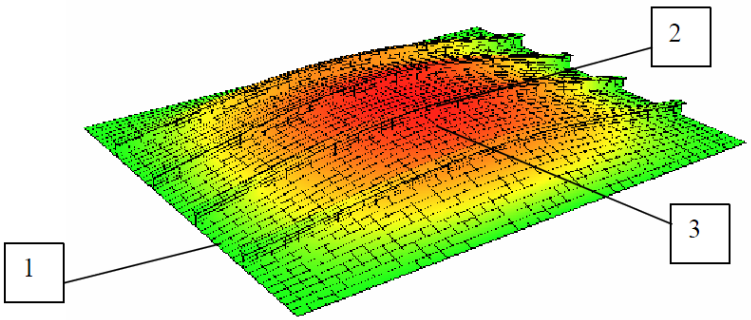


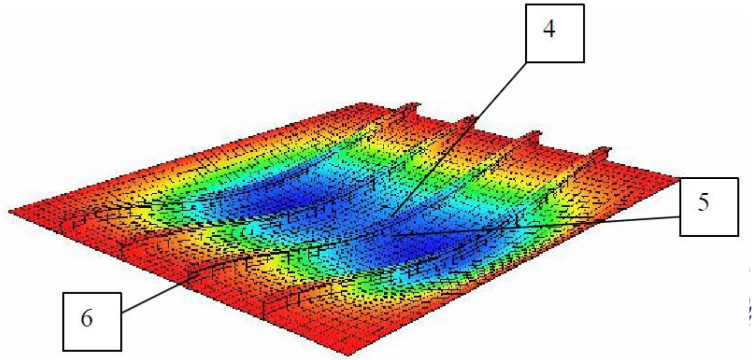


Example of Capacity Curve II

- bi-axial loading, all quadrants, thick plate







CSR Rule - PULS

Example of Capacity Curve III

- Transverse load σ_2 and shear σ_3 (τ_{12}), all quadrants.