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소성재료역학
(Metal Plasticity)

Chapter 15: Hardening

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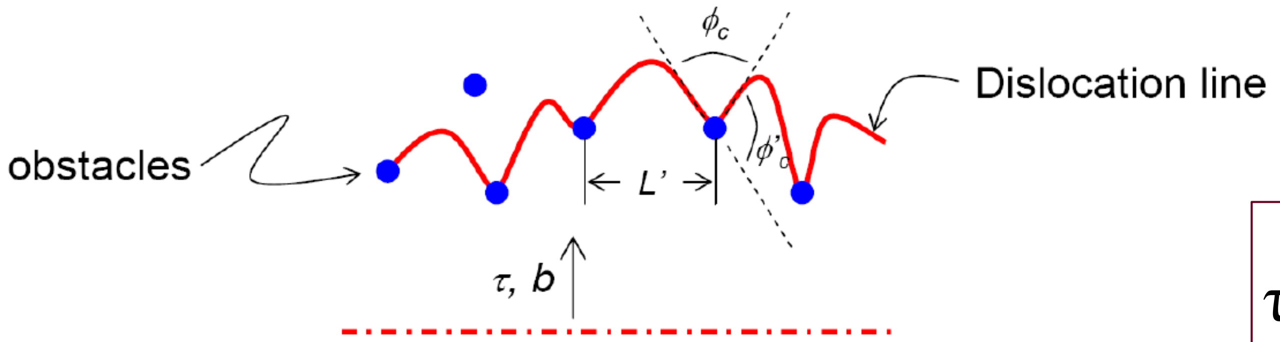
General principle for strengthening

- How to increase strength of metals?
  - Place obstacles in the path of dislocations, which inhibits the free movement of dislocations until the stress is increased to move them forward.



$$\tau \cong \frac{Gb}{L'}\cos\left(\frac{\varphi_c}{2}\right)$$

L' Effective particle spacing

ϕ'_c Critical angle to which the dislocation bends to breaking away from the obstacle

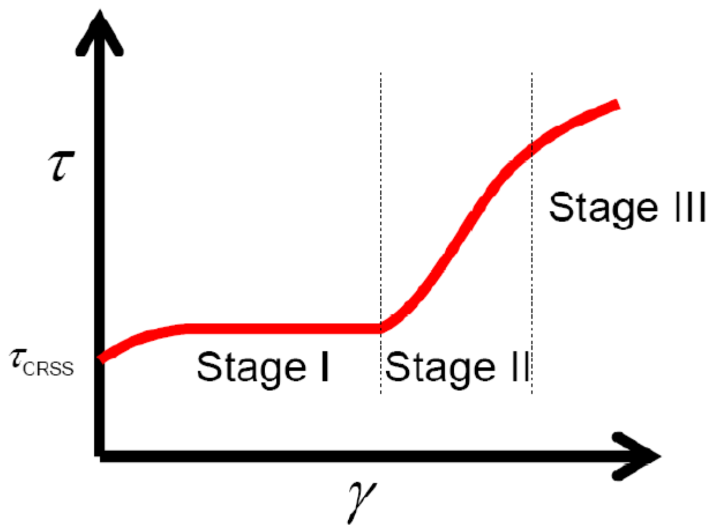
$$\varphi'_c = 180^\circ - \varphi_c$$

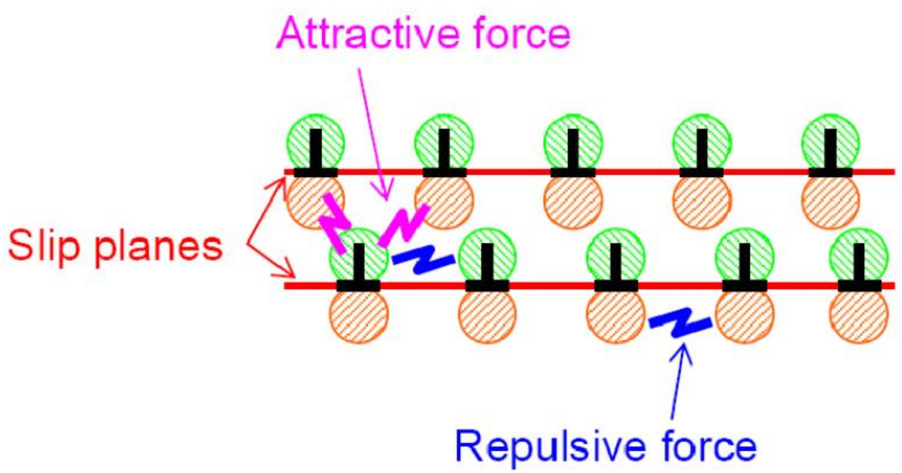
Hardening

Work hardening from physical metallurgy

- During plastic deformation, there is an increase in dislocation density. It is this increase in dislocation density which ultimately leads to work hardening
- Dislocation interact with each other and assume configurations that restrict the movement of other dislocations
- The dislocations can be either “strong” or “weak” obstacles depending on the types of interactions that occurs between moving dislocations

Work (strain) hardening of single crystal



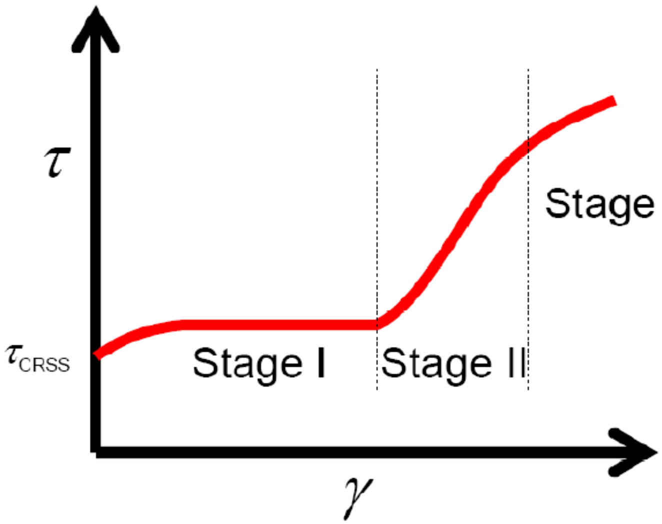


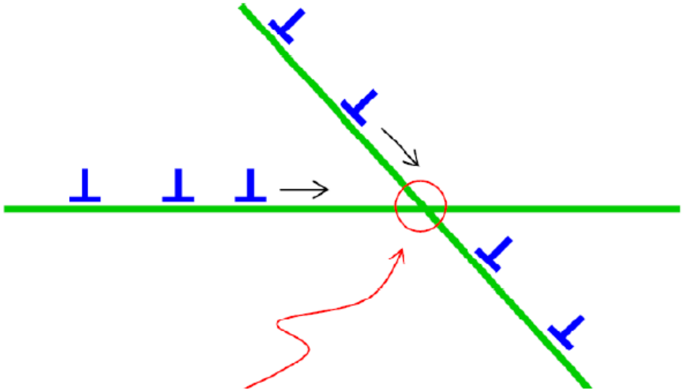
- Stage I: the stress fields interact during the early stages of deformation resulting in "weak" drag effects

- In stage I, dislocations are "weak" obstacles

Hardening

Work (strain) hardening of single crystal





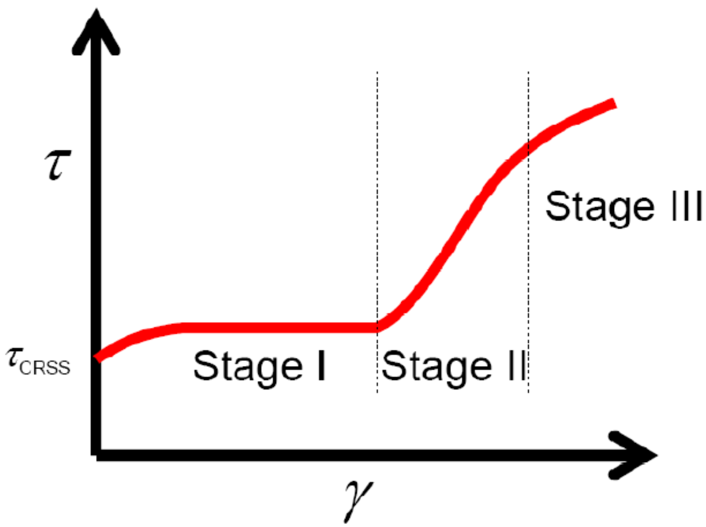
Interactions produce immobile dislocation configurations like jogs.

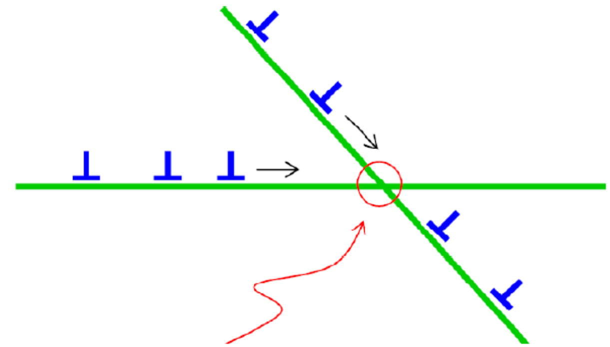
- Stage II: Linear hardening

- In stage II, work hardening depends strongly on dislocation density

Hardening

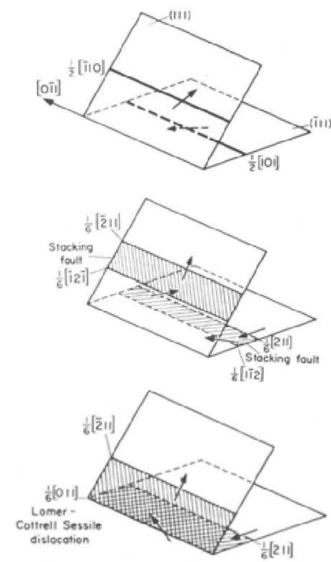
Work (strain) hardening of single crystal





Interactions produce immobile dislocation configurations like jogs.

- Stage II: Linear hardening
- In stage II, work hardening depends strongly on dislocation density
- Lomer-Cottrell locks (sessile dislocation)



Elastic Plasticity Models

Elastic-plasticity models in sheet metal forming

1) Yield function
2) Hardening model
3) Elastic modulus

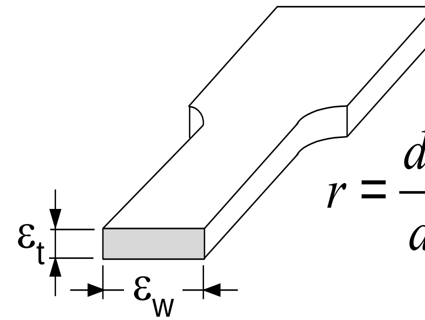
→ Accurate predictions for stress, deformation

Yield function

Elasticity models for FEM

❖ Yield functions

- Elastic vs. Plastic
- Rate of plastic strain



$$r = \frac{de_w}{de_t}$$

R-value

1D

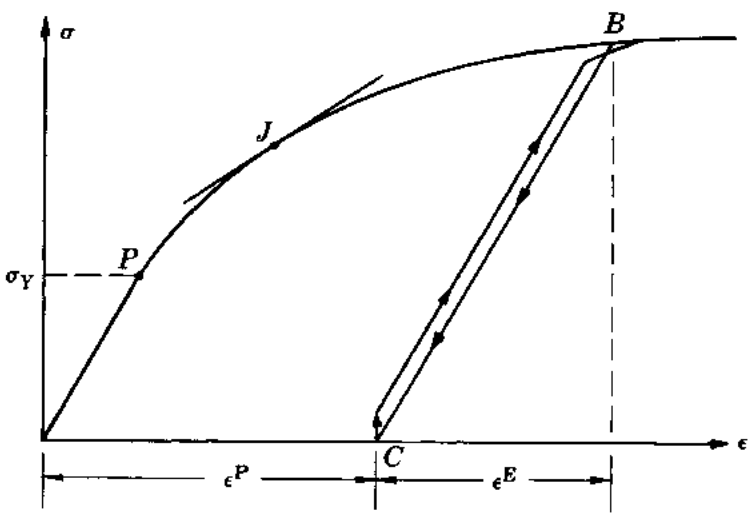
$$\sigma_Y$$

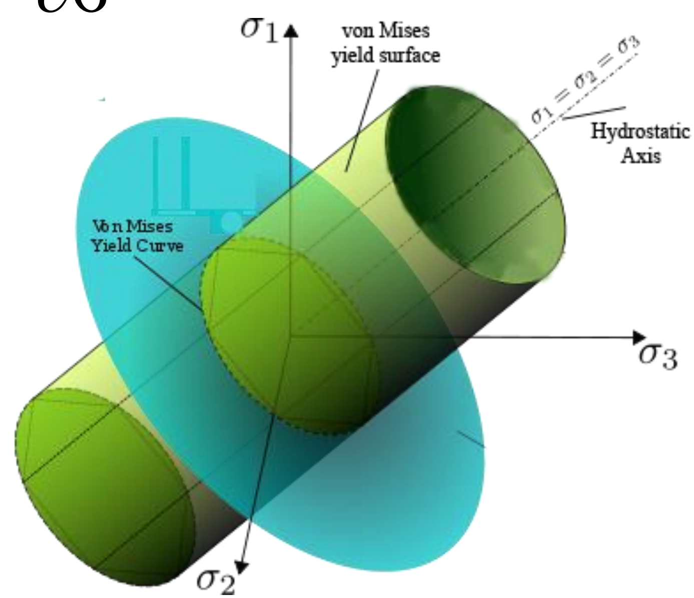
$$\varepsilon = \varepsilon^e + \varepsilon^p$$

3D (6D)

$$f(\sigma)$$

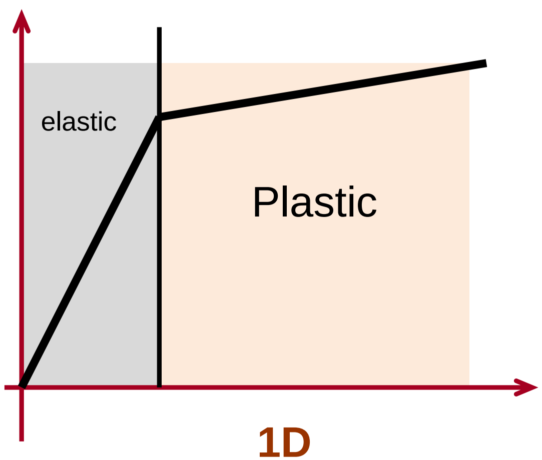
$$d\varepsilon^p = d\lambda \frac{\partial f}{\partial \sigma}$$

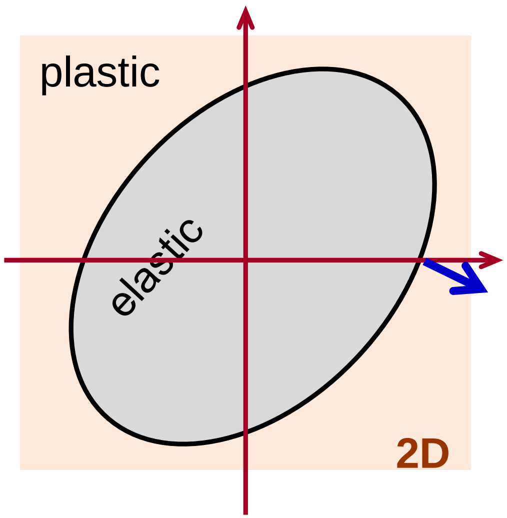


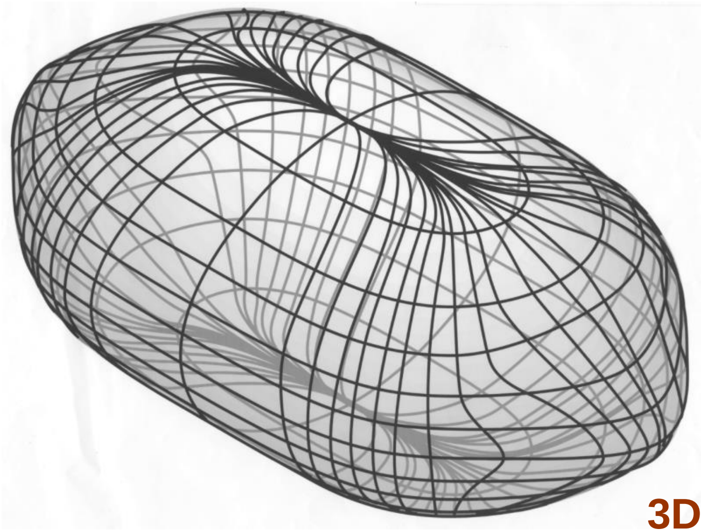


Yield functions

1-D 〉 2-D 〉 3-D (plane stress)

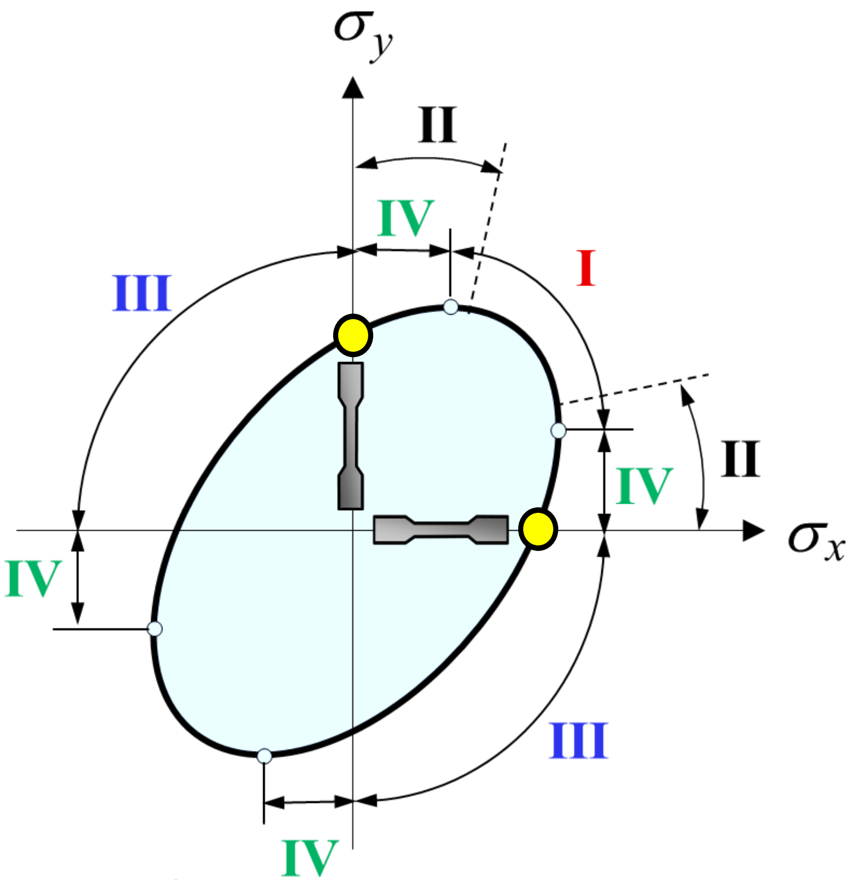


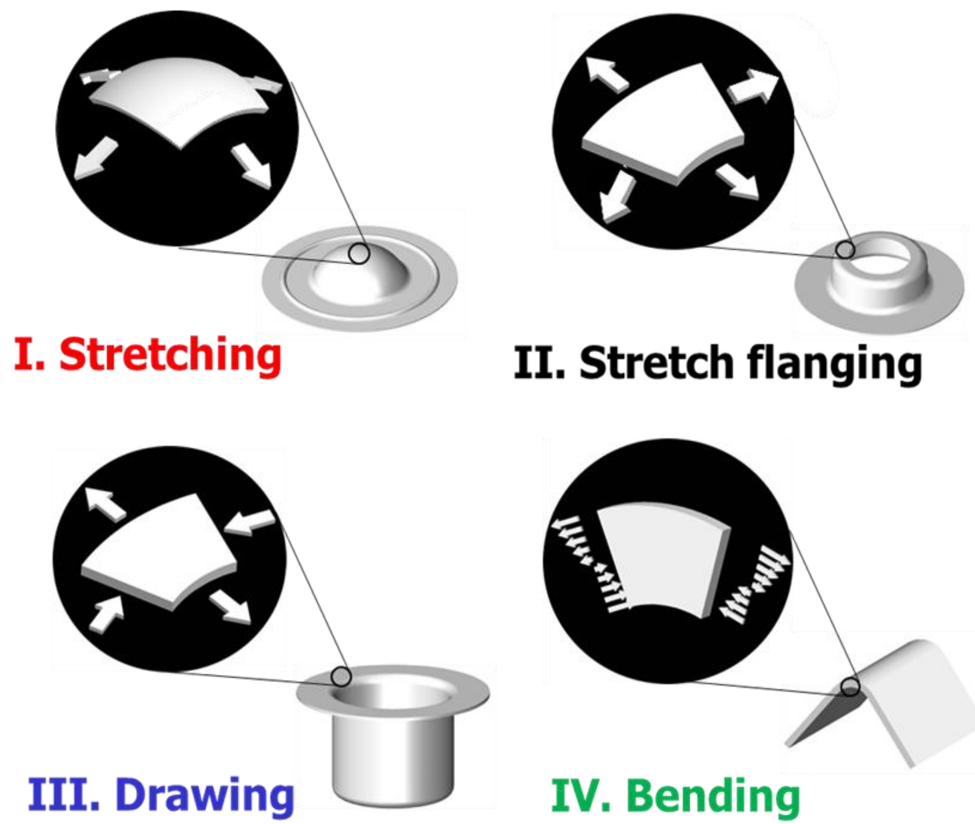




Yield function

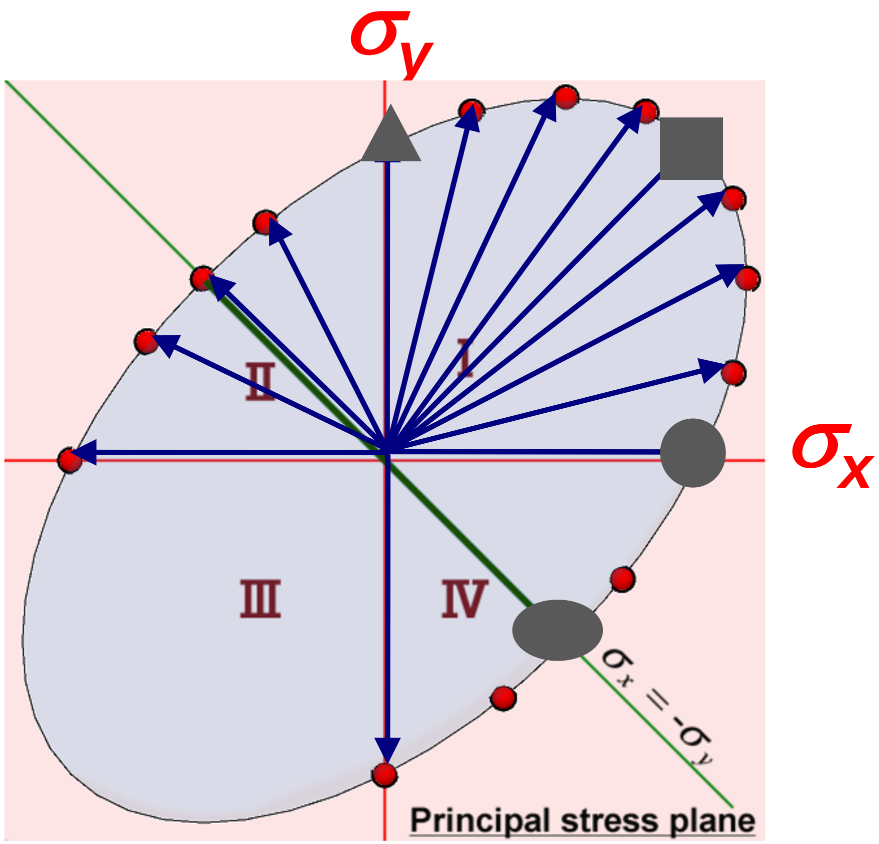
Plane stress



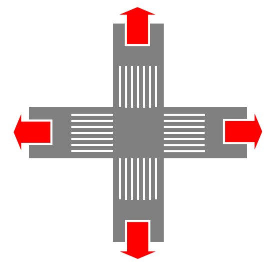


From Kuwabara

Yield function

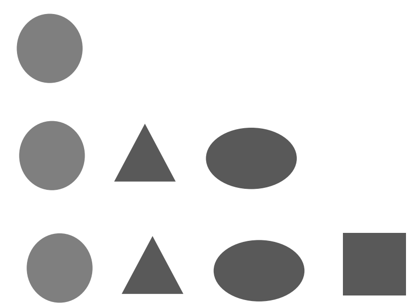


Uniaxial tension



Biaxial tension
(small strain range)

Uniaxial compression



Von Mises

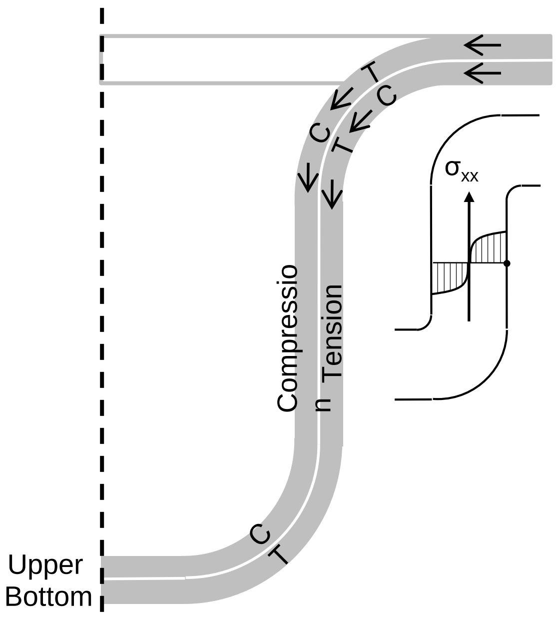
Hill 1948

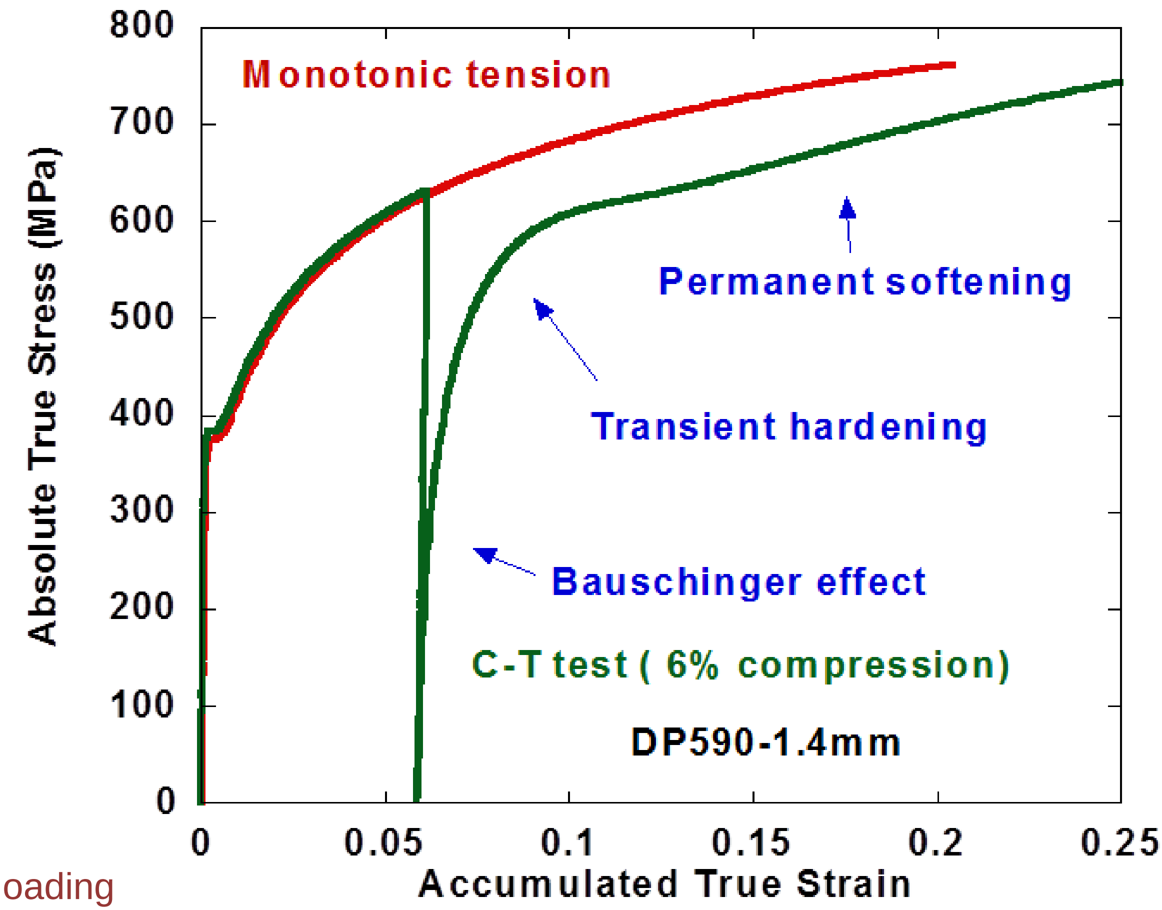
Barlat 2000-2d

From Kuwabara

Hardening law: importance

Hardening behavior





Sidewall region

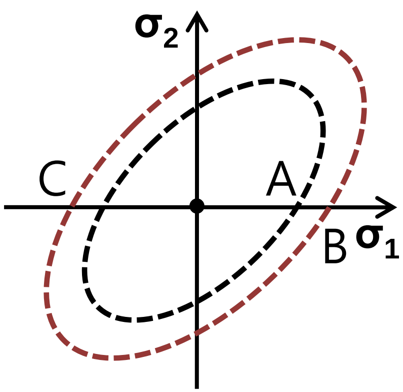
(Upper) **Tension – Compression** – Unloading
(Bottom) **Compression – Tension** – Unloading

Stress-strain behavior under reverse loading

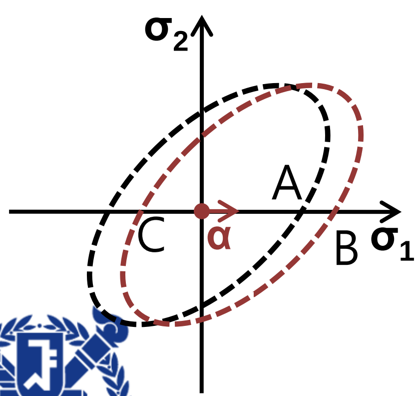
Hardening law: importance

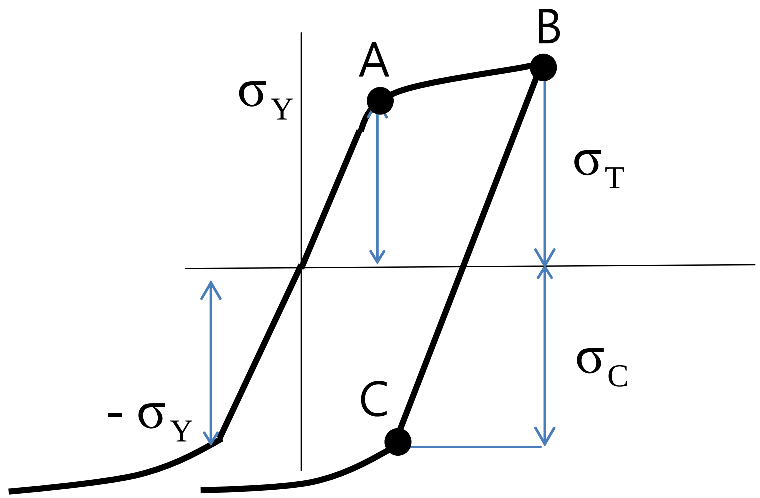
Hardening models

Isotropic hardening

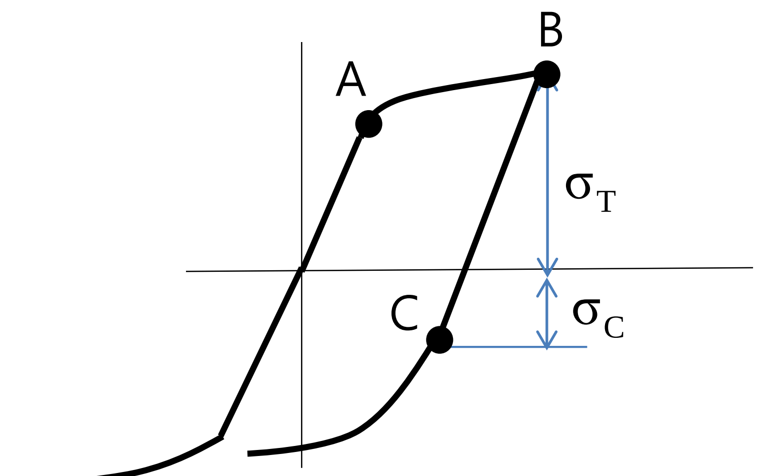


kinematic hardening





$$\sigma_T = |\sigma_C|$$

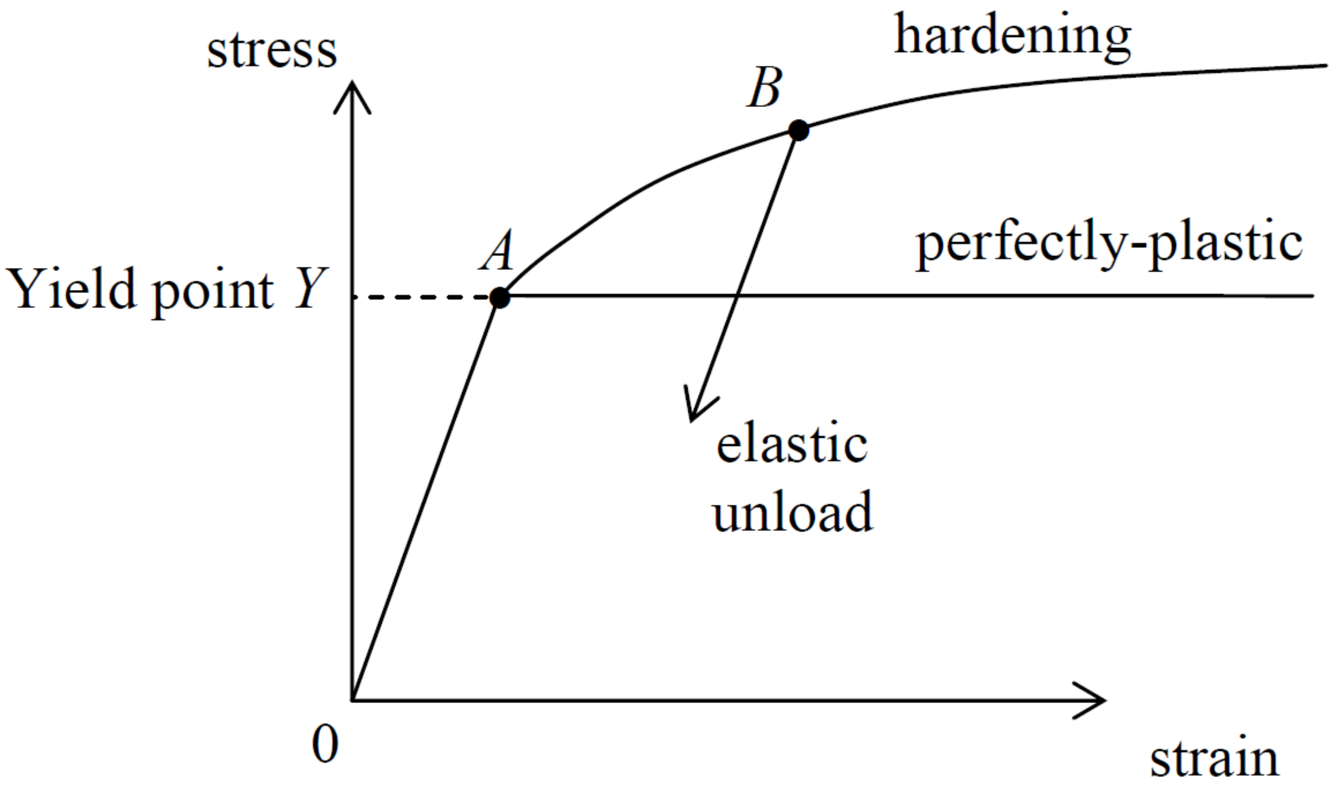


$$\sigma_T > |\sigma_C|$$

Bauschinger effect can be reproduced by kinematic hardening

Hardening law: basic

Uniaxial stress-strain curve of typical metals



Hardening law: basic

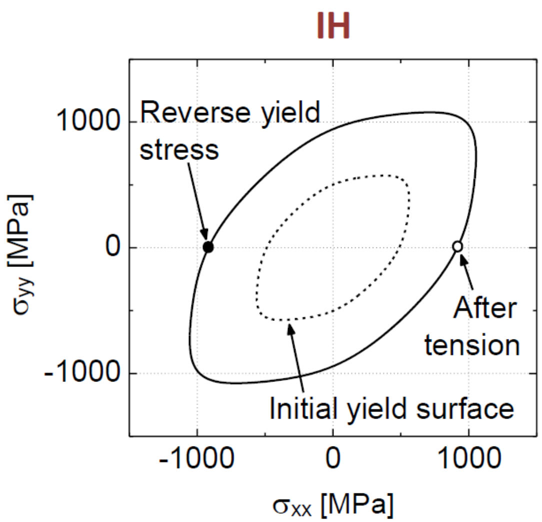
Initial yield surface

$$f(\sigma_{ij}) = 0$$

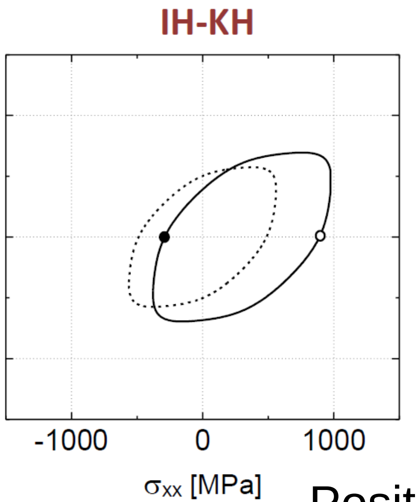
For perfect plasticity, the yield surface remains unchanged. But, in general case, <u>the size, position, shape</u> of the yield surface change

$$f(\sigma_{ij}, K_i) = 0$$

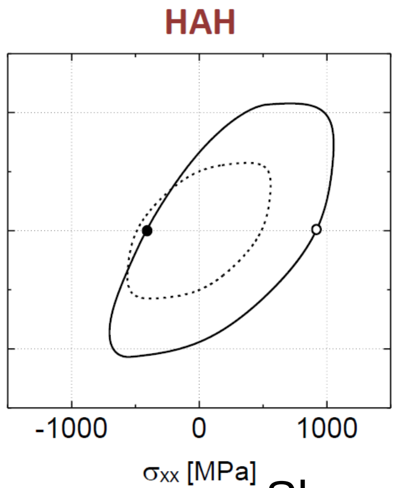
where K represents hardening parameters, which evolves during the plastic deformation



Size



Position



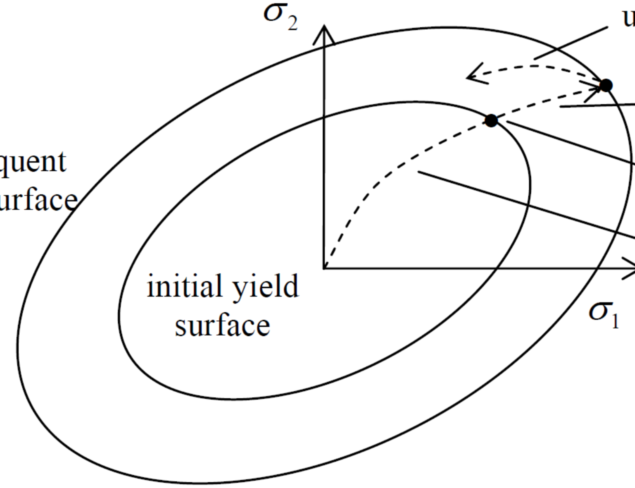
Shape

Hardening law: basic

Isotropic hardening

Yield function takes the following form: no change in position, shape, but size changes

$$\mathbf{f}\left(\boldsymbol{\sigma}_{ij}, \mathbf{K}_i\right) = \mathbf{f}\left(\boldsymbol{\sigma}_{ij}\right) - \mathbf{K} = \mathbf{0}$$



Von Mises

$$\mathbf{f}\left(\boldsymbol{\sigma}_{ij}, \mathbf{K}_i\right) = \mathbf{f}\left(\boldsymbol{\sigma}_{ij}\right) - \mathbf{K} = \sqrt{\frac{1}{2}\left[\left(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2\right)^2 + \left(\boldsymbol{\sigma}_2 - \boldsymbol{\sigma}_3\right)^2 + \left(\boldsymbol{\sigma}_3 - \boldsymbol{\sigma}_1\right)^2\right]} - \mathbf{Y} = \mathbf{0}$$

Hardening law: basic

Isotropic hardening

Size of yield surface = from a uniaxial tensile test

$$\mathbf{K} = \mathbf{C}\left(\boldsymbol{\varepsilon}^{\mathbf{p}}\right)^{\mathbf{n}}$$

Power law hardening

$$\mathbf{K} = \mathbf{Y}_0 + \mathbf{C}\left(\boldsymbol{\varepsilon}^{\mathbf{p}}\right)^{\mathbf{n}}$$

Ludwik model

$$\mathbf{K} = \mathbf{C}\left(\boldsymbol{\varepsilon}_0 + \boldsymbol{\varepsilon}^{\mathbf{p}}\right)^{\mathbf{n}}$$

Swift hardening model

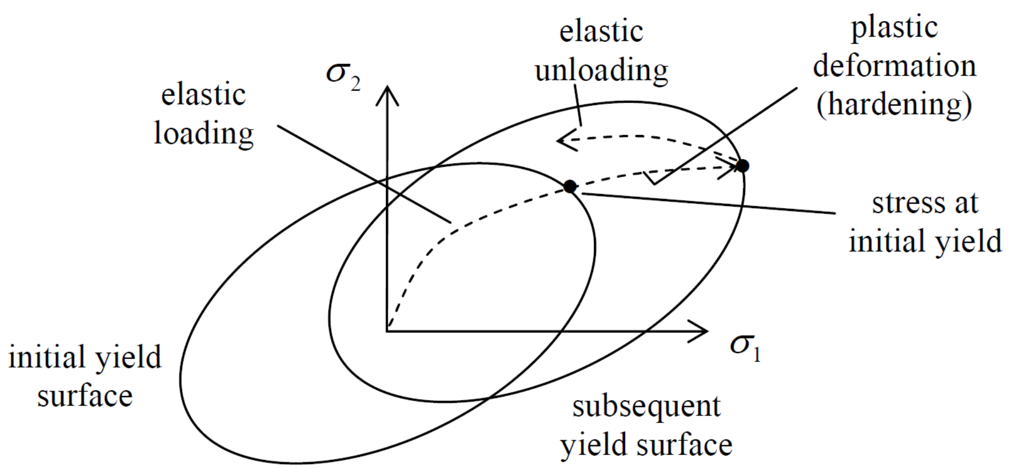
$$\mathbf{K} = \mathbf{Y}_\mathbf{0} + \mathbf{Y}_\mathbf{1}\left(\mathbf{1} - \mathbf{exp}\left(-\mathbf{C}\boldsymbol{\varepsilon}^{\mathbf{p}}\right)\right)$$

Voce hardening model

Hardening law: basic

Kinematic hardening

- Isotropic hardening: tensile yield stress = |compressive yield stress|
- No Bauschinger effect
- Kinematic hardening: Softening in the compression direction during the loading in tensile direction

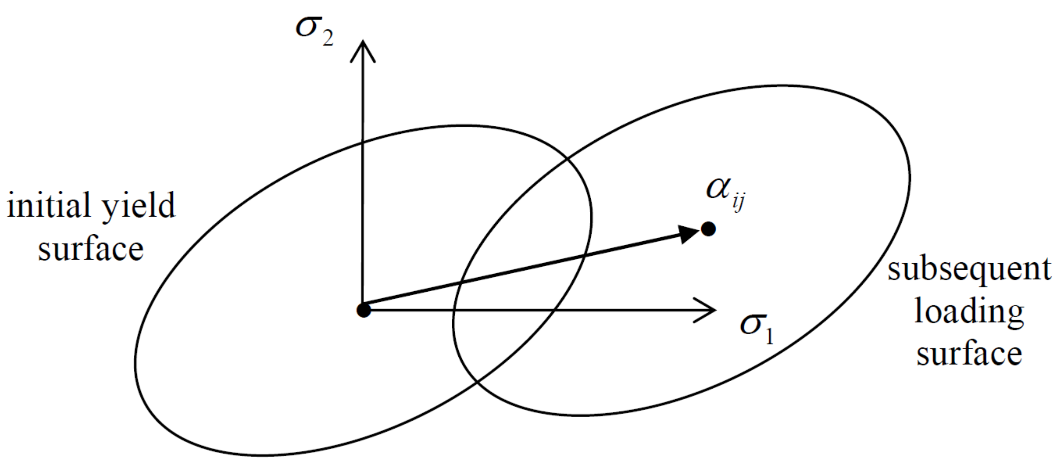


Hardening law: basic

Kinematic hardening

$$\mathbf{f}\left(\boldsymbol{\sigma}_{ij}, \mathbf{K}_i\right) = \mathbf{f}\left(\boldsymbol{\sigma}_{ij} - \boldsymbol{\alpha}_{ij}\right) = \mathbf{0}$$

The hardening parameter is called “back stress”

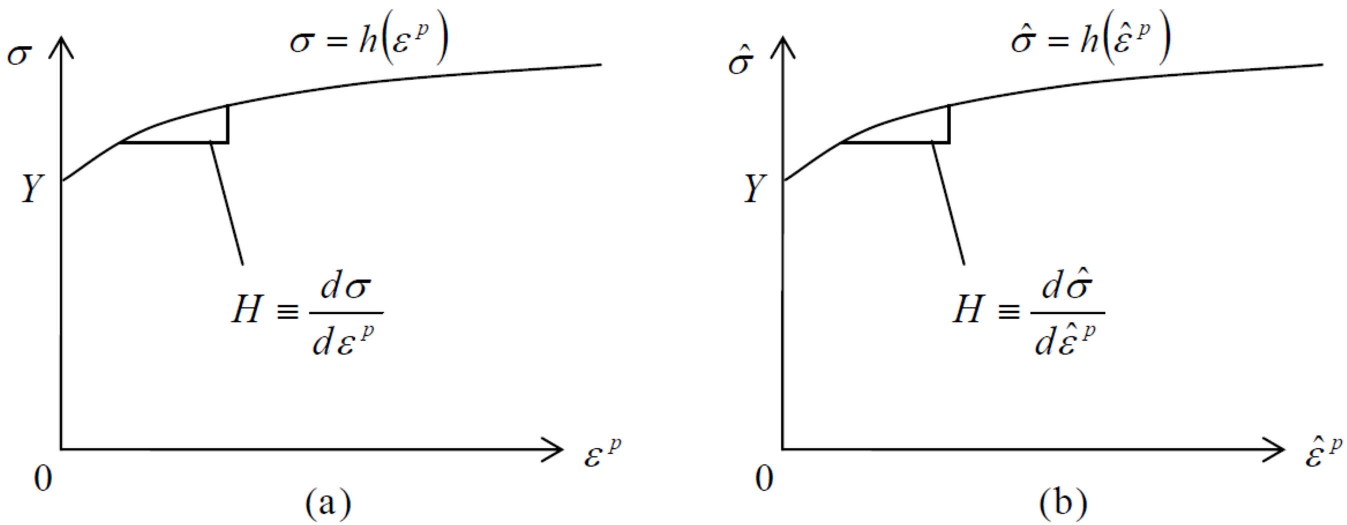


Von Mises

$$\mathbf{f}\left(\boldsymbol{\sigma}_{ij}, \mathbf{K}_i\right) = \mathbf{f}\left(\boldsymbol{\sigma}_{ij} - \boldsymbol{\alpha}_{ij}\right) = \sqrt{\frac{1}{2}\left[\left((\sigma_1-\alpha_1)-(\sigma_2-\alpha_2)\right)^2 + \left((\sigma_2-\alpha_2)-(\sigma_3-\alpha_3)\right)^2 + \left((\sigma_3-\alpha_3)-(\sigma_1-\alpha_1)\right)^2\right]}$$

Flow curve

To model three dimensional deformation behavior, a concept of effective variables, such as effective stress, effective plastic strain is introduced to control the size or position of the yield surface

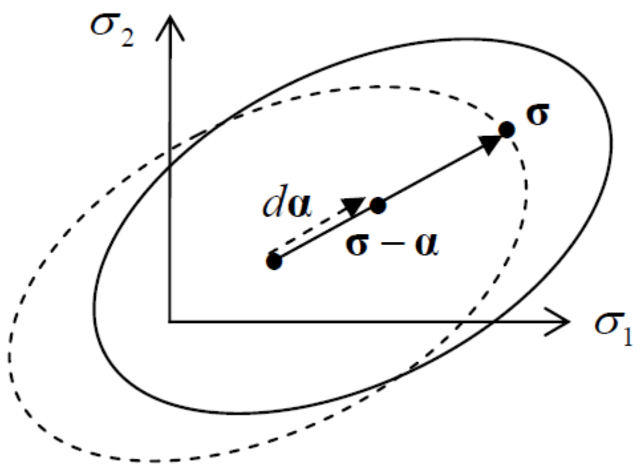


Hardening law: basic

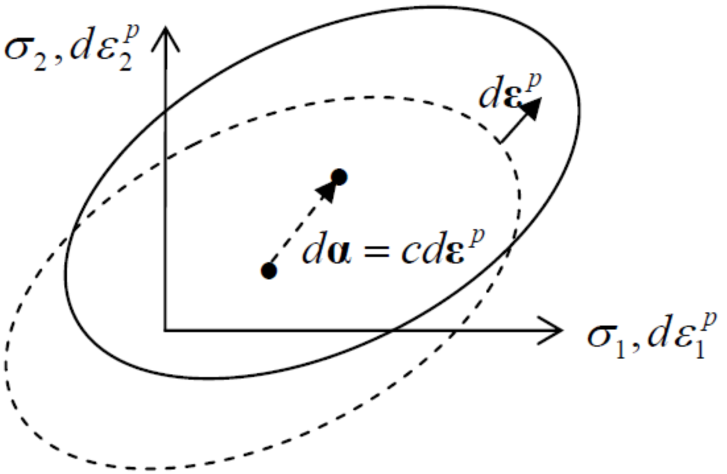
Kinematic hardening: how to define the movement?

$$\mathbf{d\alpha_{ij} \sim \sigma_{ij} - \alpha_{ij}}$$

$$\mathbf{d\alpha_{ij} \sim d\varepsilon^p_{ij}}$$



Ziegler model



Prager model

Hardening law: basic

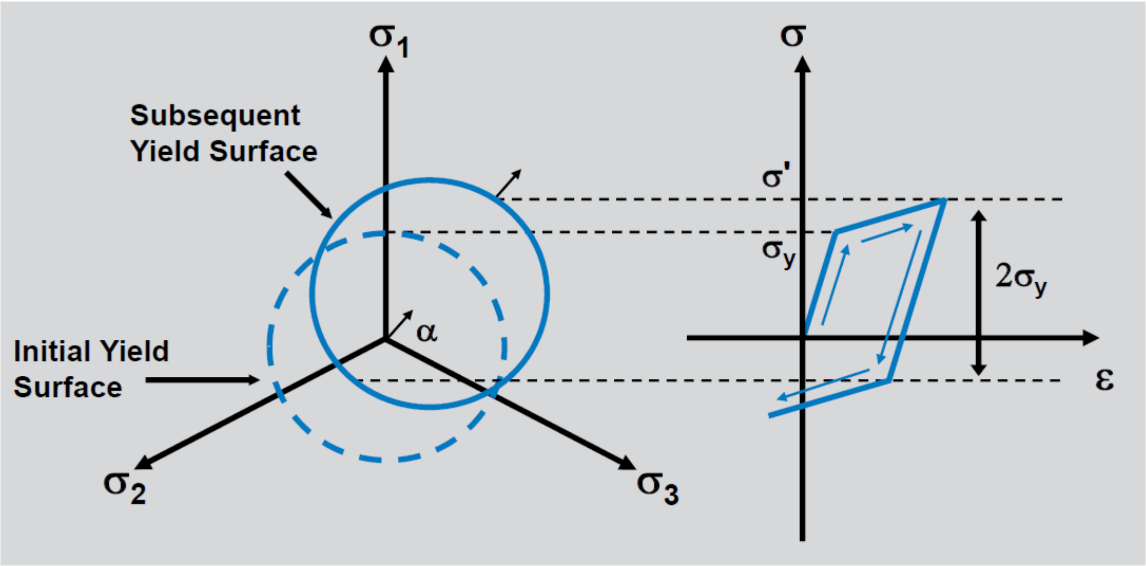
Linear kinematic hardening

$$\mathbf{f} = \sqrt{\frac{3}{2}\left(\mathbf{s}_{ij} - \boldsymbol{\alpha}_{ij}\right) : \left(\mathbf{s}_{ij} - \boldsymbol{\alpha}_{ij}\right)} - \mathbf{K} = 0$$

"Size of yield function, K=constant"

$$\mathbf{d}\boldsymbol{\alpha}_{ij} = \frac{2}{3}\mathbf{C}\mathbf{d}\boldsymbol{\varepsilon}^{p}_{ij} \quad \textit{or} \quad \Delta\boldsymbol{\alpha}_{ij} = \frac{2}{3}\mathbf{C}\Delta\boldsymbol{\varepsilon}^{p}_{ij}$$

"Translation of yield function (back stress) – linearly proportional to the plastic strain rate"



Hardening law: advanced

❖ Classical isotropic hardening

- Not so effective for non-monotonic straining
- No Bauschinger effect and transient behavior

❖ Classical combination type of iso-kinematic hardening by Prager and Ziegler

- Bauschinger effect only
- No transient behavior

❖ Combination type of the isotropic and kinematic hardening

- Chaboche, Krieg and Dafalias/Popov
- YU model
- Bauschinger effect and transient behavior

❖ Distortional hardening

- Without kinematic hardening (No back stress)

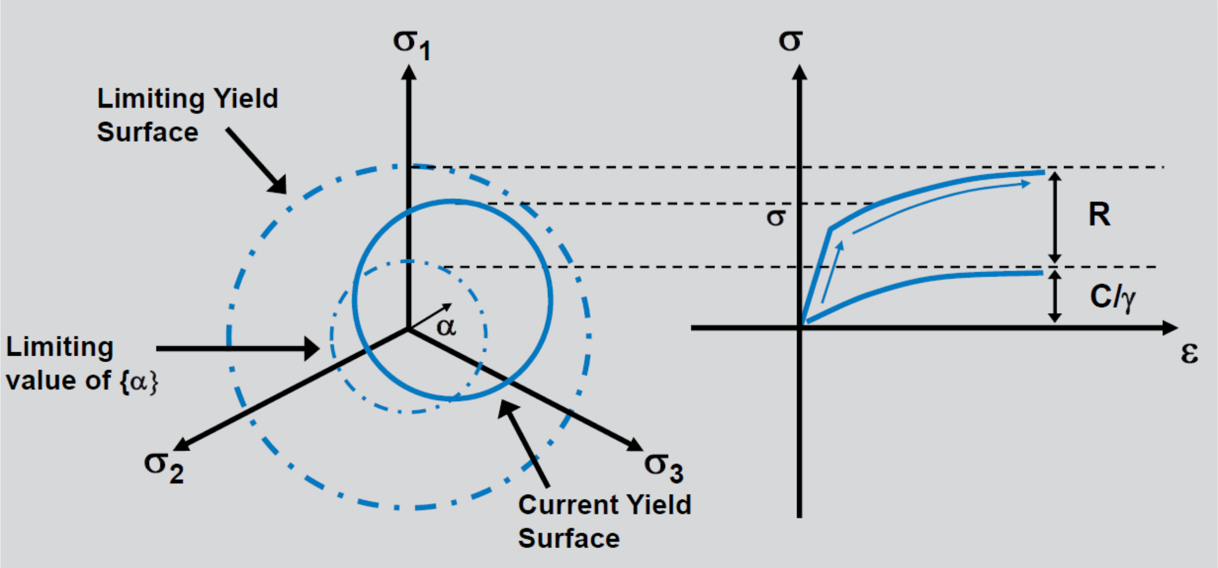
Hardening law: advanced

Nonlinear Kinematic Hardening Model (NKH)

$$\Delta\boldsymbol{\alpha} = \frac{2}{3} C \Delta\boldsymbol{\varepsilon}^{p} - \boxed{\gamma\ \boldsymbol{\alpha}\ \Delta\bar{\varepsilon}}$$

"Recall" term

$$\mathbf{f} = \sqrt{\frac{3}{2}\left(\mathbf{s}_{ij} - \boldsymbol{\alpha}_{ij}\right) : \left(\mathbf{s}_{ij} - \boldsymbol{\alpha}_{ij}\right)} - \mathbf{R} = \mathbf{0}$$

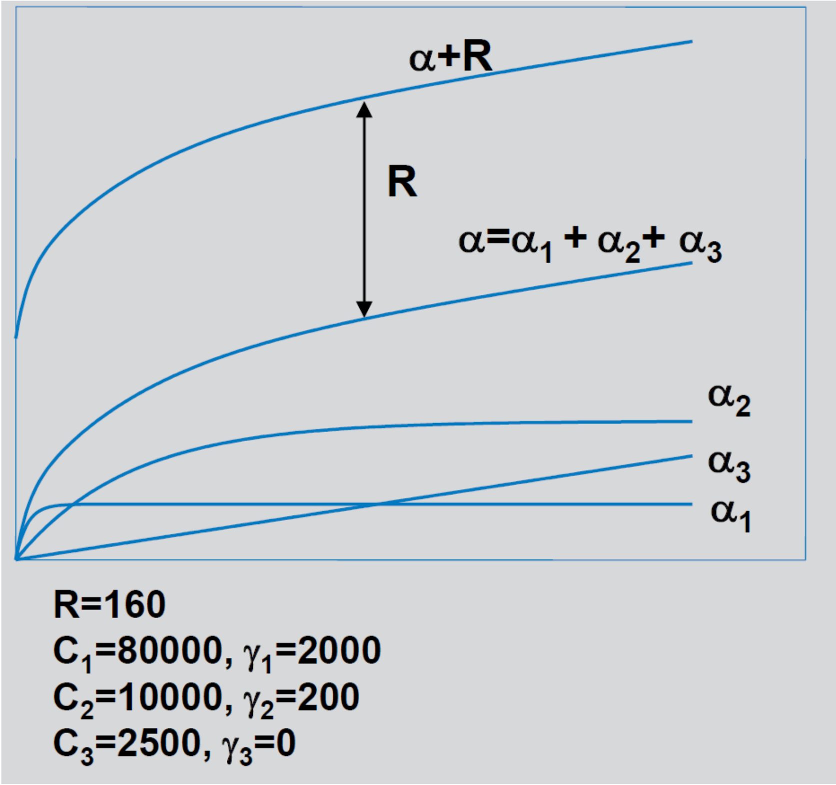


Hardening law: advanced

"Chaboche" Model

$$\Delta\boldsymbol{\alpha} = \sum_{i=1}^{n} \Delta\boldsymbol{\alpha}_i = \frac{2}{3}\sum_{i=1}^{n} C_i \Delta\boldsymbol{\varepsilon}_i^p - \gamma_i \boldsymbol{\alpha}_i \Delta\bar{\varepsilon}$$

$$\mathbf{f} = \sqrt{\frac{3}{2}\left(\mathbf{s}_{ij} - \boldsymbol{\alpha}_{ij}\right):\left(\mathbf{s}_{ij} - \boldsymbol{\alpha}_{ij}\right)} - \mathbf{R} = \mathbf{0}$$



Hardening law: advanced

❖ Two surfaces are used to represent Bauschinger and transient behaviors

- Inner(loading) and outer(bounding) surfaces

$$f(\boldsymbol{\sigma}-\boldsymbol{\alpha})-\bar{\sigma}_{iso}^{m}=0$$

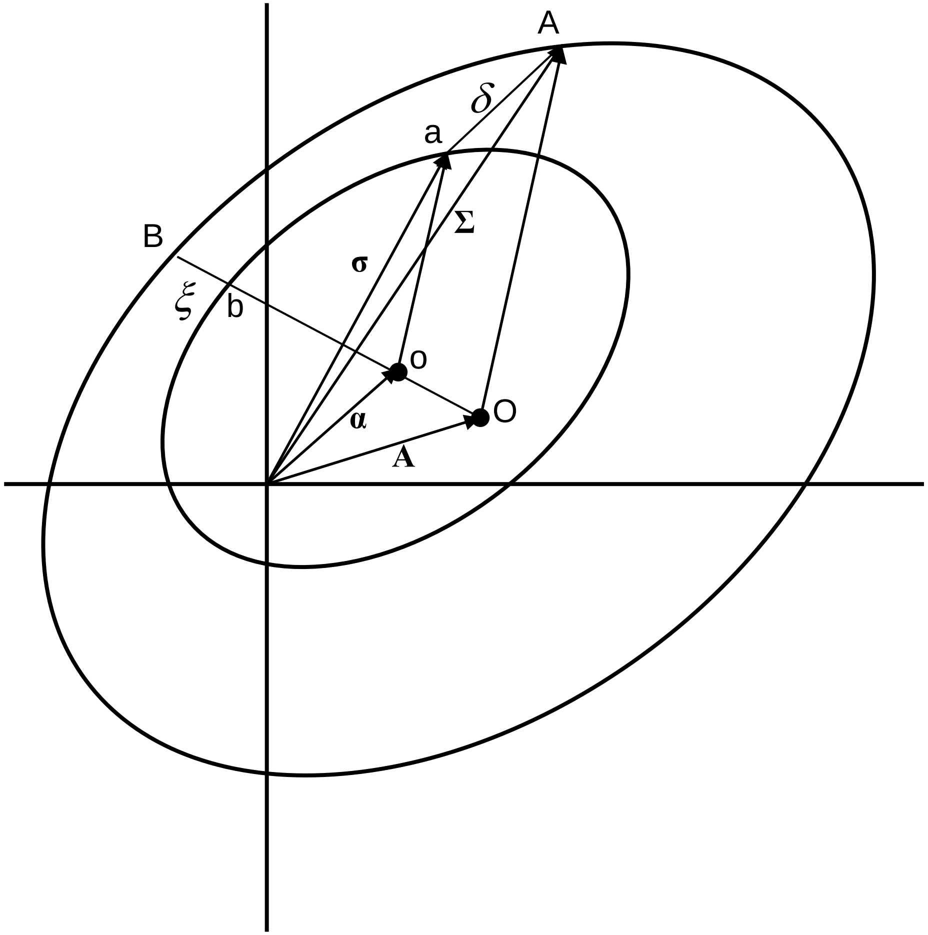
$$F(\boldsymbol{\Sigma}-\mathbf{A})-\bar{\Sigma}_{iso}^{m}=0$$

- Translation of the inner surface

$$d\boldsymbol{\alpha}=\frac{d\bar{\alpha}}{\bar{\sigma}_{iso}(\boldsymbol{\nu})}\boldsymbol{\nu}$$

$\boldsymbol{\nu}$: normalized quantity of $d\boldsymbol{\varepsilon}^{p}$ **or** $\boldsymbol{\sigma}-\boldsymbol{\alpha}$

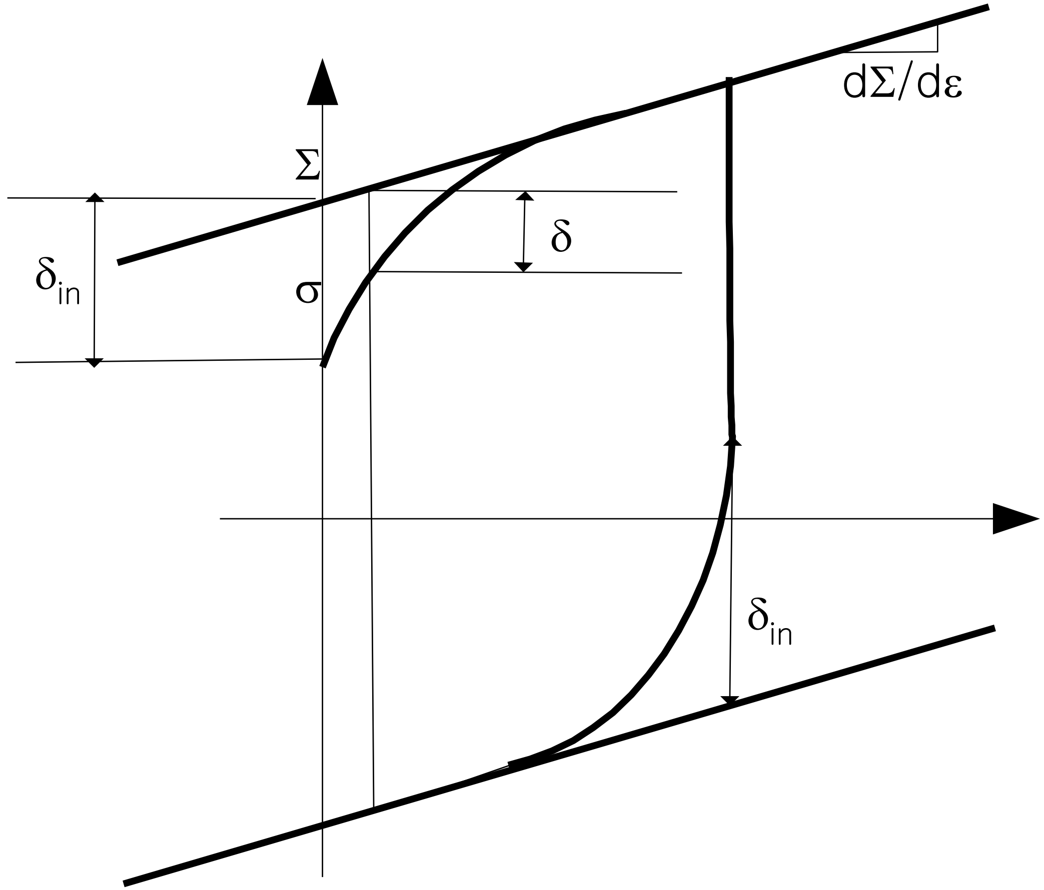
Hardening law: advanced



The hardening curve of the inner surface is newly updated every time unloading occurs, considering the gap δ

The separation of the isotropic and kinematic hardening in the inner surface should be performed considering the gaps

Hardening law: advanced



Hardening law: advanced

- In order to properly represent the transient behavior, the hardening behavior of the inner surface is prescribed every time reloading occurs

The stress relation for the 1-D tension test

$$\boxed{\frac{d\Sigma}{d\varepsilon^p} = \frac{d\sigma}{d\varepsilon^p} + \frac{d\delta}{d\varepsilon^p}}$$

This satisfies the continuous hardening slope between elastic and plastic ranges

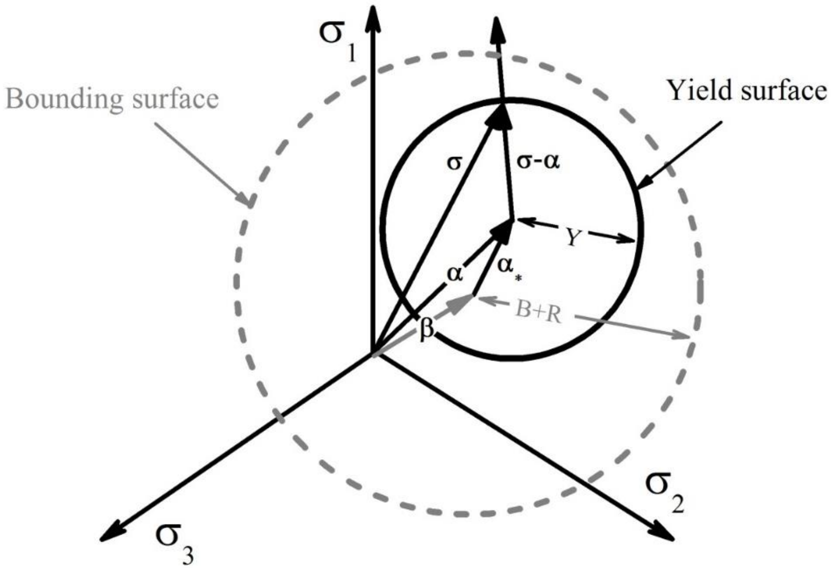
$$\frac{d\sigma}{d\varepsilon^p} = \infty \quad \textbf{for} \quad \delta = \delta_{in}$$

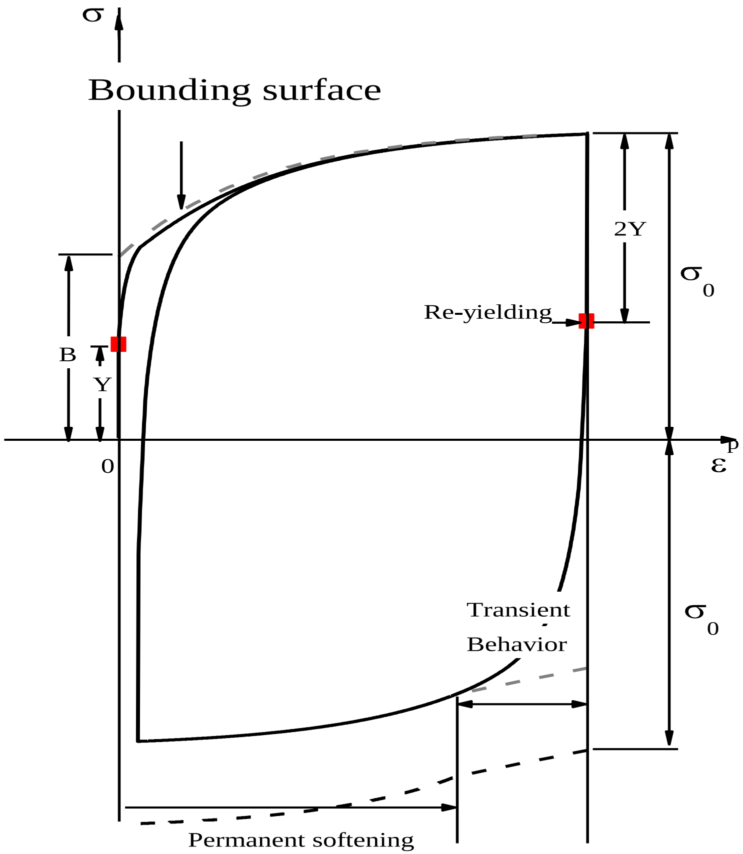
$$\frac{d\sigma}{d\varepsilon^p} = \frac{d\Sigma}{d\varepsilon^p} \quad \textbf{for} \quad \delta = 0$$

Example of gap variation between inner and outer surfaces

$$\boxed{\frac{d\delta}{d\varepsilon^p} \left(= A(\delta)\right) = -\chi(\delta_{in}) \left(\frac{\delta}{\delta_{in} - \delta} \right)}$$

Hardening model: YU Model





Basic Theory : Yoshida, F. and Uemori, T. , Int. J. Plast. 18 ,661-686, 2002

Hardening model: YU Model

Back stress of "active (real)" yield surface

$$F = f(\boldsymbol{\sigma} - \boxed{\boldsymbol{\alpha}}) - Y = 0$$

Back stress of "Bounding" surface

$$F_1 = f(\boldsymbol{\sigma} - \boxed{\boldsymbol{\beta}}) - \left(B + \boxed{R}\right) = 0$$

Isotropic part of "Bounding" surface
Control "global" hardening

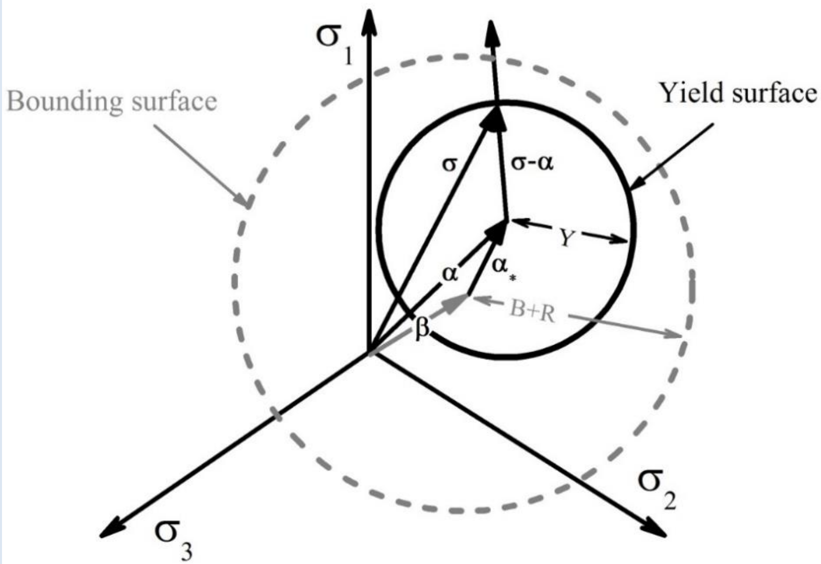
$$\boldsymbol{\alpha}_* = \boldsymbol{\alpha} - \boldsymbol{\beta}$$

$$\boldsymbol{\alpha}_* = \sqrt{\frac{2}{3}} Ca\,d\varepsilon_e^p \left(\mathbf{n}_p - \sqrt{\frac{\overline{\alpha_*}}{a}}\mathbf{n}_*\right)$$

$$a = B + R - Y$$

$$dR = m\left(R_{sat} - R\right) d\bar{\varepsilon}^p$$

$$d\boldsymbol{\beta} = m\left(\frac{2}{3} b\,d\boldsymbol{\varepsilon}^p - \boldsymbol{\beta}\,d\bar{\varepsilon}^p\right)$$



Hardening model: YU Model

Initial yield surface

$$F = f(\boldsymbol{\sigma} - \boldsymbol{\alpha}) - Y = 0$$

Bounding surface

$$F_1 = f(\boldsymbol{\sigma} - \boldsymbol{\beta}) - (B + R) = 0$$

The relative motion law

$$\boldsymbol{\alpha}_* = \boldsymbol{\alpha} - \boldsymbol{\beta}$$

$$\boldsymbol{\alpha}_* = \sqrt{\frac{2}{3}} C a d\varepsilon_e^p \left(\mathbf{n}_p - \sqrt{\frac{\overline{\alpha_*}}{a}} \mathbf{n}_* \right)$$

$$a = B + R - Y$$

Isotropic hardening of the bounding surface

$$dR = m\left(R_{sat} - R\right) d\bar{\varepsilon}^p$$

Kinematic hardening of the bounding surface

$$d\boldsymbol{\beta} = m\left(\frac{2}{3} b d\boldsymbol{\varepsilon}^p - \boldsymbol{\beta} d\bar{\varepsilon}^p \right)$$

Yoshida-Uemori Parameters

Y : Size of loading surface

B : Initial size of bounding surface

C : Parameter for the back-stress evolution

R_{sat} ,b ,m : Parameters for the size of bounding surface

h : The parameter for work-hardening stagnation or cyclic hardening characteristics.

Plastic strain dependency of Unloading modulus

$$E = E_0 - (E_0 - E_a)[1 - \exp(-\xi \bar{\varepsilon}^p)]$$

Hardening model parameter

❖ Measuring hardening, Bauschinger and transient behaviors

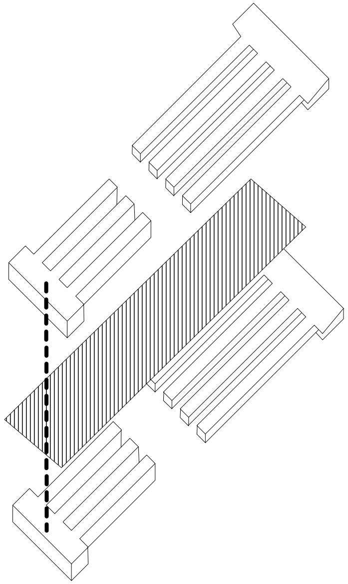
❖ Anti-buckling system

- Prevention of buckling when sheet specimen compressed
- Use of fork-shaped guides along the side of the sheet
- Large clamping force causes the fork to bend
- Difficult to obtain smooth hardening curves

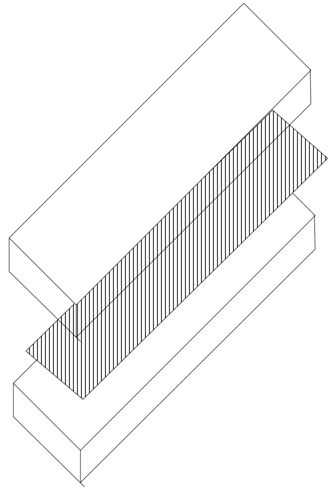
❖ Modified method using simple rigid plates

- Smooth hardening curves were obtained
- Mounted at universal testing machine

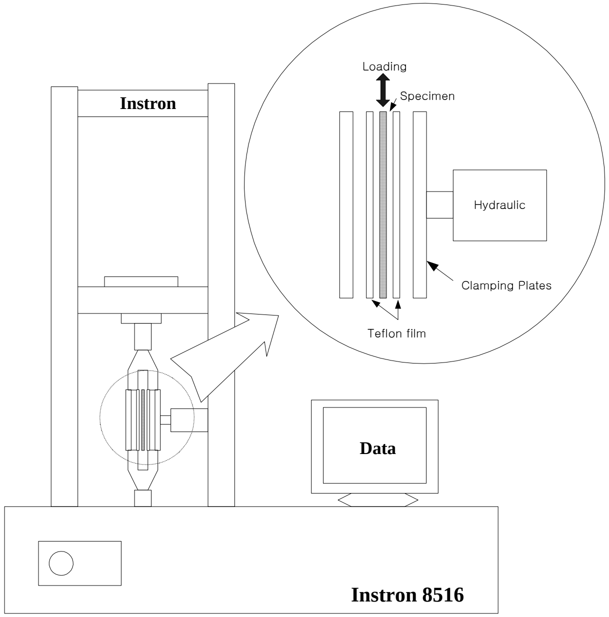
Hardening model parameter

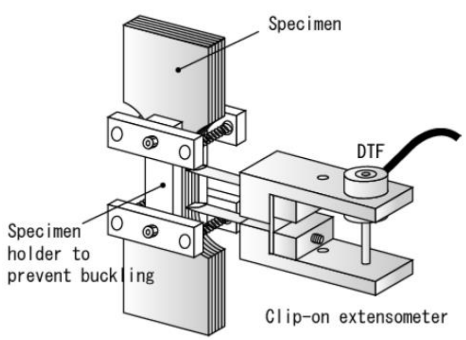


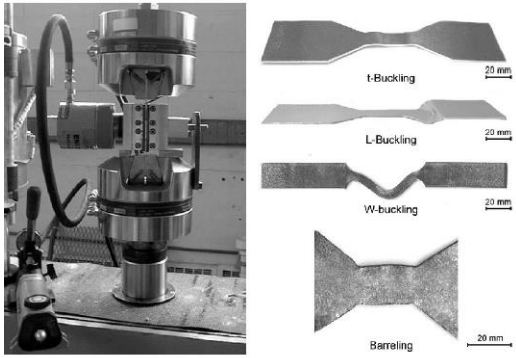
Fork device



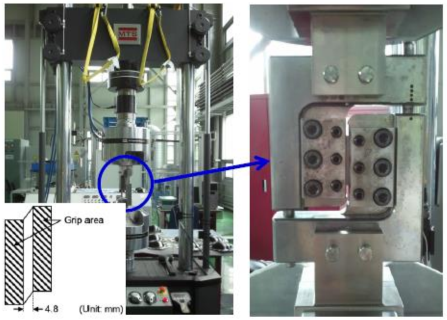
Modified device







▲Tension-compression test (Boger et al., 2005)



▲ Simple shear device connected on MTS universal testing machine

Hardening model parameter

- Under-estimate the measured stress value
- For the biaxial stress state

$$\bar{\sigma} = f^{\frac{1}{m}} = \left(\frac{\left|X_1' - X_2'\right|^m + \left|2X_2'' + X_1''\right|^m + \left|X_2'' + 2X_1''\right|^m}{2} \right)^{\frac{1}{m}}$$

$$X_1' - X_2' = L_{11}'\sigma_{xx} - L_{22}'\sigma_{zz}$$

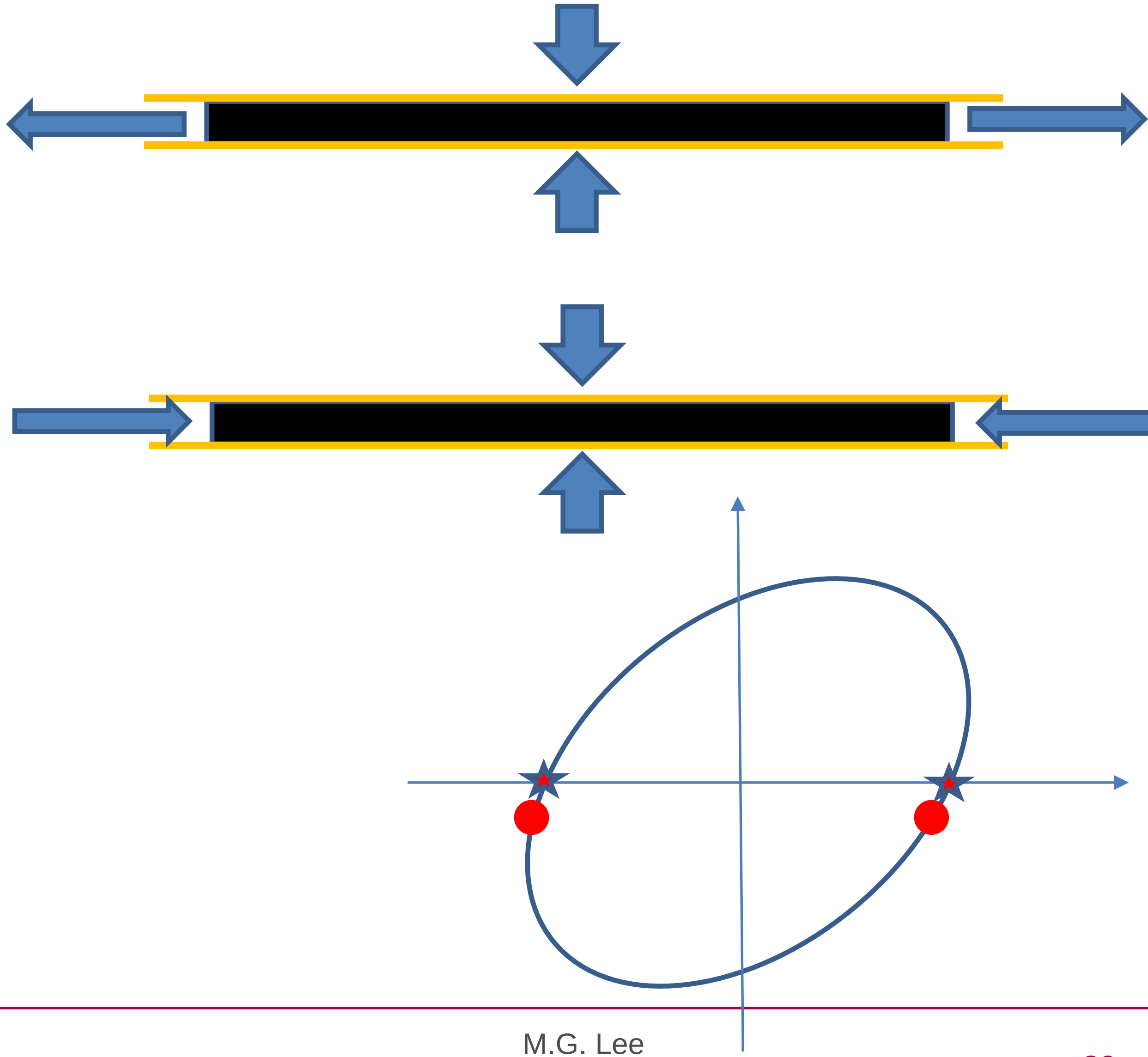
$$2X_2'' + X_1'' = (2L_{21}'' + L_{11}'')\sigma_{xx} + (L_{11}'' + 2L_{22}'')\sigma_{zz}$$

$$X_2'' + 2X_1'' = (L_{21}'' + 2L_{11}'')\sigma_{xx} + (2L_{12}'' + L_{22}'')\sigma_{zz}$$

- Using clamping force and contact area, the biaxial effect can be corrected

Hardening model parameter

Tension-compression test

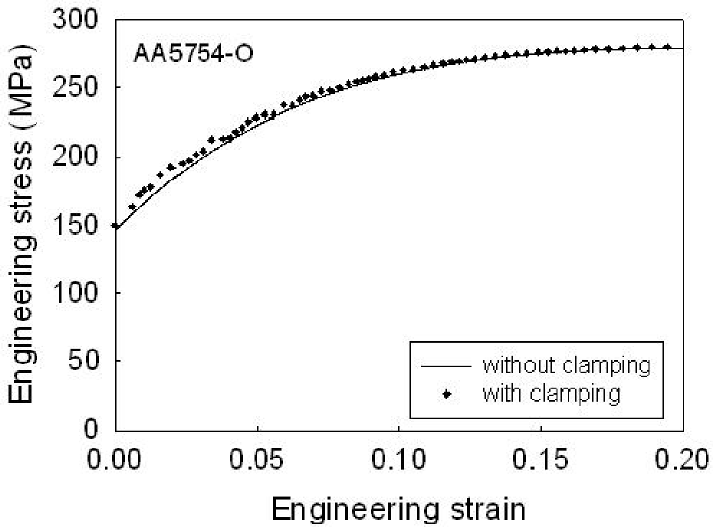


Hardening model parameter- frictional effect

- Over-estimate the measured stress value
- Indirectly evaluated by comparing the two tensile data: With/without clamping force
- Apparent friction coefficient:

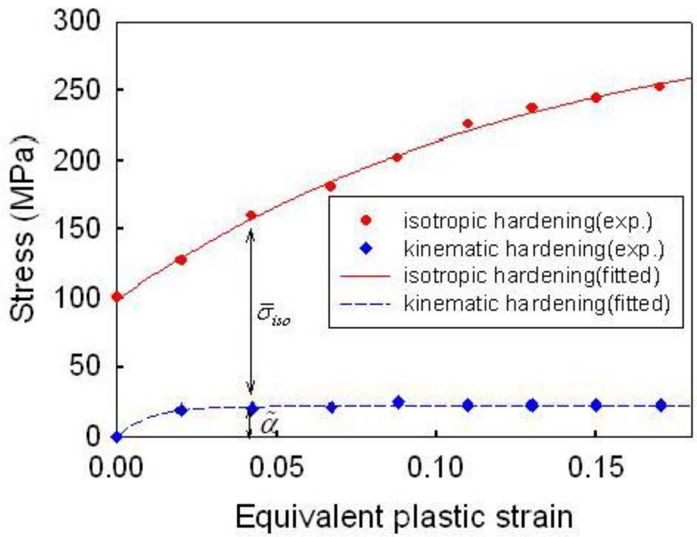
$$\mu = \frac{(F^{unc} - F^{std})}{N} = \frac{(\sigma^{unc} - \sigma^{std})A_o}{N}$$

- Apparent friction coefficient was obtained as one average value since it varies with respect to engineering strain



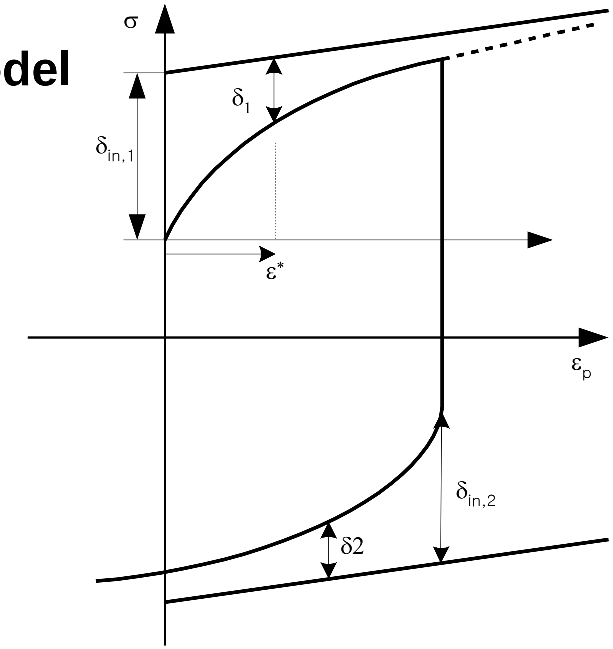
Hardening model parameter

Chaboche model



$$h_1 - h_2\bar{\alpha} = \frac{d\bar{\alpha}}{d\bar{\varepsilon}} \Rightarrow h_2 = (h_1 - \frac{d\bar{\alpha}}{d\bar{\varepsilon}})/\bar{\alpha}$$

Two-surface model



$$\frac{d\delta}{d\varepsilon^p} = h(\delta_{in})(\frac{\delta}{\delta_{in} - \delta}) = -(a + b\delta_{in})(\frac{\delta}{\delta_{in} - \delta})$$

$$h = \frac{\delta_{in}}{\bar{\varepsilon}^*}(1 - \ln(\frac{\delta_{in}}{\delta})) - \frac{\delta}{\bar{\varepsilon}^*}$$

$$a = \frac{h_1\delta_{in,2} - h_2\delta_{in,1}}{\delta_{in,2} - \delta_{in,1}} \qquad b = \frac{h_1 - h_2}{\delta_{in,1} - \delta_{in,2}}$$

Hardening model implementation

Outline of implicit integration scheme

Element equilibrium equation for a static state

$$\boxed{\mathbf{K}^e}\mathbf{U}^e = \boxed{\mathbf{F}^e}$$

$\mathbf{K}^e$: Element stiffness matrix (constitutive model)

$\mathbf{F}^e$: Surface traction + frictional force on element (friction model)

$\mathbf{U}^e$: Element nodal displacement vector

Global equilibrium equation

$$\boxed{\mathbf{K}}\mathbf{U} = \boxed{\mathbf{F}}$$

Implicit integration scheme (Newton-Raphson method)

Initially guess $\Delta\mathbf{U}$

Estimate the residual force $\Phi = \boxed{\mathbf{K}(\mathbf{U})}\mathbf{U} - \boxed{\mathbf{F}(\mathbf{U})}$

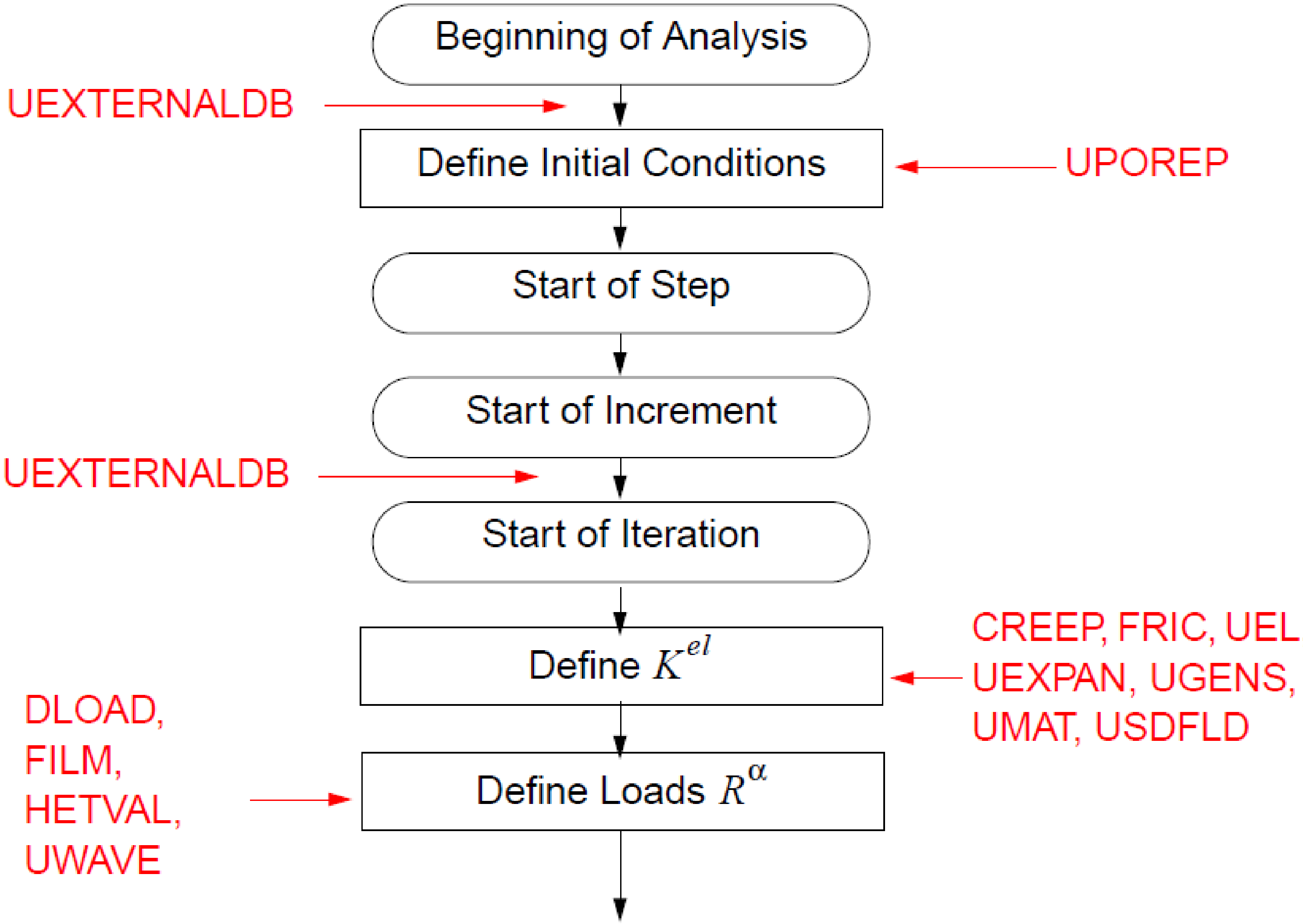
$\Phi < \mathrm{Tol}?$

yes → Proceed to next increment

no

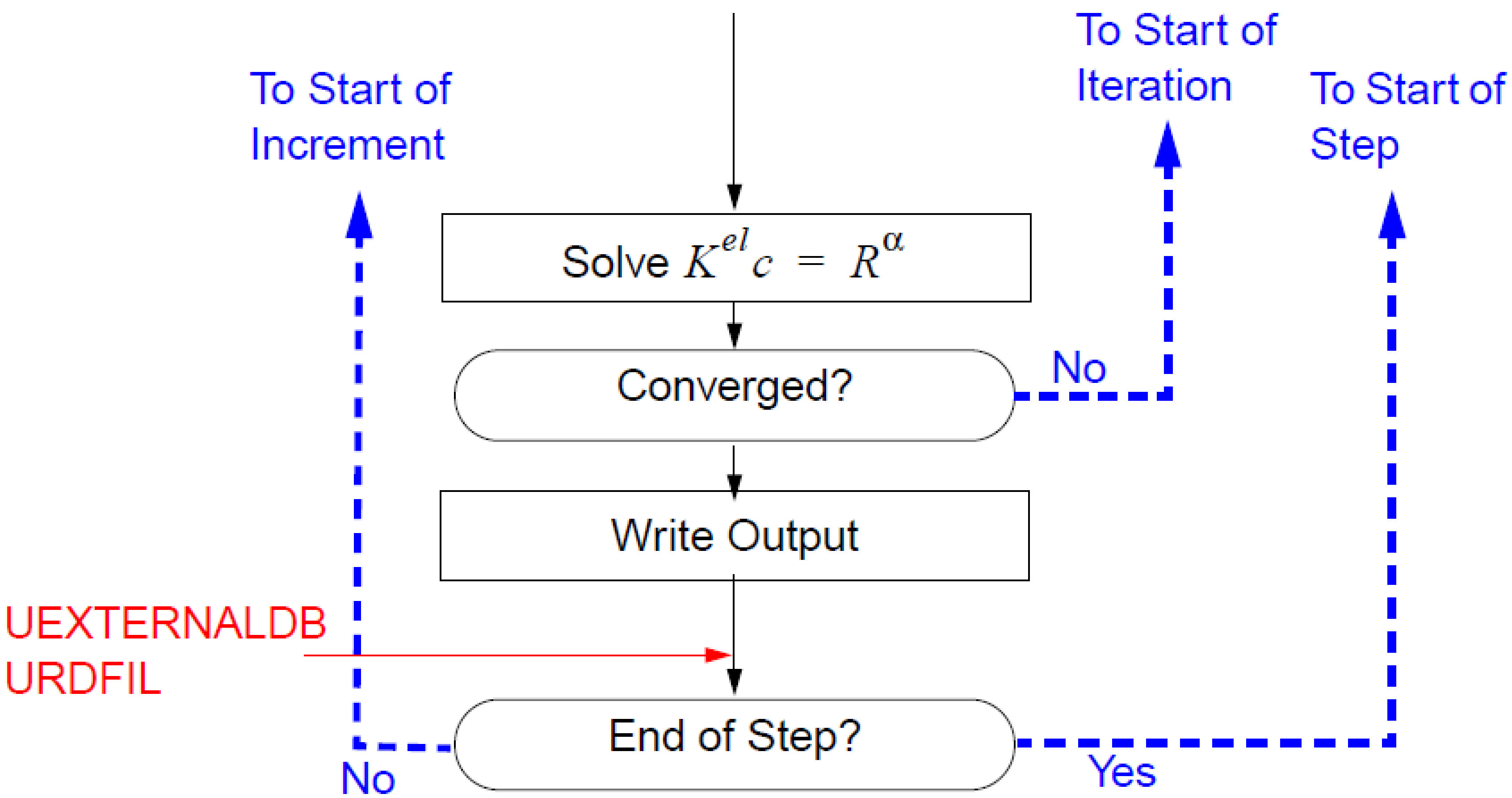
Adjust $\Delta\mathbf{U} = \Delta\mathbf{U} + d\Delta\mathbf{U}$ where $\Phi(\mathbf{U}) + \dfrac{\partial\Phi(\mathbf{U})}{\partial\Delta\mathbf{U}} d\Delta\mathbf{U} = 0$

Hardening model implementation



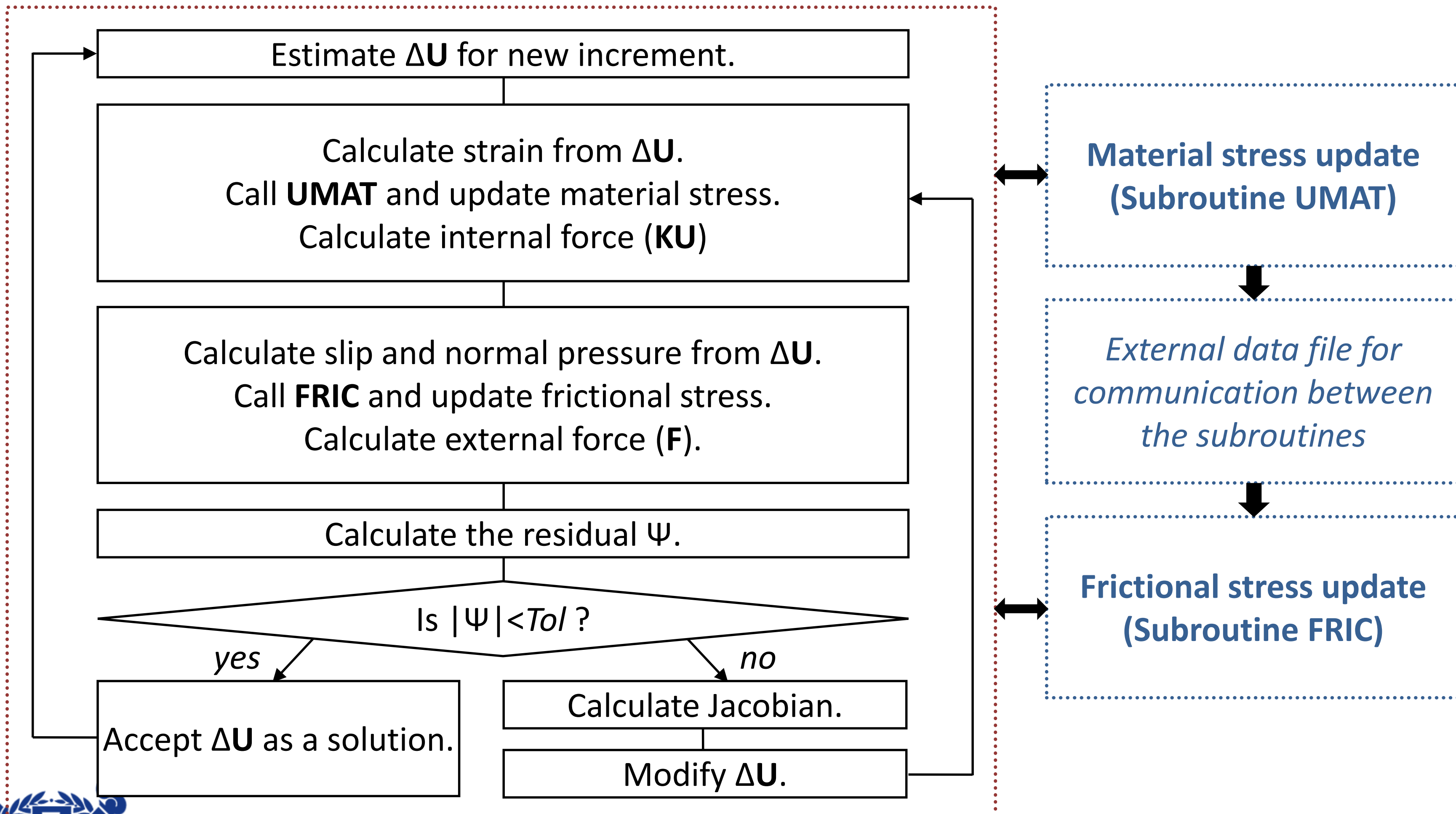
Writing User Subroutine with ABAQUS, ABAQUS

Hardening model implementation



Hardening model implementation

Integration of global equilibrium equation (Main code)



Hardening model implementation

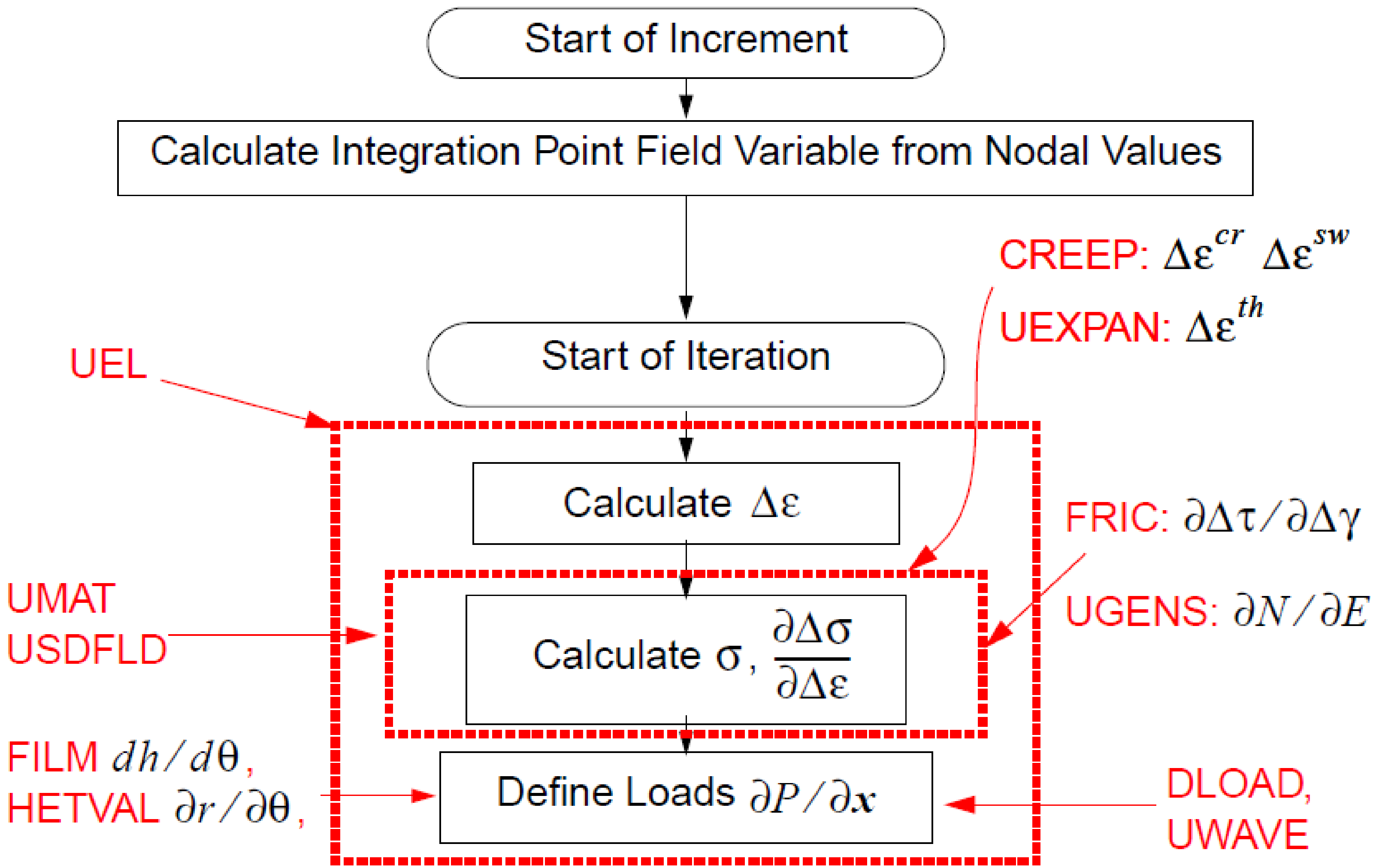
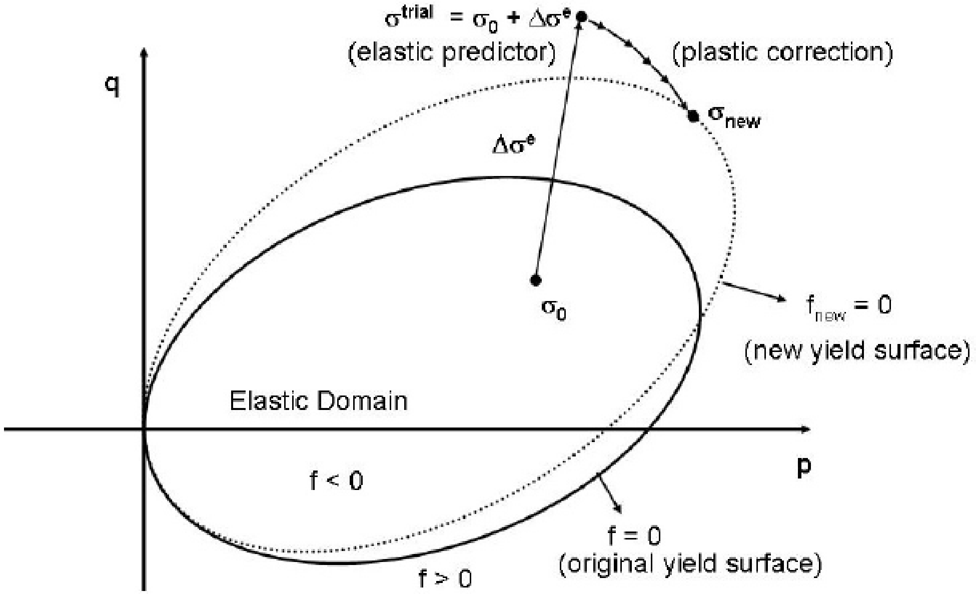


Figure 1–2. A More Detailed Flow of ABAQUS/Standard

Hardening model implementation

Stress integration: predictor-corrector



$\Delta\boldsymbol{\varepsilon}$ is given.

$$\Delta\boldsymbol{\varepsilon} = \Delta\boldsymbol{\varepsilon}^{e} + \Delta\boldsymbol{\varepsilon}^{p}$$

$$\boldsymbol{\sigma}^{T} = \boldsymbol{\sigma} + \mathbf{C}\Delta\boldsymbol{\varepsilon}$$

Specific Algorithm

$$\delta\left(\Delta\bar{\varepsilon}\right)$$

$$\Delta\bar{\varepsilon}^{new} = \Delta\bar{\varepsilon}^{old} + \delta\left(\Delta\bar{\varepsilon}\right)$$

$$\boldsymbol{\sigma}^{New} = \boldsymbol{\sigma}^{old} - \mathbf{C} : \left(\Delta\bar{\varepsilon}\frac{\partial f}{\partial\boldsymbol{\sigma}}\right)$$

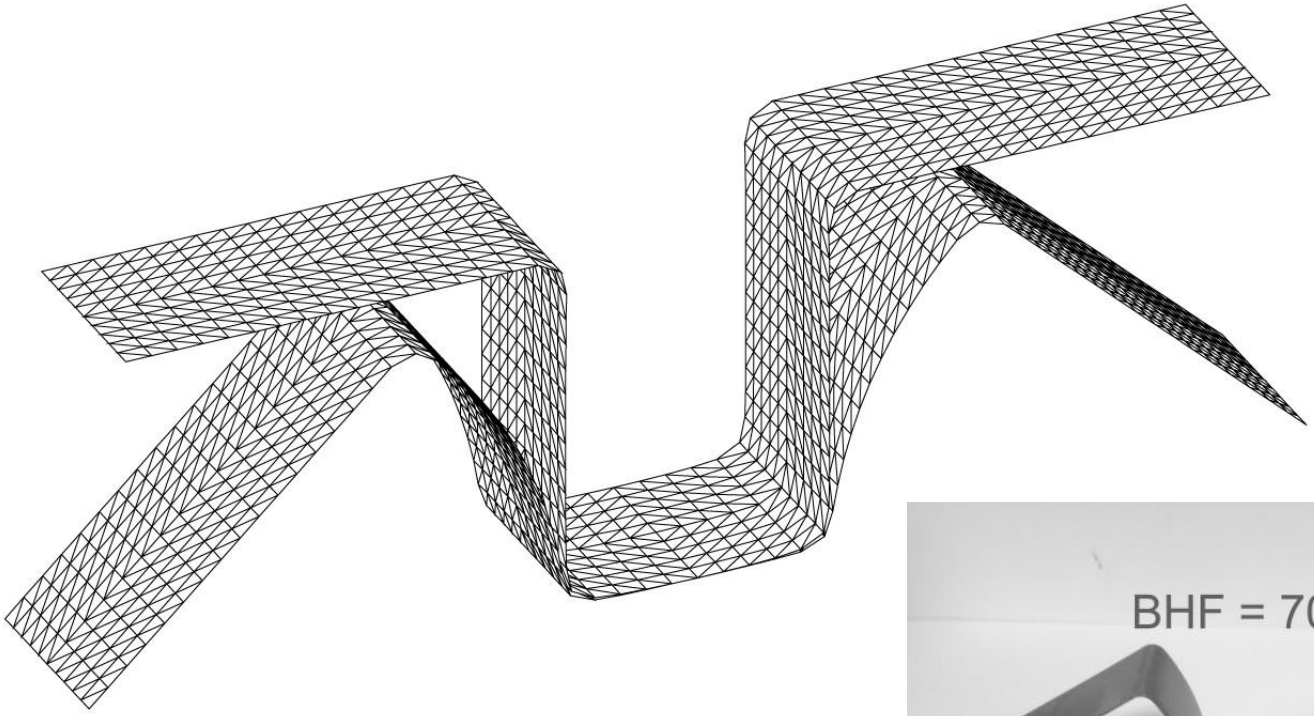
Hardening model implementation

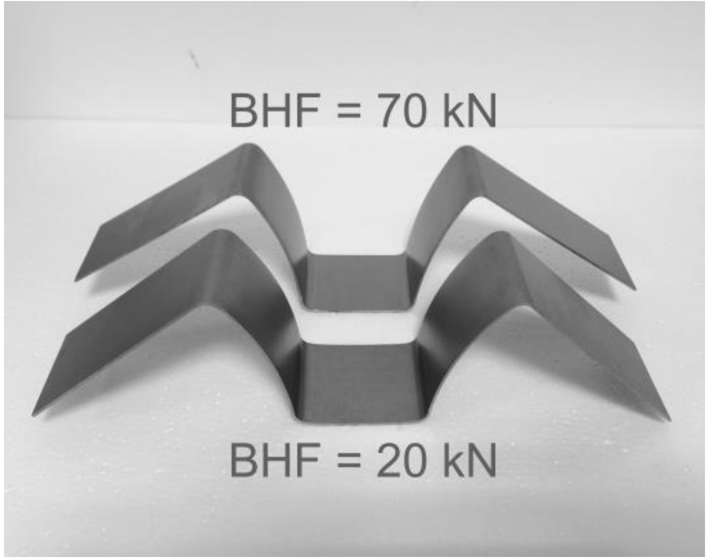
```
 SUBROUTINE UMAT(STRESS, STATEV, DDSDDE, SSE, SPD, SCD, RPL,
1 DDSDDT, DRPLDE, DRPLDT, STRAN, DSTRAN, TIME, DTIME, TEMP, DTEMP,
2 PREDEF, DPRED, CMNAME, NDI, NSHR, NTENS, NSTATV, PROPS, NPROPS,
3 COORDS, DROT, PNEWDT, CELENT, DFGRD0, DFGRD1, NOEL, NPT, LAYER,
4 KSPT, KSTEP, KINC)

C
 INCLUDE 'ABA_PARAM.INC'
C
 CHARACTER*8 CMNAME
C
 DIMENSION STRESS(NTENS), STATEV(NSTATV), DDSDDE(NTENS, NTENS),
1 DDSDDT(NTENS), DRPLDE(NTENS), STRAN(NTENS), DSTRAN(NTENS),
2 PREDEF(1), DPRED(1), PROPS(NPROPS), COORDS(3), DROT(3, 3),
3 DFGRD0(3, 3), DFGRD1(3, 3)
```

Writing User Subroutine with ABAQUS, ABAQUS

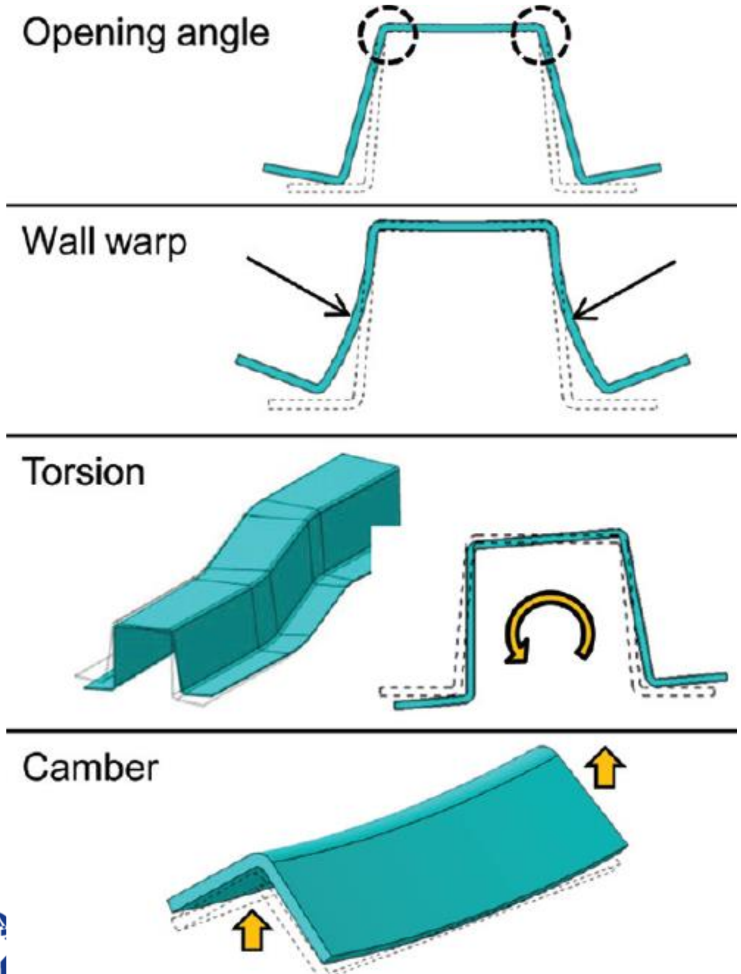
Application: springback





Application: springback

Defect in dimensional accuracy*



Elastic recovery of bending moment by uneven stress in the thickness direction after bending

Elastic recovery of bending moment by uneven stress in the thickness direction after bending & unbending

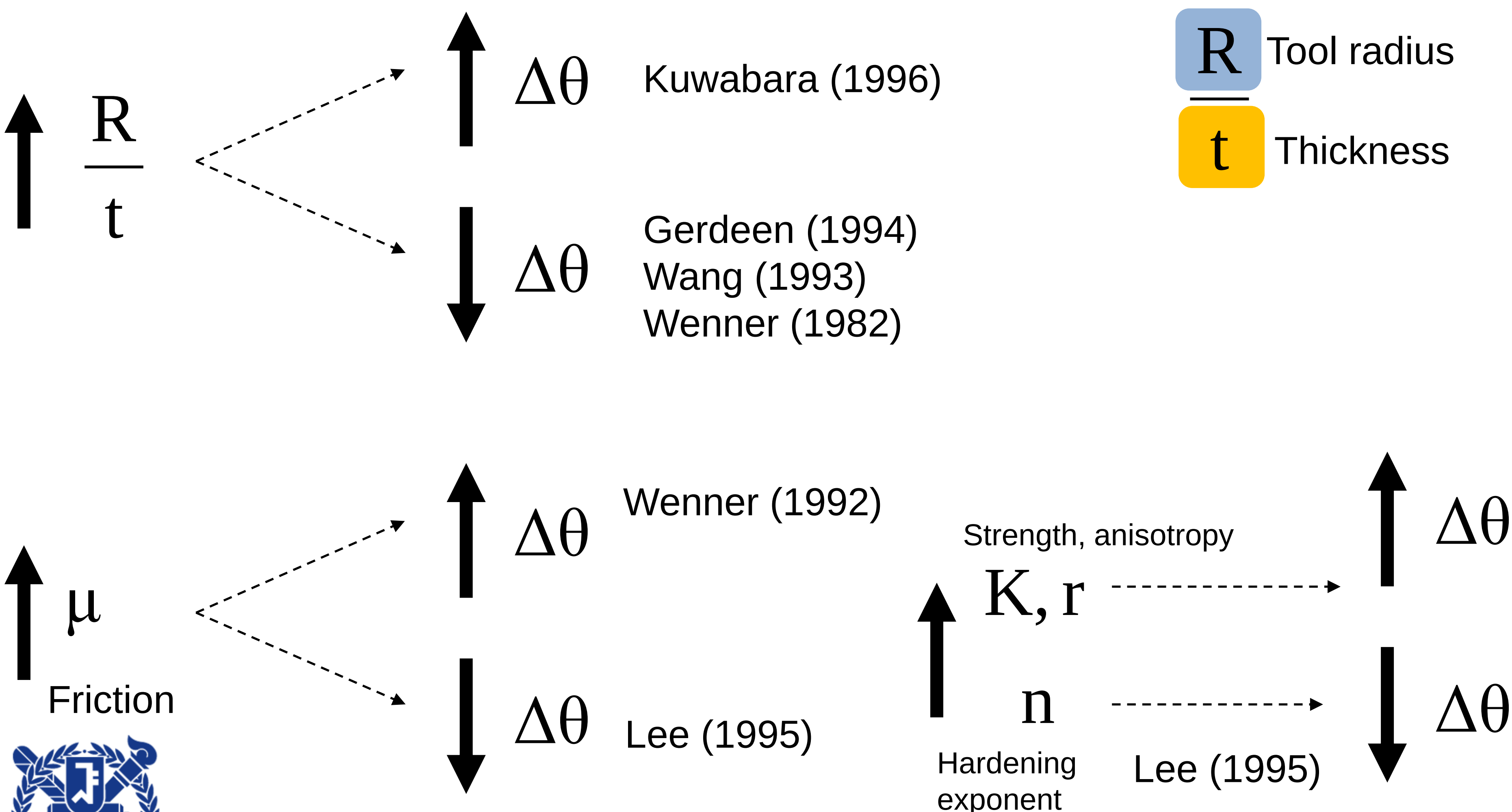
Elastic recovery of torsion moment by uneven stress in plane

Elastic recovery of bending moment along punch ridgeline by uneven stress in the thickness direction

* Yoshida & Isogai, Nippon Steel Tech Report, 2013

Application: springback

Before (selected from literature)



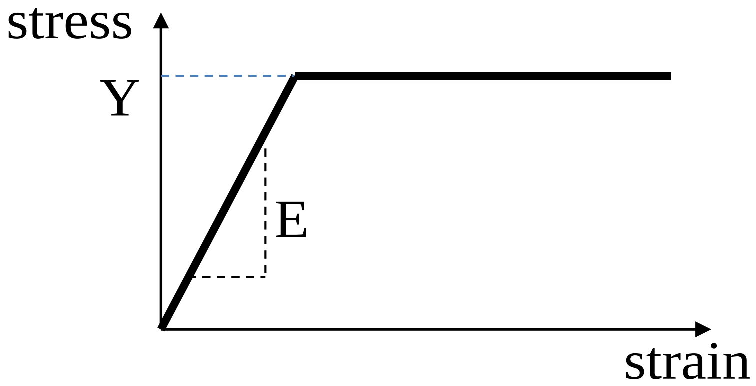
Application: springback

Idealized model

$$T = w\int_{-t/2}^{t/2} \sigma dz$$

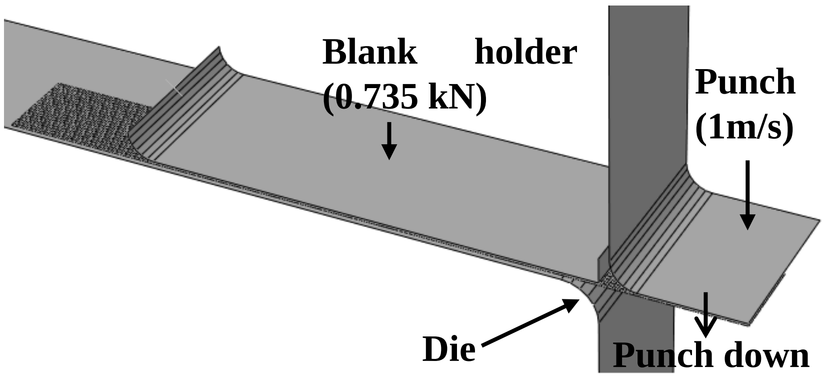
$$M = \frac{Yt^2}{4}\left(1 - \overline{T}^2\right)$$

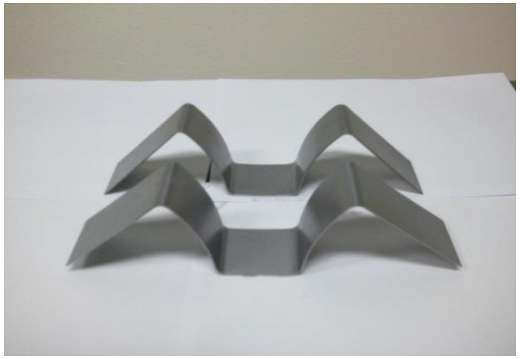
$$\boxed{\Delta\theta = \frac{1}{R} - \frac{1}{r} = \frac{3Y}{Et}\left(1 - \overline{T}^2\right)}$$

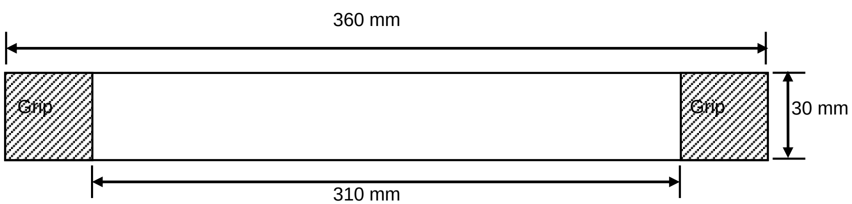


Application: springback

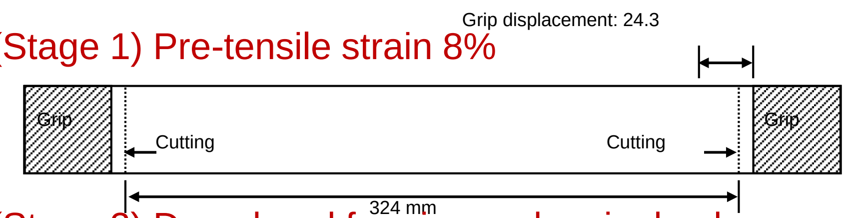
- 2D draw bending simulations





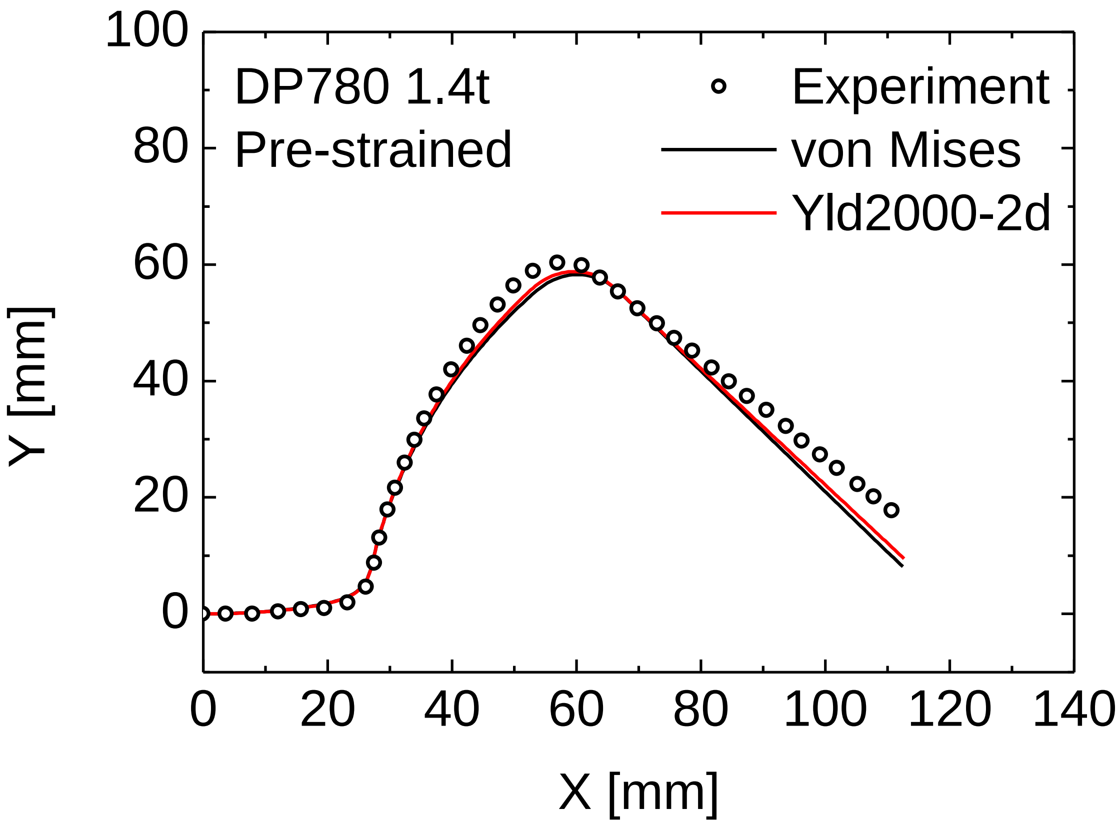


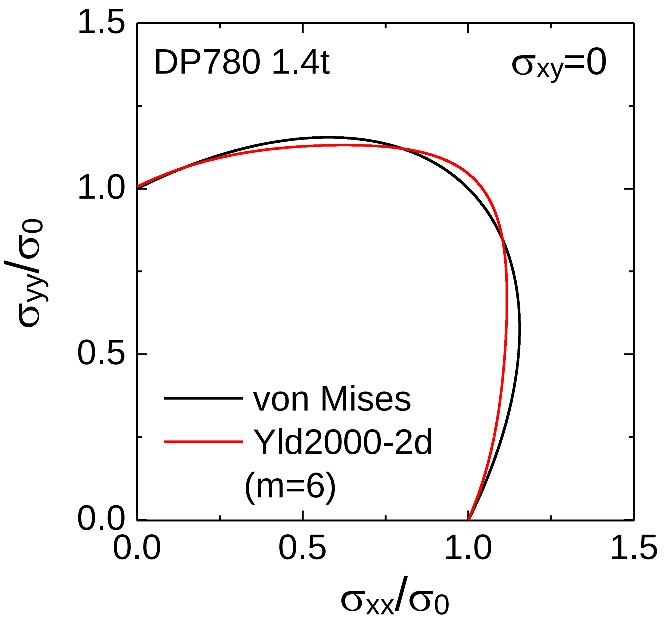
(Stage 1) Pre-tensile strain 8%



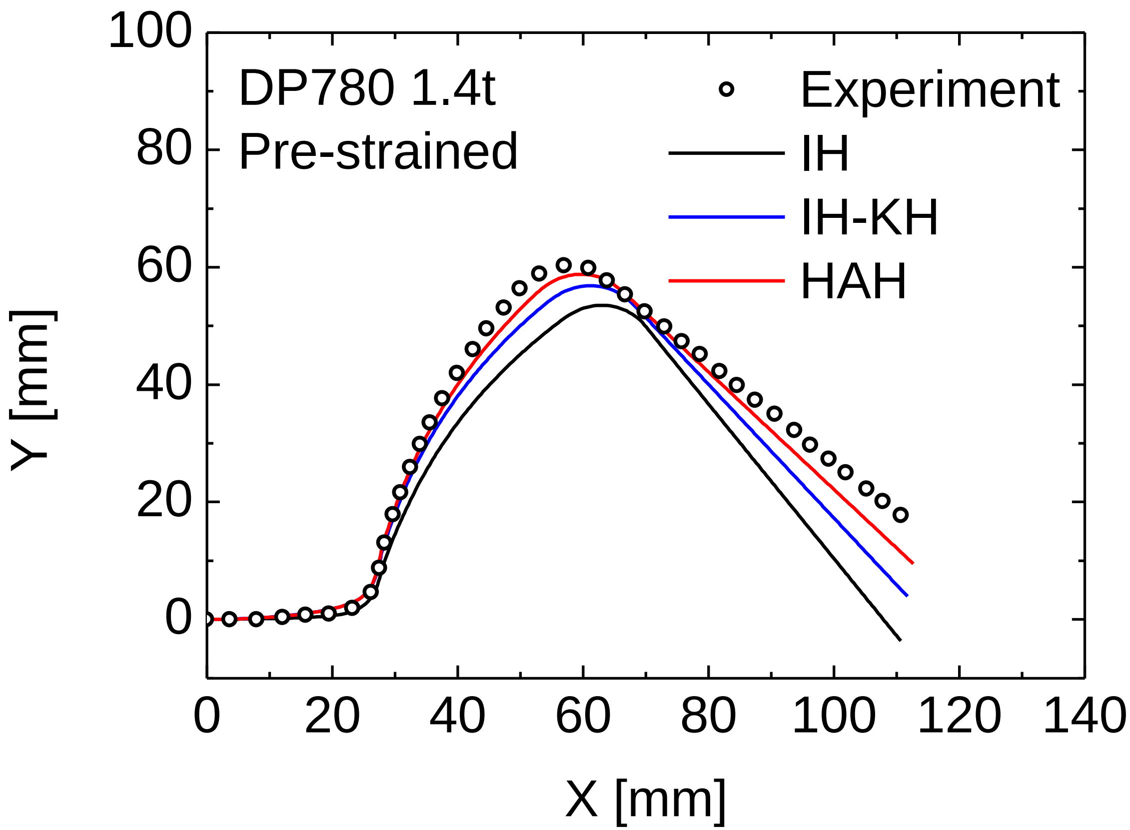
(Stage 2) Draw-bend forming and springback

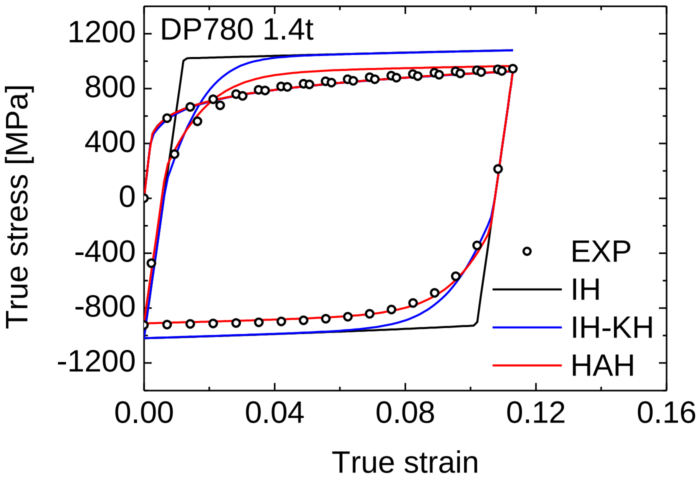
Application: springback





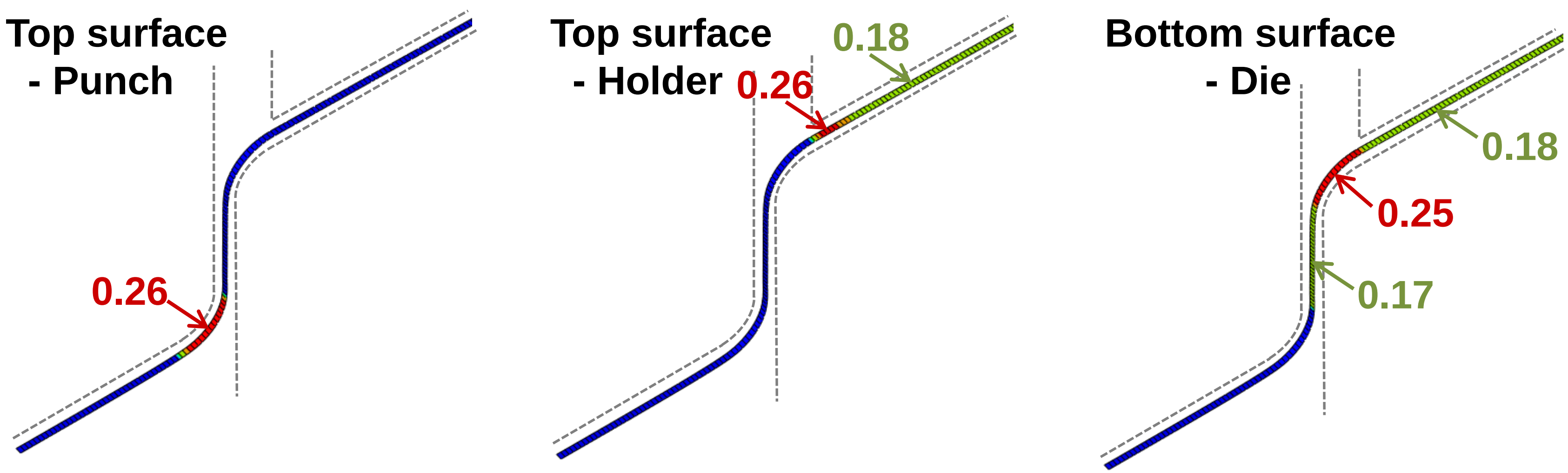
Application: springback



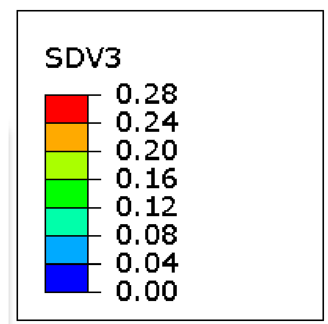


Application: issue

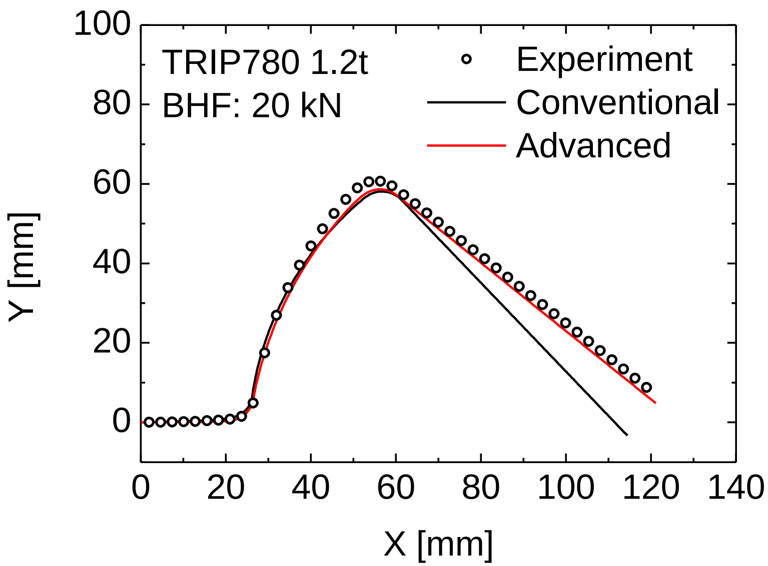
Friction coefficient

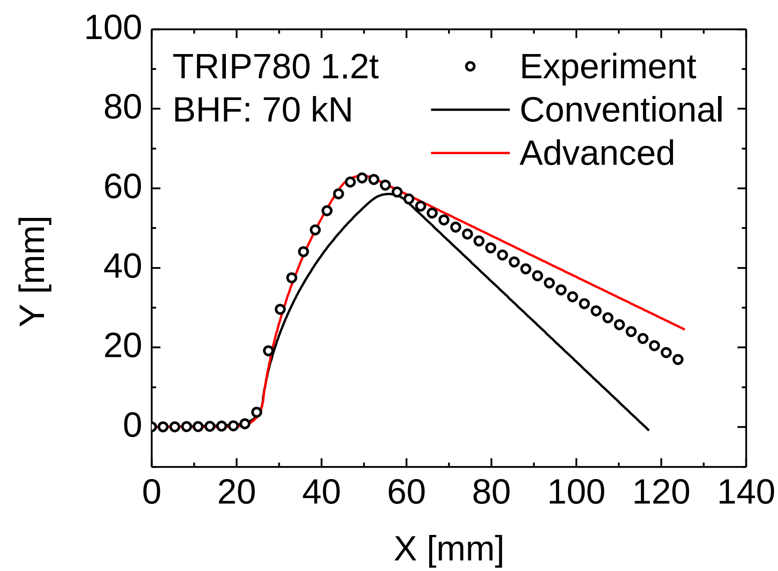


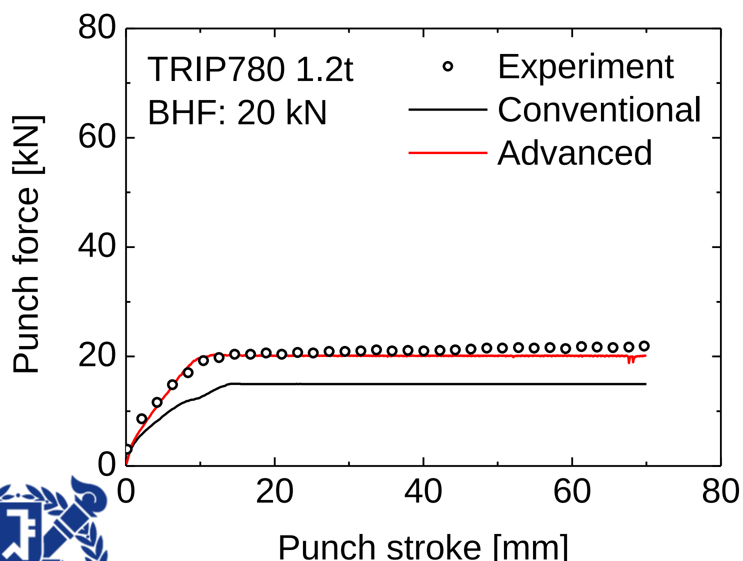
- ❖ **The friction coefficient** is **not uniform** due to different contact condition.
- ❖ A specific constant coefficient may provide similar predictions, but this value would not work if the forming condition changes.

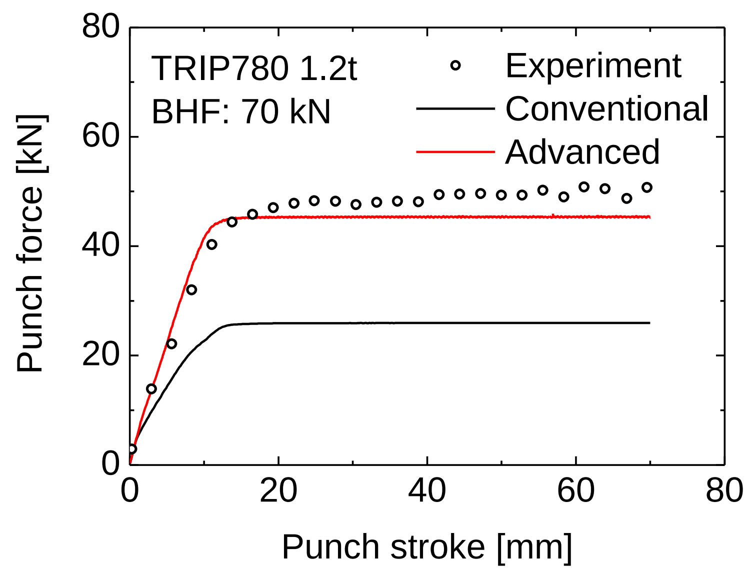


Application: issue

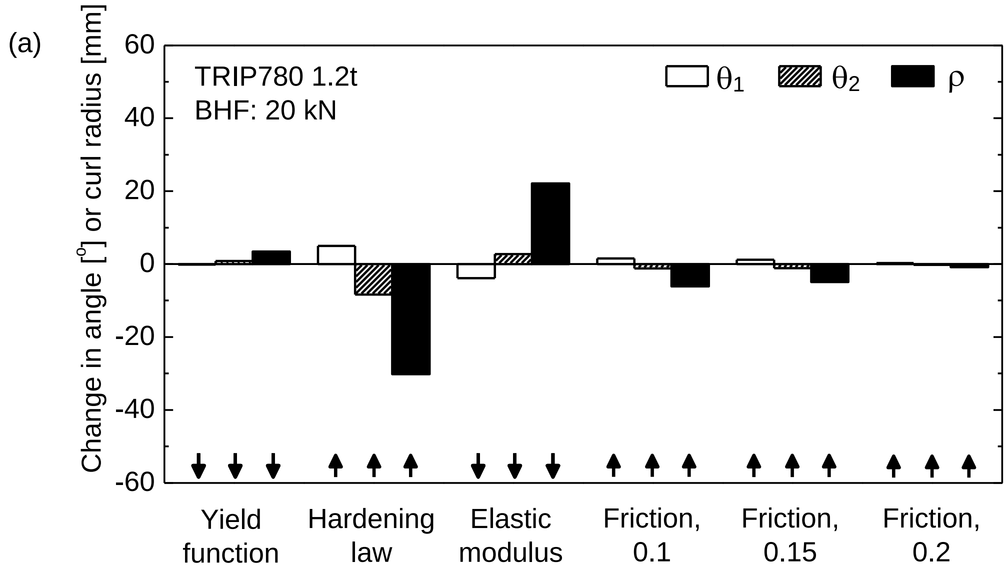


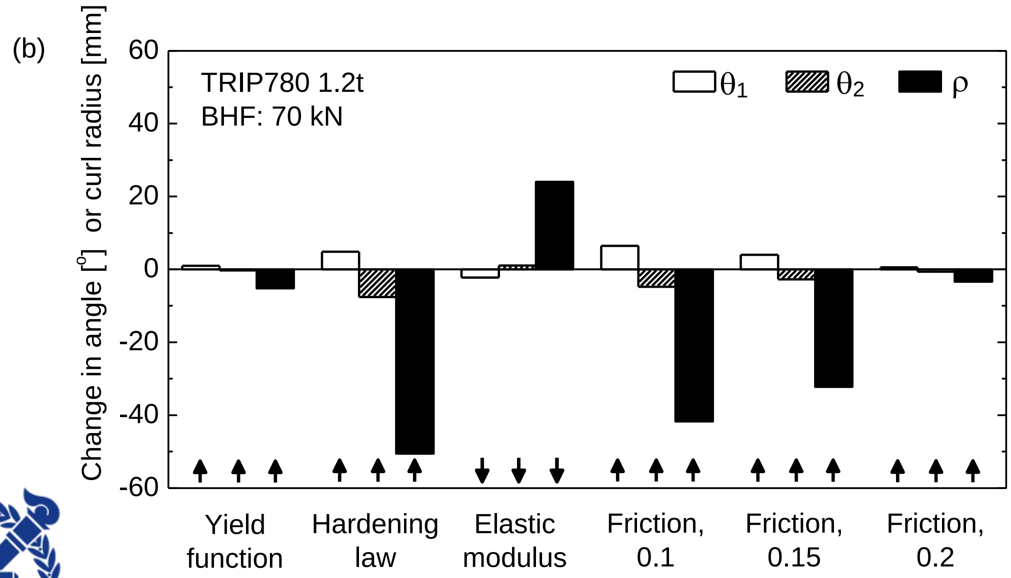






Application: issue





Sensitivity in springback

At low BHF
Hardening > Elastic modulus >> friction > yield function

At high BHF
Hardening > friction> elastic modulus >> yield function

Lee et al, IJP, 2015

Thank you.

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