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소성재료역학
(Metal Plasticity)

Chapter 2: Plasticity characteristics

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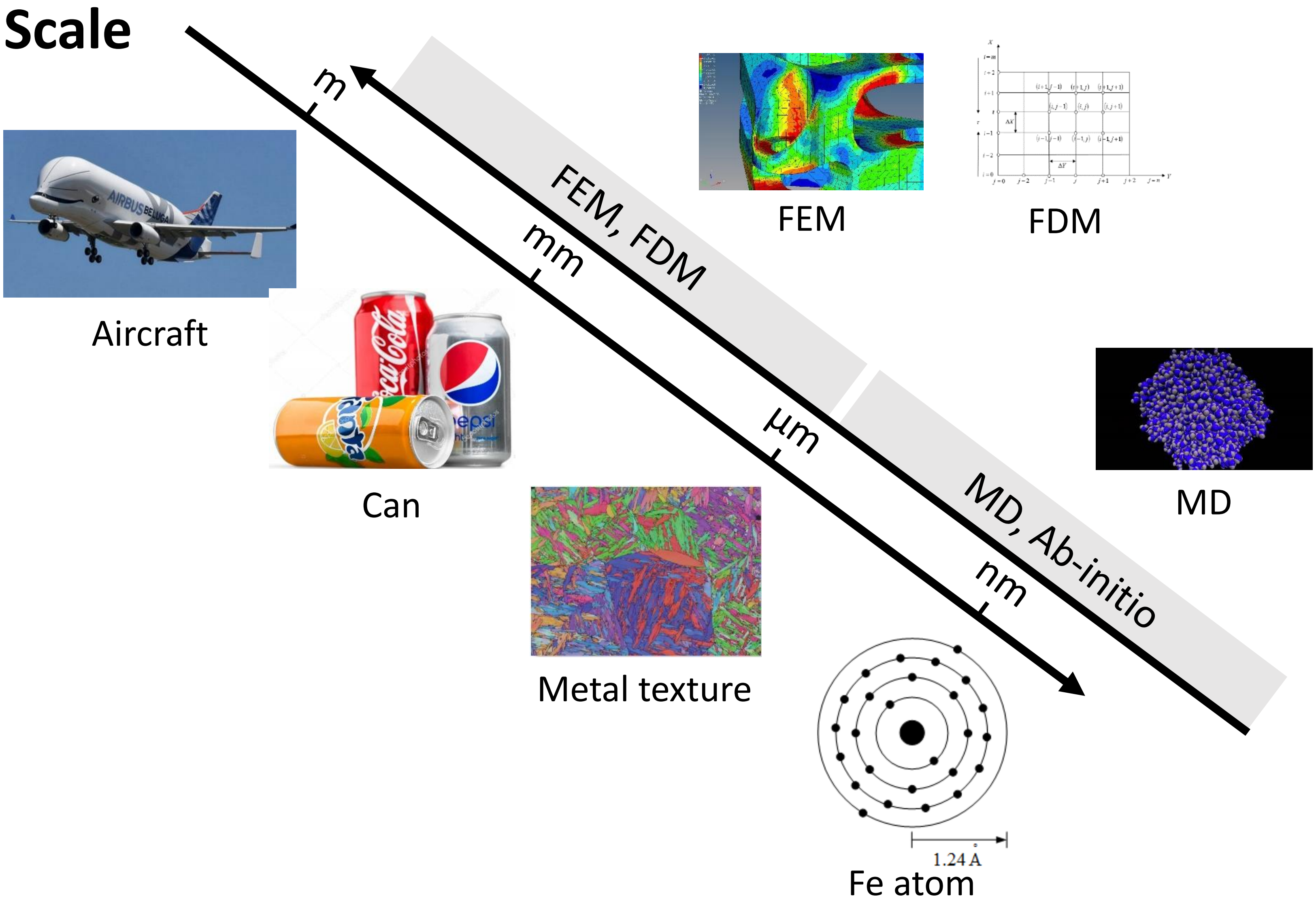
Real Word vs. Continuum Mechanics

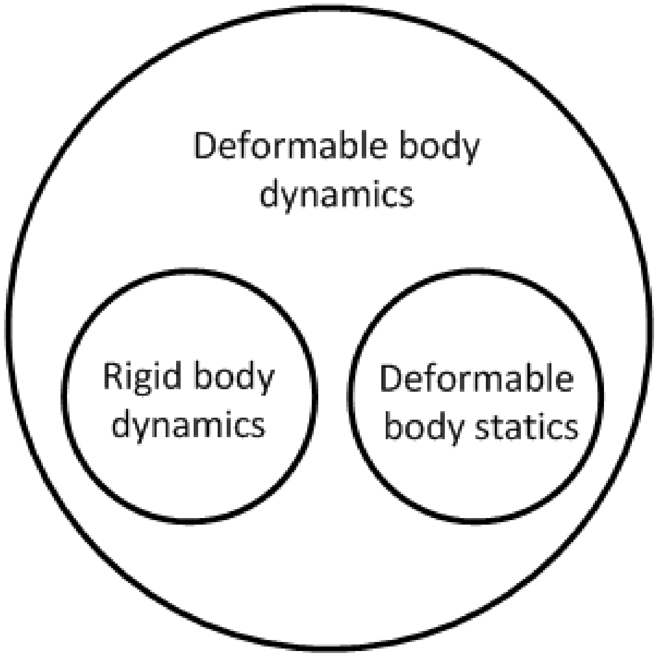
- Discrete vs. Continuum
  - ✓ Real world is discrete in atomistic scale
  - ✓ Simplification about averaged physical value : continuum scale
  - ✓ Averaged description = mathematical description = phenomenological description
  - ✓ Materials, such as solids, liquids and gases, are composed of molecules.
  - ✓ On a microscopic scale, materials have cracks and discontinuities.
  - ✓ Assuming the materials exist as a continuum, meaning the matter in the body is continuously distributed and fills the entire region of space it occupies.
  - ✓ A continuum is a body that can be continually sub-divided into infinitesimal elements with properties being those of the bulk material.

Multi-scale concept: scale bridging

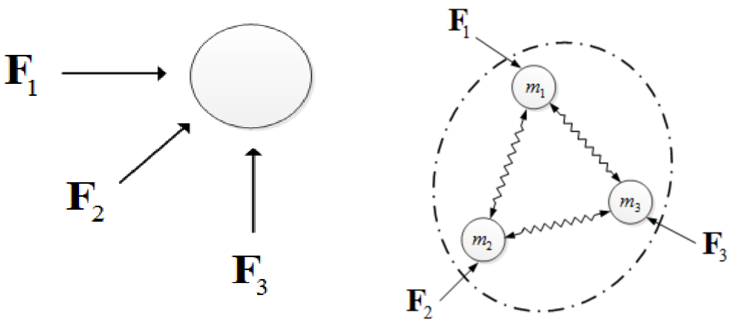
Numerical analysis



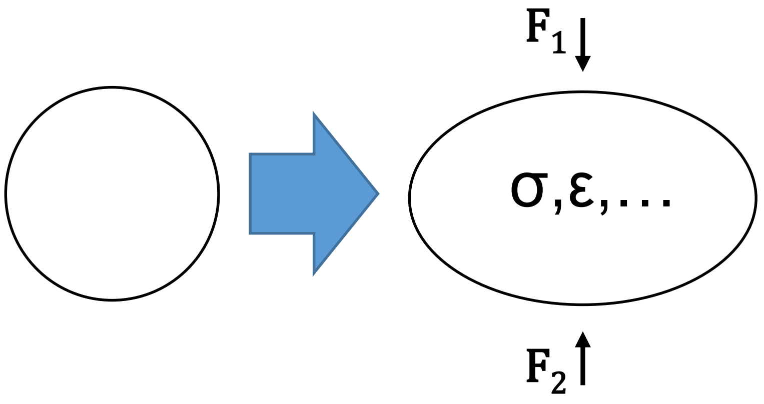
Classification of continuum mechanics



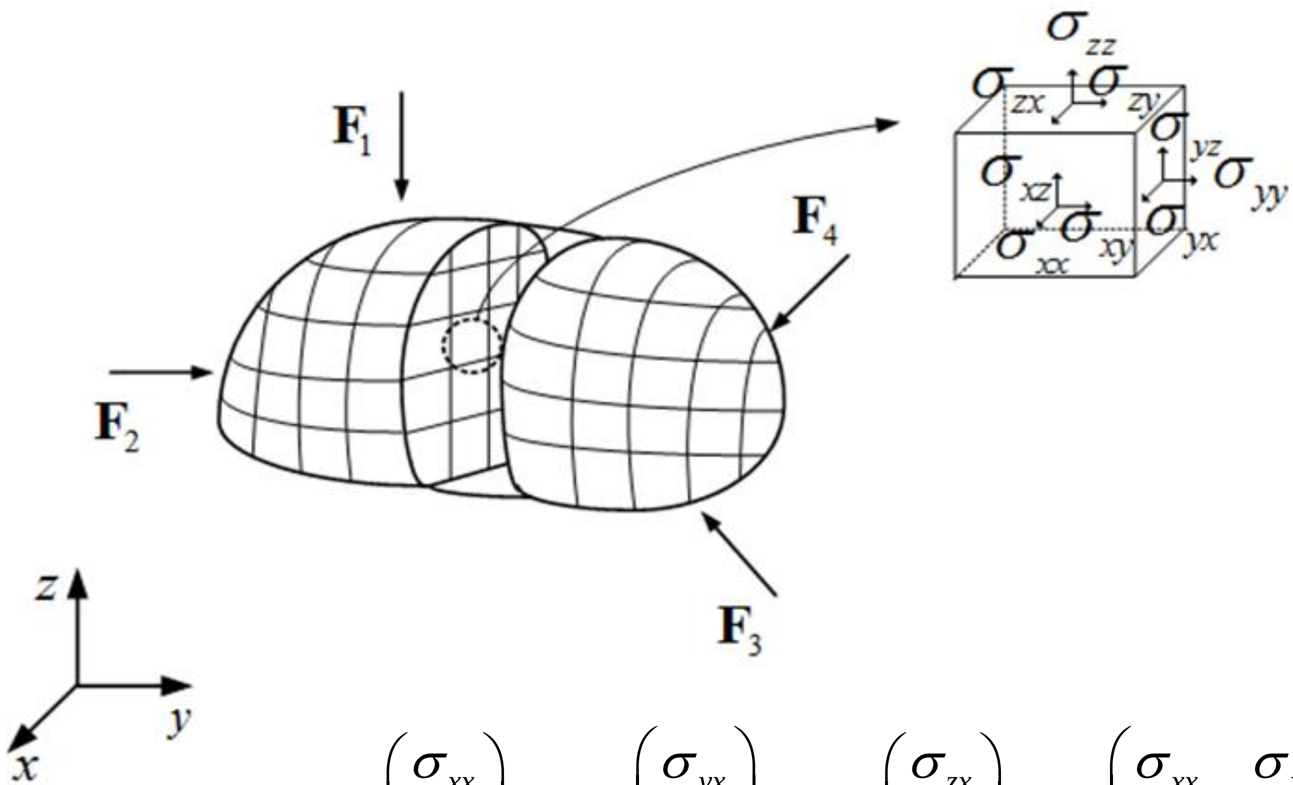
Rigid body statics & dynamics



Deformable body dynamics

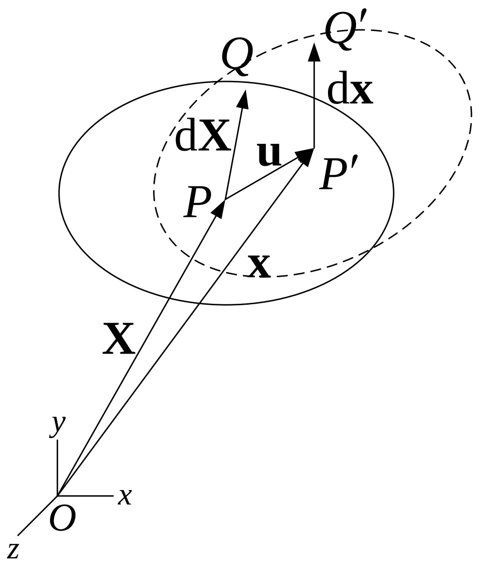


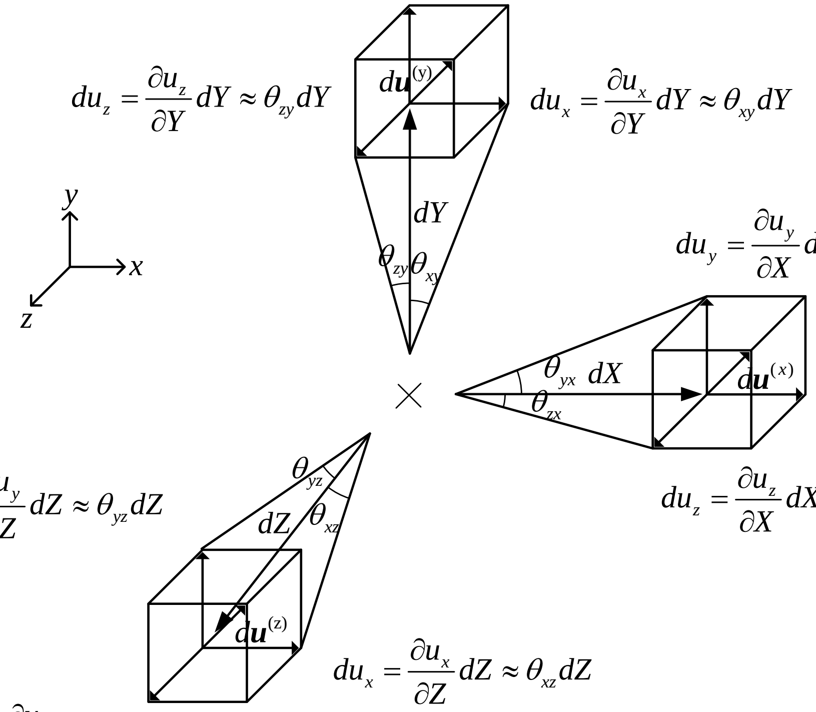
Continuum Mechanics - Stress



$$\boldsymbol{\sigma}_x = \begin{pmatrix} \sigma_{xx} \\ \sigma_{xy} \\ \sigma_{xz} \end{pmatrix}, \boldsymbol{\sigma}_y = \begin{pmatrix} \sigma_{yx} \\ \sigma_{yy} \\ \sigma_{yz} \end{pmatrix}, \boldsymbol{\sigma}_x = \begin{pmatrix} \sigma_{zx} \\ \sigma_{zy} \\ \sigma_{zz} \end{pmatrix} \quad \boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{yx} & \sigma_{zx} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{zy} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{pmatrix}$$

Continuum Mechanics - deformation





$$du_y = \frac{\partial u_y}{\partial Y} dY \approx E_{yy} dY$$

$$du_z = \frac{\partial u_z}{\partial Y} dY \approx \theta_{zy} dY$$

$$du_x = \frac{\partial u_x}{\partial Y} dY \approx \theta_{xy} dY$$

$$du_y = \frac{\partial u_y}{\partial X} dX \approx \theta_{yx} dX$$

$$du_x = \frac{\partial u_x}{\partial X} dX \approx E_{xx} dX$$

$$du_z = \frac{\partial u_z}{\partial X} dX \approx \theta_{zx} dX$$

$$du_y = \frac{\partial u_y}{\partial Z} dZ \approx \theta_{yz} dZ$$

$$du_x = \frac{\partial u_x}{\partial Z} dZ \approx \theta_{xz} dZ$$

$$du_z = \frac{\partial u_z}{\partial Z} dZ \approx E_{zz} dZ$$

Deformable body statics

- Equilibrium equations
  - ✓ 3 equations

$$\boxed{\sigma_{ij,j} + b_i = 0}$$

- Conservation of moment of momentum
  - ✓ Symmetry of stress tensor

$$\boxed{\sigma_{ij} = \sigma_{ji}}$$

- Kinematics
  - ✓ 6 equations

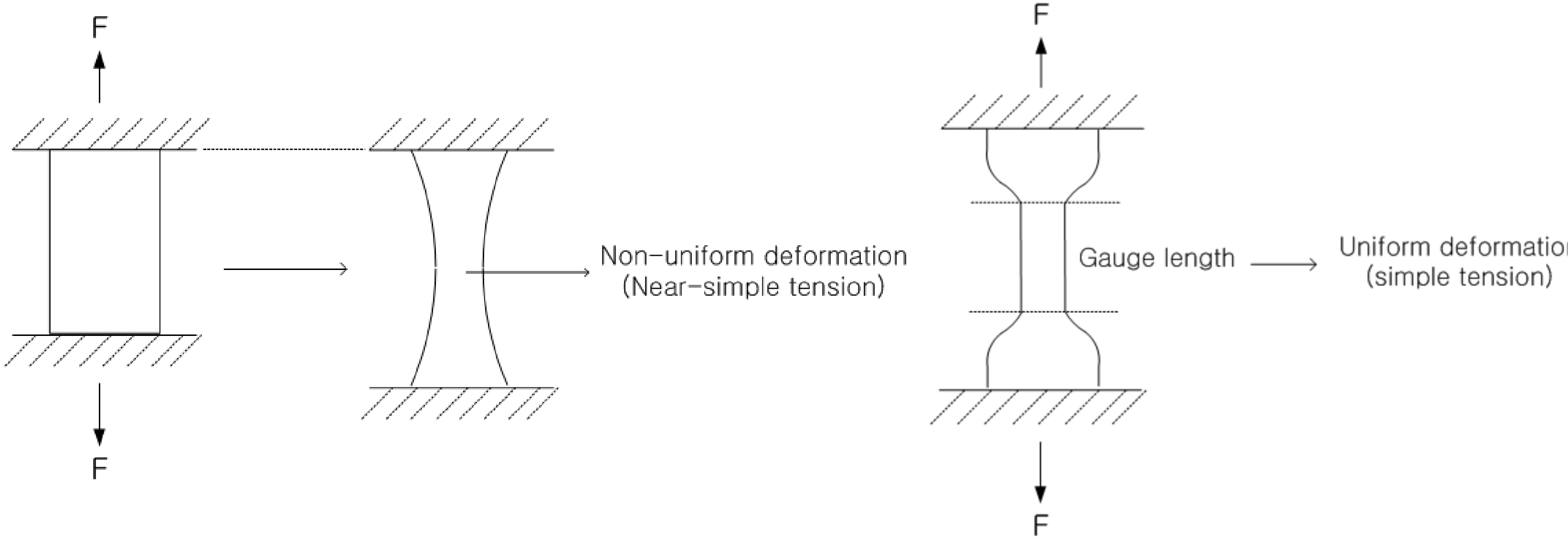
$$\boxed{\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i}\right)}$$

- Constitutive equations
  - ✓ 6 equations

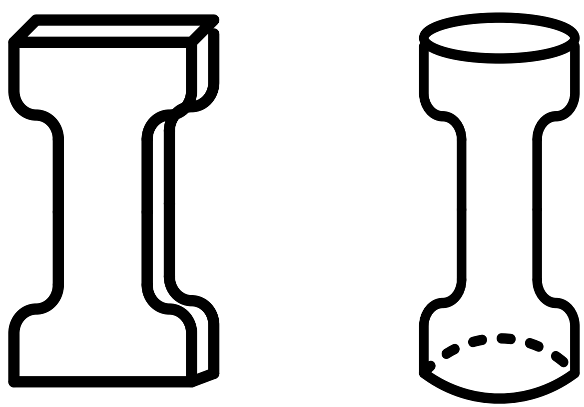
$$\boxed{\sigma_{ij} = C_{ijkl}\varepsilon_{kl}}$$

15 equations for 15 unknowns !!

Simple tension test



Sheet Bulk

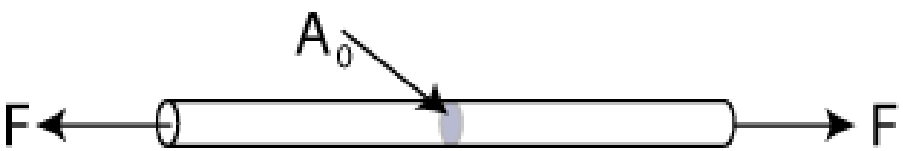


Stress in tension: 1D

- True stress $$\sigma^t = \frac{F}{A}$$

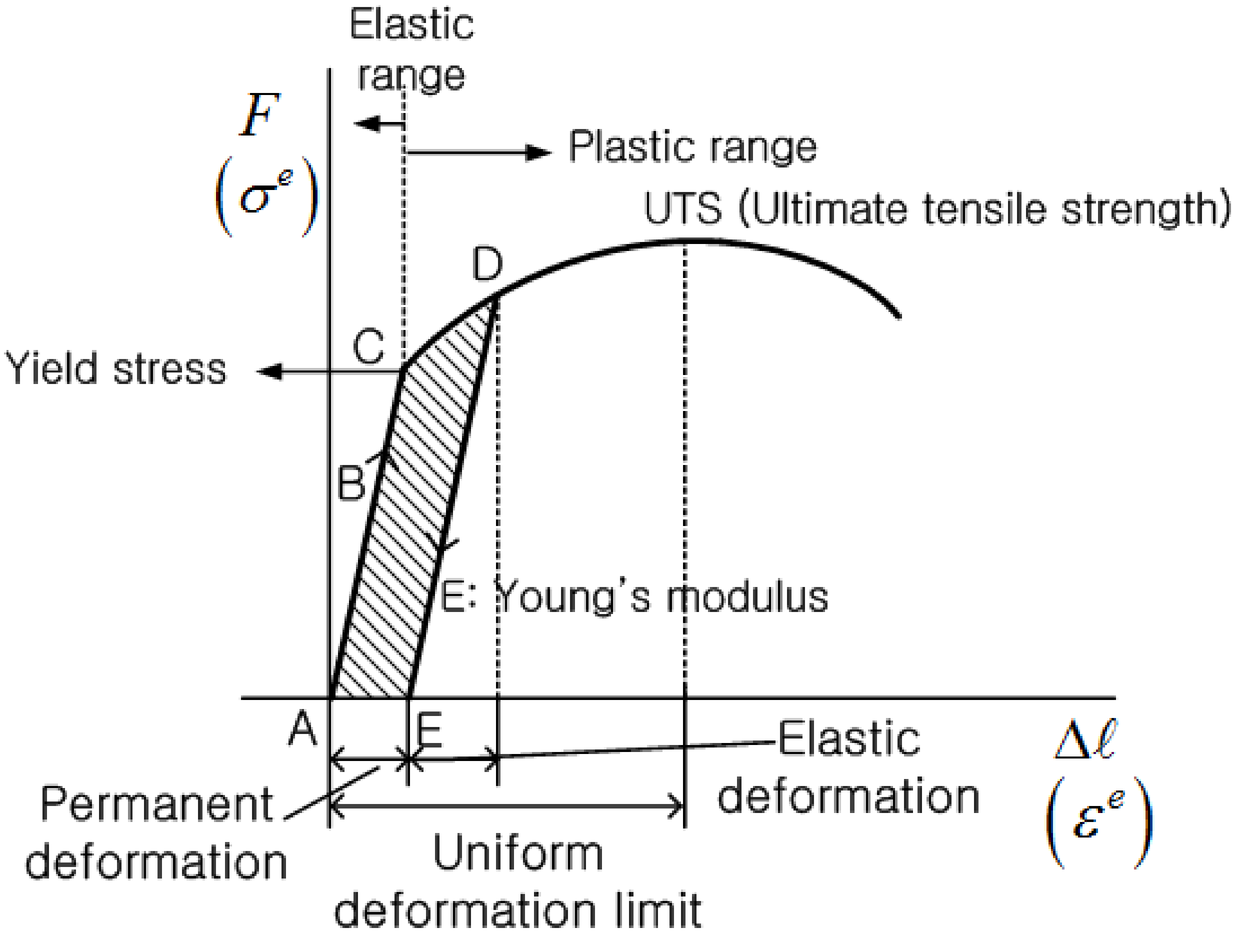
$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

- Engineering strain $$\sigma^e = \frac{F}{A_o}$$

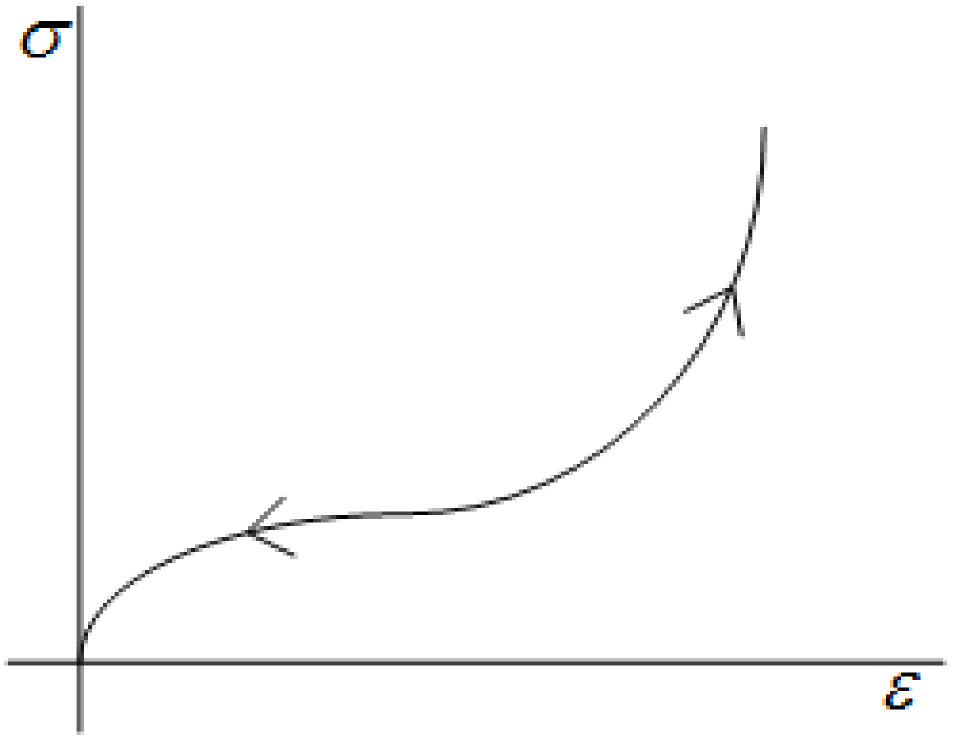


$$\text{Stress, } \sigma = \frac{\text{Force}}{\text{Cross-Sectional Area}} = \frac{F}{A_0}$$

Simple tension test
Eng. S-S curve of metals



Simple tension test
Nonlinear elastic behavior of rubber



Strain in tension: 1D

- True strain

$$d\varepsilon^t = \frac{dl}{l}$$

$$\varepsilon^t = \int_{l_o}^{l} \frac{1}{l} dl = \ln \frac{l}{l_o}$$

- Engineering strain

$$d\varepsilon^e = \frac{dl}{l_o}$$

$$\varepsilon^e = \int_{l_o}^{l} \frac{dl}{l_o} = \frac{l - l_0}{l_0}$$

Stress in tension: Eng. vs. True

- Tension case

$$\varepsilon^t = \ln \frac{l}{l_o} = \ln\left(\frac{l - l_o}{l_o} + 1\right) = \ln\left(\varepsilon^e + 1\right)$$

$$\sigma^t = \frac{F}{A} = \frac{F}{A_o}\frac{A_o}{A} = \frac{F}{A_o}\frac{l}{l_o} = \sigma^e\left(\frac{l - l_o}{l_o} + 1\right) = \sigma^e\left(\varepsilon^e + 1\right)$$

$(A_o l_o = Al : \text{volume constant})$

Stress in tension: Eng. vs. True

- Relation for small deformation

$$\varepsilon^t = \ln\frac{l}{l_o} = \ln\left(\frac{l-l_o}{l_o}+1\right) = \ln\left(\varepsilon^e+1\right)$$

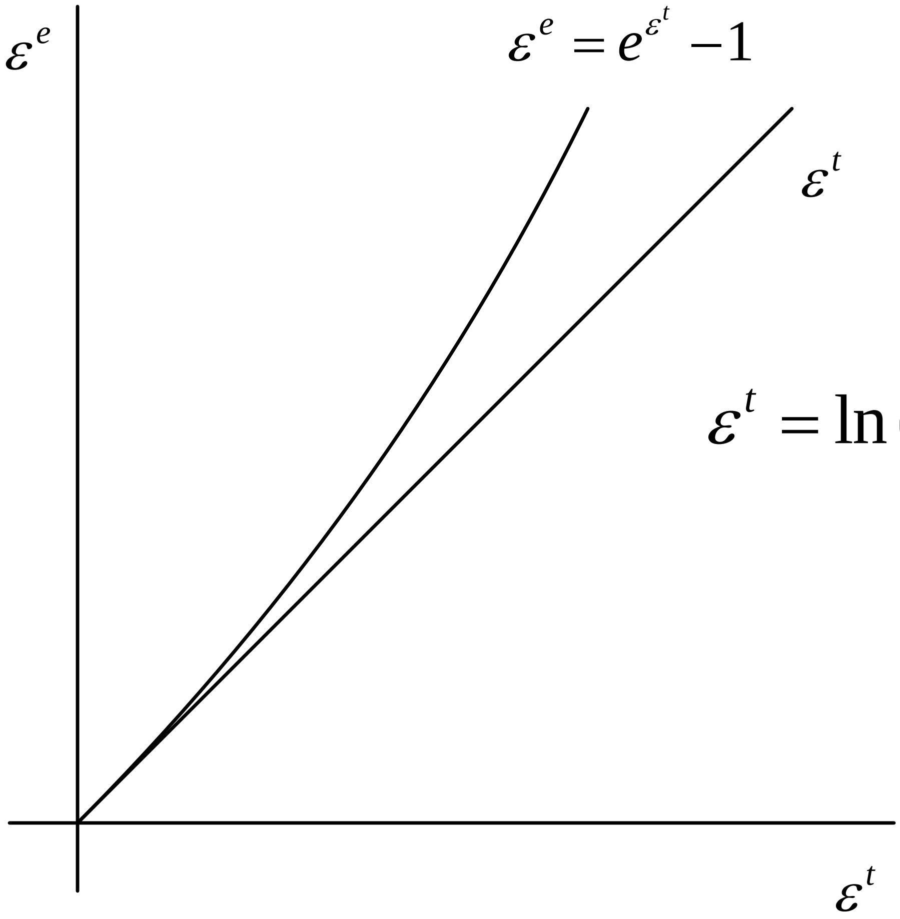
With Taylor series,

$$\varepsilon^t = \ln\left(1+\varepsilon^e\right) = \ln 1 + \left.\frac{1}{1+\varepsilon^e}\right|_{\varepsilon^e=0}\varepsilon^e - \frac{1}{2!}\left.\frac{1}{\left(1+\varepsilon^e\right)^2}\right|_{\varepsilon^e=0}\left(\varepsilon^e\right)^2 + \cdots = \varepsilon^e - \frac{1}{2}\left(\varepsilon^e\right)^2 + \ldots$$

for small deformation,

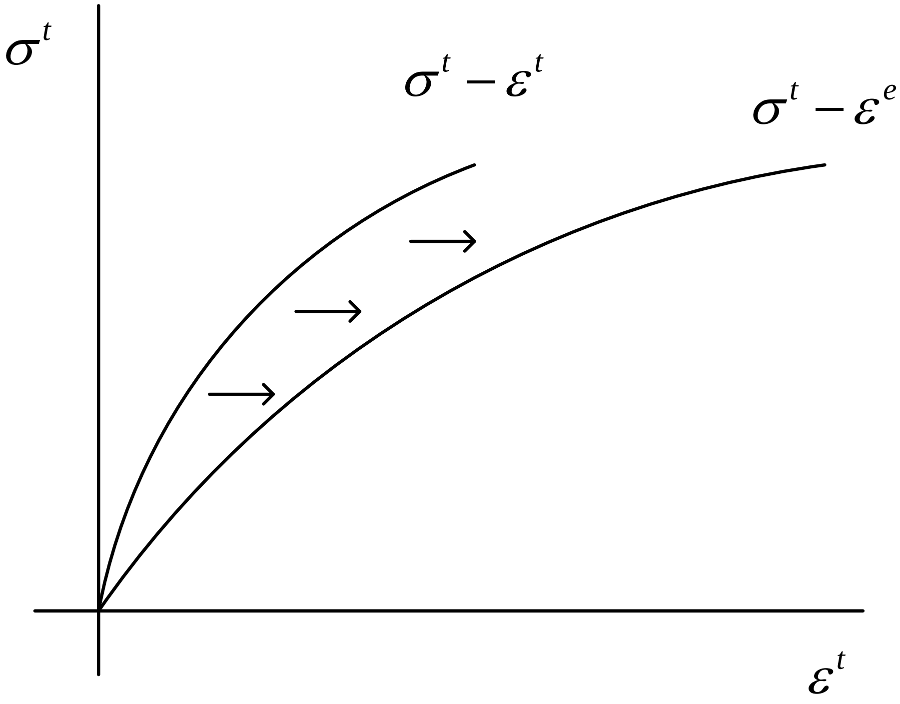
$$\varepsilon^t \approx \varepsilon^e$$

Stress in tension: Eng. vs. True



$$\varepsilon^t = \ln\left(\varepsilon^e + 1\right) \rightarrow \varepsilon^e = e^{\varepsilon^t} - 1$$

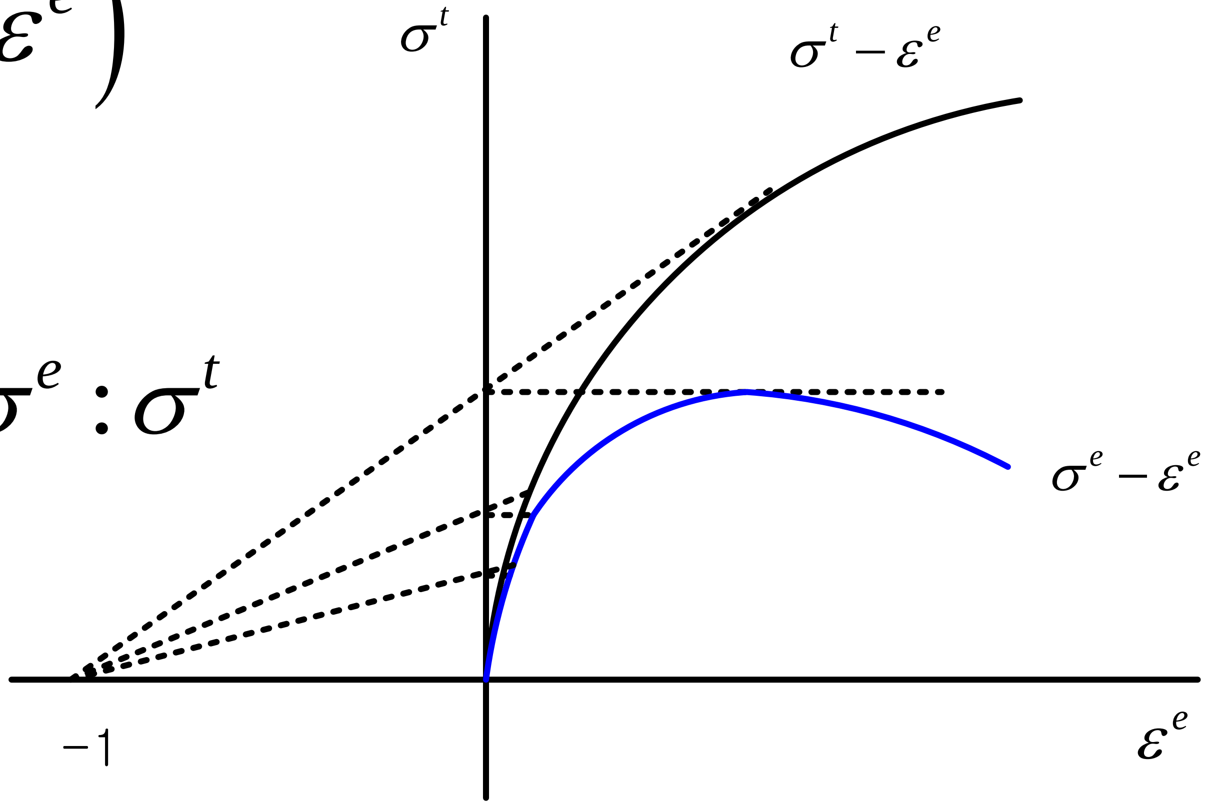
Stress in tension: Eng. vs. True



Stress in tension: Eng. vs. True

$$\sigma^t = \sigma^e \left(1 + \varepsilon^e\right)$$

$$1 : \left(1 + \varepsilon^e\right) = \sigma^e : \sigma^t$$



Stress in compression: Eng. vs. True

- Compressive case

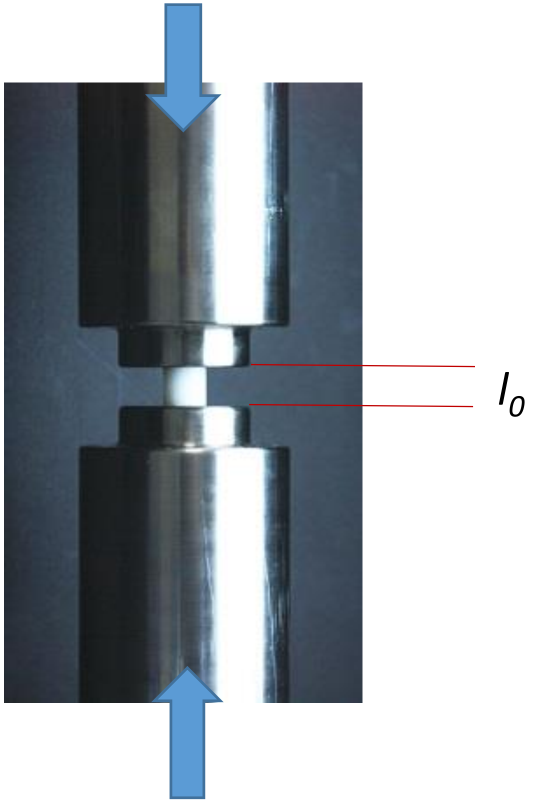
$$d\varepsilon^t = -\frac{dl}{l} \quad (\geq 0)$$

$$\therefore \varepsilon^t = \ln\frac{l_o}{l} \;\left(0 \leq \varepsilon^t < \infty\right)$$

$$\varepsilon^e = \frac{l_o - l}{l_o} = 1 - \frac{l}{l_o} \;\left(0 \leq \varepsilon^e < 1\right)$$

$$\sigma^t = \frac{F}{A} = \frac{F}{A_o}\frac{A_o}{A} = \frac{F}{A_o}\frac{l}{l_o} = \sigma^e\left(1-\varepsilon^e\right)$$

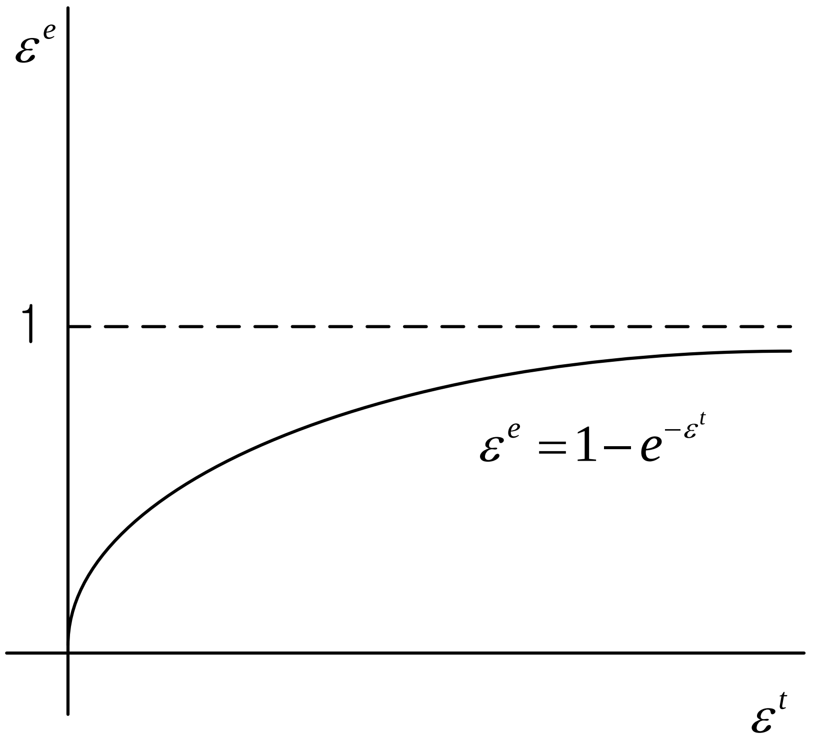
$(A_o l_o = Al$: volume constant)



Stress in compression: Eng. vs. True

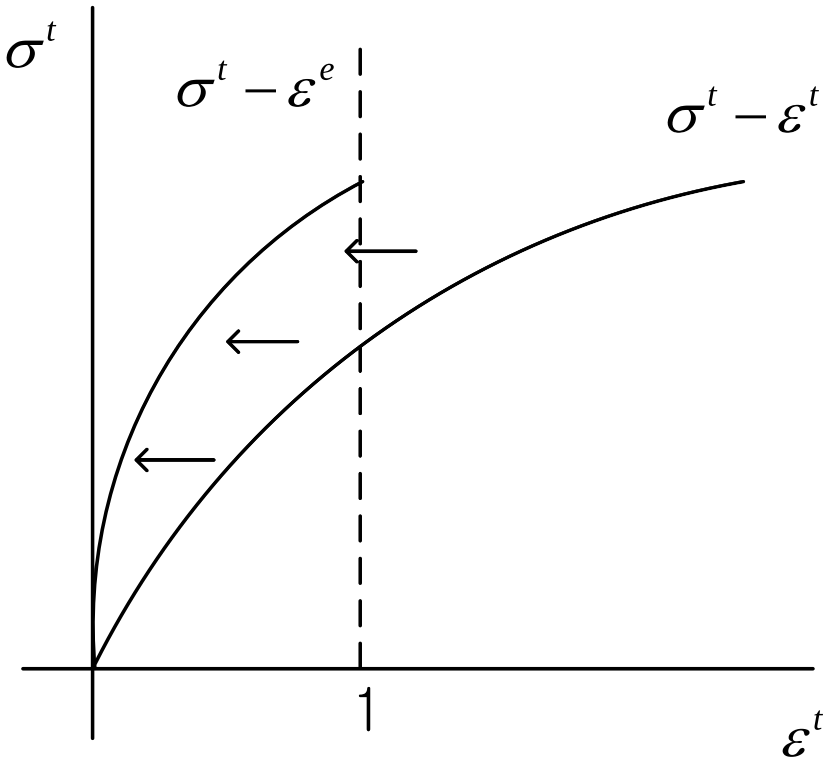
$$\frac{l}{l_o} = 1 - \varepsilon^e$$

$$\varepsilon^t = \ln\frac{l_o}{l} \Rightarrow e^{\varepsilon^t} = \frac{1}{1-\varepsilon^e} \Rightarrow \varepsilon^e = 1 - e^{-\varepsilon^t}$$



Stress in compression: Eng. vs. True

$$\varepsilon^t > \varepsilon^e$$

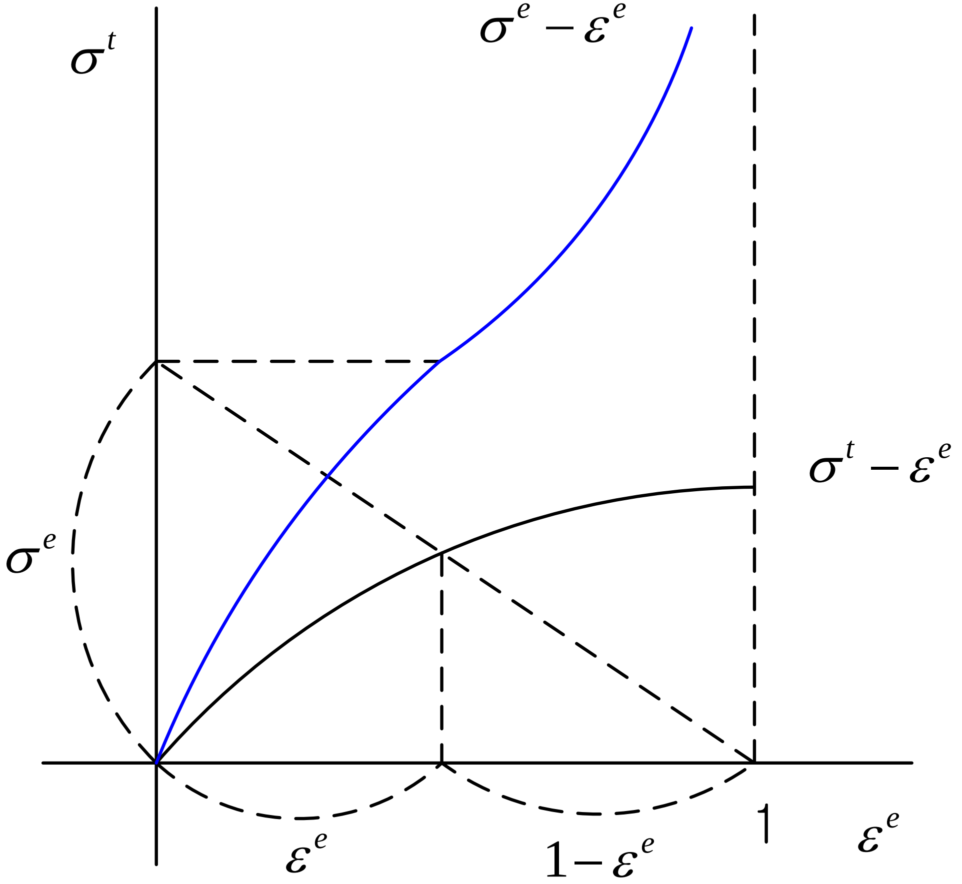


Stress in compression: Eng. vs. True

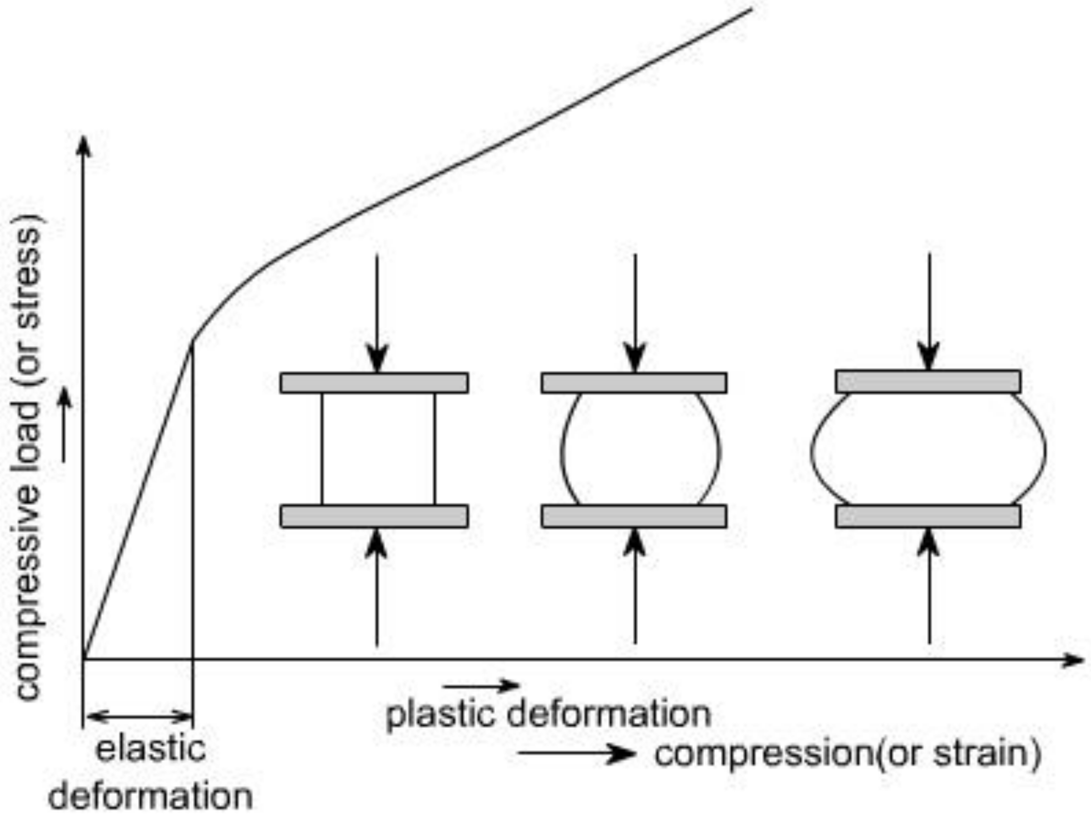
$$\sigma^t = \sigma^e \left(1 - \varepsilon^e\right)$$

$$\Downarrow$$

$$1 : \left(1 - \varepsilon^e\right) = \sigma^e : \sigma^t$$



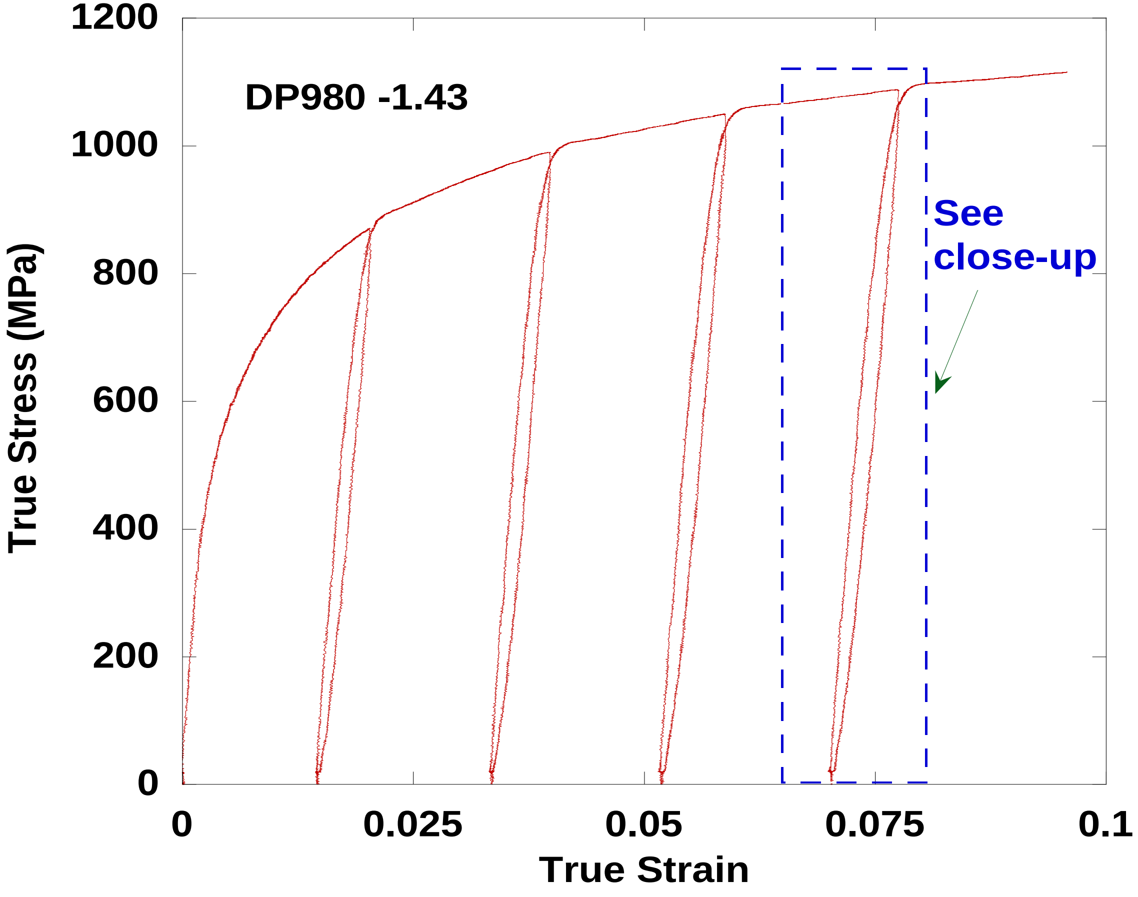
Tension vs. Compression



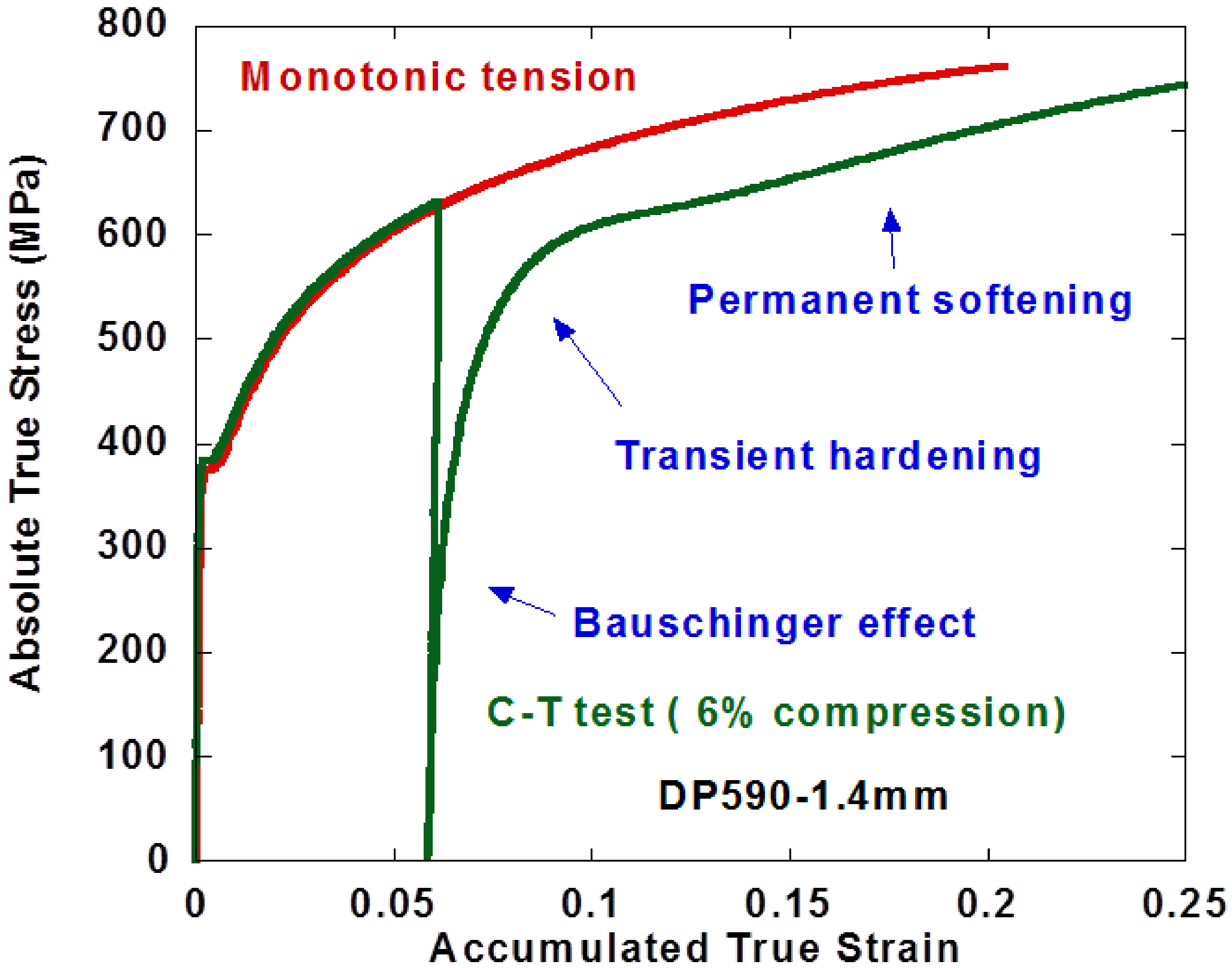
1-D (Metal) Plasticity: characteristics

- Consists of linear and non-linear regions
- Permanent deformation; vs. elasticity
- Yield stress
- Bauschinger effect
  - ✓ The yield strength of a metal decreases when the direction of strain (strain path) is changed; i.e., normally loading is reversed.
  - ✓ Mostly in polycrystalline metals.
  - ✓ The basic mechanism is related to the dislocation structure in the cold worked metals.
  - ✓ As deformation occurs, the dislocations will accumulate at barriers and produce dislocation pile-ups and tangles.

1-D (Metal) Plasticity

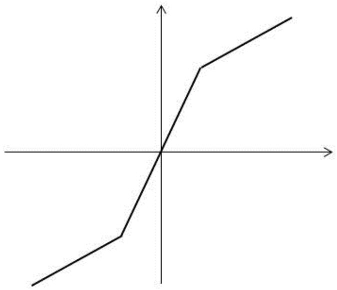


1-D (Metal) Plasticity

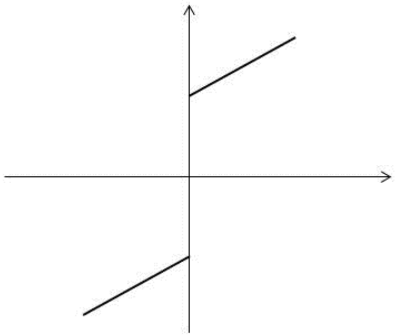


1-D (Metal) Plasticity: Simplification

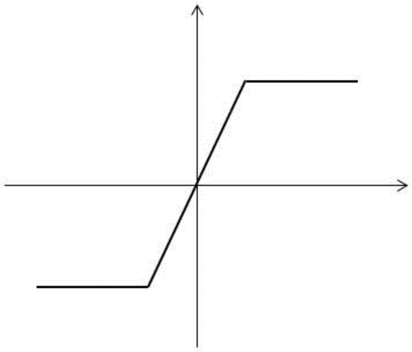
- Elastic-linear plasticity



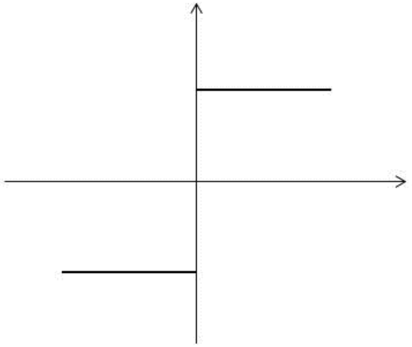
- Rigid-linear plasticity



- Elastic-perfect plasticity



- Rigid-perfect plasticity

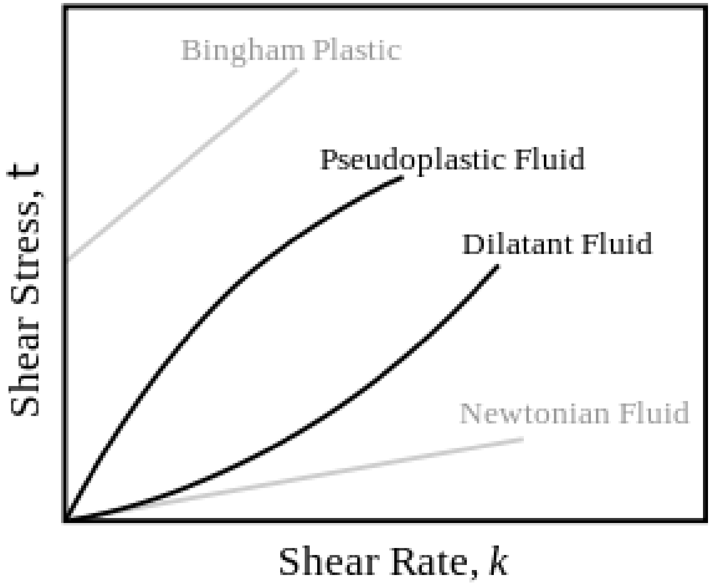


1-D (Metal) Plasticity: Simplification

- Ludwick $$\sigma = \sigma_Y + H\varepsilon^n$$
- Hollomon $$\sigma = H\varepsilon^n$$
- Voce $$\sigma = \sigma_Y + (\sigma_S - \sigma_Y)\{1 - \exp(-n\varepsilon)\}$$
- Swift $$\sigma = H(\varepsilon_S + \varepsilon)^n$$
- Prager $$\sigma = \sigma_Y \tanh\left(\frac{E\varepsilon}{\sigma_Y}\right)$$
- Ramberg and Osgood $$\varepsilon = \frac{\sigma}{E} + H\left(\frac{\sigma}{E}\right)^n$$

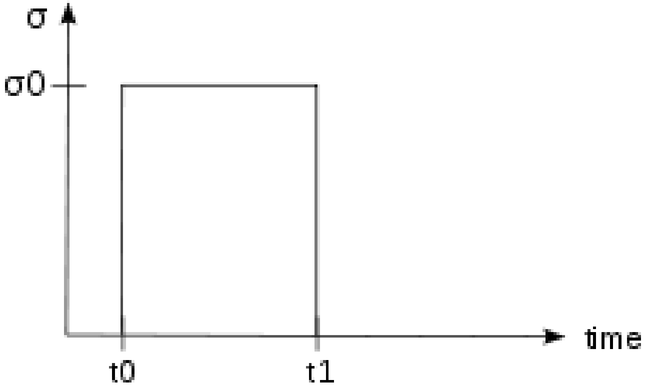
Viscoelasticity

Viscous behavior of fluid

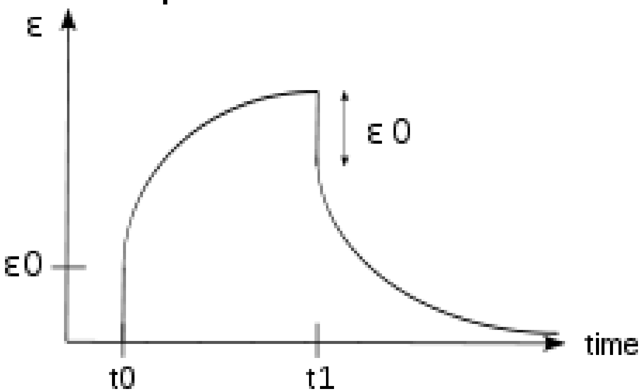


Viscoelasticity – Creep response

a) Input - stress



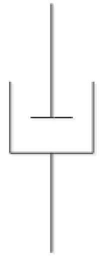
b) Ouput - strain



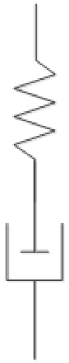
Viscoelasticity – classical models

Elastic solid

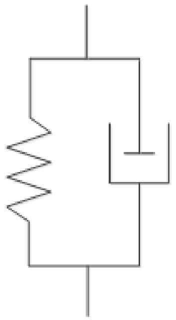
Viscous fluid



Maxwell fluid



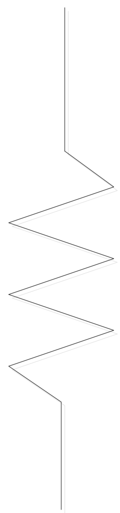
Kelvin solid (or Voigt model)



Viscoelasticity – Maxwell model

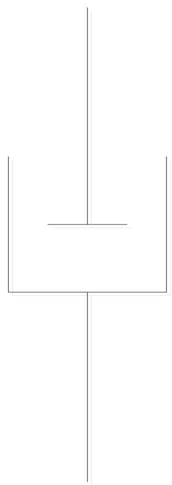
Spring

$$\sigma_s = E\varepsilon_s$$

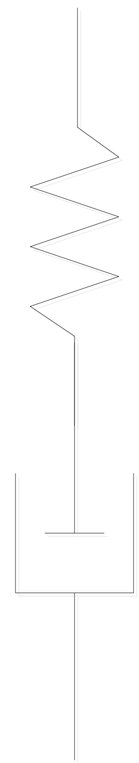


Dashpot

$$\sigma_d = \eta\dot{\varepsilon}_d$$



Maxwell fluid model



Serial connection

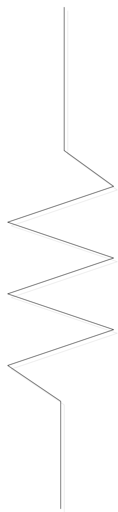
$$\sigma = \sigma_s = \sigma_d$$

$$\Rightarrow \dot{\varepsilon} = \dot{\varepsilon}_s + \dot{\varepsilon}_d = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta}$$

Viscoelasticity – Kelvin model

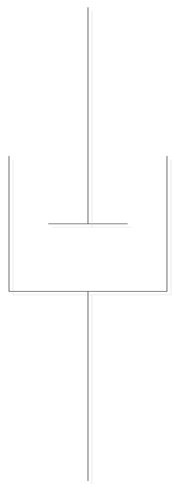
Spring

$$\sigma_s = E\varepsilon_s$$

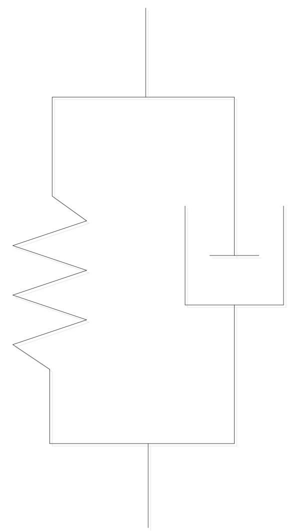


Dashpot

$$\sigma_d = \eta\dot{\varepsilon}_d$$



Kelvin solid(or Voight model)

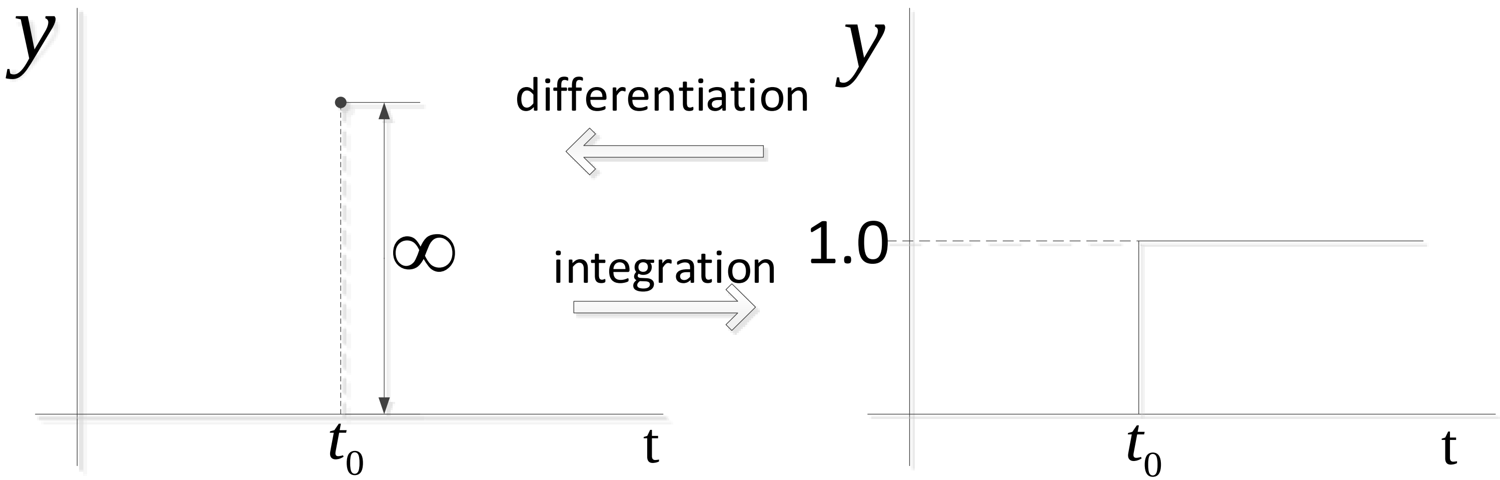


Parallel connection

$$\varepsilon = \varepsilon_s = \varepsilon_d$$

$$\Rightarrow \sigma = \sigma_s + \sigma_d = E\varepsilon + \eta\dot{\varepsilon}$$

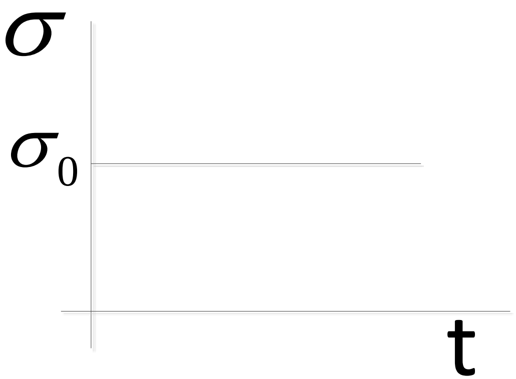
Viscoelasticity – Dirac delta function

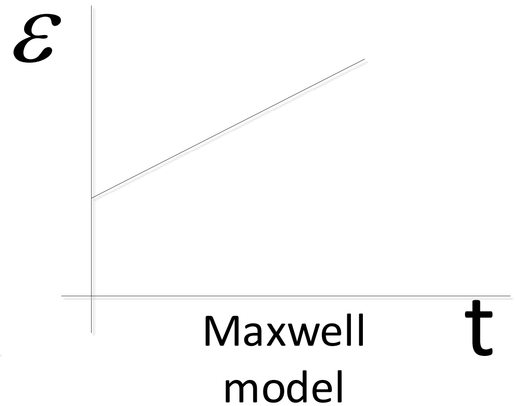


$$\delta(t-t_0)=\begin{cases}\infty & at\ t=t_0\\ 0 & elsewhere\end{cases}$$

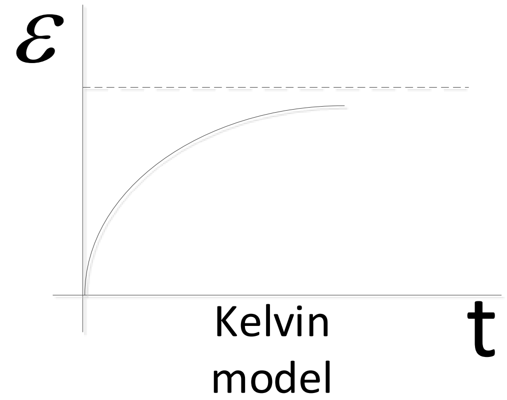
$$\int_{t^*(<t_0)}^{t}\delta(t-t_0)dt=H(t-t_0)=\begin{cases}1\ at\ \ t\geq t_0\\ 0\ \ at\ t<t_0\end{cases}$$

Viscoelasticity – Creep



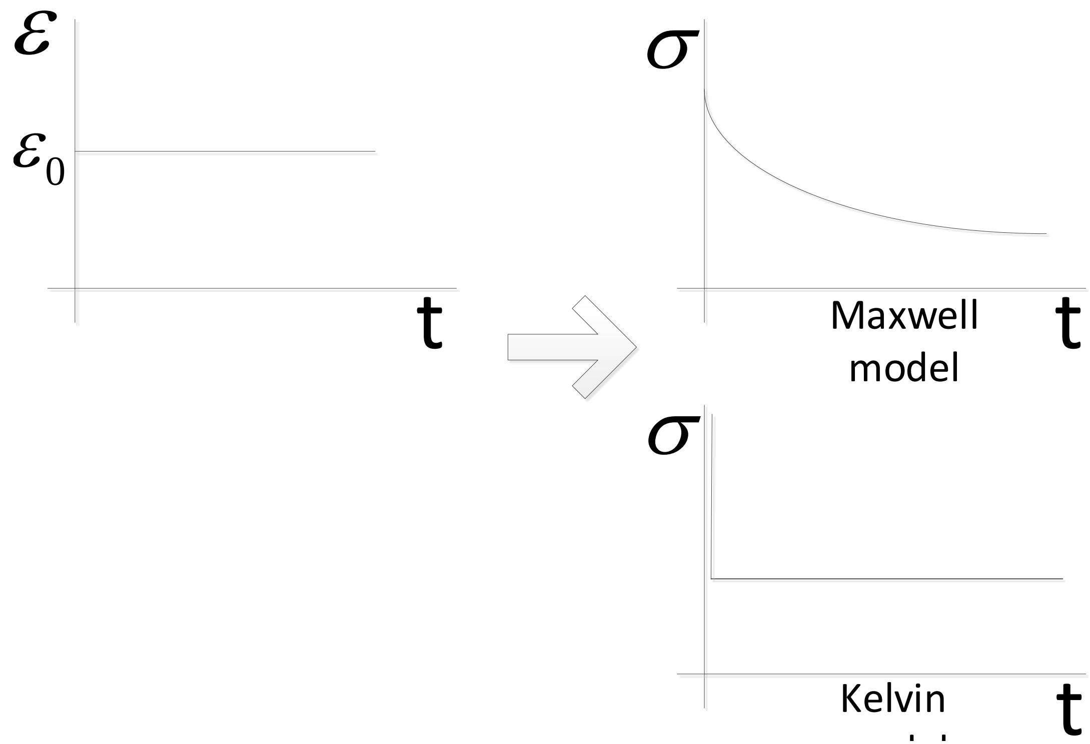


$$\varepsilon = \frac{\sigma_0}{\eta} t + \frac{\sigma_0}{E}$$



$$\varepsilon = \frac{\sigma_0}{E}(1 - e^{-\frac{E}{\eta}t})$$

Viscoelasticity – Relaxation

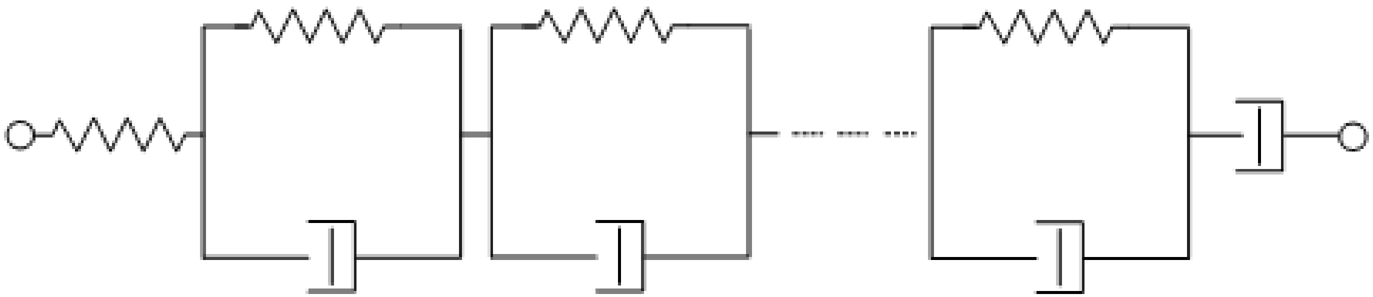


$$\sigma = E\varepsilon_0 e^{-\frac{E}{\eta}t}$$

$$\sigma = E\varepsilon_0 + \eta\varepsilon_0\delta(t=0)$$

Generalization of Kelvin/Maxwell model

Generalization of Kelvin model



Generalization of Maxwell model