Entropy equation in Chapter 4: control mass approach

- The second law of thermodynamics
  - Availability (exergy)
    - The **exergy** of a system is the maximum useful work possible during a process that brings the system into equilibrium with a heat reservoir.

\[ A \equiv E + p_0 V - T_0 S \quad (4.27) \]

- The availability function is a characteristic of the state of the system and the environment which acts as a reservoir at constant pressure \((p_0)\) and constant temperature \((T_0)\).
Entropy equation in Chapter 4: control mass approach

- Availability/exergy

\[
\delta E_{\text{in}} - \delta E_{\text{out}} = \frac{dE_{\text{system}}}{\text{Net energy transfer by heat, work, and mass}}
\]

\[
-\delta Q - \delta W = dU
\]

\[
\delta W = P \, dV = (P - P_0) \, dV + P_0 \, dV = \delta W_{b,\text{useful}} + P_0 \, dV
\]

\[
\delta W_{\text{HE}} = \left(1 - \frac{T_0}{T}\right) \delta Q = \delta Q - \frac{T_0}{T} \delta Q = \delta Q - (-T_0 \, dS) \rightarrow \delta Q = \delta W_{\text{HE}} - T_0 \, dS
\]

Carnot cycle efficiency

\[
\delta Q = -T \, dS
\]

\[
-\delta W_{\text{HE}} + T_0 \, dS - \delta W_{b,\text{useful}} - P_0 \, dV = dU
\]

\[
\delta W_{\text{HE}} + \delta W_{b,\text{useful}} = -dU - P_0 \, dV + T_0 \, dS \rightarrow W_{\text{total,useful}} = (U - U_0) + P_0(V - V_0) - T_0(S - S_0)
\]
Entropy equation in Chapter 4: control mass approach

- The second law of thermodynamics
  - From state 1 to 2
    \[ A_1 - A_2 = E_1 - E_2 + p_o (V_1 - V_2) - T_o (S_1 - S_2) \]
  - **Maximum useful work**
    \[ W_{u,\text{max} \ 1\rightarrow 2} \equiv A_1 - A_2 \]
  - **Maximum useful work** obtainable from the specified change

\[
\left( \frac{dW}{dt} \right)_{u,\text{max}} \equiv - \frac{dA}{dt} = - \frac{d}{dt} (E + p_o V - T_o S)
\]
Energy and entropy equation in Chapter 4: control mass approach

- The second law of thermodynamics

\[ A \equiv E + p_0 V - T_0 S \quad (4.27) \quad W_{u,\text{max} \ 1 \to 2} \equiv A_1 - A_2 \]

**EXAMPLE 8–7 Work Potential of Compressed Air in a Tank**

A 200-m³ rigid tank contains compressed air at 1 MPa and 300 K. Determine how much work can be obtained from this air if the environment conditions are 100 kPa and 300 K.

**Assumptions** 1 Air is an ideal gas. 2 The kinetic and potential energies are negligible.

\[
X_1 = m \phi_1
\]

\[
= m \left[ (u_1 - u_0)^0 + P_0 (v_1 - v_0) - T_0 (s_1 - s_0) + \frac{V_1^2}{2} + gZ_1^0 \right]
\]

\[
= m \left[ P_0 (v_1 - v_0) - T_0 (s_1 - s_0) \right]
\]

\[
U = \frac{3}{2} nRT
\]

**FIGURE 8–24** Schematic for Example 8–7.
Energy and entropy equation in Chapter 4: control mass approach

- The second law of thermodynamics

\[ A \equiv E + p_o V - T_o S \quad \text{(4.27)} \]

\[ W_{u,\text{max} 1 \rightarrow 2} \equiv A_1 - A_2 \]

\[ X_1 = m \phi_1 \]
\[ = m \left[ (u_1 - u_0) + P_0(v_1 - v_0) - T_0(s_1 - s_0) + \frac{V_1^2 v_0^0}{2} + g_z v_1^0 \right] \]
\[ = m[P_0(v_1 - v_0) - T_0(s_1 - s_0)] \]

\[ m_1 = \frac{P_1 V}{R T_1} = \frac{(1000 \text{ kPa})(200 \text{ m}^3)}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(300 \text{ K})} = 2323 \text{ kg} \]

\[ P_0(v_1 - v_0) = P_0 \left( \frac{RT_1}{P_1} - \frac{RT_0}{P_0} \right) = RT_0 \left( \frac{P_0}{P_1} - 1 \right) \quad \text{(since } T_1 = T_0) \]

\[ T_0(s_1 - s_0) = T_0 \left( c_p \ln \frac{T_1}{T_0} - R \ln \frac{P_1}{P_0} \right) = -RT_0 \ln \frac{P_1}{P_0} \quad \text{(since } T_1 = T_0) \]

\[ ds = \frac{dh}{T} - \frac{\nu}{T} dP \]
\[ dh = c_p \, dT \quad \nu = RT/P \]

the second T-ds relation

---

**Figure 8-24**
Schematic for Example 8-7.
Energy and entropy equation in Chapter 4: control mass approach

- The second law of thermodynamics

\[
A \equiv E + p_0 V - T_0 S \quad (4.27)
\]

\[
W_{u,\text{max} \, 1 \rightarrow 2} \equiv A_1 - A_2
\]

\[
P_0(v_1 - v_0) = P_0 \left( \frac{RT_1}{P_1} - \frac{RT_0}{P_0} \right) = RT_0 \left( \frac{P_0}{P_1} - 1 \right) \quad \text{(since } T_1 = T_0)\]

\[
T_0(s_1 - s_0) = T_0 \left( c_p \ln \frac{T_1}{T_0} - R \ln \frac{P_1}{P_0} \right) = -RT_0 \ln \frac{P_1}{P_0} \quad \text{(since } T_1 = T_0)\]

\[
\phi_1 = RT_0 \left( \frac{P_0}{P_1} - 1 \right) + RT_0 \ln \frac{P_1}{P_0} = RT_0 \left( \ln \frac{P_1}{P_0} + \frac{P_0}{P_1} - 1 \right)
\]

\[
= (0.287 \text{ kJ/kg} \cdot \text{K})(300 \text{ K}) \left( \ln \frac{1000 \text{ kPa}}{100 \text{ kPa}} + \frac{100 \text{ kPa}}{1000 \text{ kPa}} - 1 \right) = 120.76 \text{ kJ/kg}
\]

\[
X_1 = m_1 \phi_1 = (2323 \text{ kg})(120.76 \text{ kJ/kg}) = 280,525 \text{ kJ} \approx 281 \text{ MJ}
\]
Energy and entropy equation in Chapter 4: control mass approach

- Entropy change of ideal gas

From the first $T\, ds$ relation

$$ds = \frac{du}{T} + \frac{P}{T}\, d\nu$$

$$ds = c_v \frac{dT}{T} + R \frac{d\nu}{\nu}$$

$$s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} + R \ln \frac{\nu_2}{\nu_1}$$

$$s_2 - s_1 = c_{v,\text{avg}} \ln \frac{T_2}{T_1} + R \ln \frac{\nu_2}{\nu_1}$$

From the second $T\, ds$ relation

$$ds = \frac{dh}{T} - \frac{\nu}{T}\, dP$$

$$dh = c_p\, dT \quad \nu = RT/P$$

$$s_2 - s_1 = \int_1^2 c_p(T) \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

$$s_2 - s_1 = c_{p,\text{avg}} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

$$(\text{kJ/kg} \cdot \text{K})$$
Energy equation in Chapter 4: control volume approach

- The first law of thermodynamics

\[
\frac{dU^o}{dt}_{c.m.} = \left( \frac{dU^o}{dt} \right)_{c.m.} - \left( \frac{dQ}{dt} \right)_{c.m.} - \left( \frac{dW}{dt} \right)_{c.m.} \tag{4.19a}
\]

- In the control volume approach, there are flow in and out.

\[
\frac{dU^o}{dt}_{c.v.} = \left( \frac{dU^o}{dt} \right)_{c.v.} - \sum_{i=1}^{l} \dot{m}_i u_i^o \quad U^o = m u^o = m(u + \nu^2/2)
\]

- The first law

\[
\left( \frac{dU^o}{dt} \right)_{c.v.} = \sum_{i=1}^{l} \dot{m}_i u_i^o + \left( \frac{DQ}{Dt} \right) - \left( \frac{DW}{Dt} \right) \tag{4.35}
\]
Energy equation in Chapter 4: control volume approach

- Heat addition term

\[
\frac{DQ}{Dt} = \left( \frac{dQ}{dt} \right)_{c.v.} = \frac{dQ}{dt}
\]

- Work term

- Work done by the mass in the control volume + work associated with mass flow

\[
\frac{DW}{Dt} = \left( \frac{dW}{dt} \right)_{c.v.} - \sum_{i=1}^{I} \dot{m}_i (pV)_i
\]

\[
W_{\text{normal}} = P \cdot A \cdot x = P \cdot V
\]

\[
\dot{W}_{\text{normal}} = P \cdot \dot{V} = \frac{P \cdot \dot{m}}{\rho} = \dot{m} P V
\]

- Body force

\[
\vec{v} \cdot \vec{m} = -m \frac{D\psi}{Dt} = - \frac{D[m\psi]}{Dt} = -m \left( \frac{d\psi}{dt} \right)_{c.m.} = - \left( \frac{d[m\psi]}{dt} \right)_{c.m.}
\]  

(4.22c)

\[
\frac{D[m\psi]}{Dt} = \frac{d[m\psi]}{dt} - \sum_{i=1}^{I} \dot{m}_i \psi \\
\frac{D[mgz]}{Dt} = \frac{d[mgz]}{dt} - \sum_{i=1}^{I} \dot{m}_i gz
\]
Energy equation in Chapter 4: control volume approach

- Control mass

\[
\left( \frac{d}{dt} [E] \right)_{\text{c.m.}} = \left( \frac{dQ}{dt} \right)_{\text{c.m.}} + \left( \frac{dQ}{dt} \right)_{\text{gen}} - \left( \frac{dW}{dt} \right)_{\text{shaft}} - p \left( \frac{dV}{dt} \right)_{\text{c.m.}} - \left( \frac{dW}{dt} \right)_{\text{shear}} \quad (4.24c)
\]

- Control volume

\[
\left( \frac{dE}{dt} \right)_{\text{c.v.}} = \sum_{i=1}^{l} \dot{m}_i \left( u_i^0 + \frac{p_i}{\rho_i} + g z_i \right) + \left( \frac{dQ}{dt} \right)_{\text{gen}} - \left( \frac{dW}{dt} \right)_{\text{shaft}} - \left( \frac{dW}{dt} \right)_{\text{normal}} - \left( \frac{dW}{dt} \right)_{\text{shear}} \quad (4.38)
\]

- For stationary control volume

\[
\left( \frac{\partial E}{\partial t} \right)_{\text{c.v.}} = \sum_{i=1}^{l} \dot{m}_i (h_i^0 + g z_i) + \left( \frac{dQ}{dt} \right)_{\text{gen}} - \left( \frac{dW}{dt} \right)_{\text{shaft}} - \left( \frac{dW}{dt} \right)_{\text{normal}} - \left( \frac{dW}{dt} \right)_{\text{shear}}
\]
Energy equation in Chapter 4: control volume approach

- Control volume

**FIGURE 4.4** Lumped parameter control volume with a moving boundary.
Entropy equation in Chapter 4: control volume approach

- Entropy equation

\[
\left( \frac{dS}{dt} \right)_{c.m.} = \frac{dS}{dt} = \frac{dS}{dt} _{c.v.} - \sum_{i=1}^{l} \dot{m}_i s_i
\]

- For a stationary control volume

\[
\left( \frac{dS}{dt} \right)_{c.m.} = \left( \frac{\partial S}{\partial t} \right)_{c.v.} - \sum_{i=1}^{l} \dot{m}_i s_i
\]

\[
\left( \frac{\partial S}{\partial t} \right)_{c.v.} = \sum_{i=1}^{l} \dot{m}_i s_i + \dot{S}_{gen} + \frac{(dQ/dt)}{T_s}
\]
Entropy equation in Chapter 4: control volume approach

\[
\left( \frac{dW}{dt} \right)_{\text{actual}} = \left( \frac{dW}{dt} \right)_{\text{shaft}} + \left( \frac{dW}{dt} \right)_{\text{normal}}
\]

(Useful part of the actual work)

\[
\left( \frac{dW}{dt} \right)_{u,\text{actual}} = \left( \frac{dW}{dt} \right)_{\text{actual}} - \boxed{}
\]

Maximum useful work

\[
W_{u,\text{max} \ 1\rightarrow2} \equiv A_1 - A_2
\]

\[
A_1 - A_2 = E_1 - E_2 + p_o(V_1 - V_2) - T_o(S_1 - S_2)
\]
Entropy equation in Chapter 4: control volume approach

- Energy equation

\[
\left( \frac{dE}{dt} \right)_{c.v.} = \sum_{i=1}^{l} m_i \left( u_i^0 + \frac{p_i}{\rho_i} + g z_i \right) + \frac{dQ}{dt} + \left( \frac{dQ}{dt} \right)_{gen} - \left( \frac{dW}{dt} \right)_{shaft} - \left( \frac{dW}{dt} \right)_{normal} - \left( \frac{dW}{dt} \right)_{shear}
\]

- Useful work (useful part of the actual work)

\[
\left( \frac{dW}{dt} \right)_{u,actual} = \left( \frac{dW}{dt} \right)_{actual} - p_o \frac{dV}{dt}
\]

- Useful work (or useful actual work)

\[
\left( \frac{dW}{dt} \right)_{u,actual} = \sum_{i=1}^{l} m_i (h_i^0 + g z_i) + \frac{dQ}{dt} + \left( \frac{dQ}{dt} \right)_{gen} - \left( \frac{\partial(E + p_o V)}{\partial t} \right)_{c.v.} - \left( \frac{dW}{dt} \right)_{shear}
\]  \hspace{1cm} (4.44)

Partial derivative? Stationary!
Energy Equation

Entropy equation in Chapter 4: control volume approach

\[
\left( \frac{\partial S}{\partial t} \right)_{c.v.} = \sum_{i=1}^{I} \dot{m}_i s_i + \dot{S}_{\text{gen}} + \frac{(dQ/dt)}{T_s}
\]

- Multiplying \( T_0 \), then subtracting from (4.44)

\[
T_0 \left( \frac{\partial S}{\partial t} \right)_{c.v.} = T_0 \sum_{i=1}^{I} \dot{m}_i s_i + T_0 \dot{S}_{\text{gen}} + T_0 \frac{(dQ/dt)}{T_s}
\]

\[
\left( \frac{dW}{dt} \right)_{u,\text{actual}} = \sum_{i=1}^{I} \dot{m}_i (h_i^o + gz_i) + \frac{dQ}{dt} + \left( \frac{dQ}{dt} \right)_{\text{gen}} \bigg|_{\text{c.v.}} - \left( \frac{\partial (E + p_o V)}{\partial t} \right)_{\text{c.v.}} - \left( \frac{dW}{dt} \right)_{\text{shear}} \tag{4.44}
\]

- Express the actual useful work with entropy

\[
\left( \frac{dW}{dt} \right)_{u,\text{actual}} = -\left[ \frac{\partial (E + p_o V - T_0 S)}{\partial t} \right]_{\text{c.v.}} + \sum_{i=1}^{I} \dot{m}_i (h_i^o - T_0 s + gz)_i
\]

\[
+ \left( 1 - \frac{T_0}{T_s} \right) \frac{dQ}{dt} + \left( \frac{dQ}{dt} \right)_{\text{gen}} \bigg|_{\text{c.v.}} - \left( \frac{dW}{dt} \right)_{\text{shear}} - T_0 \dot{S}_{\text{gen}}
\]
Entropy equation in Chapter 4: control volume approach

\[
\left( \frac{dW}{dt} \right)_{u,\text{actual}} = -\left[ \frac{\partial (E + p_0 V - T_0 S)}{\partial t} \right]_{\text{c.v.}} + \sum_{i=1}^{l} \dot{m}_i (h^o - T_0 s + g z)_i + \left( 1 - \frac{T_0}{T_s} \right) \frac{dQ}{dt} + \left( \frac{dQ}{dt} \right)_{\text{gen}} - \left( \frac{dW}{dt} \right)_{\text{shear}} - T_0 \dot{S}_{\text{gen}}
\]

- Maximum useful work
  \[ T_0 \dot{S}_{\text{gen}} = 0 \]

- Irreversibility

\[ \dot{I} \equiv \left( \frac{dW}{dt} \right)_{u,\text{max}} - \left( \frac{dW}{dt} \right)_{u,\text{actual}} \equiv T_0 \dot{S}_{\text{gen}} \]
Energy Equation

Entropy equation in Chapter 4: control volume approach

- Irreversibility

\[ i \equiv \left( \frac{dW}{dt} \right)_{u,\text{max}} - \left( \frac{dW}{dt} \right)_{u,\text{actual}} \equiv T_o \dot{S}_{\text{gen}} \]

\[ \left( \frac{\partial S}{\partial t} \right)_{c.v.} = \sum_{i=1}^{l} \dot{m}_i s_i + \dot{S}_{\text{gen}} + \frac{(dQ/dt)}{T_s} \]

\[ i = T_o \left( \frac{\partial S}{\partial t} \right)_{c.v.} - T_o \sum_{i=1}^{l} \dot{m}_i s_i - \frac{T_o}{T_s} \frac{dQ}{dt} \]
Entropy equation in Chapter 4: control volume approach

**Example**

**EXAMPLE 8–11 Exergy Destruction During Expansion of Steam**

A piston–cylinder device contains 0.05 kg of steam at 1 MPa and 300°C. Steam now expands to a final state of 200 kPa and 150°C, doing work. Heat losses from the system to the surroundings are estimated to be 2 kJ during this process. Assuming the surroundings to be at $T_0 = 25°C$ and $P_0 = 100$ kPa, determine (a) the exergy of the steam at the initial and the final states, (b) the exergy change of the steam, (c) the exergy destroyed, and (d) the second-law efficiency for the process.

**State 1:**

<table>
<thead>
<tr>
<th>$P_1 = 1$ MPa</th>
<th>$T_1 = 300°C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1 = 2793.7$ kJ/kg</td>
<td>$\nu_1 = 0.25799$ m$^3$/kg</td>
</tr>
<tr>
<td>$s_1 = 7.1246$ kJ/kg$\cdot$K</td>
<td></td>
</tr>
</tbody>
</table>

(Table A–6)

**State 2:**

<table>
<thead>
<tr>
<th>$P_2 = 200$ kPa</th>
<th>$T_2 = 150°C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_2 = 2577.1$ kJ/kg</td>
<td></td>
</tr>
<tr>
<td>$\nu_2 = 0.95986$ m$^3$/kg</td>
<td></td>
</tr>
<tr>
<td>$s_2 = 7.2810$ kJ/kg$\cdot$K</td>
<td></td>
</tr>
</tbody>
</table>

(Table A–6)

**Dead state:**

<table>
<thead>
<tr>
<th>$P_0 = 100$ kPa</th>
<th>$T_0 = 25°C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_0 \approx u_f @ 25°C = 104.83$ kJ/kg</td>
<td></td>
</tr>
<tr>
<td>$\nu_0 \approx \nu_f @ 25°C = 0.00103$ m$^3$/kg (Table A–4)</td>
<td></td>
</tr>
<tr>
<td>$s_0 \approx s_f @ 25°C = 0.3672$ kJ/kg$\cdot$K</td>
<td></td>
</tr>
</tbody>
</table>
Entropy equation in Chapter 4: control volume approach

- Example

**EXAMPLE 8–11 Exergy Destruction During Expansion of Steam**

A piston–cylinder device contains 0.05 kg of steam at 1 MPa and 300°C. Steam now expands to a final state of 200 kPa and 150°C, doing work. Heat losses from the system to the surroundings are estimated to be 2 kJ during this process. Assuming the surroundings to be at $T_0 = 25°C$ and $P_0 = 100$ kPa, determine (a) the exergy of the steam at the initial and the final states, (b) the exergy change of the steam, (c) the exergy destroyed, and (d) the second-law efficiency for the process.

\[ A = E + p_0 V - T_0 S \quad (4.27) \quad W_{u,\text{max} \to 2} \equiv A_1 - A_2 \]

\[
X_1 = m\left[ (u_1 - u_0) - T_0(s_1 - s_0) + P_0(v_1 - v_0) \right] \\
= (0.05 \text{ kg})\{(2793.7 - 104.83) \text{ kJ/kg} \\
- (298 \text{ K})[ (7.1246 - 0.3672) \text{ kJ/kg} \cdot \text{K}] \\
+ (100 \text{ kPa})[ (0.25799 - 0.00103) \text{ m}^3/\text{kg}] \}(\text{kJ/kPa} \cdot \text{m}^3) \\
= 35.0 \text{ kJ}
\]

\[
X_2 = m\left[ (u_2 - u_0) - T_0(s_2 - s_0) + P_0(v_2 - v_0) \right] \\
= (0.05 \text{ kg})\{(2577.1 - 104.83) \text{ kJ/kg} \\
- (298 \text{ K})[ (7.2810 - 0.3672) \text{ kJ/kg} \cdot \text{K}] \\
+ (100 \text{ kPa})[ (0.95986 - 0.00103) \text{ m}^3/\text{kg}] \}(\text{kJ/kPa} \cdot \text{m}^3) \\
= 25.4 \text{ kJ}
\]

\[
\Delta X = X_2 - X_1 = 25.4 - 35.0 = -9.6 \text{ kJ}
\]

Maximum useful work
Entropy equation in Chapter 4: control volume approach

Example

**EXAMPLE 8–11  Exergy Destruction During Expansion of Steam**

A piston–cylinder device contains 0.05 kg of steam at 1 MPa and 300°C. Steam now expands to a final state of 200 kPa and 150°C, doing work. Heat losses from the system to the surroundings are estimated to be 2 kJ during this process. Assuming the surroundings to be at $T_0 = 25^\circ C$ and $P_0 = 100$ kPa, determine (a) the exergy of the steam at the initial and the final states, (b) the exergy change of the steam, (c) the exergy destroyed, and (d) the second-law efficiency for the process.

\[
\left( \frac{dW}{dt} \right)_{\text{u,actual}} = \left( \frac{dW}{dt} \right)_{\text{actual}} - P_0 \frac{dV}{dt}
\]

- $Q_{\text{out}} - W_{b,\text{out}} = \Delta U$
- $W_{b,\text{out}} = -Q_{\text{out}} - \Delta U = -Q_{\text{out}} - m(u_2 - u_1)$
  \[= -(2 \text{ kJ}) - (0.05 \text{ kg})(2577.1 - 2793.7) \text{ kJ/kg} = 8.8 \text{ kJ}\]

\[
W_u = W - W_{\text{surr}} = W_{b,\text{out}} - P_0(v_2 - v_1) = W_{b,\text{out}} - P_0 m(v_2 - v_1)
\]

\[
= 8.8 \text{ kJ} - (100 \text{ kPa})(0.05 \text{ kg})[(0.9599 - 0.25799) \text{ m}^3/\text{kg}] \left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right)
\]

\[
= 5.3 \text{ kJ}
\]

- Useful work
- Actual work
- Useful work

$P_0 = 100$ kPa
$T_0 = 25^\circ C$

$P_1 = 1$ MP
$T_1 = 300^\circ C$

$P_2 = 200$ kPa
$T_2 = 150^\circ C$
Entropy equation in Chapter 4: control volume approach

**Example**

**EXAMPLE 8–11 Exergy Destruction During Expansion of Steam**

A piston–cylinder device contains 0.05 kg of steam at 1 MPa and 300°C. Steam now expands to a final state of 200 kPa and 150°C, doing work. Heat losses from the system to the surroundings are estimated to be 2 kJ during this process. Assuming the surroundings to be at \( T_0 = 25°C \) and \( P_0 = 100 \) kPa, determine (a) the exergy of the steam at the initial and the final states, (b) the exergy change of the steam, (c) the exergy destroyed, and (d) the second-law efficiency for the process.

\[
\dot{I} = \left( \frac{dW}{dt} \right)_{u,\text{max}} - \left( \frac{dW}{dt} \right)_{u,\text{actual}} = T_0 \dot{S}_{\text{gen}}
\]

**Irreversibility**

\[
X_{\text{destroyed}} = X_1 - X_2 - W_{u,\text{out}} = 35.0 - 25.4 - 5.3 = \text{4.3 kJ}
\]

\[
\Delta X = X_2 - X_1 = 25.4 - 35.0 = \text{-9.6 kJ}
\]

**Maximum useful work**

\[
W_u = W - W_{\text{surr}} = W_{b,\text{out}} - P_0(\nu_2 - \nu_1) = W_{b,\text{out}} - P_0 m(\nu_2 - \nu_1) = 5.3 \text{ kJ}
\]

**Useful work**

\[
P_0 = 100 \text{ kPa} \\
T_0 = 25°C \\
P_1 = 1 \text{ MP} \\
T_1 = 300°C \\
P_2 = 200 \text{ kPa} \\
T_2 = 150°C
\]
Entropy equation in Chapter 4: control volume approach

Example

**EXAMPLE 8–11 Exergy Destruction During Expansion of Steam**

A piston-cylinder device contains 0.05 kg of steam at 1 MPa and 300°C. Steam now expands to a final state of 200 kPa and 150°C, doing work. Heat losses from the system to the surroundings are estimated to be 2 kJ during this process. Assuming the surroundings to be at $T_0 = 25°C$ and $P_0 = 100$ kPa, determine (a) the exergy of the steam at the initial and final states, (b) the exergy change of the steam, (c) the exergy destroyed, and (d) the second-law efficiency for the process.

\[
\dot{I} \equiv \left(\frac{dW}{dt}\right)_{u,\text{max}} \left(\frac{dW}{dt}\right)_{u,\text{actual}} \equiv T_0 \dot{S}_{\text{gen}} \quad \text{Irreversibility}
\]

The exergy destroyed could also be determined from

\[
X_{\text{destroyed}} = T_0 S_{\text{gen}} = T_0 \left[ m(s_2 - s_1) + \frac{Q_{\text{surr}}}{T_0} \right]
\]

\[
= (298 \text{ K}) \left\{ (0.05 \text{ kg}) \left[ (7.2810 - 7.1246) \text{ kJ/kg} \cdot \text{K} \right] + \frac{2 \text{ kJ}}{298 \text{ K}} \right\}
\]

\[
= 4.3 \text{ kJ}
\]
1. Introduction
2. Nonflow Process
3. Thermodynamic Analysis of Nuclear Power Plants
4. Thermodynamic Analysis of A Simplified PWR System
6. Simple Brayton Cycle
7. More Complex Brayton Cycles
8. Supercritical Carbon Dioxide Brayton Cycles
Summary of the working forms of the first and second laws

- **First law for control volume**

\[
\left( \frac{\partial E}{\partial t} \right)_{c.v.} = \sum_{i=1}^{l} \dot{m}_i (h_i^0 + g z_i) + \frac{dQ}{dt} + \left( \frac{dQ}{dt} \right)_{gen} - \left( \frac{dW}{dt} \right)_{shaft} - \left( \frac{dW}{dt} \right)_{normal} - \left( \frac{dW}{dt} \right)_{shear}
\]

- **Second law**

\[
\left( \frac{\partial S}{\partial t} \right)_{c.v.} = \sum_{i=1}^{l} \dot{m}_i s_i + \dot{S}_{gen} + \frac{(dQ/dt)}{T_s}
\]

- **Working forms**

\[
\dot{E}_{c.v.} = \sum_{i=1}^{l} \dot{m}_i (h_i^0 + g z_i) + \dot{Q} + \dot{Q}_{gen} - \dot{W}_{shaft} - \dot{W}_{normal} - \dot{W}_{shear}
\]

\[
\dot{U}_{c.v.} = \sum_{i=1}^{l} \dot{m}_i h_i + \dot{Q} - \dot{W}_{shaft} - \dot{W}_{normal}
\]

\[
\dot{S}_{c.v.} = \sum_{i=1}^{l} \dot{m}_i s_i + \dot{s}_{gen} + \frac{\dot{Q}}{T_s}
\]
 Contents

1. Introduction
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Thermodynamic Analysis of Nuclear Power Plants

- Analysis of NPPs
  - Provides the relation between the mixed mean outlet coolant temperatures (or enthalpy for BWR) of the core through the primary and secondary systems to the generation of electricity at the turbine.
  - Cycles used for the various reactor types and the methods of thermodynamic analysis of these cycles are described.
    - Rankine cycle for steam-driven electric turbines (PWR, BWR)
    - Brayton cycle can be considered for gas cooled reactor systems
    - Both cycles are constant-pressure heat addition and rejection cycles for steady-flow operation

**FIGURE 6.6** Simplified PWR plant.
Maximum temperature of the Brayton cycle ($T_3$) is set by turbine blade and gas cooled reactor core material limits far higher than those for the Rankine cycle, which is set by liquid-cooled reactor core materials limits.
Thermodynamic Analysis of Nuclear Power Plants

Efficiency to assess the thermodynamic performance of components and cycles

- Thermodynamic efficiency (or effectiveness): $\zeta$
  - The ratio between the actual useful work and the maximum useful work

- Isentropic efficiency: $\eta_s$
  - Special case of thermodynamics efficiency
  - Adiabatic case
  - Important for devices such as pump, turbine, compressor

- Thermal efficiency: $\eta_{th}$
  - The fraction of the heat input that is converted to net work output
  - The cycle efficiency that we are interested in.
Thermodynamic Analysis of Nuclear Power Plants

- Thermodynamic efficiency (or effectiveness): $\zeta$

$$\zeta \equiv \frac{\dot{W}_{u,\text{actual}}}{\dot{W}_{u,\text{max}}}$$

- Maximum useful work
  - No entropy generation

$$\left( \frac{dW}{dt} \right)_{u,\text{max}} = - \left[ \frac{\partial(E + p_0 V - T_0 S)}{\partial t} \right]_{\text{c.v.}} + \sum_{i=1}^{I} \dot{m}_i (h^o - T_0 s + gz)_i$$

$$+ \left( 1 - \frac{T_0}{T_s} \right) \frac{dQ}{dt} + \left( \frac{dQ}{dt} \right)_{\text{gen}} - \left( \frac{dW}{dt} \right)_{\text{shear}}$$

(4.46)

- Fixed, non-deformable control volume with zero shear work
- Negligible kinetic and potential energy differences

$$\dot{W}_{u,\text{max}} = - \left[ \frac{\partial(U - T_0 S)}{\partial t} \right] + \sum_{i=1}^{I} \dot{m}_i (h - T_0 s)_i + \left( 1 - \frac{T_0}{T_s} \right) \dot{Q}$$

(6.32)

where $T_s = \text{temperature at which heat is supplied}$ and $(dQ/dt)_{\text{gen}}$ is treated as part of $dU/dt$. 
Thermodynamic Analysis of Nuclear Power Plants

- Isentropic efficiency: $\eta_s$

\[
\eta_s = \left( \frac{\dot{W}_{u,\text{actual}}}{\dot{W}_{u,\text{max}}} \right)_{\dot{Q}=0}
\]

$\zeta = \eta_s$ for an adiabatic control volume

- Maximum useful work

\[
\dot{W}_{u,\text{max}} = -\left[ \frac{\partial(U - T_0 S)}{\partial t} \right] + \sum_{i=1}^{l} \dot{m}_i (h - T_0 s)_i + \left(1 - \frac{T_0}{T_s}\right)\dot{Q}
\]

\[
\dot{W}_{u,\text{max}}|_{\dot{Q}=0} = -\left[ \frac{\partial U}{\partial t} \right]_{\text{c.v.}} + \sum_{i=1}^{l} \dot{m}_i h_{is}
\]

\[
\left( \frac{\partial S}{\partial t} \right)_{\text{c.v.}} = \sum_{i=1}^{l} \dot{m}_i s_i + \dot{S}_{\text{gen}} + \frac{(dQ/dt)}{T_s}
\]

- For steady-state condition

\[
\dot{W}_{u,\text{max}}|_{\dot{Q}=0} = \sum_{i=1}^{l} \dot{m}_i h_{is}
\]
Thermodynamic Analysis of Nuclear Power Plants

- Isentropic efficiency: $\eta_s$

$$\eta_s = \left( \frac{\dot{W}_{u,\text{actual}}}{\dot{W}_{u,\text{max}}} \right)_{\dot{Q}=0}$$

$\zeta = \eta_s$ for an adiabatic control volume

- Useful actual work for an adiabatic control volume with zero shear and negligible kinetic and potential energy differences

$$\left( \frac{dW}{dt} \right)_{u,\text{actual}} = -\left[ \frac{\partial(E + p_o V - T_o S)}{\partial t} \right]_{\text{c.v.}} + \sum_{i=1}^{l} \dot{m}_i (h^o - T_o s + g z)_i$$

$$+ \left( 1 - \frac{T_o}{T_s} \right) \frac{dQ}{dt} + \left( \frac{dQ}{dt} \right)_{\text{gen}} - \left( \frac{dW}{dt} \right)_{\text{shear}} - T_o \dot{S}_{\text{gen}}$$

$$\left( \frac{\partial S}{\partial t} \right)_{\text{c.v.}} = \sum_{i=1}^{l} \dot{m}_i s_i + \dot{S}_{\text{gen}} + \frac{(dQ/dt)}{T_s}$$

- For steady state

$$\dot{W}_{u,\text{actual}} = \sum_{i=1}^{l} \dot{m}_i h_i$$

$$\eta_s = \frac{\sum_{i=1}^{l} \dot{m}_i h_i}{\sum_{i=1}^{l} \dot{m}_i h_{i s}}$$
Thermodynamic Analysis of Nuclear Power Plants

- Thermal efficiency: $\eta_{th}$
  - Cycle efficiency

  \[ \eta_{th} = \frac{\dot{W}_{u,\text{actual}}}{\dot{Q}_{in}} \]

  - For adiabatic systems, it is not useful.
Example problem (a)

- Thermal efficiency: $\eta_{th}$

$$W = 30$$

- Thermodynamic efficiency (or effectiveness): $\zeta$

$$\zeta \equiv \frac{\dot{W}_{u,\text{actual}}}{\dot{W}_{u,\text{max}}}$$

$$\dot{W}_{u,\text{max}}$$

$$\dot{W}_{u,\text{max}}$$
**Thermodynamic Analysis of Nuclear Power Plants**

- **Example problem (b)**

The plant receives $\dot{Q}_{HT} = 25 \text{ kW}$ at temperature $T_{HT} = 825^\circ \text{C}$. The plant also receives $\dot{Q}_{MT} = 50 \text{ kW}$ at temperature $T_{MT} = 240^\circ \text{C}$. The plant rejects heat to the environment at $T_0 = 20^\circ \text{C}$ and produces power $\dot{W}_{\text{out}} = 12 \text{ kW}$.

a) Determine the Second Law efficiency of the power plant.

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{u,actual}}}{\dot{Q}_{\text{in}}} \quad \zeta \equiv \frac{\dot{W}_{\text{u,actual}}}{\dot{W}_{\text{u,max}}}$$

Maximum useful work $\Rightarrow$ no entropy generation

Entropy in = entropy out

Entropy decrease in hot reservoirs

= entropy increase in cold reservoir
Example problem (b)

The plant receives $\dot{Q}_{HT} = 25$ kW at temperature $T_{HT} = 825^\circ$C. The plant also receives $\dot{Q}_{MT} = 50$ kW at temperature $T_{MT} = 240^\circ$C. The plant rejects heat to the environment at $T_0 = 20^\circ$C and produces power $\dot{W}_{out} = 12$ kW.

a) Determine the Second Law efficiency of the power plant.

$$\eta_{th} = \frac{\dot{W}_{u,\text{actual}}}{\dot{Q}_{in}} = \frac{12}{75} = 16\%$$

$$\frac{\dot{Q}_{HT}}{T_{HT}} + \frac{\dot{Q}_{MT}}{T_{MT}} = \frac{\dot{Q}_{0,\text{rev}}}{T_0} \quad \dot{Q}_{0,\text{rev}} = 35.2$ kW

$$\dot{Q}_{HT} + \dot{Q}_{MT} = \dot{Q}_{0,\text{rev}} + \dot{W}_{out,\text{rev}} \quad \dot{W}_{out,\text{rev}} = 39.8$ kW

$$\zeta = \frac{\dot{W}_{u,\text{actual}}}{\dot{W}_{u,\max}} = \frac{12}{39.8} = 30.2 \%$$
Thermodynamic Analysis of A Simplified PWR System

- First Law Analysis of a Simplified PWR System
  - One component with multiple inlet and outlet flow streams operating at steady state and surround it with a non-deformable, stationary control volume
  - Energy equation

\[
\left( \frac{\partial E}{\partial t} \right)_{\text{c.v.}} = \sum_{i=1}^{l} \dot{m}_i (h_i^0 + g_c i) + \frac{dQ}{dt} + \left( \frac{dQ}{dt} \right)_{\text{gen}} - \left( \frac{dW}{dt} \right)_{\text{shaft}} - \left( \frac{dW}{dt} \right)_{\text{normal}} - \left( \frac{dW}{dt} \right)_{\text{shear}}
\]

Negligible kinetic energy
Thermodynamic Analysis of A Simplified PWR System

- First Law Analysis of a Simplified PWR System
  - One component with multiple inlet and outlet flow streams operating at steady state and surround it with a non-deformable, stationary control volume

- Energy equation

\[
\sum_{k=1}^{l} (m h)_{in,k} - \sum_{k=1}^{l} (m h)_{out,k} = W_{shaft} - Q
\]

- Mass conservation equation

\[
m = \sum_{k=1}^{l} m_{in,k} - \sum_{k=1}^{l} m_{out,k}
\]
Thermodynamic Analysis of A Simplified PWR System

- **First Law Analysis of a Simplified PWR System**
  - **Turbine**
    - Shaft work, no heat addition ⇒ adiabatic turbine
    - Actual turbine work is related to the ideal work by the component isentropic efficiency.
    - Isentropic efficiency
      \[
      \eta_T = \frac{h_{out,s} - h_{in}}{h_{in} - h_{out}} = f(p(h_{out}), s_{in})
      \]
  - **Pump**
    - Shaft work, no heat addition ⇒ adiabatic pump
      \[
      \eta_p = \frac{\text{Ideal work required}}{\text{Actual work required}} = \frac{h_{in} - h_{out,s}}{h_{in} - h_{out}}
      \]
First Law Analysis of a Simplified PWR System

- Steam generator
  - Heat transfer between the primary and the secondary
    \[
    \sum_{k=1}^{l} (\dot{m}h)_{in,k} - \sum_{k=1}^{l} (\dot{m}h)_{out,k} = W_{shaft} - Q
    \]
  - Pinch point temperature
    - Minimum temperature difference between the primary and the secondary
    - Inversely related to the heat transfer area
      - Small temperature difference: large heat transfer area
      - Large temperature difference: small heat transfer area
    - Directly related to the irreversibility of the SG
      - Large temperature difference: large irreversibility
    - Important design choice based on tradeoff between cost and irreversibility
Thermodynamic Analysis of A Simplified PWR System

- **First Law Analysis of a Simplified PWR System**
  - **Steam generator**
    \[
    \frac{\dot{m}_p}{\dot{m}_s} = \frac{(h_{out} - h_{in})_s}{(h_{in} - h_{out})_p}
    \]
    \[\text{dh} = c_p \text{dT}\]
    \[
    \frac{\dot{m}_p}{\dot{m}_s} = \frac{h_3 - h_a}{c_p [T_5 - (T_a + \Delta T_p)]} = \frac{h_a - h_2}{c_p [(T_a + \Delta T_p) - T_6]}
    \]
  - **Reactor plant**
    - **Actual work**
      \[
      \dot{W}_{u,actual} = \dot{W}_T + \dot{W}_p = [\dot{m}_s(h_{in} - h_{out})]_T + [\dot{m}_s(h_{in} - h_{out})]_p
      \]
      \[
      = [\eta_T \dot{m}_s(h_{in} - h_{out,s})]_T + \left[ \frac{\dot{m}_s}{\eta_P} (h_{in} - h_{out,s}) \right]_P
      \]
      Positive or negative?

- **Efficiencies**
  - **Thermal efficiency**
    \[\eta_T = \frac{h_{in} - h_{out}}{h_{in} - h_{out,s}}\]
  - **Pinch point efficiency**
    \[\eta_P = \frac{h_{in} - h_{out,s}}{h_{in} - h_{out}}\]
Thermodynamic Analysis of A Simplified PWR System

- First Law Analysis of a Simplified PWR System
  - Reactor plant
    - Maximum useful work
      \[ \dot{W}_{u,\text{max}} = \dot{m}_p (h_{out} - h_{in})_R = \dot{m}_p (h_5 - h_6) \]
      - In section 6.4.2.3
    - Maximum useful work expressed with the secondary side properties
      \[ \dot{W}_{u,\text{max}} = \dot{m}_p (h_{out} - h_{in})_R = \dot{m}_s (h_{out} - h_{in})_{SGs} \]
      \[ \dot{W}_{u,\text{max}} = \dot{m}_s (h_{out} - h_{in})_{SGs} \]
    - Thermodynamic efficiency or effectiveness: \( \zeta \)
      \[ \zeta = \frac{[\dot{m}_s (h_{in} - h_{out})]_T + [\dot{m}_s (h_{in} - h_{out})]_P}{[\dot{m}_s (h_{out} - h_{in})]_{SGs}} \]
First law analysis in a simplified PWR system

- Stationary, non-deformable control volume operating at steady state

\[ \sum_{k=1}^{l} (\dot{m}h)_{in,k} - \sum_{k=1}^{l} (\dot{m}h)_{out,k} = \dot{W}_{shaft} - \dot{Q} \]

\[ \dot{m} = \sum_{k=1}^{l} \dot{m}_{in,k} - \sum_{k=1}^{l} \dot{m}_{out,k} = 0 \]

- Turbine and pump

\[ \sum_{k=1}^{l} (\dot{m}h)_{in,k} - \sum_{k=1}^{l} (\dot{m}h)_{out,k} = \dot{W}_{shaft} - \dot{Q} \]

\[ \dot{m}_{in}(h_{in} - h_{out}) = \dot{W}_{shaft} \]

- Steam generator and condenser

\[ \sum_{k=1}^{l} (\dot{m}h)_{in,k} - \sum_{k=1}^{l} (\dot{m}h)_{out,k} = \dot{W}_{shaft} - \dot{Q} \]

\[ \dot{m}_{in}(h_{in} - h_{out}) = -\dot{Q} \]
First law analysis in a simplified PWR system

- Stationary, non-deformable control volume operating at steady state

\[ m_{in} (h_{in} - h_{out}) = \dot{W}_{shaft} \]

\[ m_{in} (h_{in} - h_{out}) = -\dot{Q} \]

- Thermodynamic efficiency and cycle thermal efficiency

\[ \zeta \equiv \frac{\dot{W}_{u,actual}}{\dot{W}_{u,max}} \quad \eta_{th} = \frac{\dot{W}_{u,actual}}{\dot{Q}_{in}} \]

\[ \zeta = \frac{h_3 - h_4 + h_1 - h_2}{h_3 - h_2} \]
Thermodynamic Analysis of A Simplified PWR System

First Law Analysis of a Simplified PWR System

Example 6.4: Thermodynamic Analysis of a Simplified PWR Plant

PROBLEM

1. Draw the temperature-entropy (T-s) diagram for this cycle.
2. Compute the ratio of the primary to secondary flow rates.
3. Compute the nuclear plant thermodynamic efficiency.
4. Compute the cycle thermal efficiency.

Assume that the turbine and pump have isentropic efficiencies of 85%.

<table>
<thead>
<tr>
<th>TABLE 6.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>PWR Operating Conditions for Example 6.4</td>
</tr>
<tr>
<td>State</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
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<td>4</td>
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<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
</tbody>
</table>
Thermodynamic Analysis of A Simplified PWR System

- First Law Analysis of a Simplified PWR System
  - Example 6.4: Thermodynamic Analysis of a Simplified PWR Plant
    - Draw the temperature-entropy (T-s) diagram for this cycle.
Thermodynamic Analysis of A Simplified PWR System

- First Law Analysis of a Simplified PWR System
  - Example 6.4: Thermodynamic Analysis of a Simplified PWR Plant
    - Compute the ratio of the primary to secondary flow rates.

\[
\frac{\dot{m}_p}{\dot{m}_s} = \frac{h_3 - h_a}{\overline{c}_p[T_5 - (T_a + \Delta T_p)]} = \frac{h_a - h_2}{\overline{c}_p[(T_a + \Delta T_p) - T_6]}
\]

From steam tables:
- \(h_3 = h_g(\text{sat. at } 1124 \text{ psia}) = 1187.29 \text{ Btu/lb (2.771 MJ/kg)}\)
- \(h_a = h_l(\text{sat. at } 1124 \text{ psia}) = 560.86 \text{ Btu/lb (1.309 MJ/kg)}\)

- \(T_a = \text{sat. liquid at } 1124 \text{ psia} = 1018.8 \text{ R\(^o\) (566.0 K)}\)

Hence

\[
\frac{\dot{m}_p}{\dot{m}_s} = \frac{2.77 \times 10^6 - 1.309 \times 10^6}{5941\ [599 - (566 + 14.4)]} = 13.18
\]
Thermodynamic Analysis of A Simplified PWR System

First Law Analysis of a Simplified PWR System

- Example 6.4: Thermodynamic Analysis of a Simplified PWR Plant
  - Compute the nuclear plant thermodynamic efficiency.
  - Calculate the enthalpies of each state!

- State 1

From the steam tables:

\[ h_1 = h_{f\text{ (sat. at 1 psia)}} = 69.74 \text{ Btu/lb (0.163 MJ/kg)} \]
Thermodynamic Analysis of A Simplified PWR System

- First Law Analysis of a Simplified PWR System
  - Example 6.4: Thermodynamic Analysis of a Simplified PWR Plant
    - State 2
      - Isentropic efficiencies of 85%.
        \[ \eta_p = \frac{h_{in} - h_{out,s}}{h_{in} - h_{out}} \]
        \[ h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_p} \]
      - Isentropic process
        \[ s_{2s} = s_1 = 0.13266 \text{ Btu/lb}^o\text{R} (557 \text{ J/kg K}) \]
        \[ p_{2s} = p_2 = 1124 \text{ psia (7.75 MP}_a) \]
      - \[ h_{2s} = h(p_2, s_{2s}) \]
        \[ h_{2s} = 73.09 \text{ Btu/lb (0.170 MJ/kg)} \]
        \[ = 0.163 + \frac{0.170 - 0.163}{0.85} = 0.171 \text{ MJ/kg} \]

<table>
<thead>
<tr>
<th>State</th>
<th>Temperature °R (K)</th>
<th>Pressure psia (kPa)</th>
<th>Condition</th>
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<tbody>
<tr>
<td>1</td>
<td>—</td>
<td>1 (6.89)</td>
<td>Saturated liquid</td>
</tr>
<tr>
<td>2</td>
<td>—</td>
<td>1124 (7750)</td>
<td>Subcooled liquid</td>
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<td>1124 (7750)</td>
<td>Saturated vapor</td>
</tr>
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<td>4</td>
<td>—</td>
<td>1 (6.89)</td>
<td>Two-phase mixture</td>
</tr>
<tr>
<td>5</td>
<td>1078.2 (599)</td>
<td>2250 (15,500)</td>
<td>Subcooled liquid</td>
</tr>
<tr>
<td>6</td>
<td>1016.9 (565)</td>
<td>2250 (15,500)</td>
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</tr>
<tr>
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<td></td>
<td></td>
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<td>[ T_a + 26 ] ([ T_a + 14.4 ])</td>
<td>2250 (15,500)</td>
<td>Subcooled liquid</td>
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![Saturation line diagram](image-url)
Thermodynamic Analysis of A Simplified PWR System

First Law Analysis of a Simplified PWR System

- Example 6.4: Thermodynamic Analysis of a Simplified PWR Plant
  - State 3
    - \( h_3 = h(p_3) \) From the steam tables:
  - State 4
    - Isentropic efficiency
      \[ \eta_T = \frac{h_{in} - h_{out}}{h_{in} - h_{out,s}} \]
    - Isentropic process
      \[ s_3 = s_{4s} = \]
      \[ x_{4s} = \frac{s_{4s} - s_i}{s_{ig}} = \frac{1.3759 - 0.13266}{1.8453} = 0.674 \]
      \[ h_{4s} = h_f + x_{4s}h_{ig} = 69.74 + 0.674(1036.0) = 768.00 \text{ Btu/lb (1.79 MJ/kg)} \]
      \[ h_4 = h_3 - \eta_T(h_3 - h_{4s}) = 1187.29 - 0.85(1187.29 - 768.00) = 830.9 \text{ Btu/lb (1.94 MJ/kg)} \]

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<tr>
<td>B</td>
<td>( T_s + 26 (T_s + 14.4) )</td>
<td>2250 (15,500)</td>
<td>Subcooled liquid</td>
</tr>
</tbody>
</table>
First Law Analysis of a Simplified PWR System

- Example 6.4: Thermodynamic Analysis of a Simplified PWR Plant
  - Compute the nuclear plant thermodynamic efficiency.

\[
\zeta = \frac{h_3 - h_4 + h_1 - h_2}{h_3 - h_2}
\]

\[
\zeta = \frac{1187.29 - 830.9 + 69.74 - 73.69}{1187.29 - 73.69} \quad \text{(English units)} = 0.317
\]

\[
= \frac{2.771 - 1.94 + 0.163 - 0.171}{2.771 - 0.171} \quad \text{(SI units)} = 0.317
\]
Thermodynamic Analysis of A Simplified PWR System

❖ First Law Analysis of a Simplified PWR System

● Example 6.4: Thermodynamic Analysis of a Simplified PWR Plant

- Compute the cycle thermal efficiency.

\[ \eta_{th} = \frac{\dot{W}_{u, actual}}{\dot{Q}_{in}} \]

\[ \dot{Q}_{in} = m_s(h_3 - h_2) \]

\[ \dot{W}_{i, actual} = m_s[(h_3 - h_4) - (h_2 - h_1)] \]

- Thermal efficiency (cycle thermal efficiency)
  = plant thermodynamic efficiency

\[ \eta_{th} = \zeta \]
Thermodynamic Analysis of A Simplified PWR System

- First Law Analysis of a Simplified PWR System
  - Improvements in thermal efficiency for this simple saturated cycle
    - Pressure
      - Increase the pump outlet pressure/decrease turbine outlet pressure


**FIGURE 6.14** Thermal efficiency of Rankine cycle for a saturated turbine inlet state for varying turbine outlet pressure. Turbine inlet: 7.8 MPa saturated vapor.
First Law Analysis of a Simplified PWR System

- Improvements in thermal efficiency for this simple saturated cycle
  - Pressure
    - Increase the pump outlet pressure/decrease turbine outlet pressure

![Thermodynamic Diagram](image)

- Increase in energy supplied
- Decrease in energy rejected

- Added heat $\uparrow$
- Rejected heat $\downarrow$
- Net work $\uparrow$
- Efficiency $\uparrow$
Thermodynamic Analysis of A Simplified PWR System

- First Law Analysis of a Simplified PWR System
  - Improvements in thermal efficiency for this simple saturated cycle
    - Pressure
      - Increase the pump outlet pressure/decrease turbine outlet pressure

![Diagram showing thermodynamic analysis with annotations for added heat, rejected heat, net work, and efficiency increases.]